

**Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer**

**Prof. Saptarshi Basu**

**Department of Mechanical Engineering**

**Indian Institute of Science, Bengaluru**

**Week – 03**

**Lecture - 14**

**Imaging and Optics -6**

All right, so in this particular talk, we are going to discuss the scattering. Remember, previously we did a lot of work on the Fresnel equations and on the incidence and clear points, right? And we said, "Okay, which one is the first point of the incident?" First order refraction at this glare point, which is first order refraction, is the incident glare point of reflection, and this is how you have illuminated this particle using a monochromatic beam, for example, at an angle, so the large particle images. Point the detector to the clear point. We already did. Now let us look at scattering in terms of geometric optics. Now, say, for example, you have a particle and you have incident monochromatic light, and only part of it is shown.

And this is the surrounding media. And so, geometric optics basically describes light scattering from small particles using the laws of reflection and refraction. So this statement you should pay very clear attention to, as it describes nothing but the light scattering from small particles using the laws of reflection and refraction, which we already did in detail. So assumptions are usually made that the particle is spherical and the incident light is monochromatic, which is the case with a laser, for example.

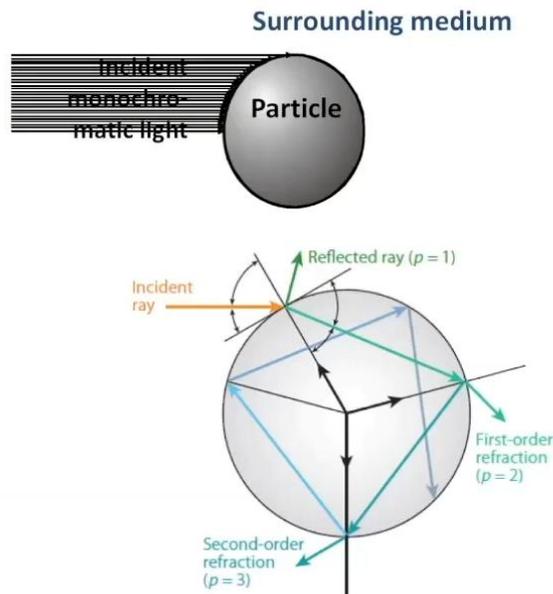
Subsequently, what you do is allow Snell's law and Fresnel equations to be used to compute the reflection and transmission angles and intensities. Snell's law and the Fresnel equations are used to compute the reflection and transmission angles. And we already saw how you can calculate the power, as well. So looking at this particular scenario, let's take one incident beam, which is the given incident ray, shown in this particular color. Now, part of it is of course reflected.

This is reflected, okay? And then you know part of it is refracted, so where it comes out, it's basically the first order refraction, and then a part of it is again reflected from this interface. Then it comes out to the other side, which is called the second order refraction, and this goes on and on. The cycle kind of goes on and on. They can understand that the incident beam, if part of it is reflected and part of it goes in, is the incident beam, and then whatever comes out on the other side is basically first-order refraction. Then whatever comes out to this side is second-order refraction, and then it goes on and on and on and on.

Each time the intensity is lowered, as you can see, this is a very prominent thing that happens. So whatever the detector measures is a combination of all these things that basically come out of this particular situation.

## Scattering According to Geometrical Optics

- Geometrical optics describes light scattering from small particles using the laws of reflection and refraction
- Assumptions are usually made that the particle is spherical and that the incident light is monochromatic.
- Snell's law and the Fresnel equations are used to compute reflection and transmission angles and intensities.



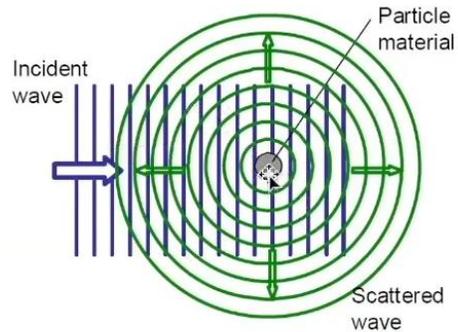
So light scattering from small particles is considered under the following assumptions: that there is a plane incident wave and that there is homogeneous particle matter. Basically, this is the particle matter, and it is a spherical particle. The exact solutions were given by me.

Or what we call the Lorenz B theory, what happens is that there is an incident wave that falls onto this particular particle, and there is a lot of scattered wave in all directions. You can see these are the scattered wave fronts in all directions. They are not equal in all directions, by the way. So there is an incident wave that lands on a particle of homogeneous material, which is spherical. And this is a planar wave incident on a plane.

# Light Scattering from Small Particles

Light scattering from small particles will be considered under the following assumptions:

- Plane incident wave
- Homogeneous particle material
- Spherical particle



Exact solutions for this problem are given by the Lorenz-Mie Theory

This gives rise to the scattered wave, which is basically scattering in all possible directions, though not equally. So the exact solutions to these problems are given by the Lorenz-Mé theory. So here you can calculate using ray optics and geometric optics. You can calculate all these, you know, how much light is reflected, refracted, et cetera, but it is much easier to use the waves and calculate it from the, you know, from the Lorenz-Mie theory. Okay, so from a geometric perspective and an optics perspective, it is easier to understand what may have gone on, why you get multiple glare points, and what is meant by first order and second order refractions, and so on.

All right, so this is what happens when you talk about Lorenz-Mie scattering: the scattering of a plane wave by a spherical particle is described fully by the Lorenz-Mie theory. So, it is basically how a plane wave is scattered by a spherical particle. The scattering is governed by what we call the "mie" parameter, which is a non-dimensional diameter. It is nothing but the diameter divided by the wavelength of the illumination, multiplied by  $\pi$ . Essentially, it is proportional to the diameter of the particle, but there is an inverse relationship with the wavelength.

And this is a function of the relative refractive index, which is  $n_p$  divided by  $n_m$ . So the scattering is characteristic of the different ranges of the me parameter. So when the me parameter is less than 0.2. So if you invert it, you will see it is less than  $\frac{\lambda}{\pi}$ , which is less than 0.

$2 \frac{\lambda}{\pi}$ . Okay, so when it is less than  $\frac{\lambda}{\pi}$ , it essentially means when the particle diameter is less

than about 0.2 times  $\frac{\lambda}{\pi}$ . Okay, so that means very small particles. Essentially, what we have is for the Rayleigh scattering.

And then we go into the me region, where the non-dimensional me parameter is between 0.2 and 10. And then we go to the geometrical optics range when  $X_m$  is much greater than 10. So if you look at it in this particular plot over here, for example, the  $\lambda$  that is used is 488 nanometers, it can be anything else. And this is the "mie" parameter that we have plotted.

So when the "mie" parameter is, you know, less than, say, about when the particle diameter is really small. So, this is the particle diameter. Because once you know the wavelength, you can also plot the particle's diameter. So when the particle diameter is about 0.

1 to 0.2 micrometers, which essentially means that it is 100 to 200 nanometers, okay, this is for this particular wavelength, you have what we call Rayleigh scattering, and this is measured at 30 degrees and at 90 degrees scattering angles. This is the measurement; as you can see, the signal is pretty low. Very, very low. The arbitrary intensity profile is somewhere around  $10^{-14}$  and ranges up to about  $10^8$ . And when you do it at 90 degrees, this is even lower.

As you can see, this goes down pretty quickly. The low particle sizes, as you can see, mean that the scattering section is pretty small, and the intensity that it scatters at different scattering angles is also pretty small. Then you actually go to the range, which is between about 0.2 and all the way up to about 10 microns in size. So, if you look at this particular range, you'll see these lobes.

We will discuss these lobes. These are due to interference. So those lobes, you get over there. It's about 10 microns in size. The intensity is pretty high, much higher.

Actually, it is about... in orders higher than this. And then, if you go to the geometric optics range, this is very high. That means up to about 100 microns. This is very high. So the idea is that the intensity scales are somewhat about  $x^6$ .

Here it is up to  $x^2$ . So this is the kind of range that we are actually looking at. Okay, so the scattering characteristics and the power are characteristic for three different ranges of the mean parameter. So this is the important part. This will come in very handy later. This is given for a particular wavelength.

Remember that, all right? So this is Rayleigh scattering, E scattering, and then you go to

geometric optics, okay? Now, if you look at this, say that there is a relative refractive index, which is about 1.4, and then you just look at the red line for the time being.

## Lorenz-Mie Scattering

Scattering of a plane wave by a spherical particle is described fully by the Lorenz-Mie Theory. The scattering is governed by the Mie parameter (non-dimensional diameter)

$$x_M = \frac{\pi d_p}{\lambda}$$

and is a function of relative refractive index  $m$

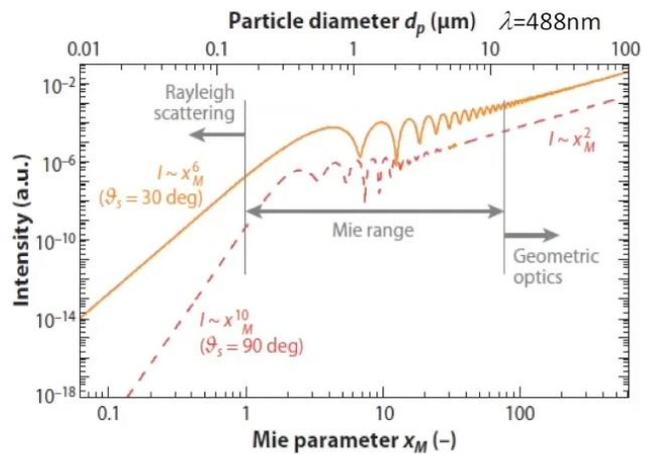
$$m = \frac{n_p}{n_M}$$

The scattering is characteristic for three different ranges of the Mie parameter.

**Rayleigh Scattering:**  $x_M < 0.2$

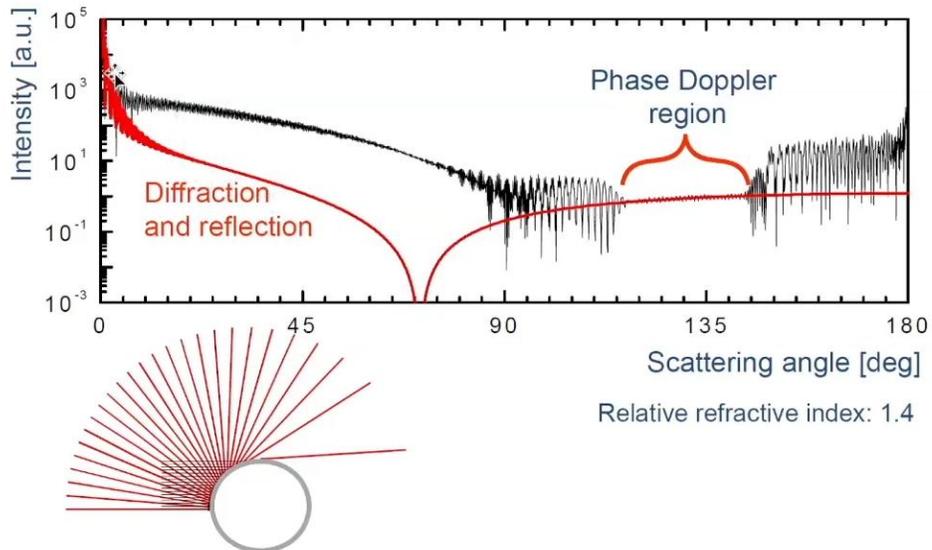
**Mie Region:**  $0.2 \leq x_M \leq 10$

**Geometrical Optics:**  $x_M > 10$



So, this is the diffraction and reflection, okay? So, as you can see over here, this is how the intensity actually varies for different scattering angles, okay? And this is the region where we will conduct the phase Doppler anemometry, which is around the scattering angle of this region. This is how the scattering, diffraction, and reflection happen. Okay, remember that this is how we are trying to understand this.

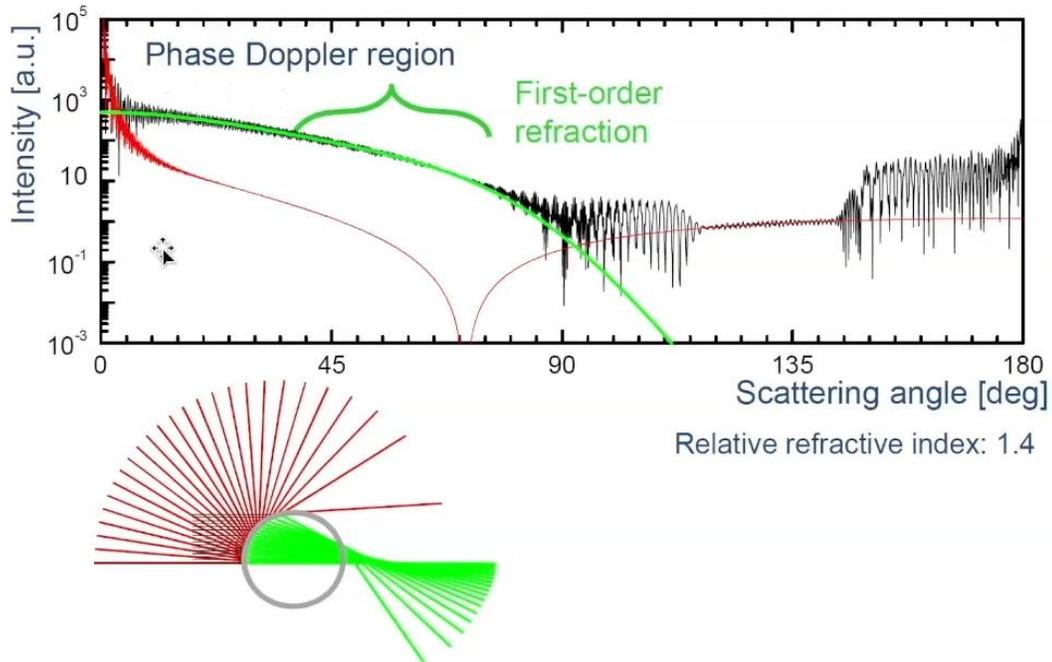
## Reflection and Diffraction



Similarly, if you look at first-order refraction, this involves reflection and diffraction. This is if you look at first-order refraction. What you can see is that this is the phase-Doppler region. And this is how refraction happens. So this is what actually comes out.

And this is the incident part. And then the relative refractive index is basically. Okay, so this is how you know the relative profile section. So, this is the reflection part. This is basically the first-order refraction. If you look at it carefully, you will see that these two are basically similar.

## First-Order Refraction



Got it? Okay. Now, if you look at it from a geometric optics perspective, the scattered light at any particular scattering angle contains different scattering orders. It contains scattered light; as we see, it has different properties. We already know from geometric optics that scattering is composed of everything: reflection, refraction, all orders of refraction, et cetera. So if you look at this, this is an incident wave, and this is a particle.

So you get this. This is the incident wave, which is in orange. And then you have a reflection, which is marked in green. Then you have first-order refraction, green, but of a different color. Then this is of second order refraction. Okay, and then you get third-order refraction.

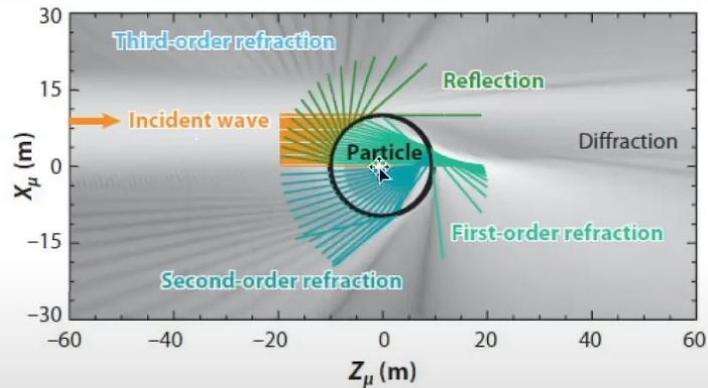
Okay, so second-order and third-order refraction. So it is composed of reflection, and then you also have diffraction because of the particles that we already covered, which gives you those airy disks, stuff like that. So reflection, first-order refraction, second-order refraction, and third-order refraction; all these things are contained when light is scattered at any particular scattering angle. It contains different scattering orders; therefore, we already saw this from the geometric optics point of view, and this is kind of summarized by this particular diagram that you see over here. So I will just hover on this for a few seconds while you can make sure that you have got everything here.

So when you talk about light scattering by small particles, this is a pretty complicated diagram that you can see over here.

## Geometric Optics

Scattered light at any particular scattering angle contains different scattering orders

- Reflection
- First-order refraction
- Second-order refraction
- Third-order refraction
- and further scattering orders...



So this is for parallel polarization, and this is for perpendicular polarization, which we already know by now. So, this is diffraction. OK, and then this is the first order refraction. This is the reflection that you see over here.

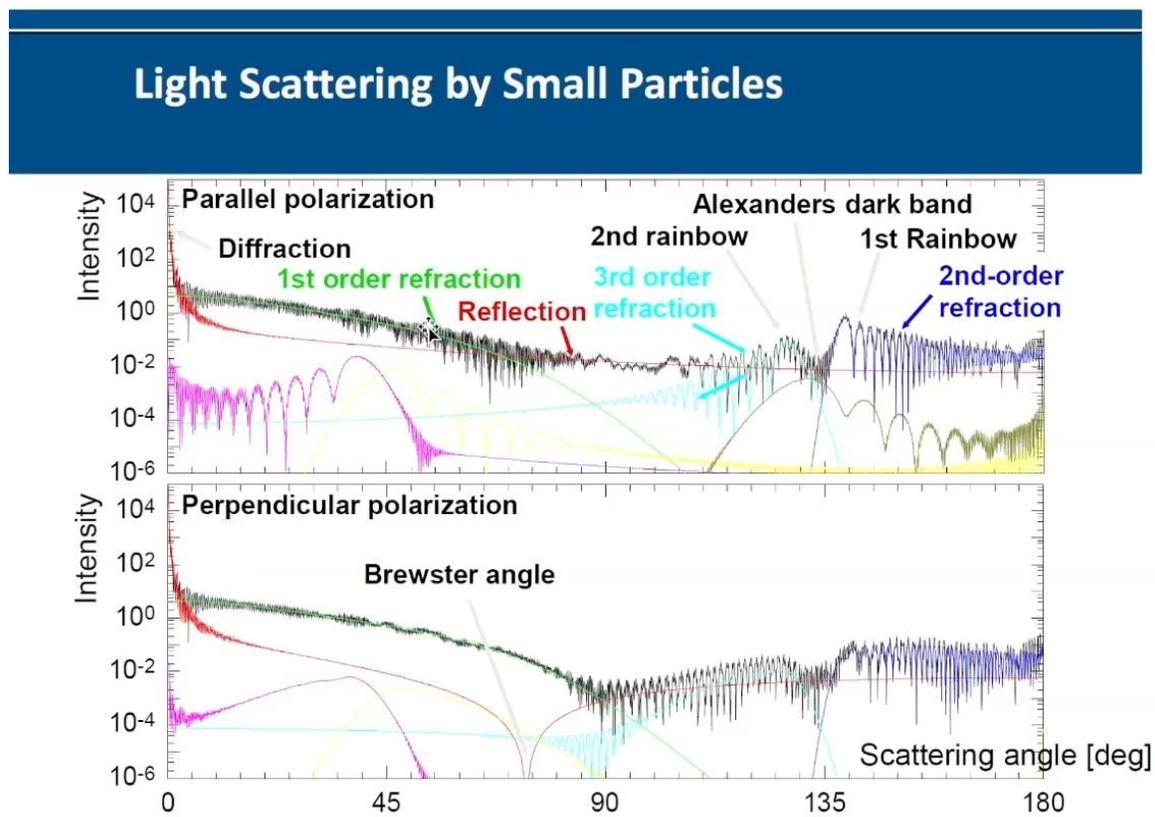
This guy that you see, this one in a kind of bluish color, is third order refraction. OK, and this is the second order refraction. And you know, it's a mess. So this is the second rainbow and the first one.

This is from monochromatic light; remember. So this is not the conventional rainbow that you know from your high school physics, perhaps. This is second-order refraction. And this is obviously Brewster's angle over here where it goes to zero. At this Brewster's angle, you can also see here where reflection is actually equal to zero. So this is the same Brewster's angle that we covered a few times in the last lecture, basically.

You can see it. So, there is this; you know, this is for perpendicular polarization. So they hear the scattering. So at different scattering angles, you get to see, you know, different dominance of different scattering modes. And you should remember that all these angle lobes that you see over here happen because of the interference among the scattering modes.

So that is also important. That's why you get these very funny-looking characteristics. This is Alexander's dark band named after Alexander the Great. As you can see, perpendicular polarization has a particular kind of scattering pattern, and parallel polarization also has a kind of pattern, and all the orders of refraction are written over here. For example, first order refraction goes down to zero after 90 degrees, which you can see right over here; reflection continues for a while, and second order refraction picks up.

After you are at a certain scattering angle. So you can see all of this kind of makes pretty common sense if you think about a particle. And if you think about it after 90 degrees, there cannot be any first-order refraction, or the probability of that will decrease quite a bit.



And this is the dependence of the relative refractive index. And you can see these look all pretty complicated, but this, so again, you know, we know once again about HAARP that this is nothing but, you know, the scattering from small particles that we are looking at over here.

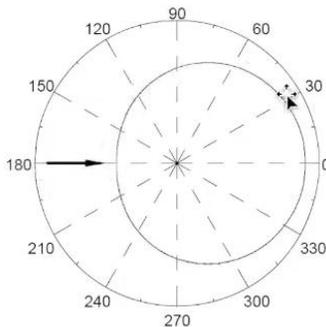
And this is once again the Lorenz-Mie scattering. If you just go back to this particular slide once again, that is what I wanted to do. So, you know, depending on which way you are actually monitoring it, it varies quite a bit. And it also depends on the particle sizes that you are trying to measure. This is the most important graph you can have. So if you do a

polar representation of the scattered light, you will find that when  $d_p$  is small, that is, it's roughly in that Rayleigh scattering limit.

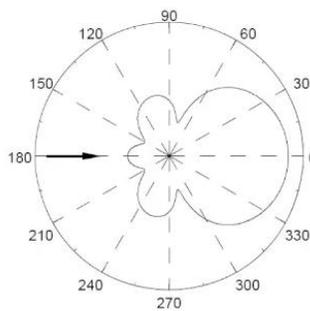
And this is the direction of illumination: the kind of scattered intensity you get. As you go to 1, regarding the wavelength, you get this kind of multi-lobe feature. As you can see, forward scattering gives you much more intensity than backward scattering. And then, if you go to 10, you get all these very intricate lobes. Here, the forward scattering is much greater than the backward scattering.

An advantage is that backward scattering, if monitored, even though it is lower than forward scattering, allows you to integrate the optics and eliminate the alignment delays. So, these are all shown on a logarithmic scale. So, the polar diagram is a function of the scattering angle. Not only the scattering angle, but the forward scattering is always more than the backward scattering. And these lobes, once again, these eight lobes we studied are due to what we call interference.

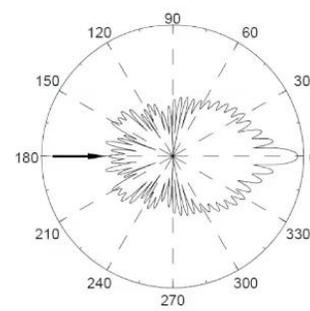
## Polar Representation of Scattered Light



$$d_p \approx 0.2\lambda$$



$$d_p \approx 1.0\lambda$$



$$d_p \approx 10\lambda$$

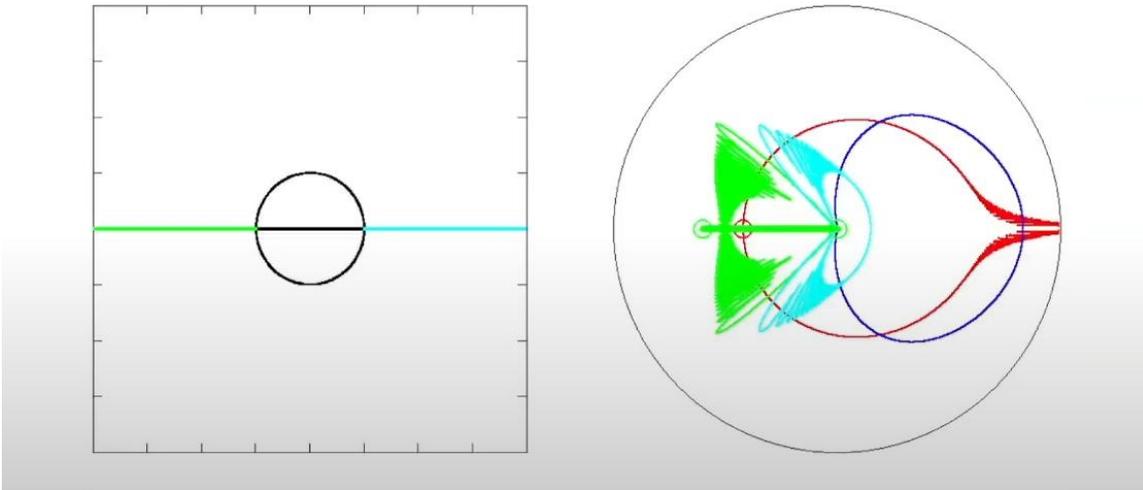
- Polar diagram of scattered intensity as a function of scattering angle
- The intensity is shown with a logarithmic scale.

Interference causes the rainbows. The explanation of rainbows lies in the fact that for higher scattering orders, the direction of the scattered wave reverses as the incident angle changes; the angle at which the reversal occurs is known as the rainbow angle. You can learn more about it, do a Google search, and learn more about this later. So again, just hold on and see that this is, once again, the same plot. I just want to re-emphasize that it is very important to know what kind of polarization you actually have. So this now moves to what

we call the fundamentals of image processing.

## Explanation of Rainbows

For higher scattering orders the direction of the scattered wave reverses as the incident angle increases. The angle at which this reversal occurs is known as the rainbow angle.



Now that we know what light propagation is, what scattering is, and a little bit about Lorentz V-theory, we understand scattering from small particles, we know geometric optics, and we are also familiar with Snell's law, the Fresnel equations, and polarization. We know all about cameras and lens systems now. Well, now it is important to know what image processing you need to do. As you know, the image detectors come in two varieties.

## Contents

- Some optical fundamentals
- Visualization
  - Hardware (Cameras, Lens systems)
- Light Propagation and Scattering
  - Light as an EM wave, Coherence, polarization
  - Snell's law, Fresnel equations
  - Lorenz-Mie theory, Geometric optics
  - Scattering from small particles
- **Image Processing Fundamentals**

One is a CCD detector; one is a CMOS detector. The CCD is a charge-coupled device. It converts the spatial arrangement of accumulated charges into an electrical signal. CMOS is a complementary metal-oxide-semiconductor with a reduced size of transistors. It's possible to give each pixel its own pre-amplifier and additional circuit. In contrast to a CCD, it is possible to read out part of the sensor without any speed loss and with the same pixel clock rate.

The high-speed camera technology is therefore dominated by CMOS. High-speed sensors are not limited by CMOS technology but by the transfer of digital image data. So these are the two types of image detectors that you have.

## Imaging Detectors

### Imaging detectors: CCD and CMOS

**CCD** (Charge-coupled device): Converts spatial arrangement of the accumulated charges into an electrical signal (read-out)

**CMOS** (Complementary metal oxide semiconductor): reduced size of transistors → possible to give each pixel its preamplifier and additional circuits

*In contrast to a CCD sensor, it is possible to read-out a part of the sensor without any speed loss and with the same pixel clock rate.*

High speed camera technologies are dominated by CMOS technology today.

High speed sensors: not limited by CMOS technology but by the transfer of digital image data.

The images are basically the grayscale images that you see over here, which have pixels that range from black to white, going through a number of intensity steps in between. So most of the images allow 256 intensity levels.

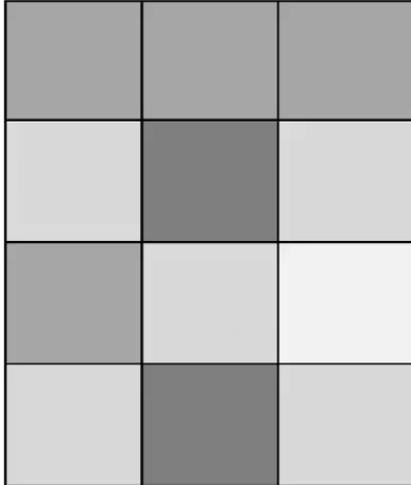
So a number arises from 256 different values that a byte can take. A byte is composed of eight bits, and such images are therefore known as 8-bit grayscale images. Some high-end scanners can also scan with finer intensity. So they can use 12-bit, 16-bit, or 10-bit to represent the intensity values. So, therefore, it makes it possible to go up to 4096, 10,

024, to 4096, or even 65536 different gray values. So that is also possible. And one bit, if you look at it, is basically two gray values, which are essentially a black and white image. So some points, when you go for 12-bit or 16-bit, are basically that the images take up more space and not all imaging software can work with this kind of arrangement. OK, so you know, you might not see any difference on the computer screen because the computers may be programmed to show only up to 256 shades of grey. So that is also another thing that one should keep in mind when one is actually doing this.

And you don't see the darkness. Basically, you don't see the dark areas of the image any better. So the number of bits that are used for intensity values is also known as the bit depth. So higher bit depth, which is what you have for 12-bit and 16-bit, is not necessarily a great

thing, but it is kind of possible too. And the pixels need not be square. They can have angular distortion and also different dimensions, the two directions, and all that stuff.

## Gray images



### Gray level

8-bit :  $2^8$ , 256 gray levels 0-255

10-bit :  $2^{10}$ , 1024 gray levels 0-1023

1-bit :  $2^1$ , 2 gray levels 0-1, **BW image**

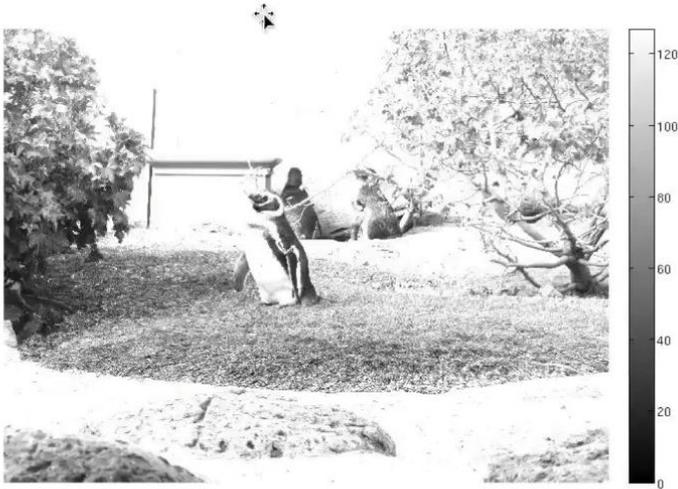
- Pixel need not be square
- Pixel can have angular distortion and different dimension in two direction
- Circular object for scaling

### Geometric and point-wise information

So this is, for example, the image on a 7-bit scale. And this is the image on an 8-bit scale.

So this is 7 bits. This is an 8-bit scale. You can see the difference.

### Image on a 7 bit scale



### Image on a 8 bit scale



A larger number of representation B-levels, not here. In image processing tools, there are quite a few. You can have MATLAB and LaVision. These are proprietary items that people normally use in their labs. There are a number of open-source codes as well, like ImageJ,

Octave, OpenCV, and stuff like that. We'll have a tutorial session on image processing later, where we will show some of these characteristics.

## Image Processing Tools

Several proprietary software

- Matlab, LaVision – DaVis, TSI - Insight, Dantec - Dynamic Studio, etc.

A large number of Open Source Codes

- ImageJ
- Octave
- GIMP
- OpenCV

Some of the basic operations are colorization, summation, multiplication, subtraction, averaging of images, thresholding, finalizing, and filters. All these things are there for gray images. For example, this is an 8-bit image; this is another 8-bit image; this is another 8-bit image. The histogram of gray levels in the image is something like this: you can see that 8-bit can have 256 levels.

## Basic Operations

- Colorizing
- Summation
- Multiplication (bit depth!)
- Subtraction
- Averaging
- Thresholding
- Binarizing
- Filters

Of gray, so you know that is what you see over here. Okay, uh, here, for example, you see an almost bimodal distribution because of less information; it is kind of a very sharp interface here. Of course, you see that because the image has many shades and many parts, there is more population in the intermediate gray levels, but the peak is still at the minimum level. And then this is a more complicated image: the car, the house, and all those things. So, the information content is very high. Here it is basically bimodal because, for example, if you put a particle, this is just the gray and the background.

## Gray images



1024 X 512, 8 bit image

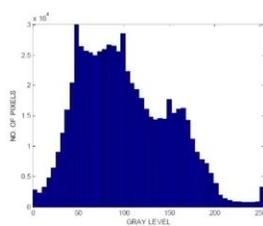
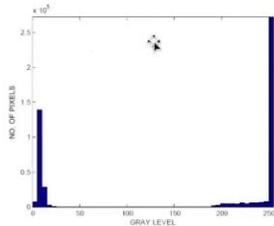


1200 X 1600, 8 bit image



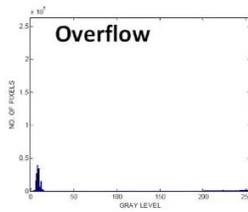
1800 X 367, 8 bit image

## Histogram of gray levels in images

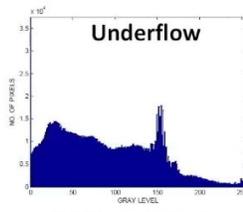


So this is kind of almost bimodal. And this has all the values in the intermediate gray sections, intermediate gray levels. The information content is very high. So what you do is that for pointwise operations, you look up a table.

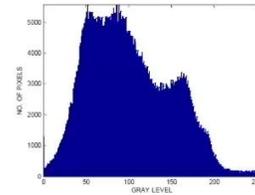
## Histogram of gray levels in images



Almost bi-modal distribution  
Less information content



More population in the intermediate gray levels  
Peak is still at the minimum gray level

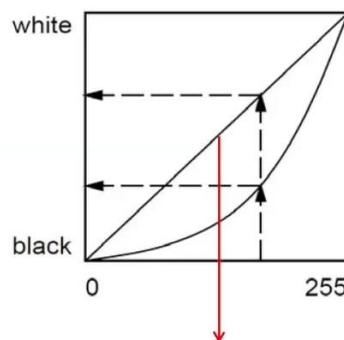


Peak at the intermediate gray level  
Information content is high

So mapping of gray values to brightness values. This is called a LUT. LUTs may be linear or nonlinear. The nonlinear LUT displayed image will not be an exact representation. So a variety of visual effects can be obtained by varying the functional form of the image. Okay, so this is, and these are other, you know, common examples of LUTs in brightness. So, you know,  $G_U$  is equal to  $U$  plus  $B$ ;  $B$  greater than zero is brightening, while less than zero is darkening.

## Point-wise operations

- Lookup Table (LUT)
  - Mapping of the gray level to the brightness value
- LUT may be linear or non-linear
- Non-linear LUT : Displayed image will not be exact representation
- Variety of visual effects can be obtained by varying the functional form of the LUT



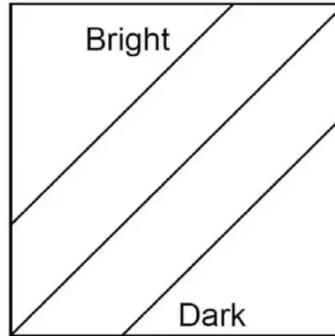
Direct mapping: unit slope, zero intercept

So you can see these kinds of examples. Okay, so this is a LUT brightness example. So this is like brightening the saturation at a higher intensity. You have more saturation and higher intensity values. So this is all that is here. This is, for example, darkening, like

saturation, lower intensity in an example. There are also pointwise operations that you can do, and a common example of an LUT will be  $u + b$ .

## Point-wise operations

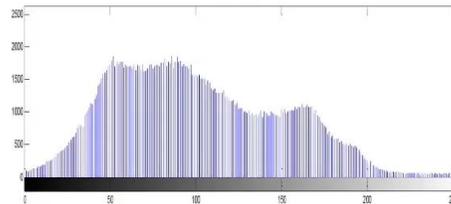
- Common example of LUT: Brightness
- $g(u) = u + b$ 
  - $b > 0$ , brightening
  - $b < 0$ , darkening
- Examples



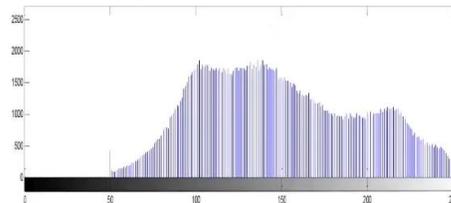
M can be high contrast or low contrast, et cetera, so that the images can be passed through multiple LUTs. So these are kinds of image processing that one can do. This is a contrasting example. So this is enhancing the contrast and saturation at limited intensity.

Okay, so these are some of the finer examples that one can use. These are more of the same. We are not going to go into detail. There are also auto-scaling examples that enhance contrast, where you set the minimum gray value as black and the maximum as white to avoid saturation of black and white. So alternatively, set 1% and 99% intensity values as black or white.

## LUT: Brightness (Example, $b=50$ )



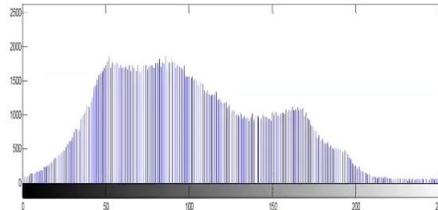
Brightening: Saturation at higher intensity



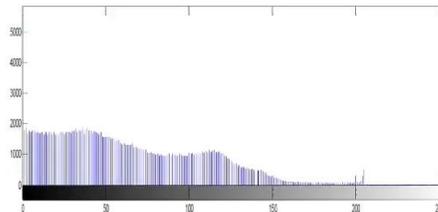
So this is an example of auto-scaling that you get. The minimum is 125, and the maximum is 2. So this is, of course, an inversion. It makes common sense what inversion means.

There are more pointwise operations. These are examples of gamma correction. It's a nonlinear LUT for contrast. So you can read more, and we will also have a demo session on this. This is like gamma correction.

### LUT: Brightness (Example: $b=-50$ )



Darkening: Saturation at lower intensity



This is enhanced contrast, no saturation at limiting. So, more of the same. This is again a gamma correction. These are different LUTs for contrast enhancement. All of these things we will show in a couple of slides over there in a definite manner. In a laboratory session.

### Point-wise operations

- Common example of LUT: Contrast
- $g(u) = m u + b$ 
  - $m \rightarrow \infty$ , High contrast
  - $m \rightarrow 0$ , Low contrast
- $b = 0.5 * (1 - m)$
- $m = -1, b = 1$  (negative / inverted image)
- Examples

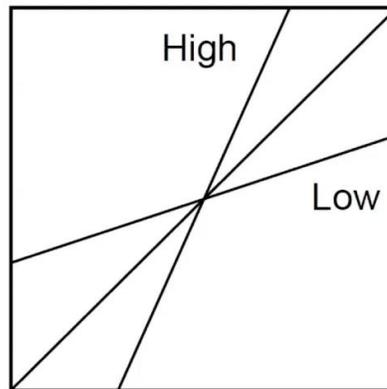


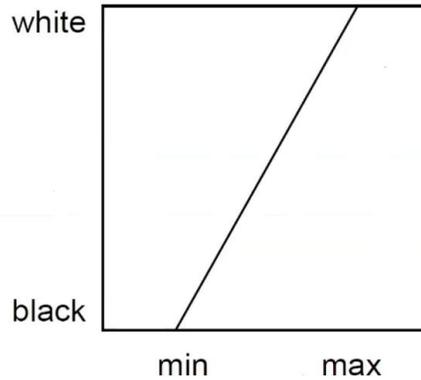
Image can be passed through multiple LUTs

So this is, for example, the threshold selection where we talk about Oates-South's algorithm. It selects a threshold that minimizes the sum of intraclass spread, which is the variance, and this is equivalent to maximizing intraclass variance. There are certain

examples. For example, this is the image. So if you took the threshold value of 60, this is the threshold value from Otsu's algorithm.

## Point-wise operations

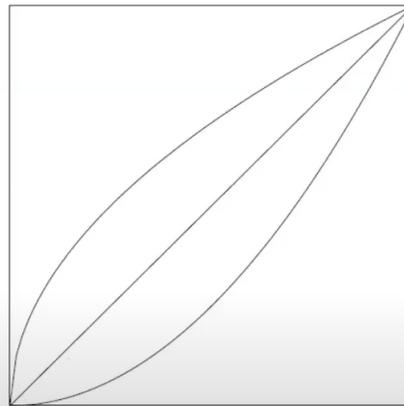
- Autoscaling (contrast enhancement)
- Set minimum gray level as black and maximum as white
- Avoid saturation at black or white
  - Few stray pixels can determine contrast
- Alternate: set 1% and 99% intensity values as black or white



So this is a very common thing: Otsu's algorithm, which is routinely used. We'll have a demonstration of it. So this is an original image. This is the grayscale image. You do the thresholding of this, and you get what you see over here.

## Point-wise operations

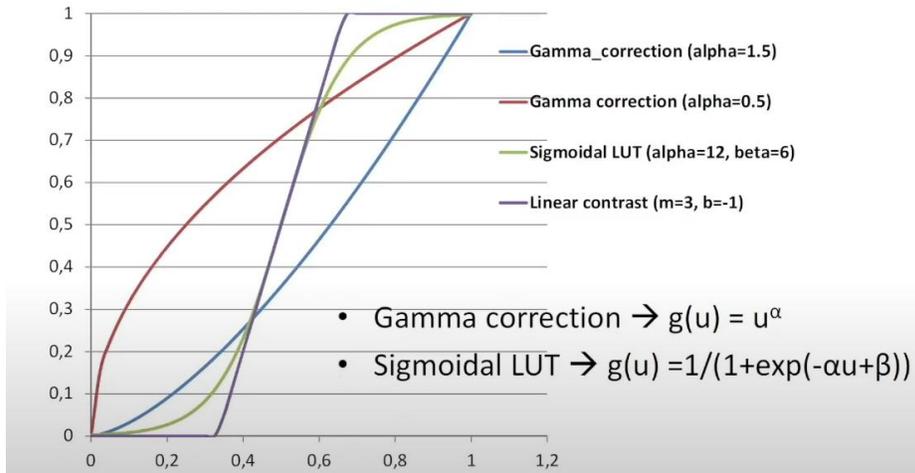
- Gamma correction (non-linear LUT for contrast)
- Linear contrast enhancement generally leads to saturation at black and white
- $g(u) = u^\alpha$ \*
- $\alpha > 1$ , Increases contrast
- $\alpha < 1$ , Reduces contrast
- Not symmetric about mid-level gray; Alternate: Sigmoidal non-linearity



\* Note that  $u$  here is normalized with the maximum intensity

In other words, image processing is important. It helps to decide the exposure time, light intensity, aperture, etc. It is essential to determine quantitative data from the images. And these are the books that you can read. This is the Springer Handbook of Fluid Mechanics and Professor Tropa.

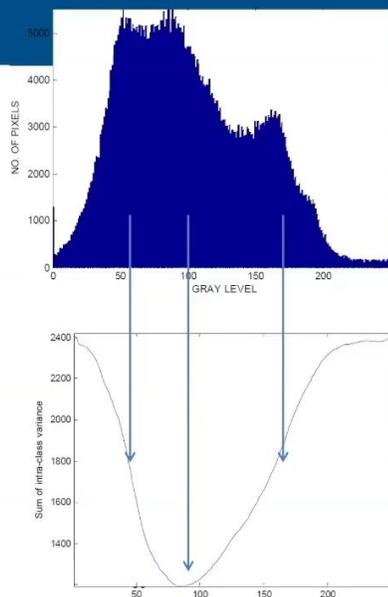
## Different LUTs for contrast enhancement



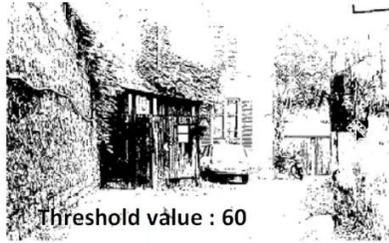
And you know, you can read all about the different comparisons of CC. Bright cameras. You can look at out-of-focus effects on particle visibility and correlation. So all of these things are here. There is a lot of material, so feel free to read it.

## Threshold selection

- Otsu's algorithm
- Selects a threshold that minimizes the sum of intra-class spread (variance)
- Equivalent to maximizing inter-class variance
- Examples



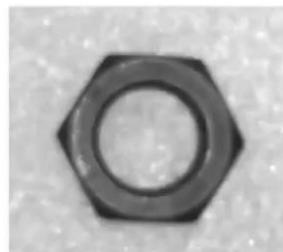
## Thresholding (Examples)



Threshold value : From Otsu's algorithm

Original Image

Gray image



Thresholded images with threshold of 0.3. 0.7. 0.56

