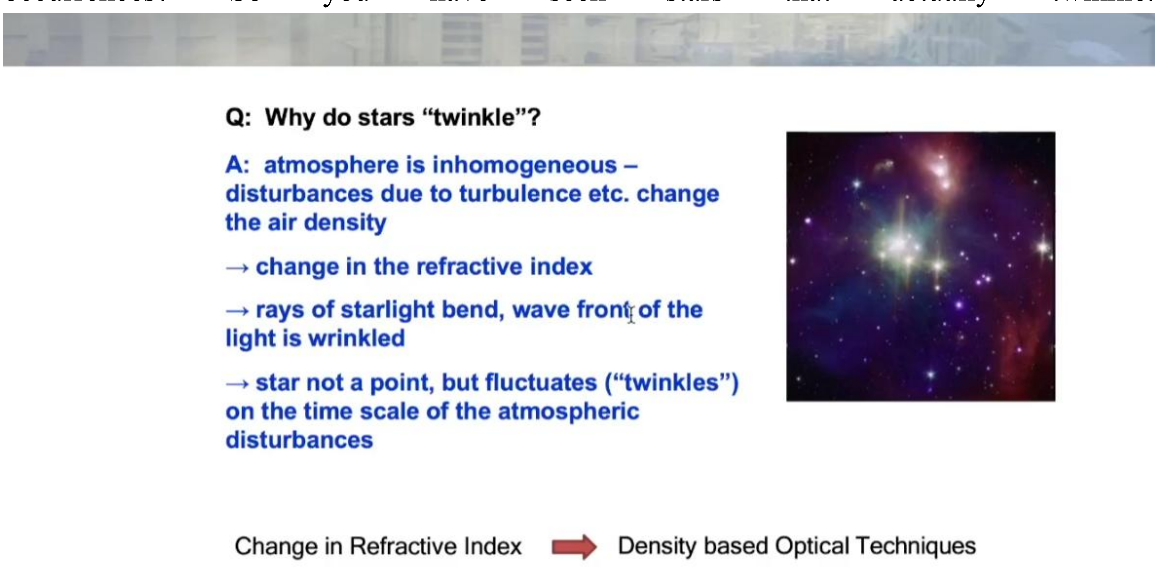
Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru
Week – 05
Lecture - 23
Schlieren and Shadowgraphy – 1

Okay, in today's lecture, we are going to cover what is called the Schlieren imaging. So Schlieren imaging is a very commonly used technique that is useful in many applications. So today's lecture notes are courtesy of Dr. L. Venkat Krishna from CSIR NAL and also from GALCIT, CALTECH. So Schlieren imaging is a very promising technique where we basically use this technique for a multitude of applications whenever inhomogeneous transparent media are involved.

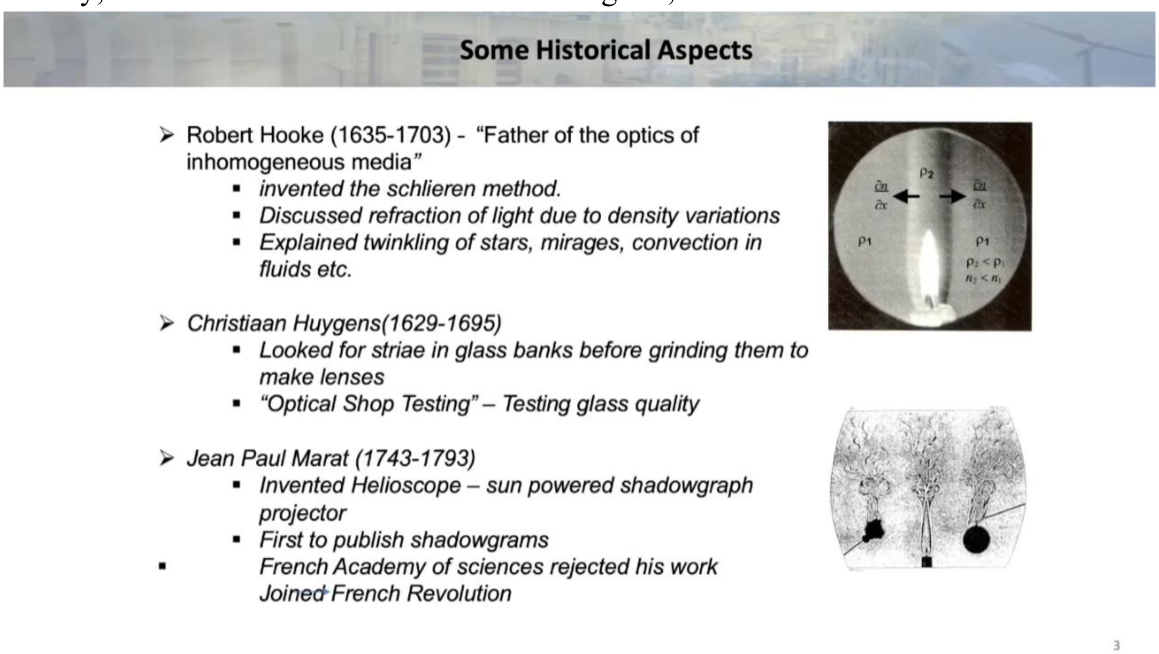
And we will see what that means during the duration of this particular lecture. Okay. But the Schlieren imaging is an extremely powerful tool. So, where do we see such occurrences? So you have seen stars that actually twinkle.



So it twinkles because the atmosphere is inhomogeneous. So what happens is that disturbances due to turbulence and changes in air density lead to a change in the refractive index, and because there is a change in the refractive index, the rays of starlight actually bend, and the wavefront of the light is wrinkled. As a result of that, fluctuations start to happen. So on the time scale of atmospheric disturbances, the fluctuations appear. Therefore, you see the stars twinkling.

But this is a celestial event. And because it's a celestial event, can't we do the same thing and use it for our own purposes? So can we use the change in refractive index, which is linked with the density change, and develop density-based optical techniques for a multitude of applications? But that is, I think, what the main crux of the story is going to be for the next few lectures. So this is a celestial event that occurs very commonly. So if you look at a slightly historical aspect of this, Robert Hooke was a guy who is called the father of optics in inhomogeneous media. He basically invented the Schlieren Method.

He discussed that the refraction of light due to density variations causes the twinkling of stars, mirages in the desert, and convection in fluids. So, let's take a look at this candle flame. There is a candle that is burning. As a result of that, the air around the candle gets heated, a common occurrence, and then you form a plume. That means the air loses its density, becomes lighter, and rises.



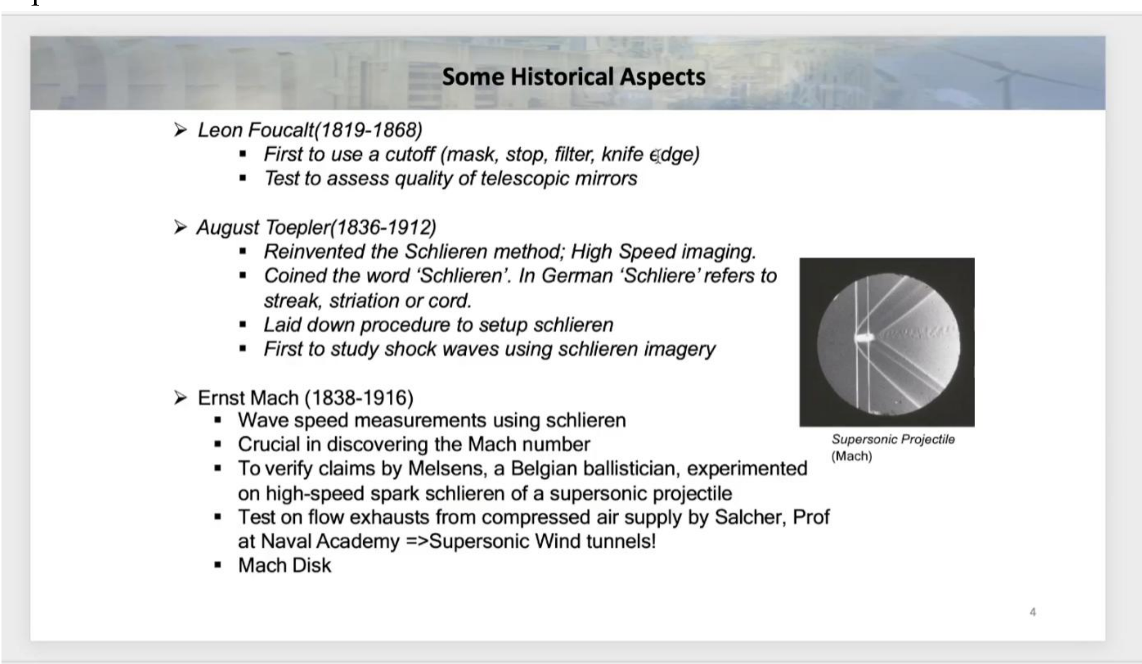
So the rest of the fluid, which is right around here, is basically fluid that is at room temperature. As a result, what happens is that if you go from the center to the edge and beyond, in both directions—right side or left side—you have a change in density. As you can readily see, the density of row two is less than that of row one because row two is lighter and has a lower density. That's why it rises; that's why you have a plume. So naturally, you have a change in refractive index from the center all the way to the edge, because there is a change in density that has led to a change in refractive index, which is written as the partial differentiation of n with respect to x.

Now the refractive index, N_2, actually has a lower refractive index than that of N_1. That means the colder fluid, which is the fluid that has a higher density, therefore has a higher refractive index as well. So these are the relationships that are presented here. This can also have a dependence in the x direction if x is into the board. Into the plane, so it can have a variation in that direction also for a 3D structure.

So, this is the crux of the story. What Christian Huygens did was look for striae and glass blanks, and he ground them to make lenses, which is the most important discovery because without these lenses and mirrors, we would not have any optical measurements at all, and he had an optical shop that tested the quality of this glass. Now, John Paul Murat. He invented what we call the Helioscope, which is a sun-powered shadowgraph projector. They're nice images that you see.

And he was the first to publish these shadowgraphs. So unfortunately, the French Academy of Sciences rejected his work, and he joined the French Revolution. Science faces a lot of problems, so to speak. So this problem is naturally one of them. And it is not something that one can do, so there are a lot of hurdles in anything that you do.

So, these are some of the historical aspects. So then, of course, there was Leon Foucault. He was the first to use a cutoff, mask, stop, filter, knife edge, and basically brought more quantification to the Schlieren aspect. And he was also the first to test the quality of the telescopic mirrors. Foucault, you have also heard the name from his pendulum experiment.

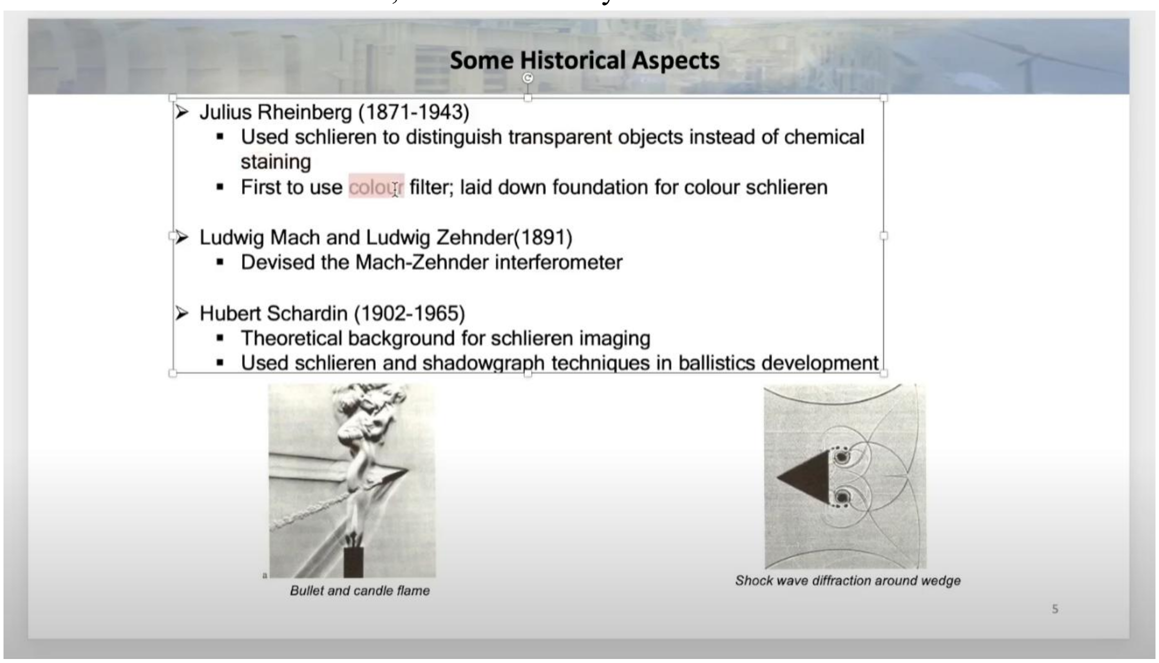


Then August Topler reinvented the Schlieren method. What he did was use high-speed imaging and coin the word Schlieren. In German, Schlieren refers to streak, creation, or cord. And he also laid down the procedure for setting up the Schlieren. And he was also the first to study shock waves using Schiller.

For example, this is an image of a shock wave. This is a supersonic projectile. And you can see that there is an attached shock in front of it. So this clearly says that there are sharp shock waves that do have hard density changes; as a result of that, there are changes in the refractive index—not large, but that is what is shown over here. Then, of course, there was Ernest Mark.

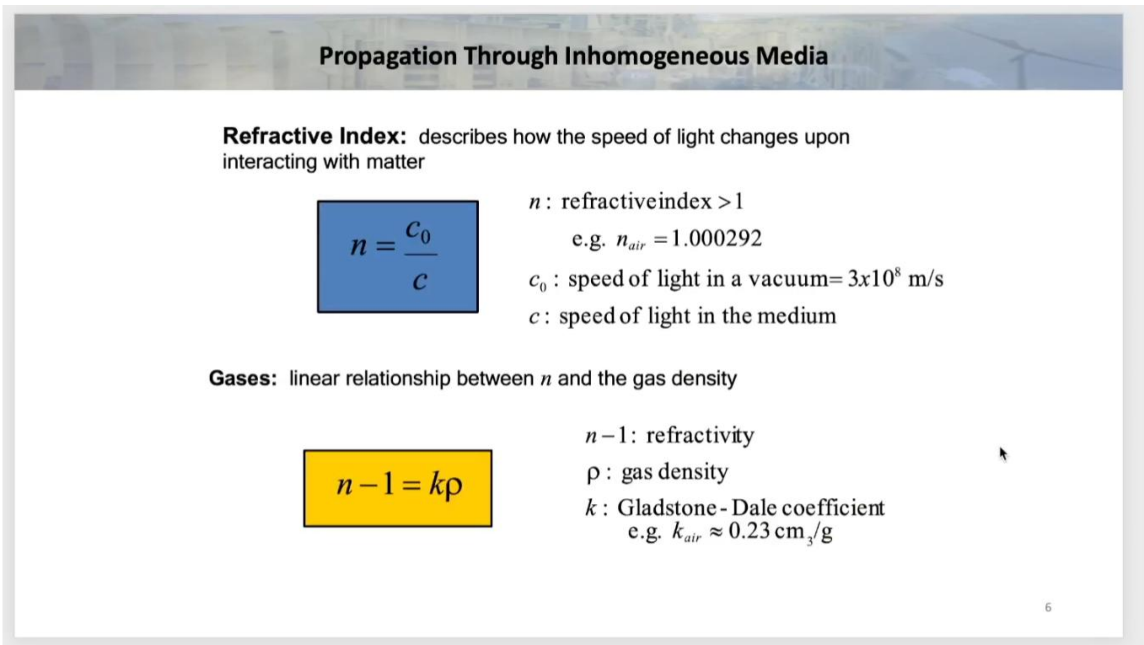
He used Schlieren to measure the wave's speed. And he made very crucial discoveries in pointing out the Mach number, which is basically the ratio of the velocity of the projectile divided by the velocity of sound. So naturally, if this number is less than one, it means subsonic. And if it is greater than one, it means it is supersonic. That means the velocity of the projectile or the object is much greater than the velocity of sound.

Now, to verify these claims, there has been work on high-speed spark shullering of a supersonic projectile. There has been a test on the flow of exhausts and also measurements in supersonic tunnels. The Mach disk and other things came into the picture right at that point. So some more aspects are that Julius Reinberg used Schlieren to distinguish between transparent objects instead of chemical staining. And he was the first one to use a color filter, which basically laid the foundation for color Schlieren.



Then, of course, there were Mark and Snyder, and they devised the interferometer, which basically uses optical path difference. And then Hubert did the theoretical background for Schlieren imaging, and he used it for different ballistic developments. So you can see a bullet penetrating through a candle flame here. You can see the large-scale structure of the candle flame and the very fine structures of the bullet, or the trail that is left behind by the bullet. So you can see the granularity of the details that we can extract just by looking at them.

On the other hand, you can see the shock wave in diffraction around a wedge, for example. Here, you can also see large-scale structures and small-scale structures. All these things are clearly visible when you deal with this kind of framework. All right, so one can see that it has developed over time; it has developed through multiple people, and we have gotten better with the advent of high-speed imaging, with the advent of, you know, more sophisticated optics and higher quality optics, I might add. We have come a long way from where we were, okay, from two or three centuries ago.



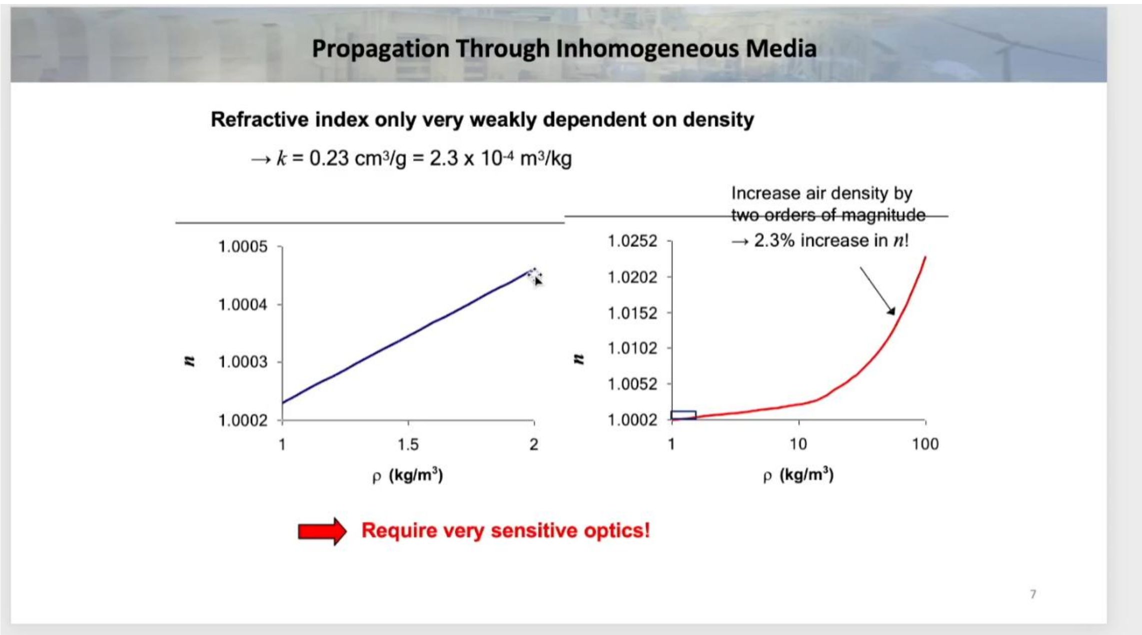
So now we go into a little bit of mathematical detail. So, how do we define the refractive index? So the refractive index is nothing but the velocity of sound divided by the velocity of light in vacuum, divided by the speed of light in the medium. The velocity of light in a vacuum is three times ten to the power of eight meters per second. And the speed of light in the medium is whatever it is going to be. Now, normally the refractive index will be greater than one because the highest velocity that you can have in a particular medium is in a vacuum.

The air, for example, has a value that is 1.000292. So the first significant digit comes from the fourth place of the decimal. So this is the speed of light in air. So the good part is that, for gases, there exists a linear relationship between the refractive index and the gas density.

So they are related as n minus one is equal to k times rho, where one is subtracted just because of this vacuum issue. So n is the refractive index of that particular medium, and n minus one is called refractivity. And ρ is the gas density. And k is the Gladstone-Dale coefficient, which is about 2.

23 cm^3/g. This is cubic centimeters. So you can readily understand that there is a linear relationship. So you think that, oh, things are very good because a linear relationship is what helps, as you don't want relationships to be power laws and stuff like that. You want them to be linear so that you can extrapolate to higher values or interpolate them.

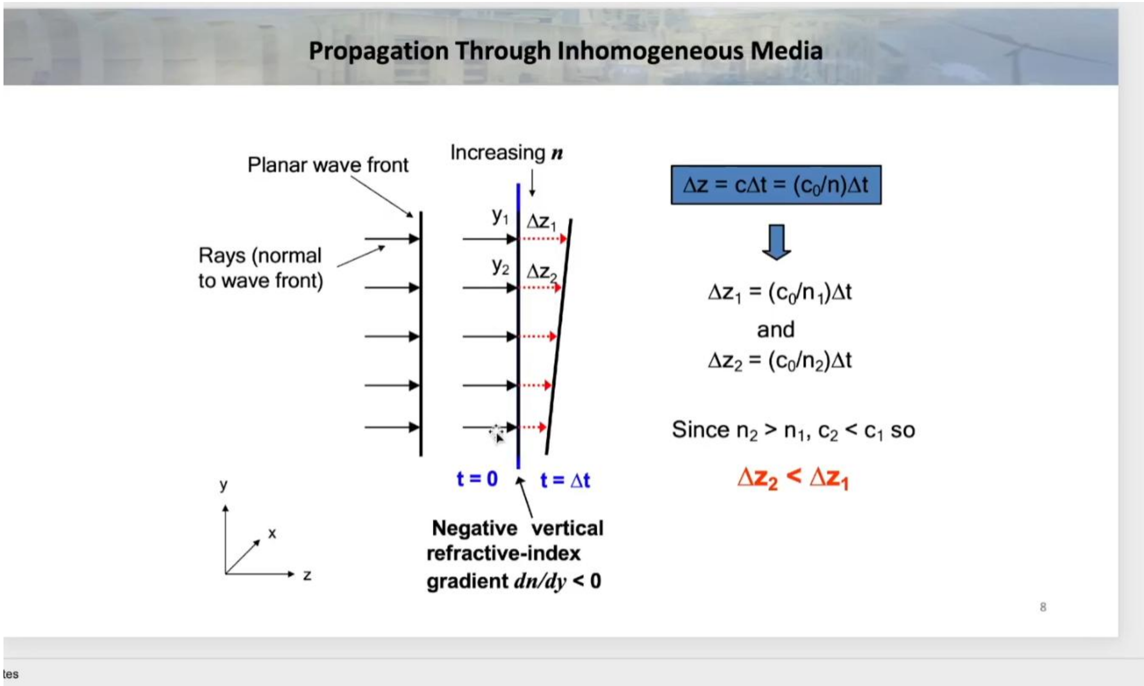
You can do all kinds of things. So this is good news that you have a direct correlation between density and the refractive index. And the refractive index is all about the velocity of light in the two media concerned. But to make the story a little bit more difficult. The refractive index is a very weak function of density. That means you can see here that the density increases from one to two, and the refractive index hardly shows any increase.



It goes to about, say, 1.004 from about 1.0029 or 22. So this is a very minuscule increase in the refractive index for a twofold increase in density. To make matters worse, if we now plot a large variation in density, you can see that the density is plotted from one all the way up to 100. So there is a two-order change in the value of density, but this two-order change in the value of density only leads to about a 2.

3% increase in N. The N increases by only 2.3%. So even for two orders of change in density. Now, this poses a problem because you have a very large variation in one parameter and a very small variation in the refractive index. Density variation by two orders is not obvious in any, for example, in a candle flame where you have this plume that rises along with it, around the candle.

That does not have a density variation of two orders. So there is a minuscule change in refractive index or a very large change in density. So this means that you need to have very sophisticated and sensitive optics. So let's look at the next one. How the light actually propagates through inhomogeneous media, for example, is that there is a planar wave front, which means all the rays are actually parallel to each other, and not only parallel; they traverse the same distance over a certain period of time.



This means this particular material. does not have any change in refractive index. That means there is no change in density. This impinges. And then this is where, in this particular medium, right after this boundary, you start to have a change in the refractive index. So, how does the refractive index change? It changes in a negative way.

That means the gradient of the refractive index with respect to y is less than zero, which, in other words, means that as you decrease y, your refractive index n is actually going up. So the refractive index here, in the lower part of this particular figure, is actually higher than the refractive index at the top. So that means this particular medium is highly homogeneous and has a refractive index change given by this gradient. Now, if you look at it, the top ray, or any ray as a matter of fact, traverses some distance, which is Δz, in a small period of time, which is Δt.

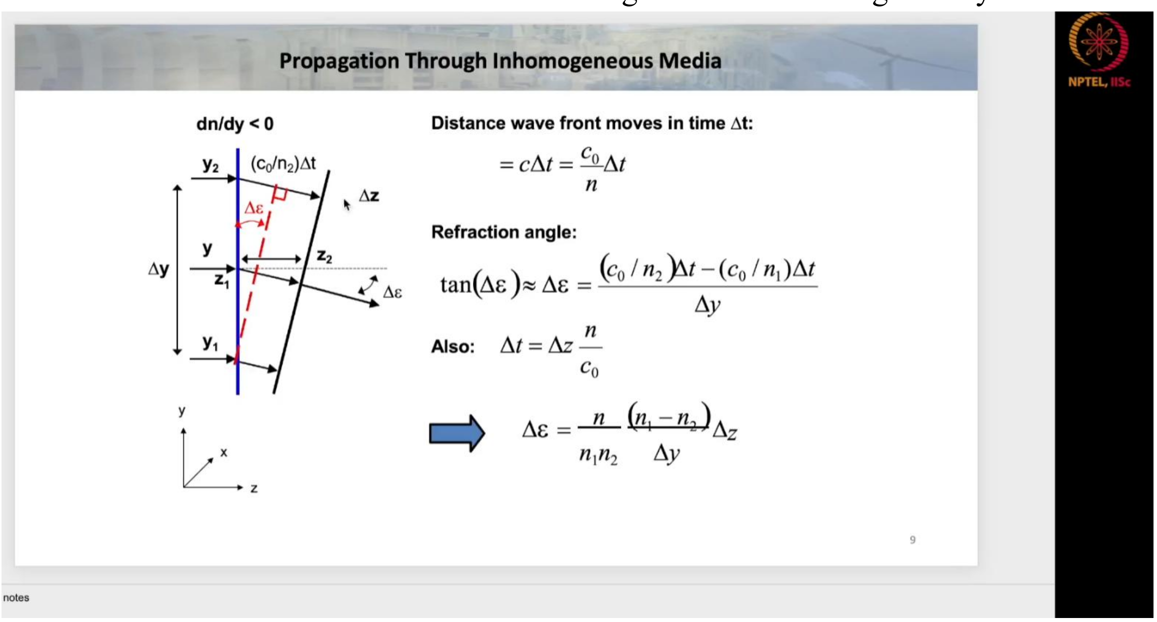
Now, Δz is given as cΔt. Where c is the local velocity of light in that medium at that particular location. In other words, you can say that c is c_0/n, where n is the local value of the refractive index, and c_0 is the velocity of light in a vacuum. This is multiplied by Δ t. Now, if we consider it for two rays, which are y1 and y2, you can see Δz_1 is given by c_0/n_1 into Δt, and Δz_2 is given as c_0 by n_2 into Δt. Now, since your n_2 is greater than your

n_1, and that's what we said—that the refractive index actually increases—that means the medium is lighter here, which means less density here and more density here.

That is what it means. So n_2 is greater than n_1. Therefore, c_1 must be greater than c_2. All right, so light travels faster in a lighter medium or in a medium that has a lower density. Therefore, it traverses a slightly greater distance compared to y_1. The ray y_1 traverses a little bit further distance compared to ray y_2.

So, in other words, Δz_2 is less than Δz_1. So, therefore, if you now draw a front where these rays were after the same amount of time Δt compared to this, you'll find that they have traversed different distances. As a result, their wavefront is no longer planar; instead, it exhibits an angle. So this is at t_0, this is at t equal to 0 plus Δt. So, as you can see, it traverses different distances, and because of this gradation of refractive index, you can imagine this to be like a candle plume or something else that is lighter on the top and colder at the bottom.

That is what it is. All right. So next, if you consider the same scenario, you will see that this is a highly exaggerated view. Remember, the actual change is not that great. Again, we say that the refractive index gradient is less than zero. So, it's a negative gradient. That means it's an increase with the gradient increasing as y decreases.

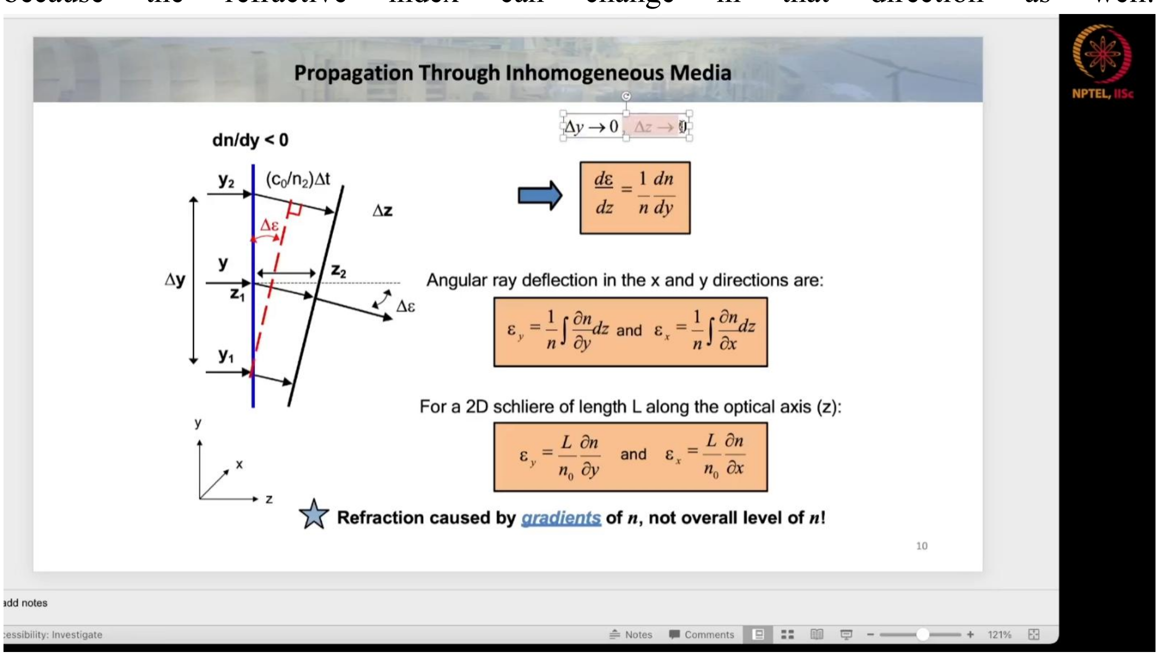


So in other words, again, look at this: this is y_2; this is y_1. Now, if you see that this wavefront, if you draw a parallel line to this, you'll see that it is tilted by an angle which is Δε. So, this is a highly exaggerated view, remember. The actual Δε is a very, very low number. So the distance the wave moves in time Δt is given as again cΔt, which is $c_0/n\Delta t$.

Now if you want to consider the refraction angle, you will find that it is basically the tangent of this angle. But because this angle is very small, and that's what we showed, n changes by a very small magnitude for a large change in density. Therefore, this is given as Δ ε. So, Δ ε, how do you calculate it now? You basically consider this value, which is $c0n_2\Delta t$.

So this is the distance that it traverses. And then you subtract c_0 divided by n_1 Δt, which is this particular distance that it traverses. You basically take this out, so you are left with this. And then you divide it by Y, the ΔY. The moment you do that, you will find that your Δt is nothing but your Δz multiplied by n/c_0, so once you perform the mathematical manipulations, you'll find that this Δ change is with respect to the change in Δz. This expression shows that the number (n_1-n_2) is very small, and (n_1-n_2) is a very small number because we say that the refractive index changes by only a minuscule amount; therefore, this makes Δε also very small to begin with.

So, this is nice and easy. Now, if we go to more of a limit in the case where Δy --> 0 and Δz --> 0, the expression that we wrote earlier can now be reduced to ΔyThis is the $\frac{\partial \varepsilon}{\partial z}$., given as $\frac{dn}{ndy}$. So dn by dy is the rate of change; it is the spatial gradient of the refractive index divided by one over the refractive index. And this is the corresponding change in angle of the tilt with respect to z. Now, recognize that though we have taken the variation in the y direction, there could be variation in the x direction, which is into the board, because the refractive index can change in that direction as well.

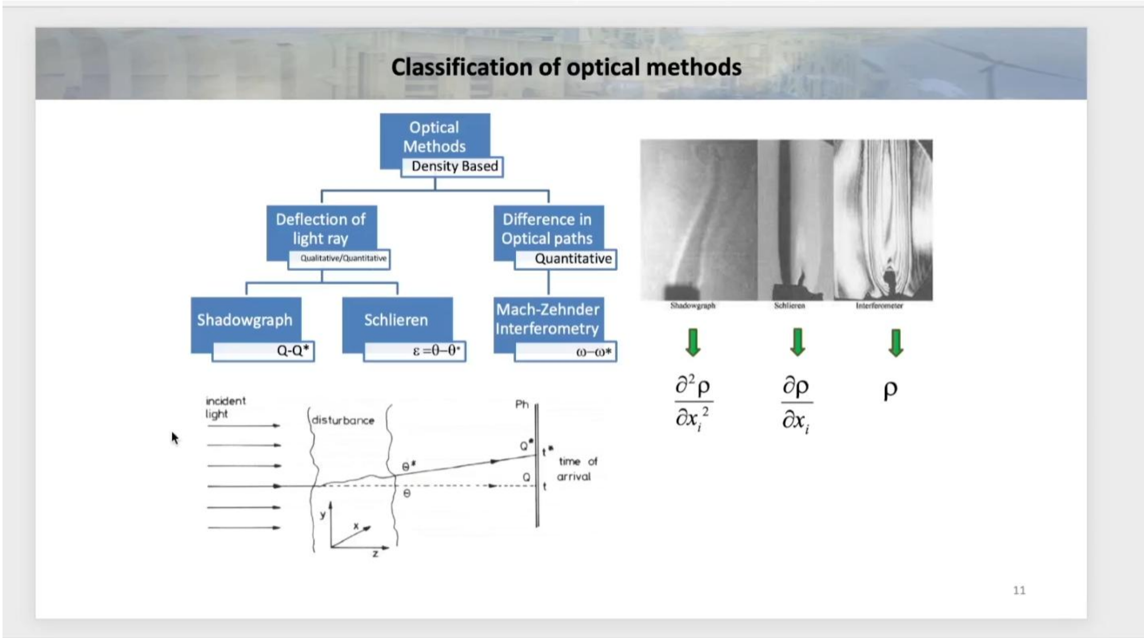


As a result of that, you see that your total change in ε in y, the y direction, is given as the integral of the relative change of n with respect to y multiplied by dz, similar with respect to x. That means if you are considering the component into the plane, the tilt of the wave front is also given in the same way, except that you have now. A change in refractive index with respect to x, and in both these cases, you are integrating it across the z. So if we have a 2D chiller of length l along the optical axis z, you are going to integrate it out, assuming that this quantity is no longer dependent on the z. So what you have is basically L by n_0 multiplied by the gradient of the refractive index in the x direction and in the y direction.

Now, this L that you see makes it a line of averaged type of measurement because you are integrating it along the Z axis. You are integrating out the effect in the Z axis, and you are only concerned with the density gradient or the refractive index gradient in the X and Y directions. So this is an important thing that you should note: this is therefore along a length L on the optical axis. The longer the length L, the greater this ε_y and ε_x will be. Therefore, you need a larger optical path if you want to achieve more sensitivity in your measurements.

And also, refraction is caused by the gradients of n and not by the overall value. So you can have a very dense medium, and you can have a very light medium. Your chiller is not going to work if there are no inhomogeneities in the medium. So the medium has to be transparent, and it has to exhibit inhomogeneity. When you have a homogeneous medium where there is no change in density, this method will not work.

Okay, so shock has a change in density; the candle flame has a change in density. Everywhere there is a slight change in density that leads to an even slighter change in the refractive index, and that leads to a little bit of a tilt of your wave fronts. Those tilts are basically given by these expressions, which are functions of the. The gradients of the refractive index, rather than their absolute values, and the length scale—the length across which you are doing this measurement—also matter quite a bit.



add notes

So now there is a classification of optical methods. You have density-based methods. And then, density-based methods are basically of two kinds. One is the deflection of the light ray that we just saw. These measurements can be qualitative or quantitative. So under the category of deflection of light rays, you have shadowgraphy and Schlieren.

And when there is a difference in optical paths, we get something called interferometry. Interferometry is quantitative. This deflection can be qualitative or quantitative. It will give you a picture. But in order to know exact information out of the picture, you need to do something else.

As you can see, if there is a disturbance like this and there is an incident beam of light, the light beam gets deflected, as we saw. Because there is an inhomogeneous medium, the line gets tilted a little bit. Those tilts are the things that we calculated here. Okay, these are the tilts.

So the Schlieren will measure those tilts and give you some idea. Whereas, on the other hand, the shadow graphs will measure that there is a change in the intensity as well. So that will give you a shadow. If you see, the shadow graph is dependent on the second-order derivative of the density with respect to x. So this change translates to a change in the gradient of n as well, because they are related in a linear fashion. And in Schlieren, of course, you have the first-order derivative of the density with respect to xy.

And essentially, this also means that, with respect to this, this is the gradient of the refractive index with respect to x and y. Interferometry, obviously, will have just the absolute value. All right, so what we will do in this particular section is stop here, and we will take it up in the next class. So, as you can see, whatever we saw from a historical perspective, Schlieren, shadowgraphy, or whatever you call it, depends on this deflection of light because there is a change in the medium. And because there is a change in the medium or the homogeneity of the medium, the rays get tilted by a little bit, very, very little, so if you have sensitive optics, you can still measure them, and you showed some examples of a bullet, you know, of other types of things, of a candle, so we can do all this kind of stuff.

So you can see that these are the shadow graphs, these are the chillerine, these are the interferometry; they all, I mean, except for the interferometry, are all the gradients. Of the density, maybe the first-order derivative or the second-order derivative, they are never wholly dependent on the density as such. So you need a variation in density or a variation in the refractive index. But the good part is that the density and the refractive index are related in a linear fashion.

So that is a good part of the business. Right, so this is a very initial discussion about how, uh, axillary in principle can work, but of course, this is a lot of work. You need very good quality mirrors and lenses, as you will see. That's why the historical perspective was so important. Okay, all right, so we will see you in the next class.