Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
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Week – 03
Lecture - 13
Imaging and Optics -5

Welcome to this particular class, and today we are going to do a little bit more on wave propagation. Remember, we did polarization in our last class, so wave propagation at an interface is very typical refraction that you already know about. And so this is refraction that you know already. This is an incident angle, and there's a part of it that is actually reflected. This is the interface, and a part of it goes inside that is transmitted. So i is incidence, r is reflected, and t is transmitted.

And the angles of reflection and incidence are the same. And the law of refraction states that $n_1 \sin\theta_i = n_2 \sin\theta_r$. This is something that we already know. This is high school physics.

Wave Propagation at an Interface

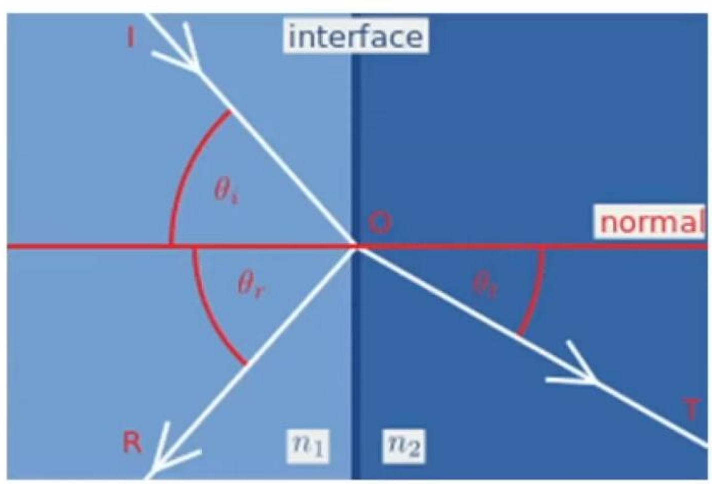
θ_i (incident angle), θ_r (reflection angle), θ_t (refraction angle)

Law of reflection

$$\theta_i = \theta_r$$

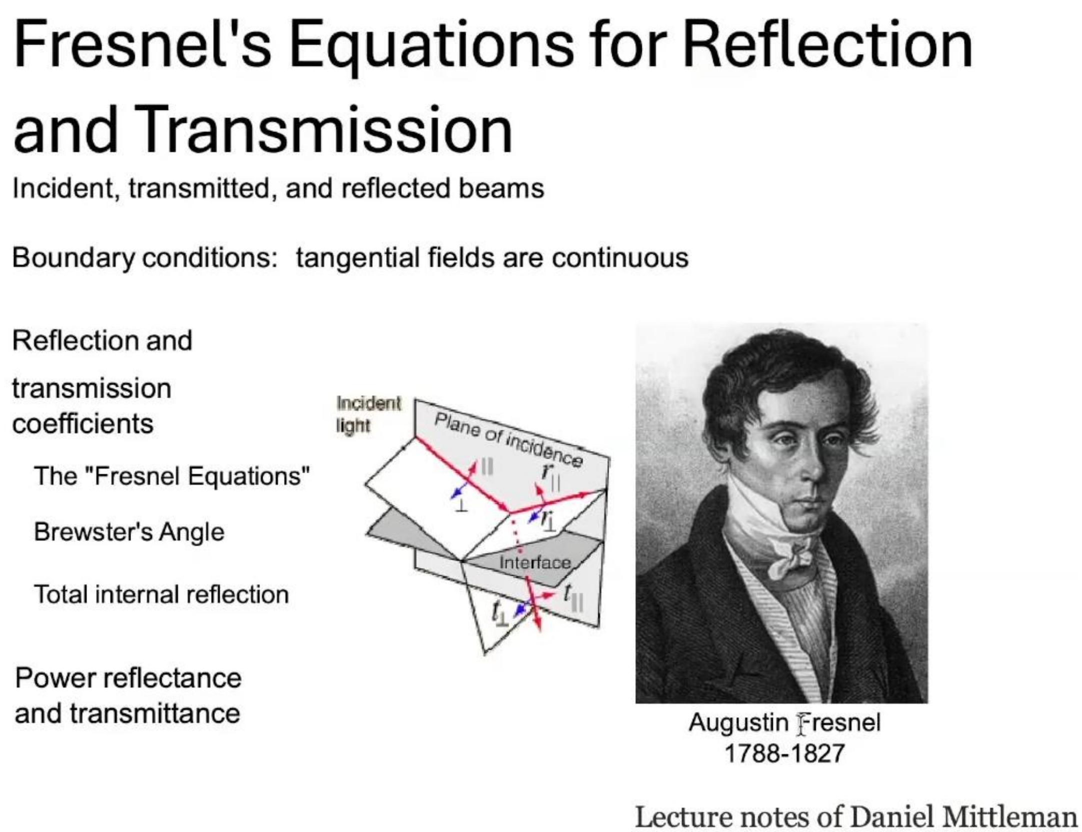
Law of refraction

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$



All right, now let us look at the notes from Daniel Mittelman of Brown University, and this is taken from there. So we are going to talk about the Fresnel equations for reflection and transmission. So, what do we have? In this case, if you look at it in slightly 3D, what we will have is that there is incident light, then there are these sheets, the incident light, this is the plane of incidence, where you see, and then this is the reflection and this is the transmitter. And this is the interface, this slightly gray-colored boundary. So, we're going to talk a little bit about what reflection and transmission coefficients are, the Fresnel equations, what we call Brewster's angle and total internal reflection, and the power reflectance and transmittance.

These are the things that we are going to cover. Much of this is indebted to Augustin Fresnel.

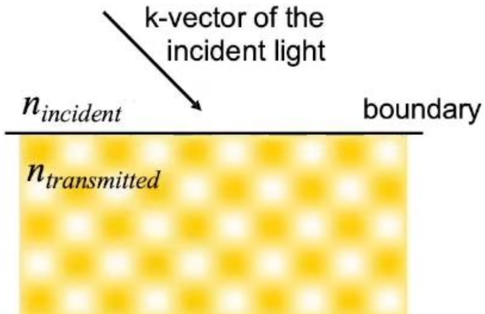


So what is the problem? The problem occurs when light propagating in a uniform medium encounters a smooth interface. So this is a smooth interface, which is the boundary between the two mediums with different refractive indices. So this is the refractive index of the incident medium.

This is the refractive index of the transmitted medium. And this is the K-vector of the incident light. Okay, so this is the posing of the problem.

Posing the problem

What happens when light, propagating in a uniform medium, encounters a smooth interface which is the boundary of another medium (with a different refractive index)?

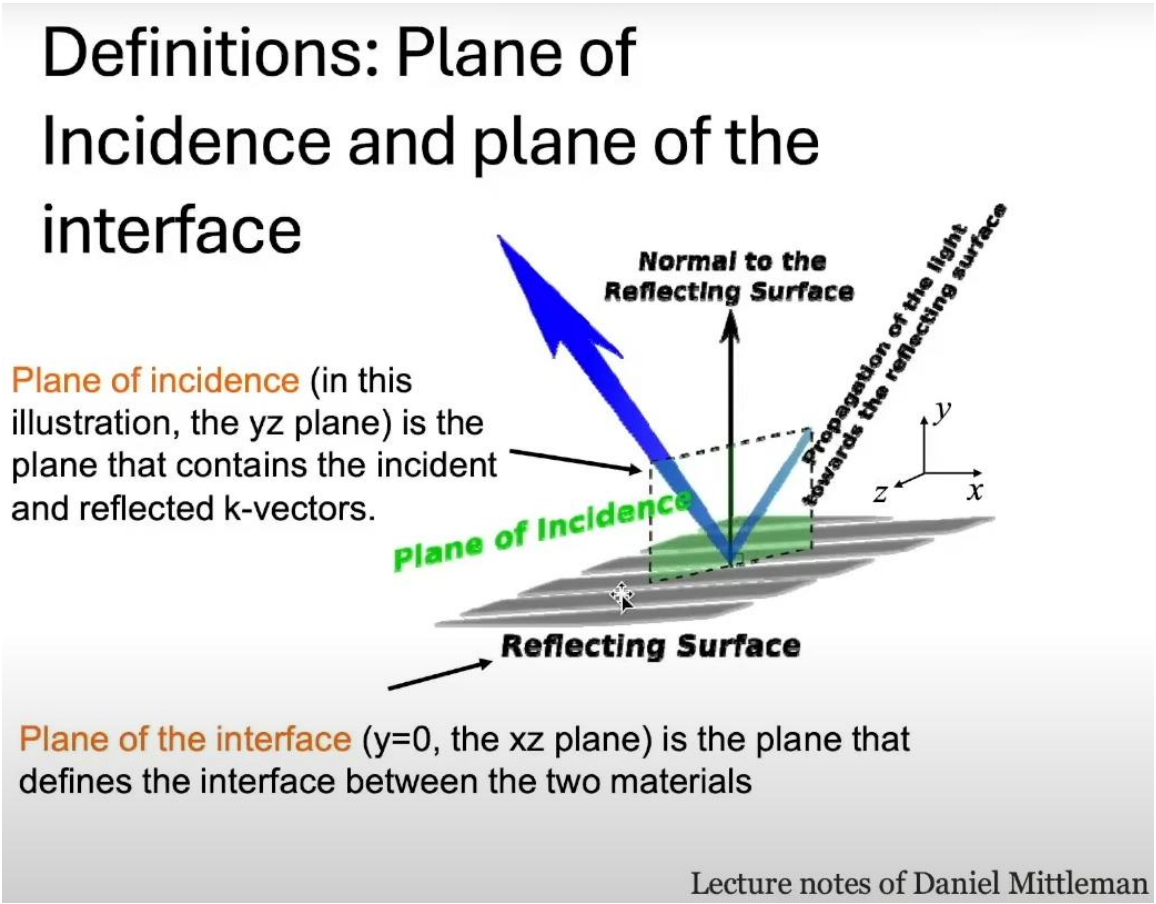


Now, if you look at it carefully, you will see that this is the XYZ coordinate system. So the plane of incidence in this illustration is basically the YZ plane.

So this is the plane of incidence. As you can see here, this is the plane of incidence. And that contains the incident and the reflected k-vectors. So this is the k-vector of incident light. This is the k-vector of the reflected light.

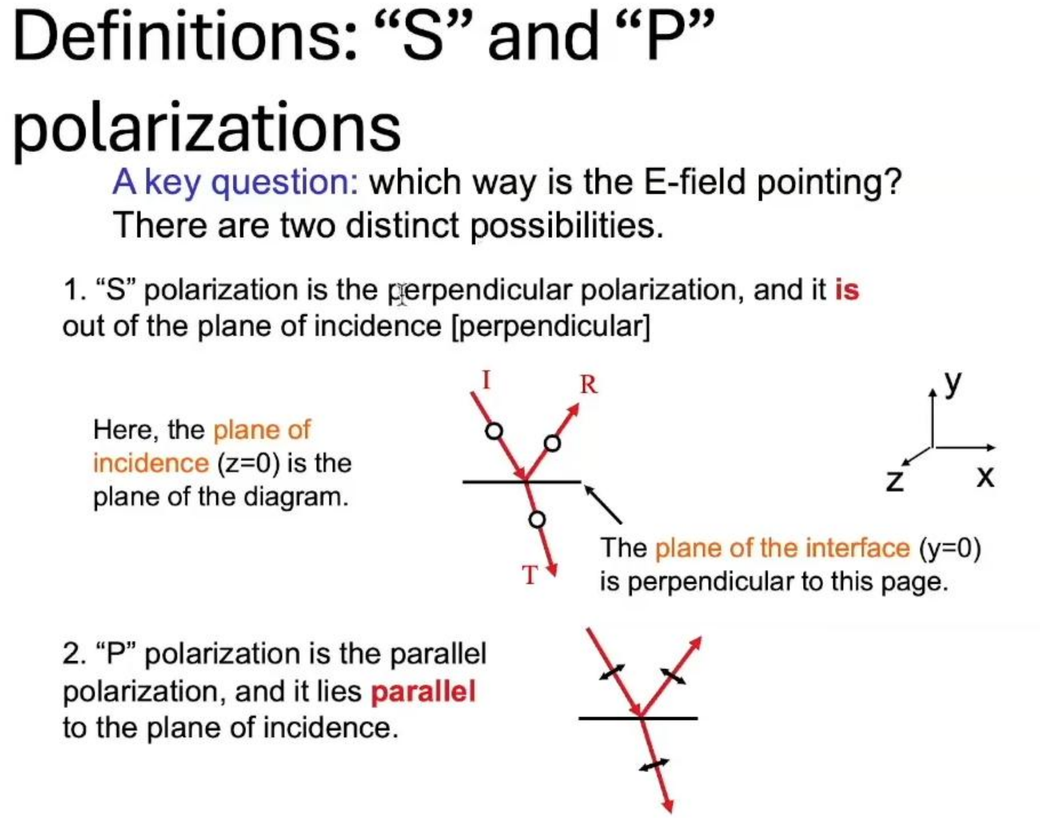
And this is basically the reflective surface. So the plane interface, or the plane of the interface, which is at y equal to zero, is that particular point, and it is basically the exit plane. So this is the exit plan. If you look at it, this is the exit plane.

All right. So this is the posing of the problem. So these are the definitions of the plane of incidence and the plane of interface. So you can see that they are perpendicular to one another.



All right. Now, if you look at it, we examine the definition of what we now call the S and P polarizations.

So the key question is which way the E field is pointing? So there are two possibilities. When we talk about S polarization, and remember the plane of incidence is Z equal to zero, that is the plane of this particular diagram. So the electric field is basically out of the plane, which is perpendicular. All right. OK. And this is basically your plane of the interface, which is y equal to zero and is perpendicular to this page. So you can understand now that in polarization, the electric field is coming out of the plane. When you look at the p-polarization, the electric field lies parallel to the plane of incidence. So this is the plane of incidence. The electric field is contained in the plane of incidence.



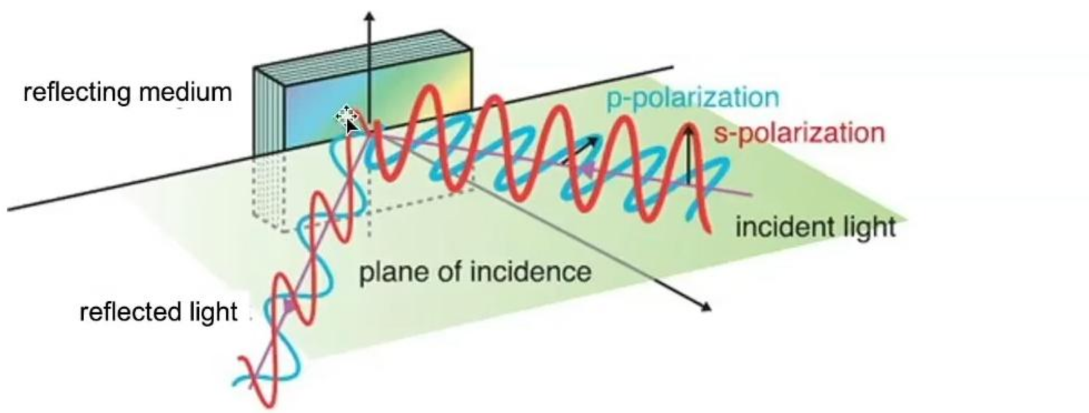
Lecture notes of Daniel

So these are the S and P polarizations that we are talking about. Remember, the plane of the interface is always at y equals zero. Now let us look at the definitions of S- and P-polarizations. Here, you can see it. So this is a reflective medium.

And this is the plane of incidence, this green-colored boundary, that green-colored plane, and this is basically the interface. As you can see, if you look at the red-colored line and the red-colored wave, this is basically the red-colored wave that you see over here; if you follow my cursor, this is the S polarization, where you can see that it is perpendicular. It is perpendicular to the incident plane. The p polarization is parallel, or it is contained within the incident plane, which is the blue colored line. The amount of light that is reflected or transmitted is different for two different incident polarizations; the s polarization is perpendicular, as we stated here, out of plane, and the p polarization lies parallel to the plane of incidence, and that is exactly what we.

Shown over here. So, the blue-colored lines are basically parallel to the plane of incidence, and the red-colored lines represent the S polarization, which is perpendicular to the plane of incidence. But the amount of reflected and transmitted light is different for two different incident polarizations. Right, and this is the reflecting medium, or the medium through which the light is going to go; part of the light is actually going to go through. Okay, so the Fresnel equations, if you look at them for perpendicular E field, okay, so Augustine Fresnel was the first to do this calculation. So we basically treat the S polarization first, which is the perpendicular E field.

Definitions: "S" and "P" polarizations



The amount of reflected (and transmitted) light is different for the two different incident polarizations.

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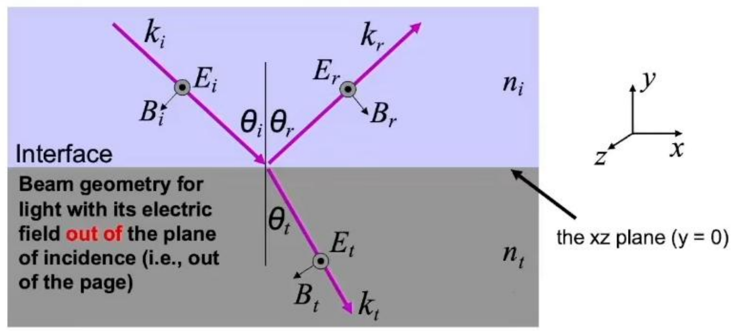
Okay, if you recall, this is perpendicular; this point that you see over here is basically the perpendicular point. This indicates a direction that is out of plane. It is out of this page, essentially. So this is E_i, which is the incident electric field. This is the reflected electric field.

And this is the transmitted electric field. This is the corresponding magnetic vector, which is B_i, incident, reflected, and transmitted. Okay, so this is the X, Y plane, X, Z plane rather, so this is Y equal to zero, right? And this is the refractive index of the incident medium divided by the refractive index of the transmitted medium. So this should all be very clear to you.

Fresnel Equations—Perpendicular E field

Augustin Fresnel was the first to do this calculation (1820's).

We treat the case of s-polarization first:



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All right, now the boundary conditions for the electric field at an interface with S polarization.

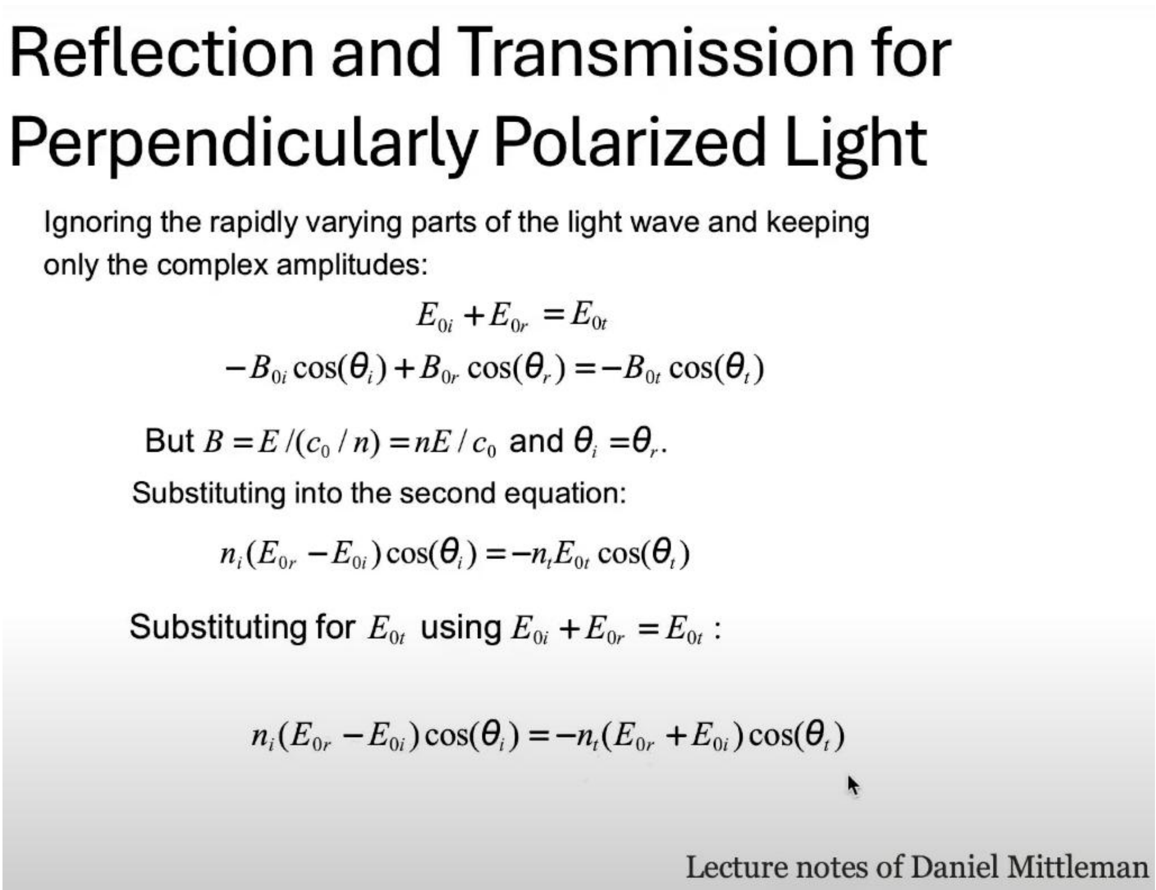
OK, what we can say over here is that the component of the E field that lies in the exact plane is continuous as you move across the plane of the interface. Here, all the fields are in the z direction, which is in the plane of incidence, so if you say E_i at y equal to zero, okay, that is where plus E_r at y equal to zero is equal to the E_t at y equal to zero. So we are not explicitly writing the x, y, z, and t dependence, but they are all there when you write about the electric field, right? Now, if we look at the boundary condition for the magnetic field, the total magnetic field, the total B field in the plane of the interface is continuous. Hence, all the B fields are in the xy plane. So we would take the xy-components.

This is B_i; this is the x component at y equal to zero times $\cos\theta_i$. If you look at this, this is the angle; this is the $cos\theta_i$ angle. So, that will be the x component; this is the theta in, sorry, this one, and this is the component that we are taking $-B_{0i}cos\theta_i + B_{0r}cos\theta_r = -B_{0t}cos\theta_t$. And that is the transmitter. So it is really tangentially B by μ, but we are using that $\mu_i = \mu_t = \mu_0$ in this particular case.

All right. So that is good. So now we go to the reflection and transmission of perpendicularly polarized light. Ignoring the rapidly varying parts of the light wave and keeping only the complex amplitudes, what we have is $E_{0i} + E_{0r} = E_{0t}$, and then similarly, .

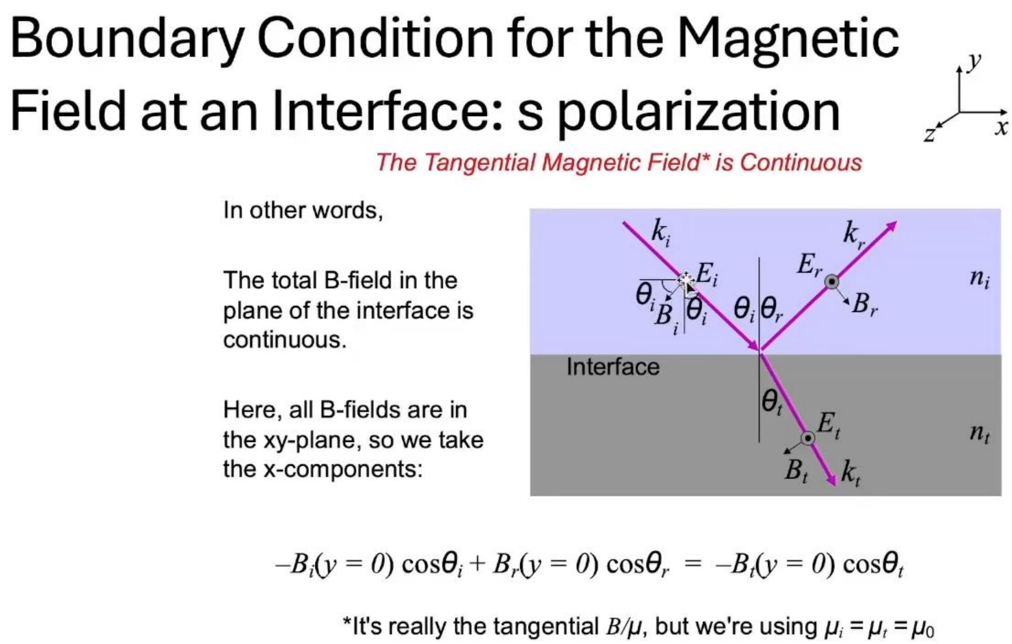
But B, as we know, is E divided by c_0 by n, which basically translates to n into E divided by c_0, and $\theta_i = \theta_r$. So, substituting this into this particular equation, which is the second equation, we have $n_i(E_{0r} - E_{0i})cos\theta_i = -n_t(E_{0t})cos\theta_t$.

Substituting for E_{0t} and using this expression, we finally have this expression with us. That is $n_i(E_{0r} - E_{0i})cos\theta_i = -n_t(E_{0r} + E_{0i})cos\theta_t$. So this is how this equation comes into the picture. So these are basically the Fresnel equations. So if we rearrange all these terms and solve for E_{0r} by E_{0i}, this gives you nothing but the reflection coefficient in the perpendicular direction.



This is the reflection coefficient, which is nothing but the electric field of the reflected wave divided by E_{0i}, which is the incident field. And if you take the ratio of the two by using these equations, you get this expression, which is ni times cos theta i minus nt times cos theta t divided by ni times cos theta i plus nt times cos theta t. Analogously, if we do the transmission coefficient now, which is E_{0t} by E_{0i}, everything is normalized with respect to the incident amplitude, incident complex amplitude. And so the transmission coefficient in the perpendicular direction, or perpendicular for perpendicularly polarized light, is actually given by this, which is nothing but *$2n_i \cos\theta_i$ divided by $n_i \cos\theta_i$ plus $n_t \cos\theta_t$*. So the lower part is basically the same for the reflection coefficient and the transmission coefficient for perpendicularly polarized light.

Remember, for perpendicularly polarized light. All right. So this solves this particular issue. OK, these are all of Fresnel's equations. Similarly, there is a parallel electric field now for p polarization.



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OK, now when you actually have p polarizations, you look at it so that the electric field is contained in the plane or parallel to the plane of incidence, if you remember. OK, and the magnetic field is basically what is coming out of the plane. So here you remember that the reflected magnetic field must point into the screen to achieve this. OK, where X is X, with the circle meaning into the screen. X with the circle means this on the screen.

So that is what it is. Now, the reflection and transmission coefficients for a parallel polarized beam are B0I minus B0R.

Reflection & Transmission Coefficients for Perpendicularly Polarized Light

Rearranging $n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t(E_{0r} + E_{0i})\cos(\theta_t)$ yields:

$$E_{0r}\left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right] = E_{0i}\left[n_i\cos(\theta_i) - n_t\cos(\theta_t)\right]$$

Solving for E_{0r} / E_{0i} yields the reflection coefficient:

$$r_\perp = E_{0r} / E_{0i} = \left[n_i\cos(\theta_i) - n_t\cos(\theta_t)\right] / \left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right]$$

Analogously, the transmission coefficient, E_{0t} / E_{0i}, is

$$t_\perp = E_{0t} / E_{0i} = 2n_i\cos(\theta_i) / \left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right]$$

These equations are called the Fresnel Equations for perpendicularly polarized (s-polarized) light.

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So now the magnetic field summation that we are talking about is equal to B0T. And again, I mean, you just invert the order, essentially. So

$$E_{0i}\cos\theta_i + E_{0r}\cos\theta_r = E_{0t}\cos\theta_t$$

Now, if you solve for E0R by E0I again, which yields the reflection coefficient for parallel polarized or p-polarized light, you get this.

Reflection & Transmission Coefficients for Parallel Polarized Light

For parallel polarized light, $B_{0i} - B_{0r} = B_{0t}$

and $E_{0i}\cos(\theta_i) + E_{0r}\cos(\theta_r) = E_{0t}\cos(\theta_t)$

Solving for E_{0r} / E_{0i} yields the reflection coefficient, $r_{||}$:

$$r_{||} = E_{0r} / E_{0i} = \left[n_i \cos(\theta_t) - n_t \cos(\theta_i)\right] / \left[n_i \cos(\theta_t) + n_t \cos(\theta_i)\right]$$

Analogously, the transmission coefficient, $t_{||} = E_{0t} / E_{0i}$, is

$$t_{||} = E_{0t} / E_{0i} = 2n_i \cos(\theta_i) / \left[n_i \cos(\theta_t) + n_t \cos(\theta_i)\right]$$

These equations are called the Fresnel Equations for parallel polarized (p-polarized) light.

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So the lower part looks very suspiciously similar. Not exactly similar, so this is the expression that you get. So this is a little bit of a cross. As you can see, $n_t \cos \theta_t$ is the transmitted angle, and this is $n_i \cos \theta_i$, which is the incident angle. The transmitted transmitter, the refractive index of transmission, or the refractive index of the transmitted medium is multiplied by the cosine of the incident angle.

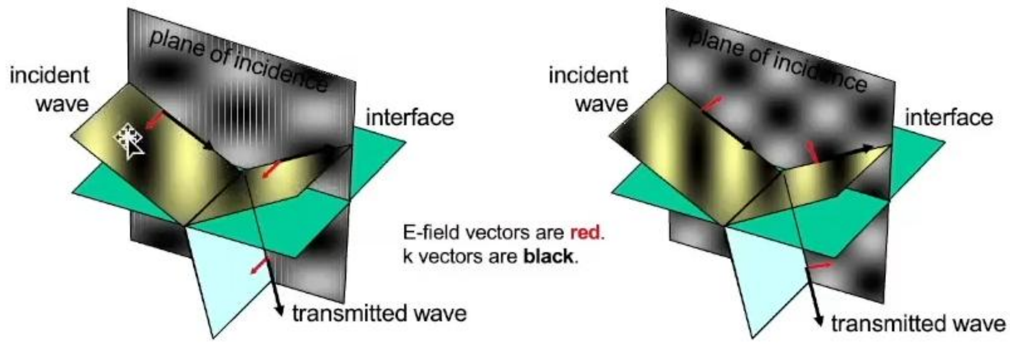
Analogously, your transmission coefficient for parallel polarized light and depolarized light is given by this suite of four equations, which now that you have got tells you about the reflection and transmission coefficients for parallel polarized light. You can put all of them in a summary plot like this, where you can see that, okay, this is the plane of incidence. This is the incident wave. This is basically the reflected wave. And you can see which direction the E vectors are, basically, in red.

The K vectors are in black. And this is basically the transmitted wave. Whereas on the other hand, if you see, so the, This is the transmitted wave now. And this is the incident wave. And this is, again, the k-vectors. So, for s-polarized light, these are the expressions.

For the p-polarized light, this is the expression. For both polarizations, ni sine theta i is equal to nt sine theta t. So this part comes from that law of refraction that we all already know. So you see these expressions now, therefore, you know, so this is how these things actually work. So you can see in this S-polarized light that this was perpendicular and this is contained in the plane of interest, the difference.

Between the two. The e vectors are always the same. The k vectors are always the same.

To summarize...



s-polarized light:

$$r_{\perp} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_{\perp} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

p-polarized light:

$$r_{\parallel} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

$$t_{\parallel} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

And, for <u>both</u> polarizations: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

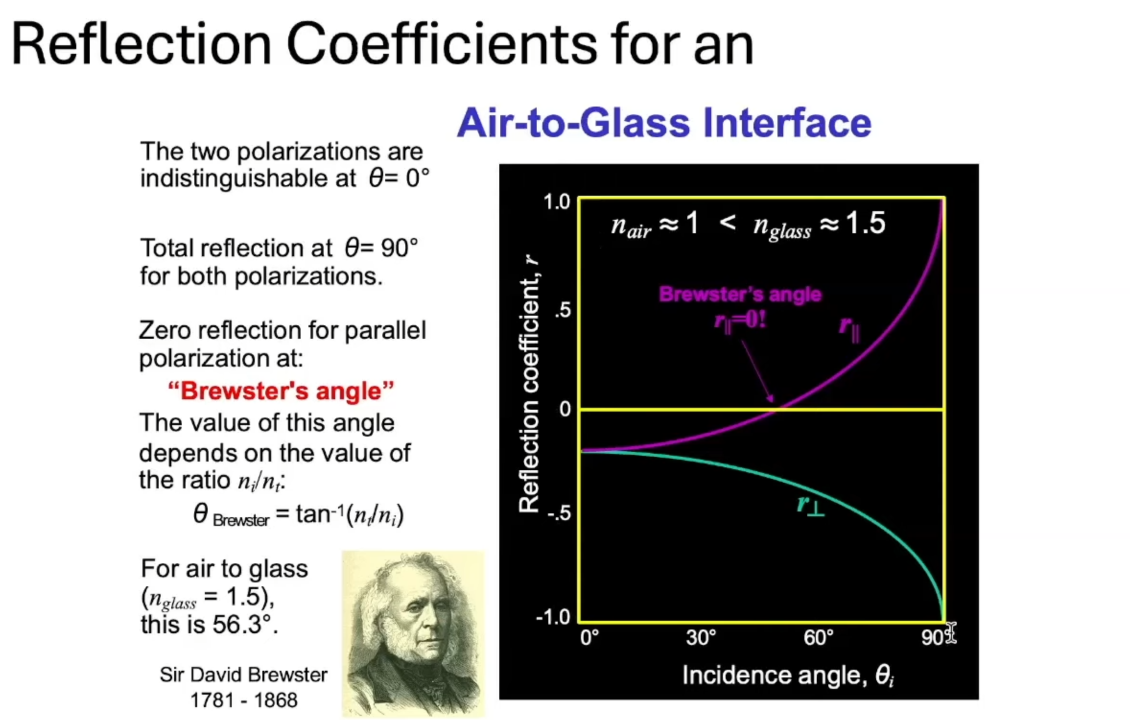
All right? Now you can plug all these things together. Now, you can use it for the reflection coefficient for, say, an air-to-glass interface. I think you understood that in this case, again, just to summarize, go back and summarize, because I think this has to be made clear: in the S-polarized light, this is perpendicular.

The E vector is perpendicular to the plane of incidence. Here, the E vector is contained in the plane of incidence, parallel to the two differences. Okay, so at the air-to-glass interface, if we look at it now, two polarizations are present. So, this is the air-to-glass interface, and what we are plotting is the reflection coefficient and the incident angle. And so this is for parallel polarization, and this is for perpendicular polarization.

So S and P. And as you know, the refractive index of air is about one, and that of glass is about 1.5. So, two polarizations are basically indistinguishable when you actually have theta equal to zero. So it's like vertical polarization. Incident total reflection happens at 90 degrees for both polarizations; zero reflection for parallel polarization occurs at Brewster's angle.

So for parallel polarization, the reflection is zero at Brewster's angle. So the Brewster's angle depends on the values of ni and nt. Brewster's angle is basically the inverse of nt by ni, which is roughly for air to glass; this is about 56.3 degrees, courtesy of Sir David Brewster. So here, the zero reflection for parallel polarization, that is, for the key polarization, happens at around 56.

3 degrees. This is called the Brewster. And for both, the reflection coefficient, as you can see, is that they are indistinguishable here. And both actually total reflection happens at 90 degrees. All right. So at around 90. So now, if you calculate the reflection coefficients for the glass-to-air interface, n glass is greater than n air, which we already know.



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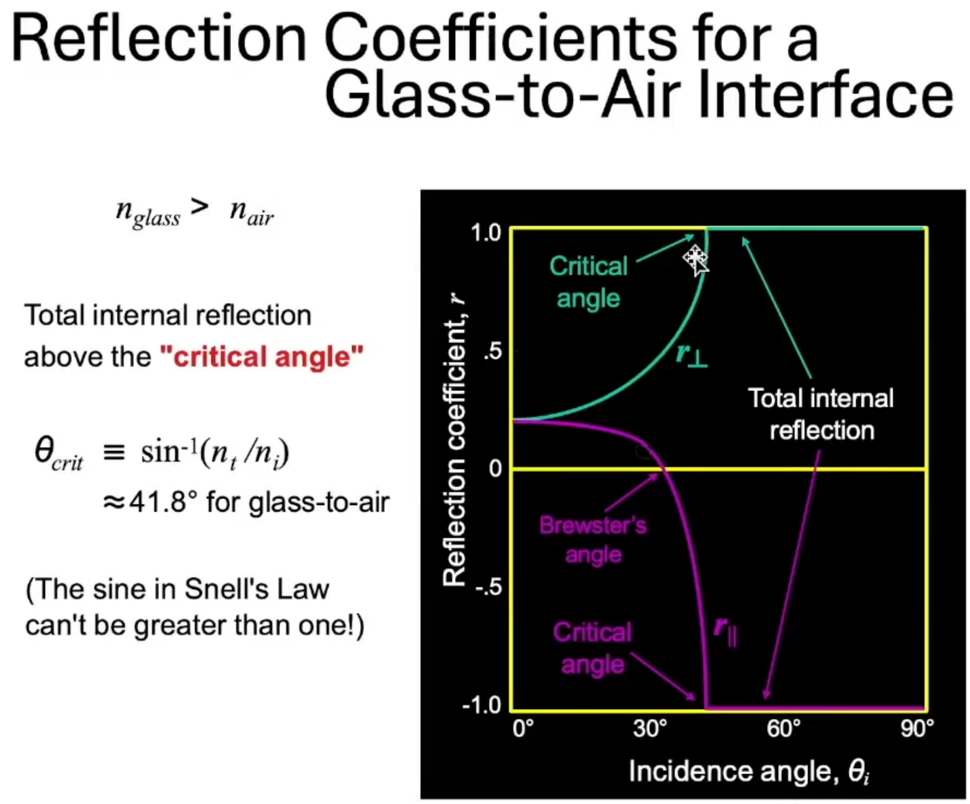
This is air to glass, and so this is glass to air. So total internal reflection occurs at a certain critical angle, which is this one. This angle is more or less the same.

It's the same for both. So this is about 41.8 degrees. This is the sine inverse of nt divided by nr. A sine in Snell's law cannot be greater than one. So this is what you can see. This is again Brewster's angle for parallel polarization.

It happens around here. And this is the point of total internal reflection. Total internal reflection occurs after the critical angle. Everything is total internal reflection. No, nothing can be; you cannot distinguish between reflection, and you cannot distinguish between the two polarizations at an incident angle of zero.



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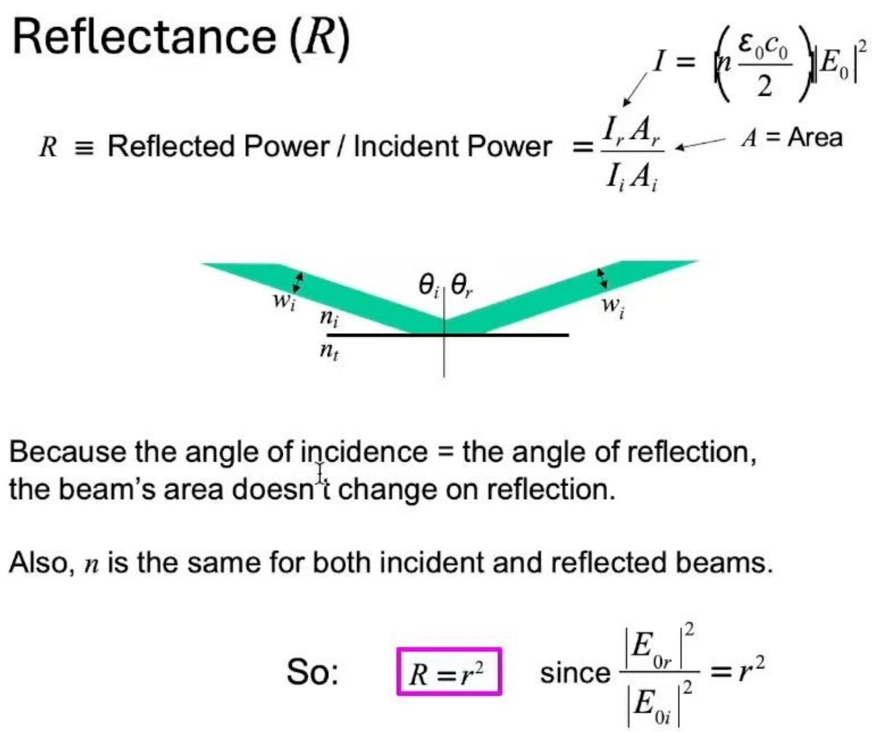
So this is all done. And I mean, these are the kinds of nice little curves that you see. So the reflectance is basically the reflected power divided by the incident power. So how is this calculated? It is calculated as $\frac{I_r A_r}{I_i A_i}$.

A is nothing but an area. The "I" is given by this:

$$I = \frac{n c_0 \varepsilon_0}{2} E_0^2$$

So this is how it goes. Because the angle of incidence is equal to the angle of reflection, the beam area does not change upon reflection. The n is the same for both the incident and the reflected beams. Therefore, $R = r^2$, since $\frac{E_{0r}^2}{E_{0i}^2} = r^2$.

So the reflectance is the reflected power divided by the incident power. It is equal to small r squared, which is the reflection coefficient.



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All right, now for the transmittance, it is called I; again, it's the transmitted power by the incident power. Now, since it is the transmitted power by the incident power, it is I_t divided by I_iA_i.

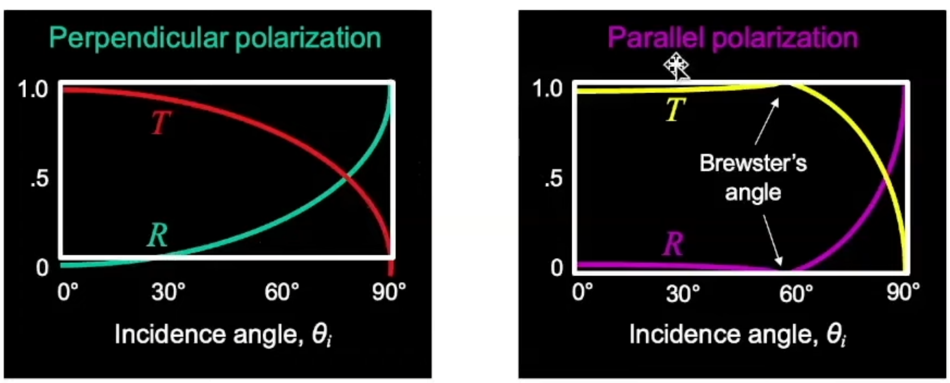
There is a... there is a... These are a ratio, and this is the same. I the same. The BIM width, of course, has changed. W_i is not the same as W_t, but we can calculate them. A_t by A_i is equal to W_t by W_i and is nothing but the cos $(\frac{\theta_t}{\theta_i})$. So the beam expands or contracts in one dimension when you actually have refraction.

If you do these calculations over here, and as we know that $\left(\frac{E_{0t}}{E_{0i}}\right)\left(\frac{E_{0t}}{E_{0i}}\right) = T^2$, that is the transmission coefficient squared, the transmittance is given by this factor, which is $\frac{n_t cos\theta_t}{n_i cos\theta_i}$. And you multiply it by T^2, so this is the only change that basically accounts for the change in the area because the area was the same in the case of the reflected and incident waves; this is not the same for the transmitted wave, so this is the only change, nothing much. Then the reflectance and transmittance for the air-to-glass interface. Now this is, uh, you know, transmittance, and this is reflectance.

So, as you can see, this is how it goes. And this is a case of parallel polarization. This is perpendicular polarization. This is Brewster's angle. This is Brewster's angle for parallel polarization. And that's where the reflectance is actually equal to zero and the transmitter is equal to one.

So the general rule is that it is not always true that the reflection coefficient plus the transmission coefficient is equal to 1. But it is always true that the reflectance plus the transmittance is equal to 1. So that is the thing you should keep in mind. Similarly, if you consider the glass to air interface, the reflectance plus the transmittance is still equal to one.

Reflectance and Transmittance for an Air-to-Glass Interface



Note that it is NOT true that: $r + t = 1$.

But, it is ALWAYS true that: $R + T = 1$

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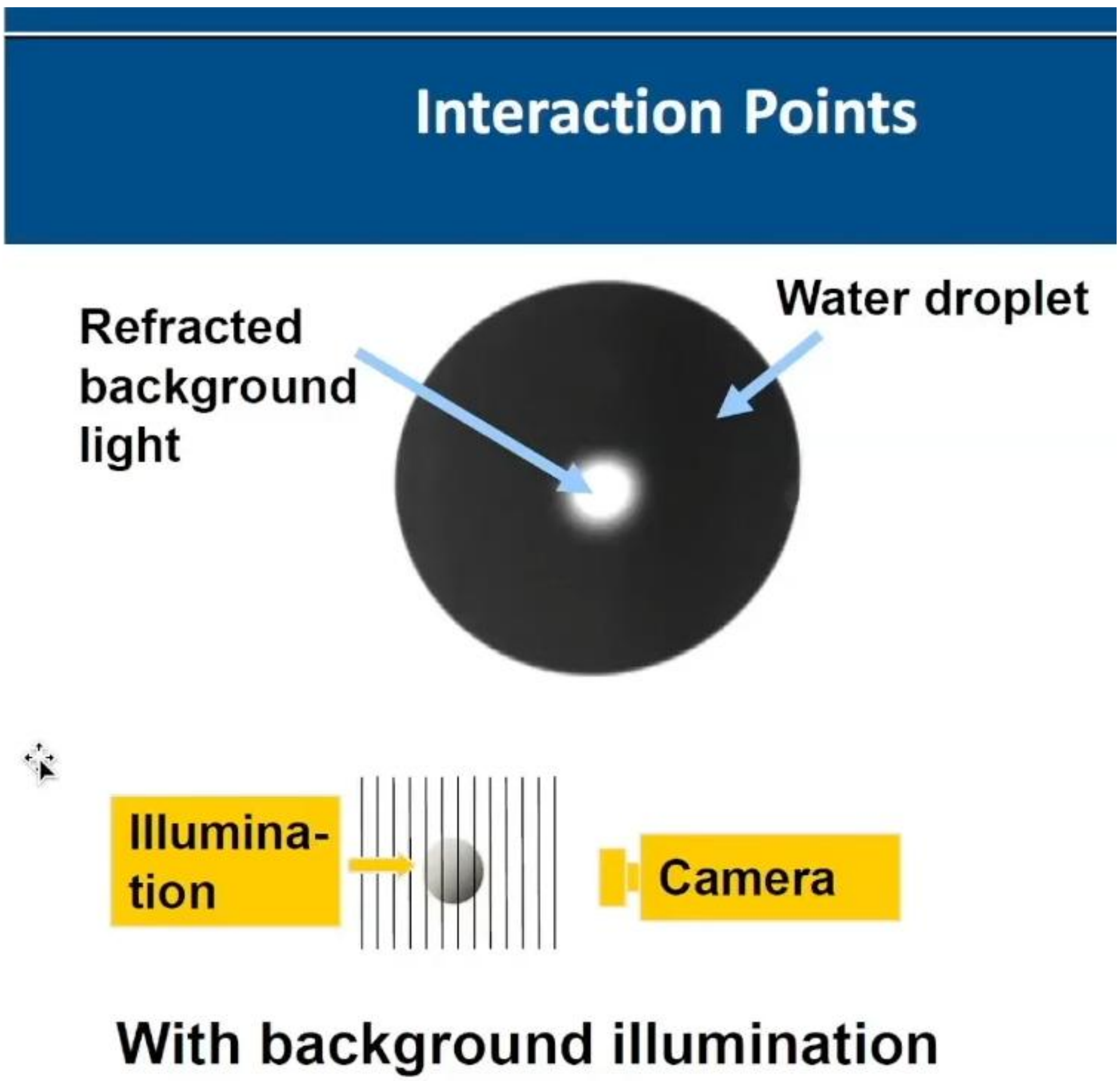
So this is, again, perpendicular polarization. This is parallel polarization, as you can see. And this is how, so here, of course, you get total internal reflection from this point. This is a critical angle. And transmittance basically goes to zero for those values.

So this is how this is pointed out. And this is the same thing that has been plotted. This is also from Tropea. So this reflectance plus transmittance is equal to one. And you know, the intensity of the reflected and the transmitted wave is therefore given by this.

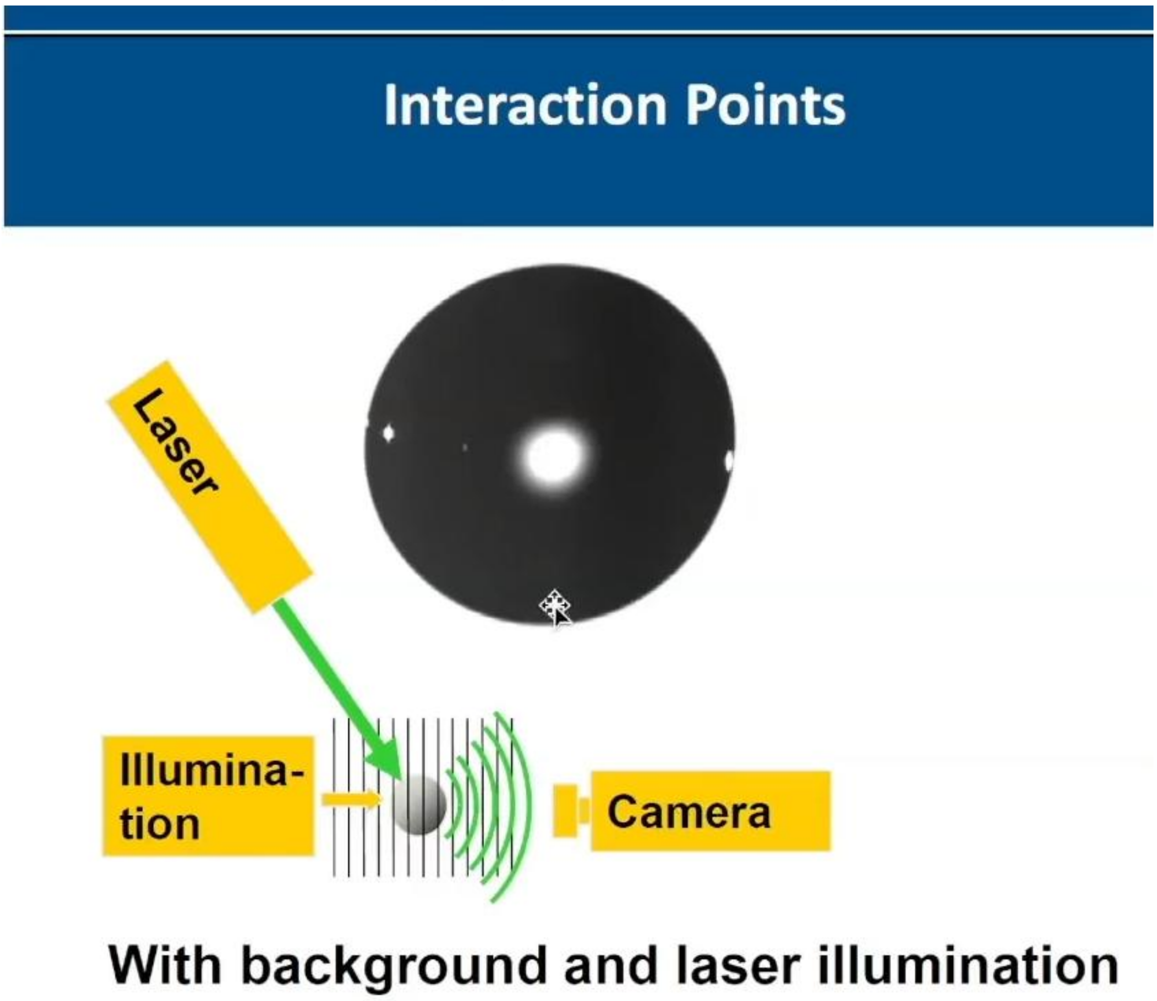
And this is another, more detailed plot. Okay. What I showed was a cartoon. This is a more detailed plot. And mu1 is equal to mu2. And this is what you get at the end of the day. So the intensity of the reflected and transmitted wave is given by this. These are the Fresnel equations that we covered in detail, and you showed whatever it is.

Okay. Now, just to give an idea before we go on to a few more things: what is the use of all these things here? Now, if you have a droplet that is illuminated and then you have a camera in front of it, this is back illumination and this is what the camera is looking at. You see a shadow of the droplet, and you get a refracted background light, which is right there at the center. This is what you see, okay?



Now, if you take a laser and illuminate the same, here, of course, you don't see a diffraction pattern, the fringes, because this is sufficiently large. As we noted earlier, the smaller the obstacle, the greater. Is the effect of the diffraction so large that you don't see it, even if it is there? You don't see that much; you just see this glare, this background light, which you can see over here.

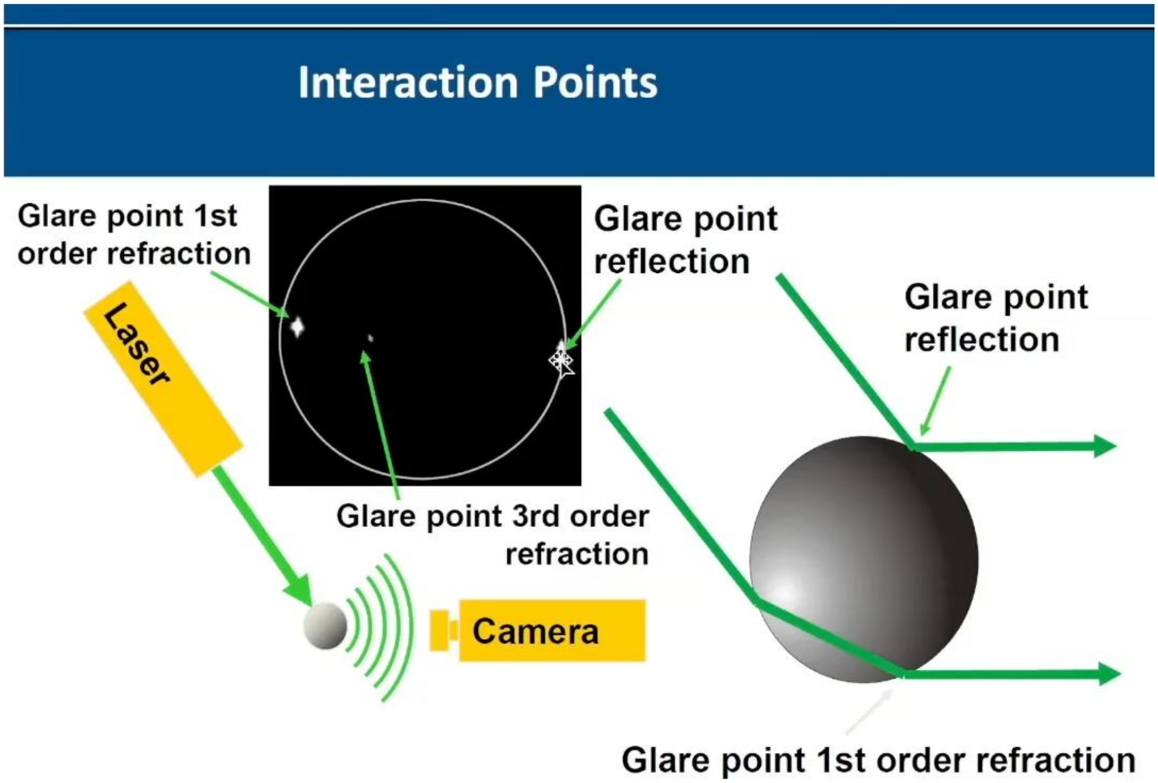
Okay, so if you now illuminate it with a laser, you see a few spots, as you can see: one, two, three, four types of spots over here. The background illumination gives rise to this, you know, the central uh. You know, there is a bright spot, and then you get a few other spots over here, which is what you get. So these are basically the interaction points.



This glare, this particular spot, is called the first order refraction. Okay, this is the third order refraction, and this is the glare point reflection. So glare point reflection is something like this. The light beam hits and moves away. So it gives rise to the first glare point.

And the glare point first-order refraction is something like this. The light enters through here and exits through there. So this is the first order of refraction that we are talking about. All right, okay, so all these things are very important, and here, of course, you just have the laser, so there is no shadow. This white circle that you see is just to mark that there is a droplet or a particle, and it is just to show that the particle was there, so it is not meant for taking it.

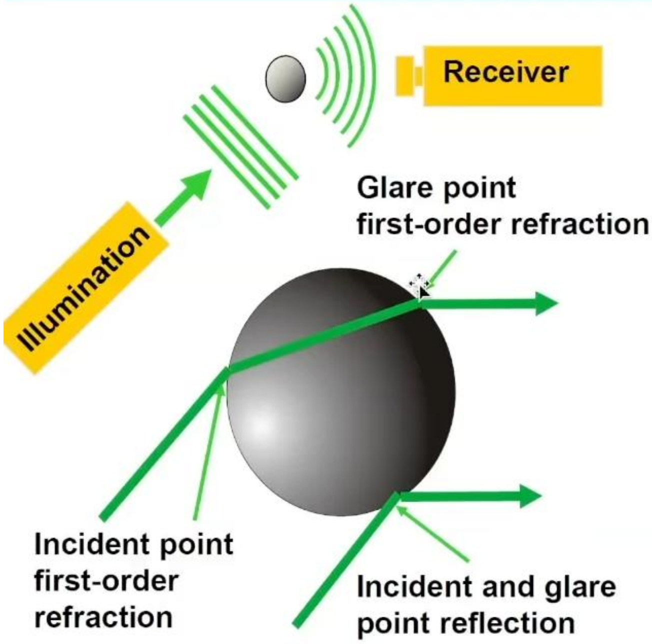
You don't get this shadow anymore. The shadow that you see over here, you don't see there anymore, okay?



So essentially, it is that when you illuminate it and there is a receiver and a particle, you see this glare from first-order refraction points, incident first-order refraction, and you'll see incident and glare point reflection. The path from the incident beam to the detector is unique for each scattering mode. Large particle images, you know, incident at the point onto the detector through the glare point. So both points are unique. For each scattering mode, which we will cover in the next class, both points lie on the scattering plane.

So what you see are basically nothing but first-order refraction and incident clear points. There are second orders, too, as we will see later. So, in the next class, we are going to start with scattering. And according to geometric optics, what are the different types of scattering that we will see in the next class?

Incidence and Glare Points



- The path from incident wave to detector is unique for each scattering mode
- Large particle images incident point onto detector through glare points
- Both points are unique for each scattering order/mode
- Both points lie in the scattering plane