Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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Week – 03

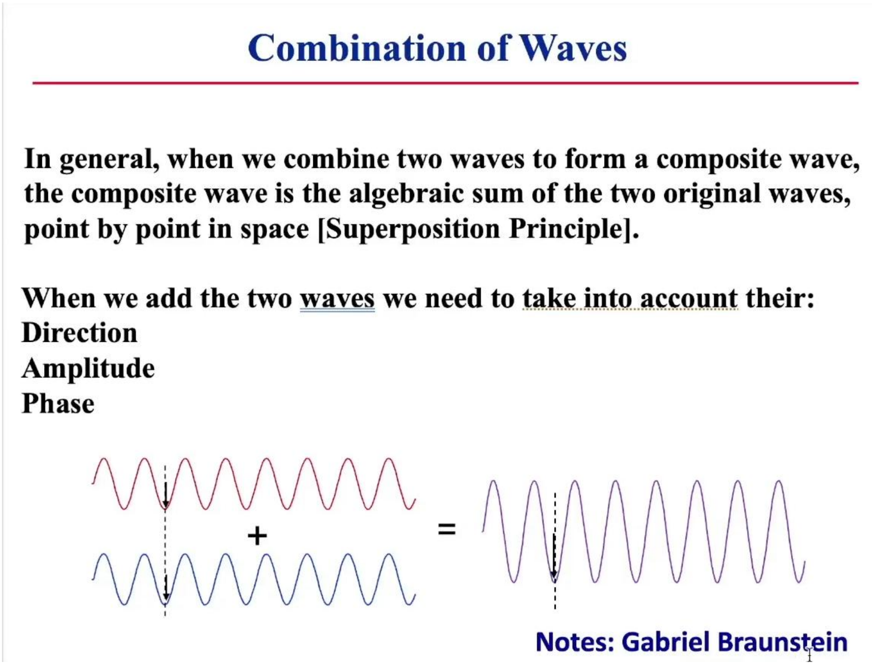
Lecture - 15

Imaging and Optics -7

We will give a small lecture on the combination of waves. Now, these are taken from the notes of Gabrielle Brownstein from the University of Central Florida. So this gives rise to what we talk about: a little bit on interference and a little bit on coherence. So in general, what happens is that, I mean, this is high school physics; it has been reiterated here for completeness. When we combine two waves to form a composite wave, the composite wave is basically the algebraic sum of the two original waves point by point in space. This is called the superposition principle.

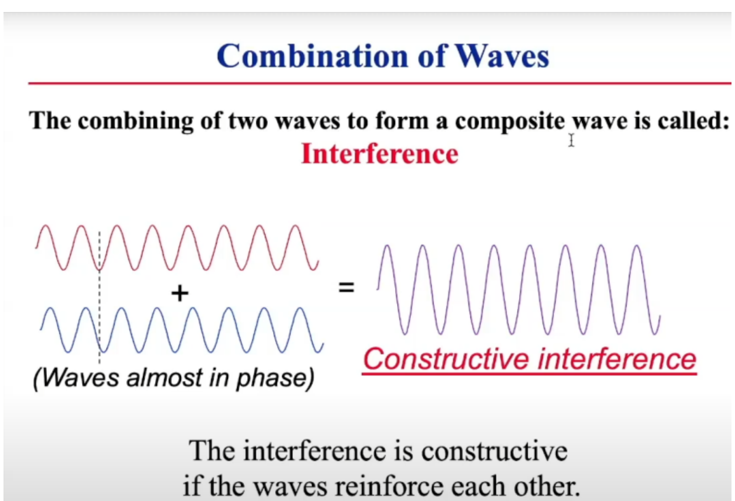
So when we add the two waves, we obviously need to take into account their direction, their amplitude, and their phase. So, for example, here is a red wave, and the blue is another wave. These waves are in phase with each other, as we can see. They have almost the same amplitude and the same direction.

As a result, the resultant wave is amplified. It's basically the algebraic sum of the two original waves, where the amplitude is doubled, and surely the phase remains the same, and the direction also remains the same. So this is what you get when you apply the principle of superposition. It is called the principle of superposition, which is nothing but the algebraic sum of the two waves. All right.



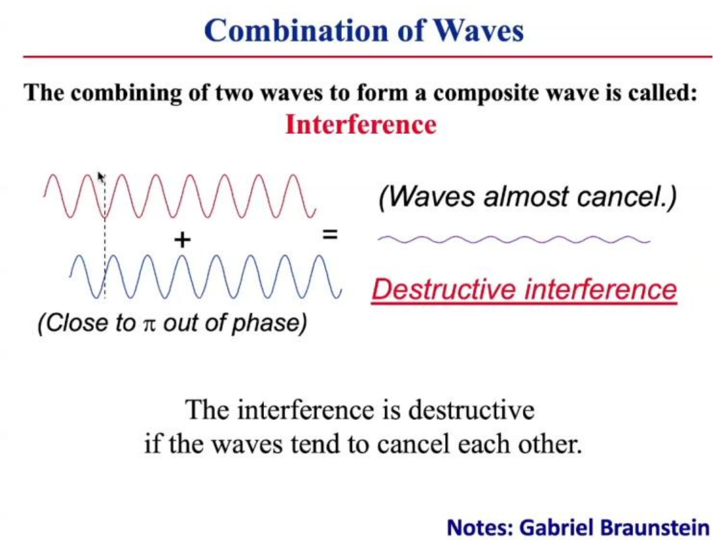
Now, when the two waves combine to form a composite wave in optical parlance, it is called interference. So here you can see that there are two waves that are almost in phase: the red and the blue. What we showed almost in phase is a little bit of phase jitter, as you can see. So the result is basically what we call constructive interference, in which the waves reinforce each other. You had two smaller waves, and they kind of give rise to a very large wave, okay? So this is constructive interference.

And the phenomenon of combining the two waves is called interference, in general, all right?



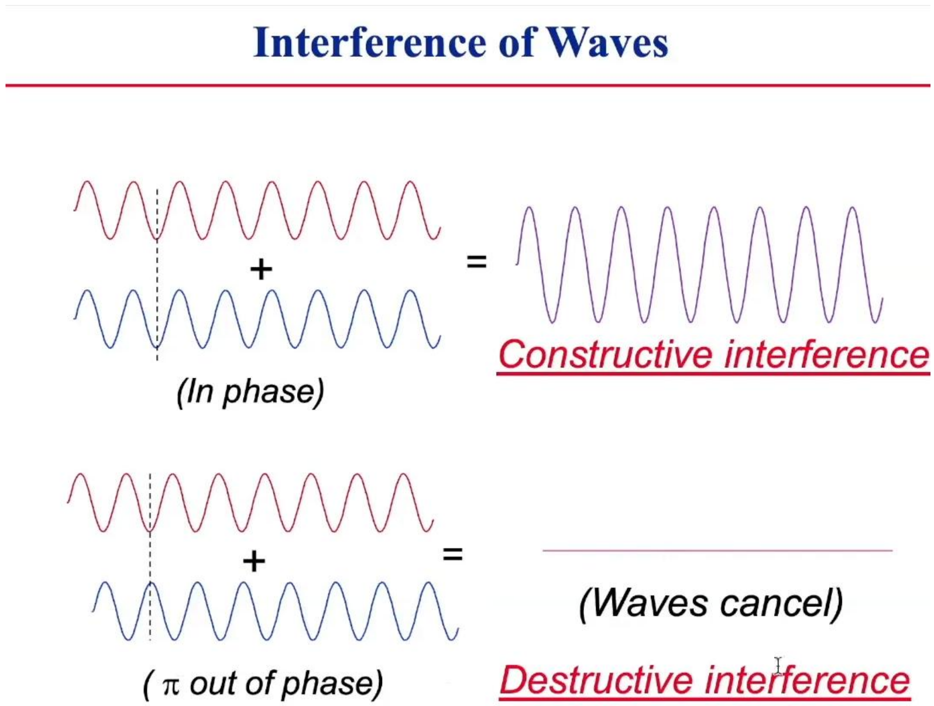
So based on this, if we move along, if the two waves are like pi out of phase, okay? So they are π out of phase. So, as you can see, the trough of this wave is actually like kind of half of this when it is rising. So this is shifted. There is a phase shift, right? Ideally, if this were here, the trough of the blue wave would be at this particular dotted line, then they would be in phase. Otherwise, now they are pi out of phase.

As a result, the waves almost cancel each other. So what we call a destructive interface. When the interference is destructive, the waves cancel each other; it's as simple as that. Cancellation, okay? Like you had an addition effect here; it isn't cancellation, right?



So when they are in phase, they do give rise to constructive interference, as you can see. When they are π out of phase, the waves cancel each other, and you get what we call destructive interference.

So this is very simple logic and very simple math. Now, this is what we are going to use in the PDPA, LDV, and all these other cases.

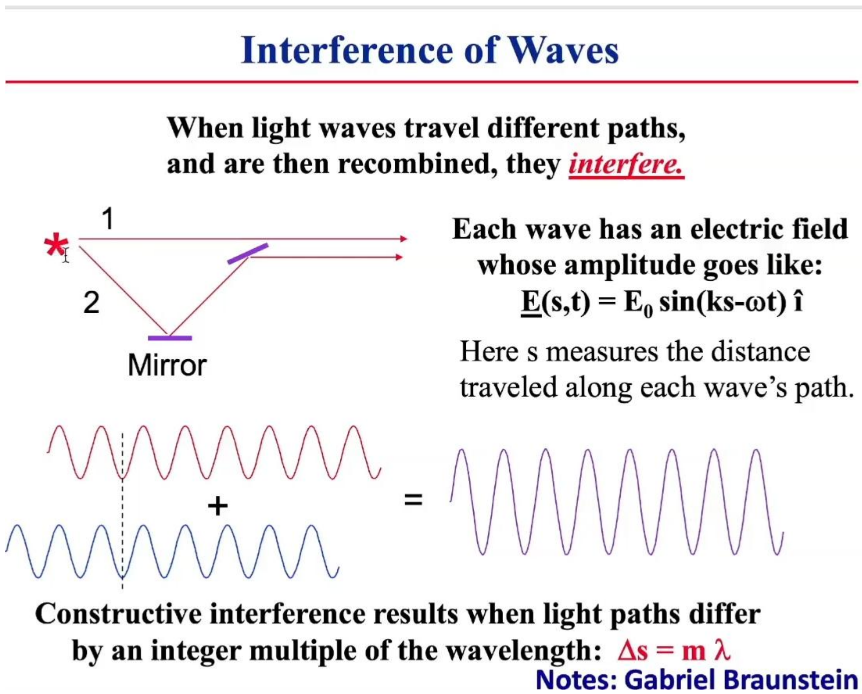


So, how is this achieved? So when light waves travel in different paths and then are recombined, they interfere. For example, this is a common light source, and one beam of light passes directly like this. The other is reflected and ultimately brought in the same direction to the detector, but it passes through a couple of mirrors.

As a result of that, it traverses a little extra in terms of the path that it takes. Here, the path is from A to B. It takes A, C, D, and then B. So, if each wave has an electric field whose amplitude is equal to E naught times the sine term, then this is the vector; this is the unit vector. The S measures the distance traveled along each wave's path.

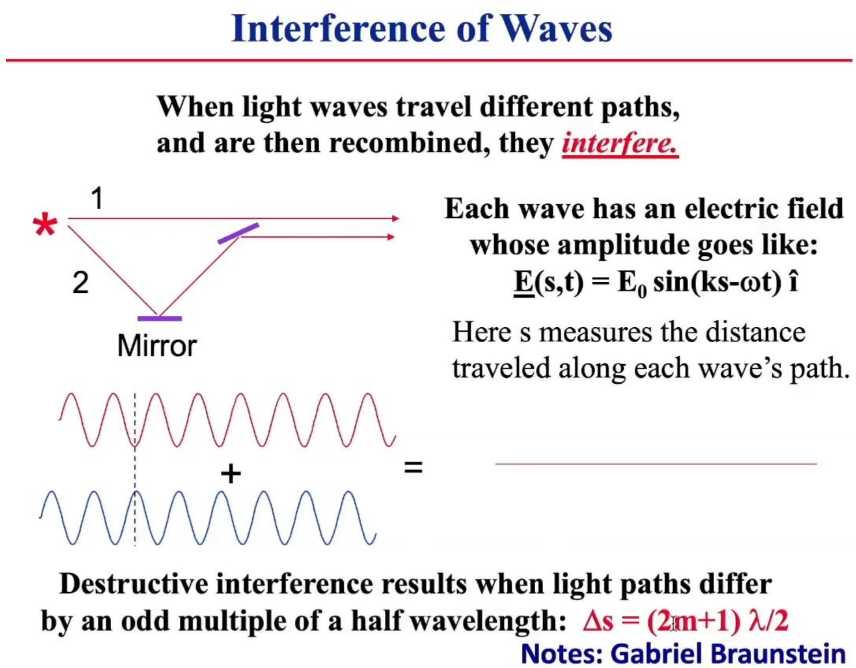
So, what happens is that when, in this case, these two waves actually recombine with each other. So if the light paths differ by an integer multiple of the wavelength, that is this delta S, this additional path that the wave travels, one wave travels more than the other. So if S is the path, the number two ray travels a little bit more than number one. So there is a delta S difference in the light path. So this ΔS difference, if it is an integral multiple of the wavelength, which is lambda, then you get constructive interference.

So no matter what, the waves may vary by quite a bit in path length, okay, or several, you know, but these are still small because you see lambda is small. Lambda is a small number usually associated with light.



So if they vary by some integer multiples, even when these waves actually combine with each other, you tend to get what we call constructive interference. So the path length, or the difference in path length, has to be an integral multiple of lambda. That is what you need to take into consideration.

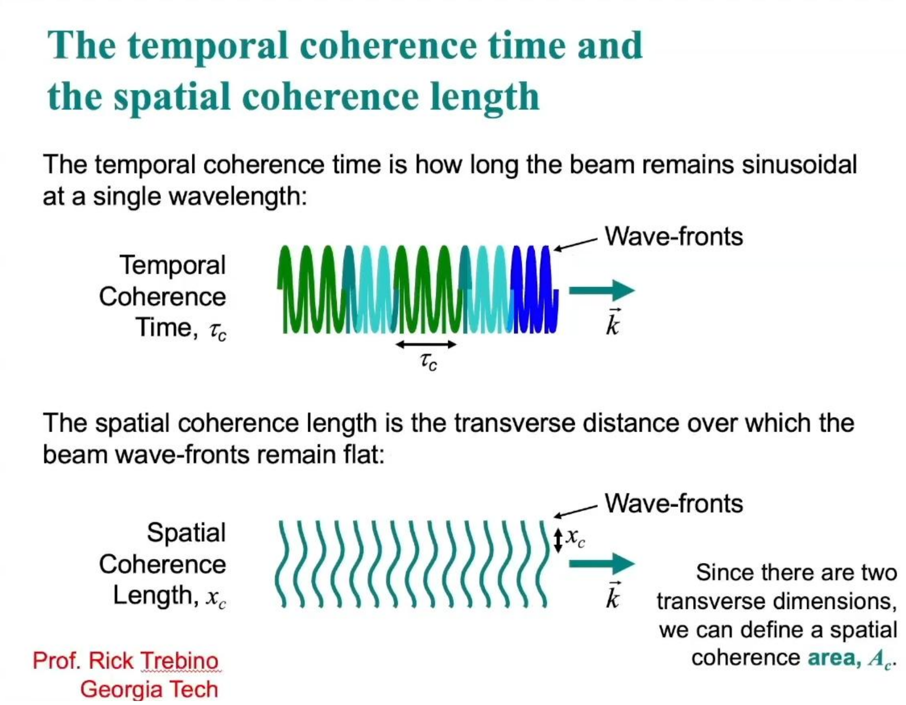
So if that is the case, then you get this constructive interference. If that is not the case, then you get what we call destructive interference, which occurs when the light paths differ by an odd multiple of half the wavelength. So if this Δ S, which is again the path difference between beam 1 and rays 1 and 2, is like an odd multiple, that is why you have 2m plus 1 of lambda over 2, you get destructive interference. So this is the logic. Once you get the destructive interference, you get this.



Means nothing, right? So let's talk a little bit about temporal coherence time and spatial coherence length. This is taken from the notes of Rick. All right, so you can see that this was taken from Professor Rick Trevino's lecture at Georgia Tech. So the temporal coherence time is how long that beam remains sinusoidal at a single wavelength. This is what temporal coherence is all about.

So the temporal coherence time is, say, tau c. This small period is the time that the beam remains sinusoidal at a single wavelength. So, this is the wavefront, and this is the direction of propagation. So the temporal coherence time is the short time during which the beam remains completely sinusoidal at a single wavelength. Spatial coherence length is basically the transverse distance or the distance over which the beam wave fronts remain flat.

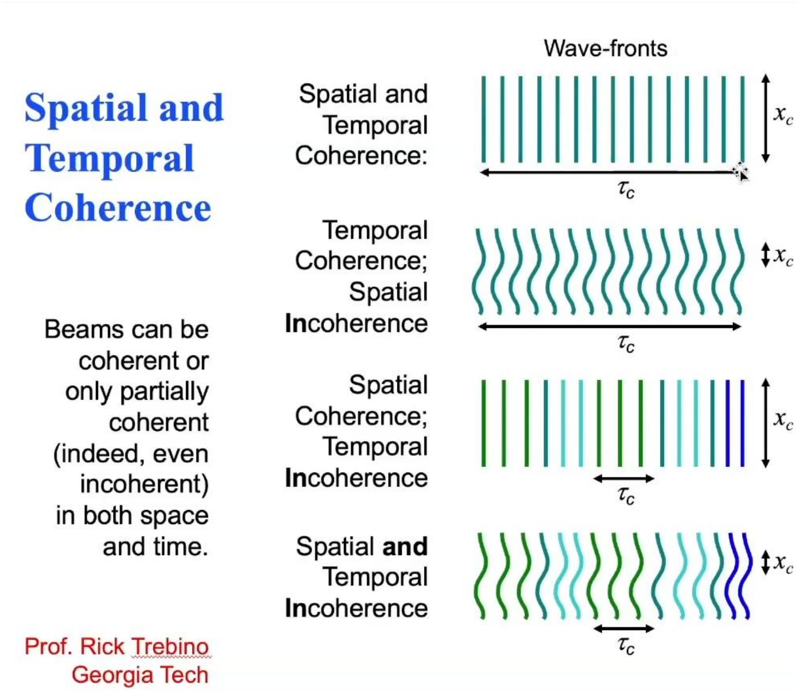
Okay, so this is the spatial coherence length xc. So this is the transverse distance; understand this transverse distance over which this beam fronts. This is a beam front, for example, these wiggly lines that you see over which the beam fronts remain flat. But there are two transverse dimensions, okay, because x and y, and therefore we can define a spatial coherence area, which is called ac, as well. So temporal coherence is the time that the beam remains sinusoidal, okay, at a single wavelength, and spatial coherence is the distance in the transverse direction over which the wave front remains flat.



So these are the two definitions of this. So if you look at spatial and temporal coherence, you will see that this particular front, for example, is spatially and temporally coherent. Because this is the τ_C, this is the X_c. This is the time at which the wavefront remains sinusoidal at a particular wavelength of X_c. Now, if you look at this particular picture, you will see that this is temporarily coherent but spatially incoherent, because the spatial incoherence is just a small length, and this is the temporal coherence.

So it maintains that. And then you have spatial coherence but temporal incoherence. So, as you can see, this particular frequency changes. So it cannot maintain; it only maintains a single sinusoidal wave for a very short duration of time before it changes. But the X_c is temporarily coherent with the wavefront. And this is both spatial and temporal incoherence, where what you have actually varies.

After a short time and distance. So the beams can be coherent, partially coherent, or even incoherent in both space and time. These are some examples. These are the wavefronts, and this is how we have explained it.



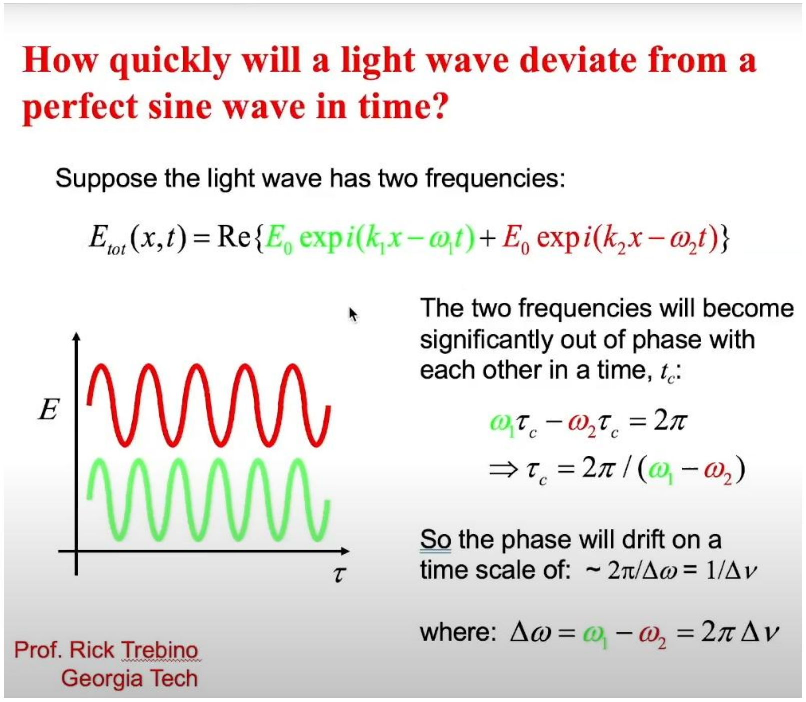
So how quickly will light deviate from its perfect sine wave over time? Okay, so let us assume that the light wave has two frequencies.

One is ω_1, one is ω_2. Okay, so let's assume that these are the two frequencies you have, all right? Okay, so this is the total E, which is the real part of this. So, the two frequencies will become significantly out of phase, remember? They will become; these are basically the two waves. Okay, so they have a little bit of a phase difference to start with, right? Omega one and omega two. So, the waves have different frequencies, sorry.

They have different frequencies to start with, all right? They do not necessarily have an initial phase, but they have different frequencies. They all start from the same point, but they have different frequencies. Now these two frequencies will become significantly out of phase with each other over a time period, say τ_C, . Okay, this is τ_C, or tau c, so this is basically $\omega_1$multiplied by τ_C, minus ω_2 multiplied by τ_C, which is equal to 2π, because that is the definition of significantly out of phase. So, in other words, τ_C, becomes 2πdivided by $\omega_1 - \omega_2$, so the phase will drift.

The phase will drift on a time scale of $\frac{2\pi}{\Delta\omega}$, which is nothing but 1 over the frequency, the delta frequency, because $\Delta\omega$ is nothing but $2\pi\Delta f$. So the two waves will become significantly out of phase over a time scale given asτ_C, which is nothing but 2πtimes the frequency difference between the two waves. All right. So this is how quickly a sine wave deviates from a perfect sine wave over time. OK, so assume that it starts as a perfect sine wave in a very short time because it has two frequencies.

It will drift. OK, the phase will drift. So this is exactly what happens in this particular case.



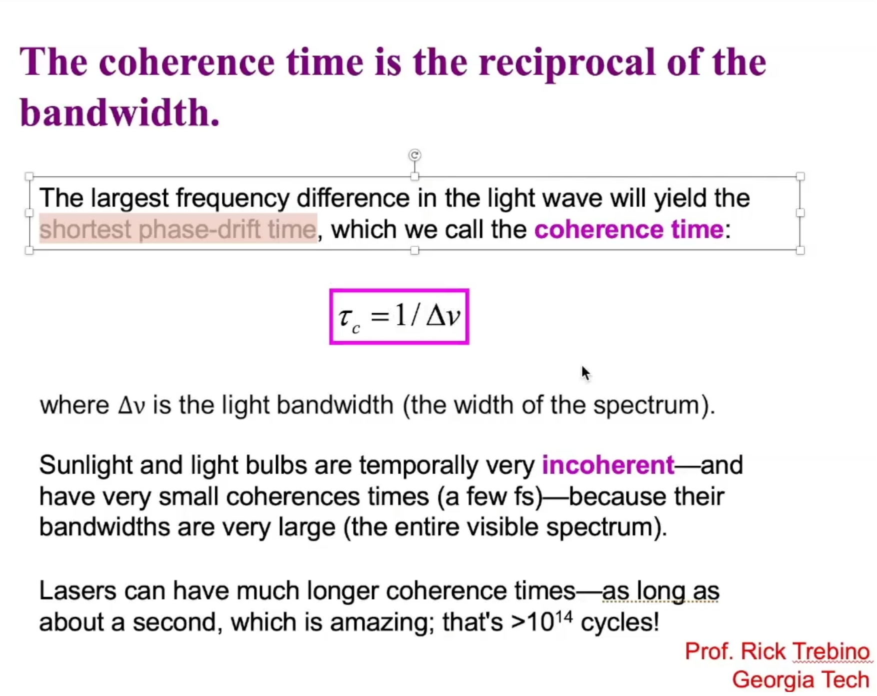
So the coherence time is therefore the reciprocal of the bandwidth. Or in other words, the largest frequency difference in the light wave will yield the shortest drift time because it is inversely proportional to the frequency difference. The larger the frequencies in the light wave, the more the drift in time will be.

Or the shortest will be the drift time. So this coherence time becomes very small if the frequency drift or the largest frequency difference between the light waves is very large. So delta frequency is a light bandwidth, which is the width of the spectrum. So you can see the sunlight and light bulbs, which have all kinds of frequencies, are temporally very incoherent. Okay, there are very, very small coherence times because the bandwidths are very large, as it is the entire visible spectrum.

So the entire visible spectrum, if you take it, is like several hundred nanometers. Okay, so if you divide and convert that to the corresponding frequency scale, you get a coherence time that is very, very short. On the other hand, the lasers have a very low frequency difference because they are monochromatic to begin with; the delta frequency change is very, very, very small. As a result, the coherence time is rather large; it is as big as a second, which is amazing because this is more than 10 to the power of 14 cycles. For example, up to 10^{14}cycles, you can actually maintain the coherence of a laser beam.

Understand? So two things that you should understand here: the coherence time is the reciprocal of the bandwidth for lasers because the bandwidth is very small. This is not a problem. You can actually have a very high coherence time, whereas for light bulbs, sunlight, etc., all other routine sources of light that you get are actually very incoherent and they have very.

.. All coherence times, all right? And how did we deduce it? We deduced it by taking the maximum out-of-phase frequencies, okay, when the frequencies become completely out-of-phase.

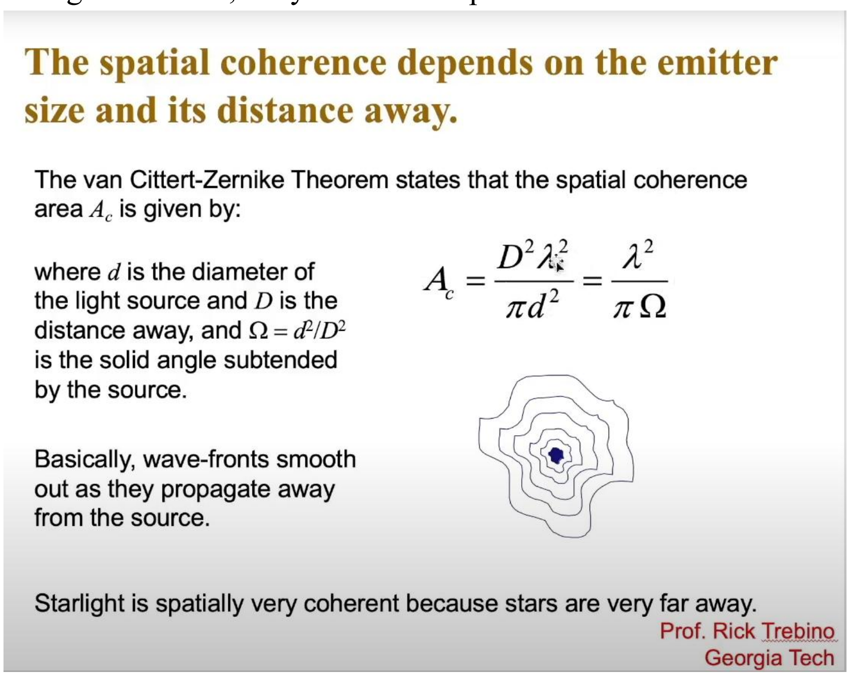


All right, now the spatial coherence depends on the emitter's size and the distance. Remember that we conceived the idea of a spatial coherence area. So this is given by the Van Sitter-Zernike theorem, which states that the spatial coherence area is determined by this particular parameter. Now, if you look at this parameter, d, the small d is the diameter of the light source, right? And capital D is the distance.

Which you are monitoring, and this ω that you see over here is $\frac{d^2}{D^2}$, so it's basically the light source divided by the distance at which you are kind of trying to monitor it. So it is basically nothing but the solid angle subtended by the source. It's a solid angle that is subtended by the source. So, if you look at AC now, if you look at this, you get lambda squared divided by pi times the solid angle. So basically, what happens here, if you look at it carefully, is that the wavefronts smooth out as they propagate away from the source.

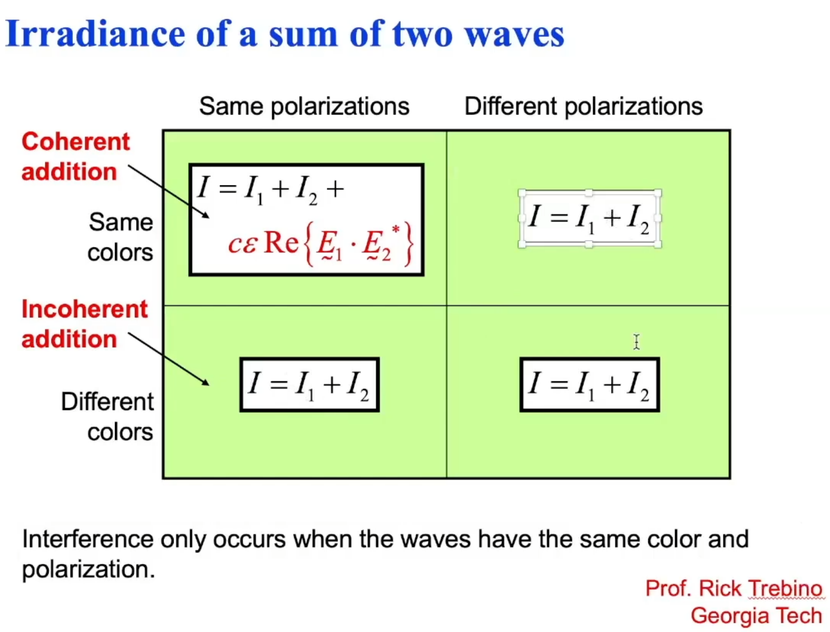
That means, you know, what happens is that as this solid angle becomes larger and larger, okay, the solid angle—sorry, the other way around. So you can see that as the distance increases, you know, as you continue moving, that is D, what happens is that this solid angle becomes smaller and smaller, right? As a result of that, this particular spatial coherence area becomes larger and larger. The other way of saying this is that the wave fronts actually become very smooth as they propagate away from the source. It is also a function of the wavelength; that means the larger the wavelength, the more the spatial coherence area. Another way of telling is that the greater the distance from the light source, the greater the coherence area.

This is precisely what happens with starlight. They are spatially very coherent because the stars are very far away. Okay, very, very far away means this d is very, very large. And this small d is, of course, like the star, regardless of the dimension of the star, like the light source. So, in other words, you don't even have to go to the star. If you are sufficiently far away, your spatial coherence is very high, okay? And also, you know, the smaller the light source is, okay? More is a spatial coherence.



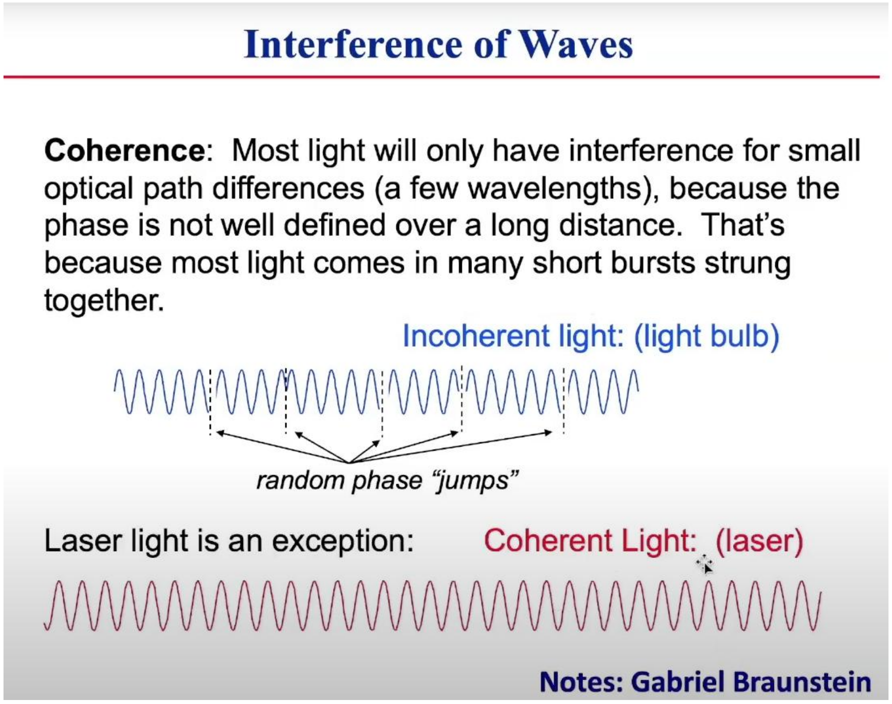
So all these things are captured on this particular slide. So, two other things that, when you actually have, you know, the sum of two waves, which we were kind of doing, is interference, which we talked about earlier. So if we look at this particular part, okay, you will find that if I make it red, it will be the second color. So if we look at this, we will see that when you have the same polarizations (we already talked about polarizations earlier), this is coherent addition; same colors, so same polarization, same color, you get an addition. And then you actually have the other term, right? It's not just an algebraic sum, but it is also the other terms that come into the picture; it's when you have different colors.

When you have incoherent addition and different colors, it's just the sum. And when you have different polarizations, there is also the sum; when you actually have different polarizations, different colors, you have just a simple sum. So interference occurs only when the waves have the same color and polarization. This is also to be kept in mind that when it has the same color and the same polarization, then only will the interference occur. So, the interference of waves means that most light will have interference for very small optics.



So coherence is that most light will have interference for a small optical path difference. Q wavelength; remember, it's an integral multiple of the wavelength because the phase is not well defined over long distances. So that is why most light comes in short bursts, strung together. So this is, for example, the important light from a light bulb, and you can get these random phase jumps because the phase is not well defined over very long distances.

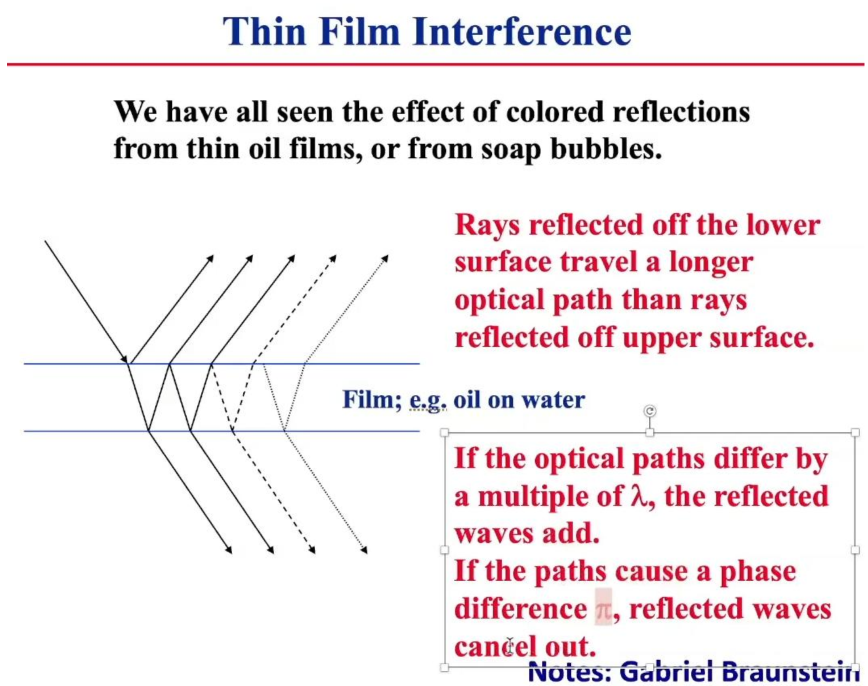
All right. So this is more about the interference and the coherence. The laser is, of course, an exception. It's a coherent light source. So we already know. And this is also going to be very useful when you do the PDPA and the LTV.



We look at a little bit about what we call thin film interference. We have seen the effect of different colored reflections from thin oil films or from soap bubbles. For example, this is a film. This is oil on water, for example.

So there is water, then there is oil, and there is light. So part of it gets reflected, part of it gets refracted and comes out, and then gets refracted. Part of it gets refracted, then it undergoes reflection, then it undergoes refraction again and stuff like that. So the rays that are reflected off the lower surface, which is this lower part, travel a longer optical path than the rays that are reflected from the upper surface. This is known. So if the optical paths differ by a multiple of lambda, the reflected waves add.

And if the paths cause a phase difference of π, the reflected waves will cancel out. So this is the fundamental of thin film. So, for example, here, ray 1 has a phase difference of π upon reflection.

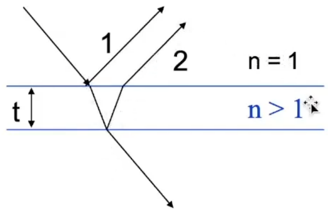


Ray 2 travels an extra distance of 2t. There's 1t and then 2t. Twice, the normal incidence approximation applies; for example, oil on water or an optical film on a glass soap bubble. So constructive interference occurs when rays one and two are in phase; in other words, the 2t that it experiences is an integral multiple of the lambda n plus this half lambda n. So lambda n is nothing but lambda divided by the refractive index. So this $2nt = \left(m + \frac{1}{2}\right)\lambda$.

So, this is the constructive interference theory. All right, so this is the constructive interference theory, and, uh, as you can see, this undergoes a phase change of π, and then it has the 2t; that's what we are kind of adding over here. Okay, so the destructive interference part will happen when the two rays are π out of phase. In other words, $2tn = \lambda m$, which gives rise to 2nt into. m lambda, so this is the thin film interference profile

Thin Film Interference

Ray 1 has a phase change of π upon reflection
Ray 2 travels an extra distance 2t (normal incidence approximation)



oil on water
optical film on glass
soap bubble

Constructive interference: rays 1 and 2 are in phase

$\Rightarrow 2t = m\lambda_n + \frac{1}{2}\lambda_n \Rightarrow \boxed{2nt = (m + \frac{1}{2})\lambda}$ $[\lambda_n = \lambda/n]$

Destructive interference: rays 1 and 2 are π out of phase

$\Rightarrow 2t = m\lambda_n \Rightarrow \boxed{2nt = m\lambda}$

When ray 2 is in phase with ray 1, they add up constructively, and we see a bright region. The different wavelengths tend to add constructively at different angles, and we see bands of different colors. Also, oil on water and optical film on water, the thin films work with even low-coherence light as the path lengths are really short. When ray 2 is π out of phase, the rays interfere destructively, and that's how anti-reflection coatings actually work. So this is the anti-reflection coating that is used routinely. So you see that there are optical explanations behind each and every one of them.

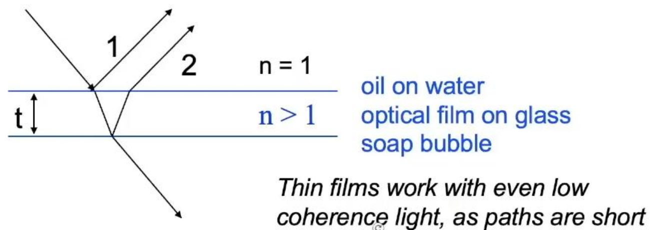
And this is very common because when you do thin-film interferometry, this is the technique that is used. This is used, for example, when you have a droplet impinging on a pool of liquid or a droplet impinging on a surface. And we can see very intricate features because they are also dynamic.

Here, of course, it's a static example. The film is static. It's not moving. But if the film is also moving and if you have a high-speed camera to continuously monitor it or high-speed detectors, you can actually find out the dynamics of this thin film, which may not be possible with a normal camera.

Thin Film Interference

When ray 2 is in phase with ray 1, they add up constructively and we see a bright region.

Different wavelengths will tend to add constructively at different angles, and we see bands of different colors.



When ray 2 is π out of phase, the rays interfere destructively. This is how anti-reflection coatings work.

Notes: Gabriel Braunstein

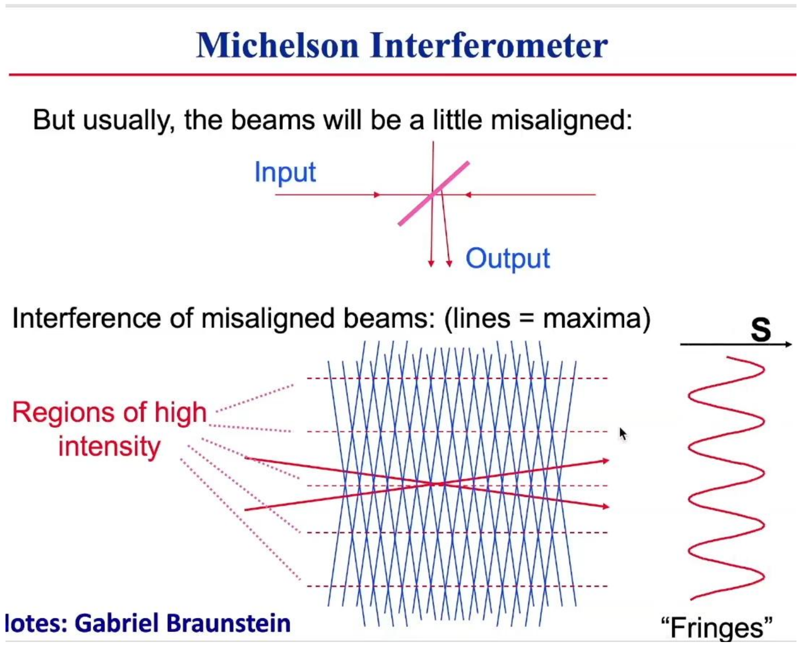
Then we talked a little bit about the Michelson interferometer. So it basically uses a beam splitter to create two optical paths.

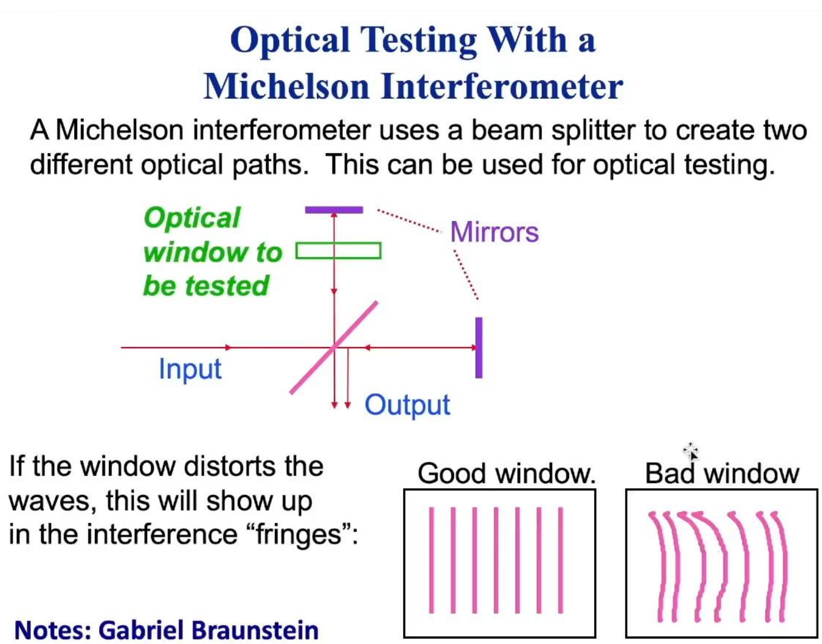
This is the input beam, for example. And then there are two mirrors. So part of the beam; this is a partially coated mirror. So part of the beam goes to one mirror, and part of it gets transmitted to the second mirror. Then they get reflected, recombine, and that is what you see as the output. So this is like a beam splitter. The output beams are perfectly aligned; they will interfere uniformly, giving either a bright or a dark output depending on their relative phases.

So these two mirrors are at two different path lengths of tension. But usually, the beams are a little bit misaligned to begin with. So this is the input. So this is the output. So, the interference of misaligned beams, we already know, represents the maxima of these lines. And this is the beam for the other beam, which leads to the lines with maxima, and as they kind of, uh, kind of interfere, what you will have is regions of high intensity, okay? Unlike the perfectly aligned situation, okay, you get these fringes that you get.

So with this, you get the fringes. This is already something that we know. So you can perform optical testing with a Michelson interferometer. It uses a beam splitter to create, as we see, two optical paths. The same can be used for testing optical windows. So you can choose an optical window.

If the window distorts the waves, they will appear as interference fringes. So this is a good window. This is a bad window, for example. Right. This is also Newton's ring that you can see over here.





And, you know, Newton's rings are also, it's almost like a thin film. So you actually have a curved interface right now. So the constructive interference with an integral number of half wavelengths occurs between the two surfaces. OK, so this is basically the back surface reflected beam.

This is the reflected beam from the front surface. This is the incident beam. So you will see color lambda when constructive interference actually occurs. And this is what you see. You'll see bold colors with n equal to 1. Otherwise, the variation with lambda is fast.

This effect also causes colors in bubbles and oil films on puddles. So you can see these are real-life examples where we can see why the Newton's ring, I mean, this is called a Newton ring, but it's essentially also a thin film interference, which we already talked about in these previous slides. In the Michelson interferometer, you can also do one thing. You can change.

You can also jitter the mirror. That can also give rise to changes and variations. You can test Windows. You can use this. For detecting very small faces.

And this is, as I said, routinely used for droplet impingement. So we finished the lecture. This covers a lot on interference and coherence, just to give an idea of what things are. Of course, we have skipped the math as much as possible, but we have given a little idea of what this is. So this will be handy when you do PDPA, LTV, and any other measurements, as a matter of fact.