Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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Week – 03

Lecture - 12

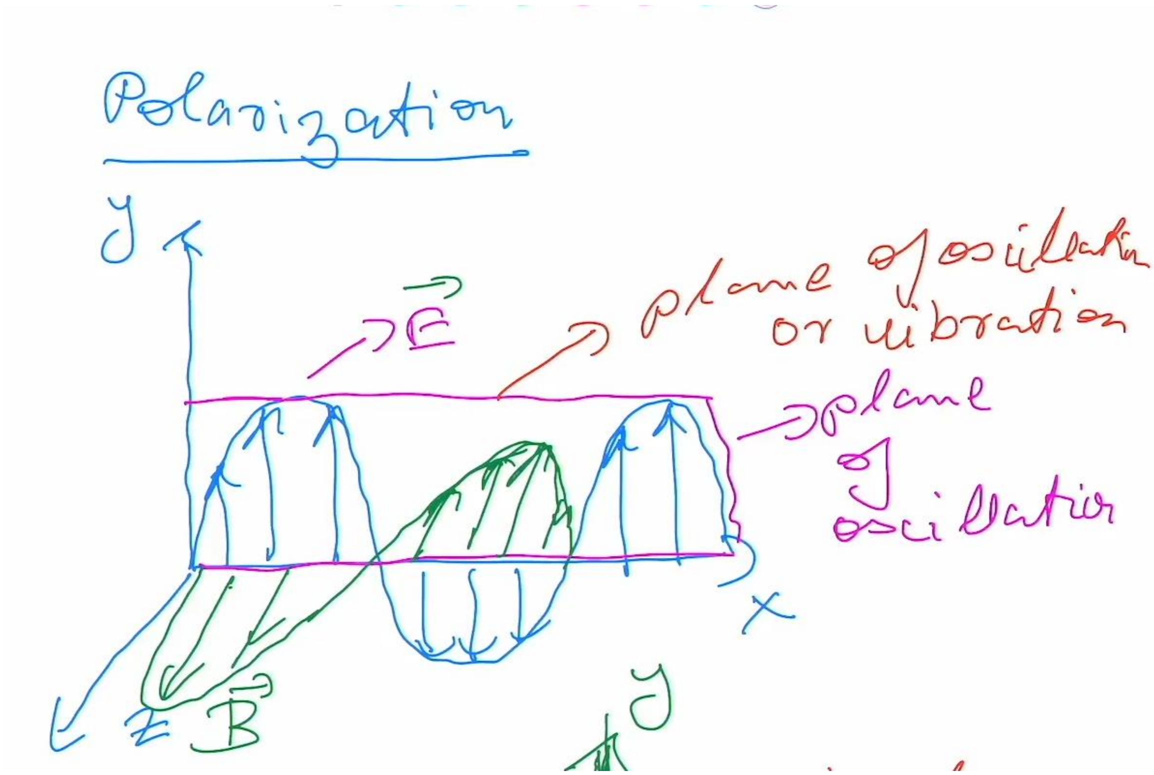
Imaging and Optics -4

So, in this particular section, we talk about light in general and what light is. So, let's look at something called polarization. Okay, what is polarization? Polarization is, for example, something we should examine. Yeah, forgive my drawing. So this is Y, X, and Z. All right.

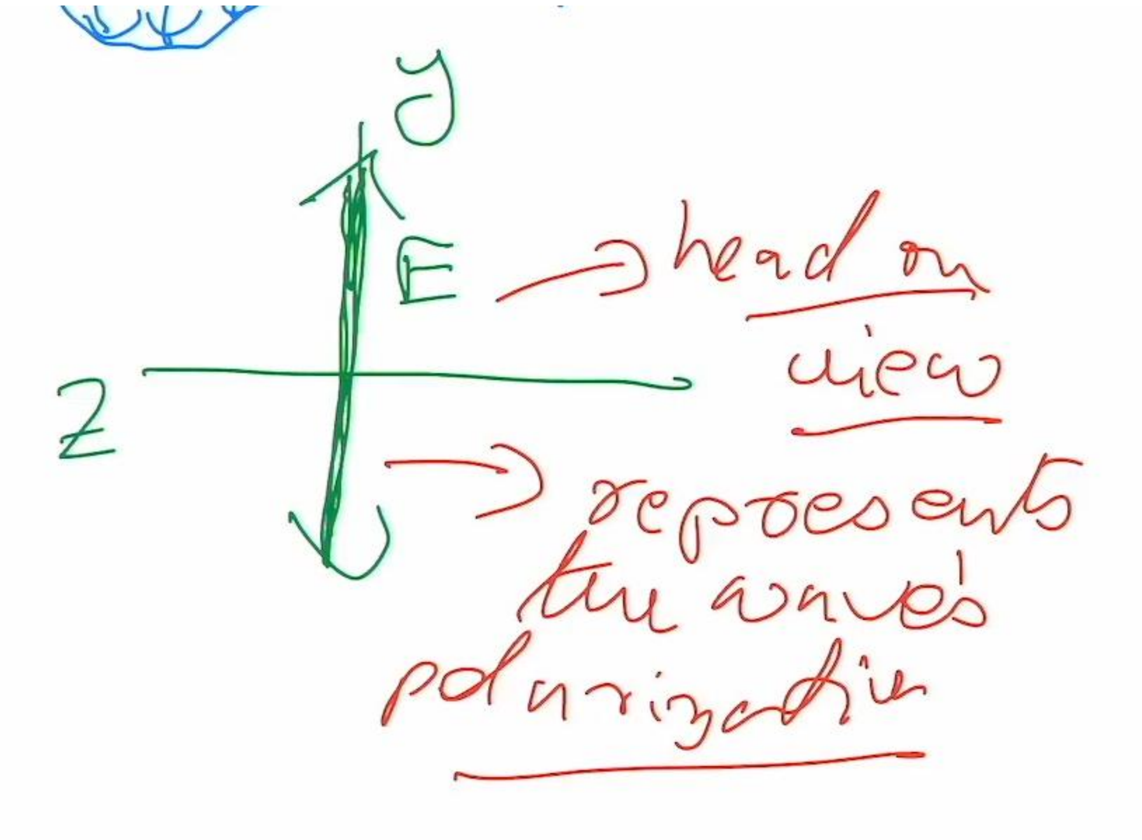
So, this is a sample of an electromagnetic wave. The electric field. I mean, this is actually symmetrical. It does not come out that way in my drawing.

So, okay, this is the plan. Of oscillation, this is the electric field E. Okay, there is obviously the magnetic field as well, which let me represent it like this: that is B, a magnetic field. And this is the electric field. So, if you look at this particular situation, you will see that this is Z and Y, and the electric field is something like this.

Back to E. Right, so this is like the plane of oscillation, the plane. Uh, so this particular statement says that the plane containing the electric vectors is called the plane of oscillation or vibration. The plane of oscillation or vibration, right? Okay, so we represent the wave's polarization by showing the extent of its electric field oscillations in a head-on view. So this is the head-on view. So this shows the wave's polarization from a head-on view of the plane of oscillation.



So this is the head-on view that shows the wave polarization; that is what is represented in this particular view. All right, so we know what we are doing, therefore. Okay, so now let us consider two orthogonal components. optical disturbances.



Okay, so one is saying Ex, and both are propagating in the Z direction, remember? This $E_x = \hat{\imath} E_{0x} \, cos(-\omega t + kx)$.The other one, if you look at it, is given as Ey; that E is given as $E_y = \hat{\jmath} E_{0y} \, cos(-\omega t + kx + \varepsilon)$. So, what is epsilon here? Epsilon is equal to the relative phase difference between the waves, both of which are traveling in the same direction, so that is what you have. So in this particular case, if you look at it, these are two waves that are at different phases; perhaps the relative phase between the two is along different axes. So the resultant optical disturbance is given as $\vec{E}(z,t) = E_x \hat{\imath} + E_y \hat{\jmath}$; so if epsilon is equal to zero or an integral multiple of plus or minus two pi, the waves are in phase; in that case $\overrightarrow{E} = (E_x \hat{\imath} + E_y \hat{\jmath}) \, cos(-\omega t + kz)$. In this particular case, this is what it has become.

Two orthogonal optical disturbances

$$\vec{E}_x(z,t) = \hat{i} E_{0x} \cos(kx - \omega t)$$

$$\vec{E}_y(z,t) = \hat{j} E_{0y} \cos(kx - \omega t + \varepsilon)$$

ε : relative phase difference between the waves both of which are traveling in z-direction

So, obviously, the resultant wave is also linearly polarized. And the resultant electric field oscillates along a tilted line. So, that is what you get at the end of the day. So, in other words, if we have to put this pictorially, we must present it visually. So let's try to do this.

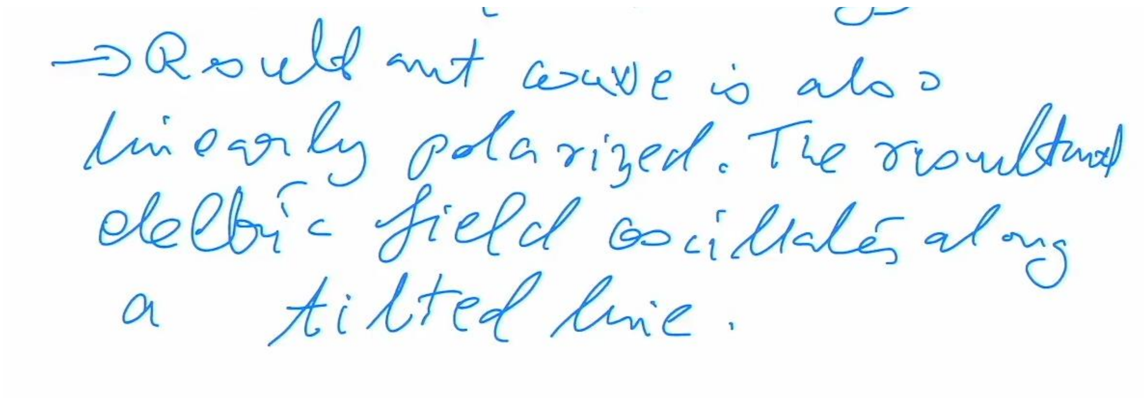
Resultant optical disturbance

$$\vec{E}(z,t) = \vec{E}_x(z,t) + \vec{E}_y(z,t)$$

If $\varepsilon = 0$ or integral multiple of $\pm 2\pi$, the waves are in phase. In that case

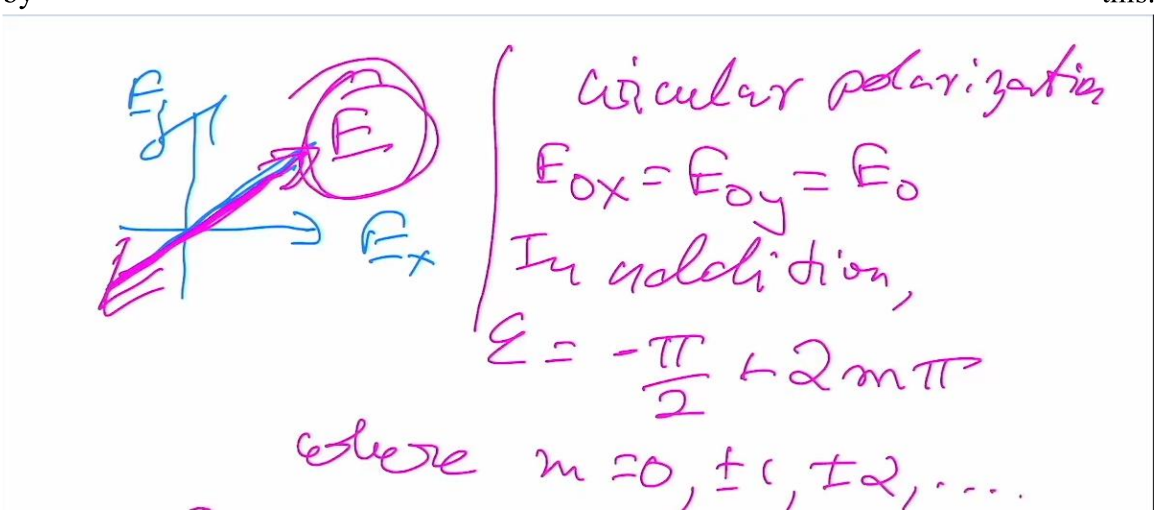
$$\vec{E}(z,t) = (\hat{i} E_{0x} + \hat{j} E_{0y}) \cos(kz - \omega t)$$

There can be different combinations.



Larger and smaller, for example, can have different directions of light in this field. Depending on the angles and strength, it can vary accordingly. So this is the line along which the resultant E actually oscillates.

So, you can also have what we call circular polarization now. What does circular polarization do? You can have another particular case where $E_{0x} = E_{0y} = E_0$. In addition, you have $\varepsilon = \frac{-\pi}{2} + 2m\pi$, where m is equal to zero, plus or minus one, plus or minus two, and so on. So you can see that $E_x = \hat{\imath}E_{0x}\,cos(-\omega t + kz)$, $E_y = \hat{\jmath}E_{0y}\,cos(-\omega t + kz)$so the resultant wave is given as follows: the two resultants. So, the resultant wave is given by this.



So this is what we call a right circular light. So, this is circular polarization. So the resultant electric field E is rotating clockwise at an angular frequency of ω. Therefore, this is called right-handed circular polarization. So a linearly polarized wave can be synthesized from two oppositely polarized circular waves of equal amplitude.

So this is another thing that one should note: a linearly polarized wave can be synthesized from two oppositely polarized circular waves. Therefore, that is what you get.

$$\vec{E}_x(z,t) = \hat{i} E_0 \cos(kz - \omega t)$$

$$\vec{E}_y(z,t) = \hat{j} E_0 \cos(kz - \omega t)$$

Resultant wave

$$\boxed{\vec{E} = E_0\left[\hat{i}\cos(kz-\omega t) + \hat{j}\sin(kz-\omega t)\right]}$$

$\vec{E}$: rotating clockwise at an angular frequency of 'ω'

→ right circularly polarized

Let's do one thing. So this is the important part: it can be synthesized from two oppositely polarized circular waves of equal amplitudes. Lastly, you have this elliptical polarization.

→ A linearly polarized wave

can be synthesized from two oppositely polarized circular waves of equal amplitude.

Elliptical polarization. We consider a general case: recall. A general case, for example, is $E_x = \hat{\imath} E_{0x}\, cos(-\omega t + kz)$, $E_y = \hat{\jmath} E_{0y}\, cos(-\omega t + kz + \varepsilon)$.

Eliptical polarization

$$E_x = E_{0x} \cos(kz - \omega t)$$
$$E_y = E_{0y} \cos(kz - \omega t + \varepsilon)$$

So expanding the expression for E_y: if you expand the expression for E_y,

$$\frac{E_y}{E_{0y}} = cos(kz - \omega t)\, cos(\varepsilon) - sin\,(kz - \omega t) sin(\varepsilon)$$

$$\frac{E_x}{E_{0x}} = cos(kz - \omega t)$$

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\, cos(\varepsilon) = -\, sin\, (kz - \omega t) sin(\varepsilon)$$

So now we can say from this particular equation that, if you look carefully at this particular equation, $sin(kz - \omega t) = [\,1 - (\frac{E_x}{E_{0x}})^2]^{(1/2)}$

Expand expression for E_y into

$$\frac{E_y}{E_{0y}} = \cos(kz - \omega t) \cos\varepsilon - \sin(kz - \omega t) \cdot \sin\varepsilon$$

$$\frac{E_x}{E_{0x}} = \cos(kz - \omega t)$$

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos\varepsilon = -\sin(kz - \omega t) \sin\varepsilon$$

$$\sin(kz - \omega t) = \left[1 - \left(\frac{E_x}{E_{0x}}\right)^2\right]^{\frac{1}{2}}$$

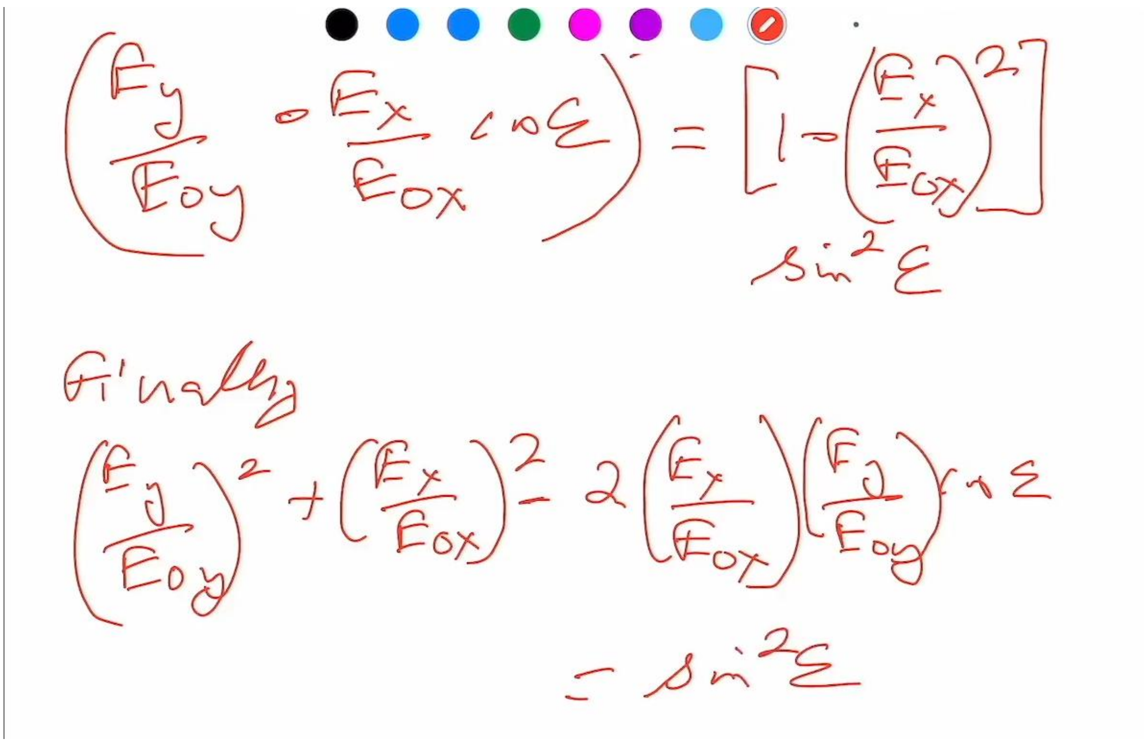
Okay, what you are going to get, this also leads to, if you look at this expression now,

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \, cos(\varepsilon) = [\ 1 - (\frac{E_x}{E_{0x}})^2]^{(1/2} sin^2 \, \varepsilon$$

Finally, if we rearrange all the terms, we get

$$(\frac{Ey}{E_{0y}})^2 + (\frac{Ex}{E_{0x}})^2 - 2 \, \frac{E_y}{E_{0y}} \frac{E_x}{E_{0x}} \, cos(\varepsilon) = sin^2 \, \varepsilon$$

Okay, this is the equation of an ellipse making an angle alpha with the ex and ey coordinates.



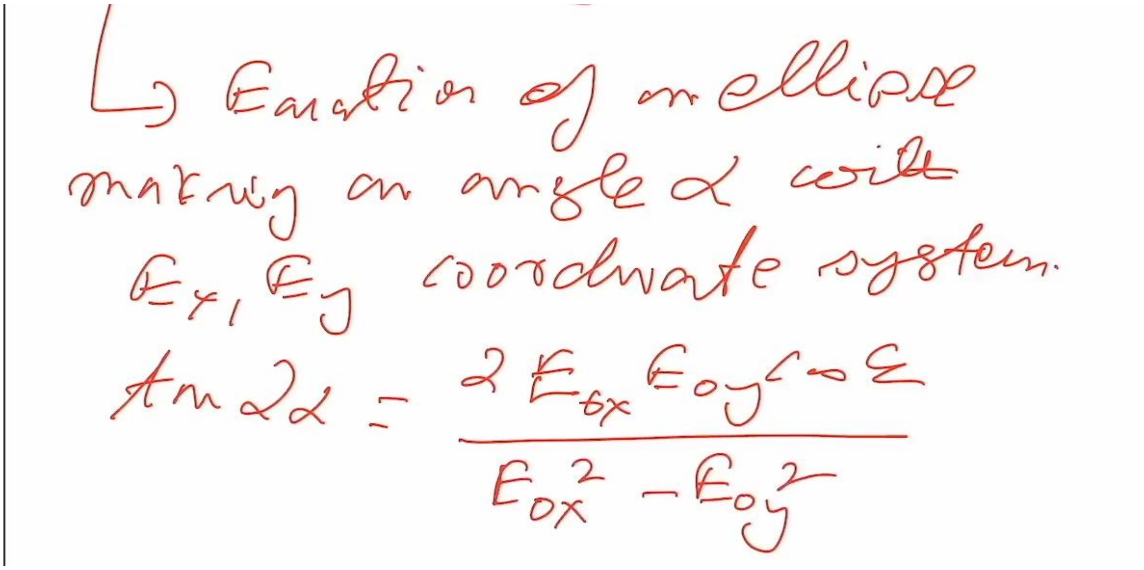
So the equation becomes:

$$tan(2\alpha) = \frac{2E_{0x}E_{0y} \, cos[\varepsilon]}{E_{0x}^2 - E_{0y}^2}$$

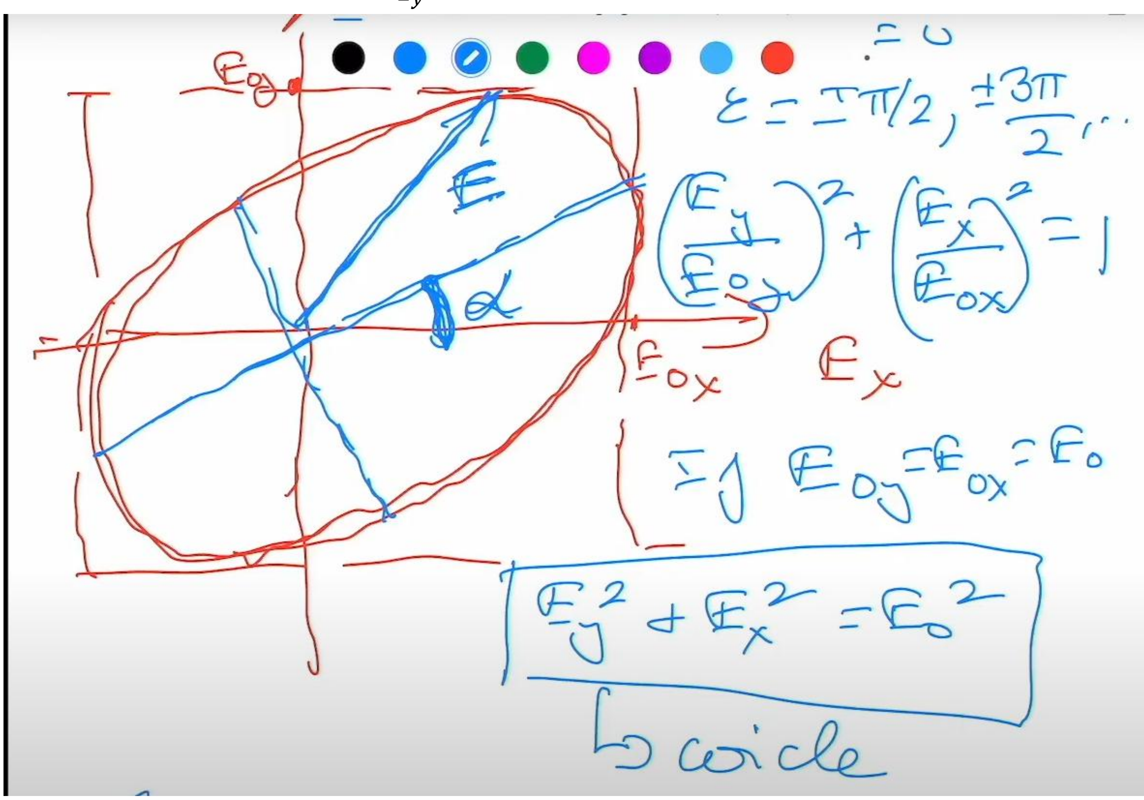
In other words, this is like that. If I try to draw it properly, then maybe I can.

These are the X.U.I.s. And you know, this is like, you know, why? Consider a box in it. Those are here. And this particular ellipse is something like this. So, this is E major.

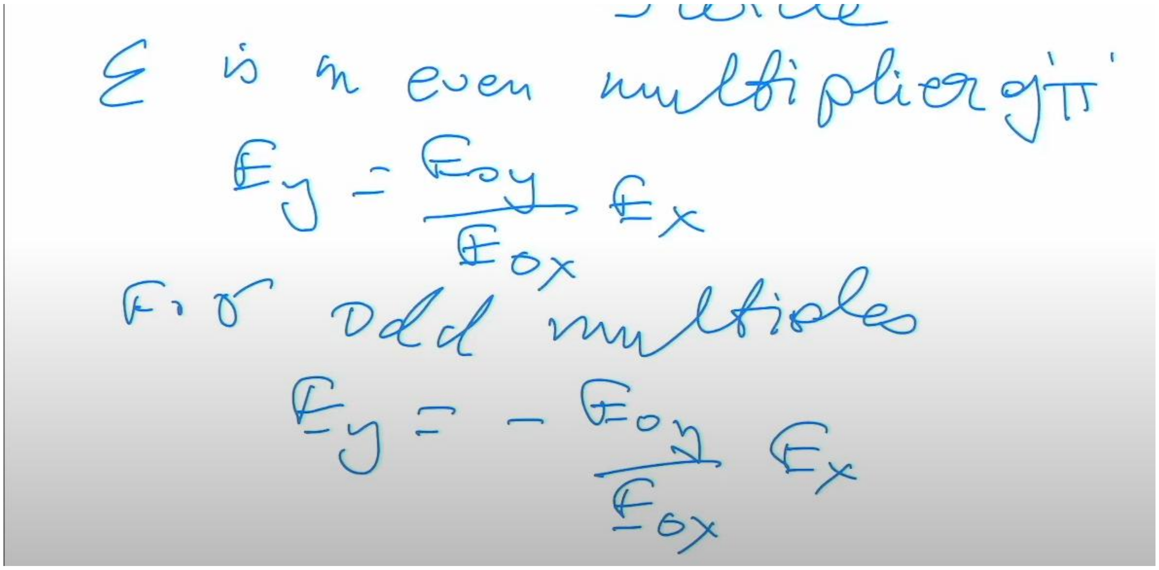
This is the minor ellipse. This is Alpha. And this is the alpha-axis ellipse. Okay, so if alpha is equal to zero, epsilon is equal to plus or minus pi over two, plus or minus three pi over two, we will have this very familiar form: $(\frac{Ey}{E_{0y}})^2 + (\frac{Ex}{E_{0x}})^2 = 1$. Furthermore, if your $E_{OY} = E_{OX} = E_0$, then this can be reduced to $E_{0x}^2 + E_{0y}^2 = E_0^2$. So clearly, this has become a circle.



So if epsilon is an even multiplier of pi, then this results in $\frac{E_y}{E_{0y}} = \frac{E_x}{E_{0x}}$.And for odd multiples, this is equal to $\frac{E_y}{E_{0y}} = -\frac{E_x}{E_{0x}}$. Okay. So these are both straight lines with slopes of plus or minus $\frac{E_{0y}}{E_y}$. Otherwise, we have linear lighting.



So both linear and circular light may be considered special cases of elliptically polarized light. All right? So both of these, you know, linear and circular lights, are special cases. Elliptically polarized light. So there are different configurations and types of ellipses that you can use. So you can have those kinds of configurations, too.



That is given. But you should also see that, you know, in this particular example, sorry, the one that you mentioned, these are both straight lines. They are sloped. Slopes are plus or minus zero y over zero x; that's why we know that there are linear and circular flights in general. So this is, in a nutshell, what the ellipse will be, what the ellipse is, and how the polarization actually occurs. So basically, it is determined by the orientation of the final vector E over there.

Polarization is an important component in dealing with light. Similarly, you can, you know, have wave propagation at an interface; then you have the Fresnel equation and many things that you will see in the next few lectures. But this is particularly important when we talk about polarization. So polarization is basically a plane wave that is propagating in the z direction and can have two field components perpendicular to the propagation. As a result of that, when you actually interpret it as a sum of the two independent waves, which are Ex and Ey, what you get at the end of the day is a polarization that is given by the orientation of the vector E.

And then, of course, there is this relative... The angle between the two means the relative phase between them, so when the phase is basically zero, the two waves are in phase, or if they are integral multiples of plus or minus two pi. We have looked at circular polarization, we have looked at linear polarization, and we have looked at, you know, Elliptical polarization shows why it is the key factor here, meaning all our special cases of how things can be determined from elliptical polarization.

In the next class, we are going to look at it. wave propagation at an interface. That is another important thing. And that wave propagation at the interface, looking at the Fresnel equations, examining the Fresnel equations in detail, and considering first-order, second-order, and third-order refraction points, refraction modes, and similar concepts.

And then finally going on to scatter. So, these are important things. And these are important, especially when you have to look at the lights. Propagation and scattering. So light is an electromagnetic wave, which we showed; therefore, we need to know coherence, polarization, Snell's law, Fresnel equations, Lorentz theory, geometric optics, and scattering from small particles before we go to the fundamentals of image processing and conclude this particular section. Geometric optics and scattering from small particles before we go to the image processing fundamentals and end this particular section.

So thank you for your time, and this is where we end this particular lecture.