Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru
Week – 03
Lecture - 10
Imaging and Optics – 2

All right, so in this particular lecture, we look at some of the phenomena that are called diffraction. So, in order to cover this phenomenon, let us look at something that you are very familiar with. Let's consider a slit. which has a finite width, which is a. So a is the width of the slit, and there are a bunch of rays, collimated rays, which are coming and are incident on this slit. Now, each point in the slit acts as a source for light waves.

So each point in the slit that you can see over here, marked as 1, 2, 3, 4, 5, acts as a source for the light waves. So there is a bunch of parallel light beams that are incident on a slit, which has a finite width a, and each point in the slit acts as a source of secondary waves. And these different light waves therefore interfere. And this is a narrow slit, but it has a finite width.

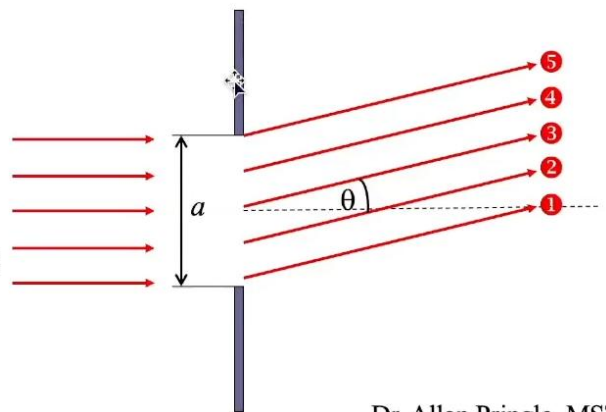
So this is the first premise that one should understand: there is a parallel beam of light, a parallel ray of light, which is incident on the slit. And each point in the slit acts as a source of light waves. And these light waves, therefore, start to interact with each other or interfere with each other.

Single Slit Diffraction

Now: consider the effect of finite slit width

Single slit:

- each point in slit acts as source of light waves
- these different light waves interfere.



Dr. Allan Pringle, MST

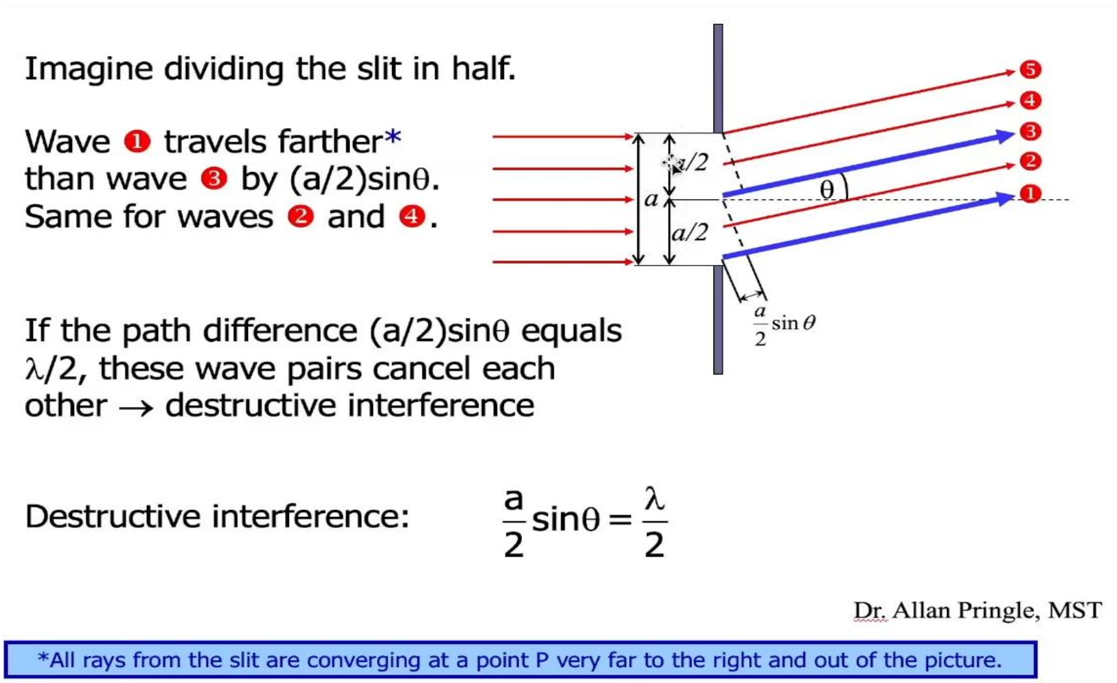
Okay, so what happens is that if you imagine, let us imagine that you have divided the slit in half, which had a total width of a, so the half width was A divided by two. Now, let us take a look at the wave that is marked in blue, which is labeled as one.

Okay, so this wave, okay, travels further than wave three. Right, just because if you drop a perpendicular from here, this particular small distance, which is nothing but $\frac{a}{2sin\theta}$ (theta being the angle). Okay, so if you look at these two beams or these two light rays, one and three, you will see that one has traveled a distance greater than ray three, and that greater distance is nothing but a by two into sine theta. All right, the same is valid if you look at rays two and four. Okay, it is the same thing; they travel that same small distance.

Okay, so this path difference, which is nothing but $\frac{a}{2sin\theta}$, if this path difference $\frac{\lambda}{2}$, where λ is nothing but the wavelength of the incoming beam, then these two wave pairs cancel each other, and you get what we call destructive interference. So for this destructive interference to happen, your $\frac{a}{2sin\theta} = \frac{\lambda}{2}$, half the wavelength. So you understood that initially the waves were all nice and horizontal; now they are deflected. And that deflection angle is kind of θ. Now, if this slit has a width of $\frac{a}{2}$, then the ray which is at the center travels a little bit of a shorter distance than the ray at the extreme end.

So at the extreme end, this particular ray travels a little bit more distance than the central ray. So this is the central ray that is deflected. This is the edge ray, which is also deflected. So this edge ray reaches the imaging plane okay after and actually travels a larger distance, and that larger distance is nothing but $\frac{a}{2sin\theta}$ whereas the ray at the central or optical axis actually travels a smaller distance, which is given by the path of this ray three. So if this path difference actually equals $\frac{\lambda}{2}$, then the waves actually destruct each other or they cancel each other, which we call destructive interference.

And the equation is nothing but $\frac{a}{2sin\theta} = \frac{\lambda}{2}$. So this is the premise. So all rays from the slit are converging at a point P and at a point very far to the right and out of the picture.

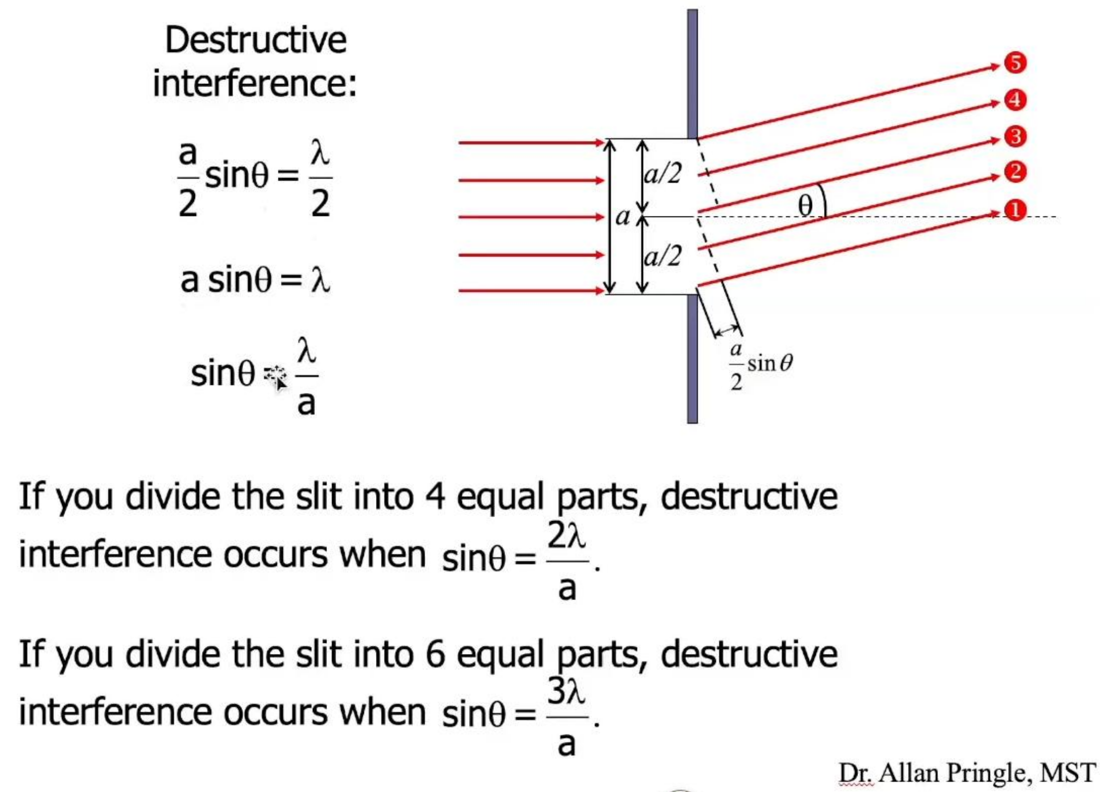


Now, for destructive interference, therefore, we already saw that it's $\frac{a}{2sin\theta} = \frac{\lambda}{2}$. So, therefore, it's $sin\theta = \frac{a}{\lambda}$.

Now, instead of two parts, if we divide the slit into four equal parts, four equal parts. Then the interference will occur, or destructive interference will happen when $\frac{a}{2sin\theta} = \frac{\lambda}{2}$, because we have now divided it into four parts. If you divide it into six equal parts, destructive interference will occur when $sin\theta = \frac{3\lambda}{a}$. You can understand that what we did in this particular case was divide it into two parts. So, therefore, the difference was by $2sin\theta$.

If we divide it into four parts, you can imagine that this will become like a divided by 4.

As a result of that, you will get $\frac{sin\theta}{1} = 2\frac{\lambda}{a}$. If you divide it into six parts, it will become $\frac{sin\theta}{1} = 3\frac{\lambda}{a}$. So this is the criteria for destructive interference. We showed it for two. Now we are claiming that 4 will be like this; 6 will be like this.



In general, destructive interference occurs when $a\,sin\,\theta = m\lambda$. m is 1, 2, 3, 4, 5, 6. So this gives the positions of the dark fringes. The dark fringes will only occur when this relationship is valid; therefore, this gives a position for the dark fringes.

There is no dark fringe for m equal to zero. Okay, so the bright fringes are approximately halfway in between the dark fringes. Right, so the dark fringes formula is therefore quite easy, as you can see that if, uh.

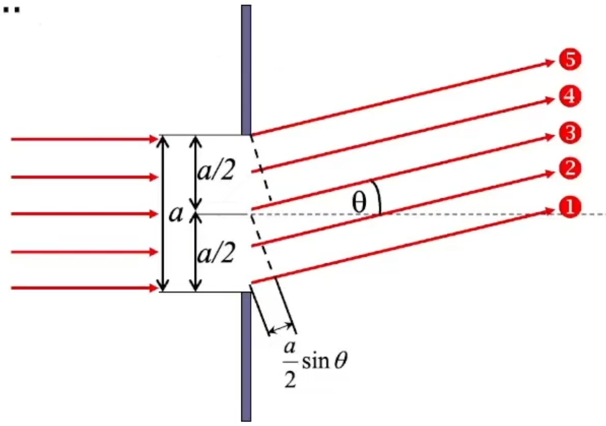
.. 4 gives to 2. So this is what you would actually get. $a\,sin\,\theta = m\lambda$. m equals 1, 2, 3, 4, 5, 6. So this gives the position of the dark fringes. Remember the dark fringes.

And there is no dark fringe for m equal to 0.

In general, destructive interference occurs when

$$a \sin\theta = m\lambda \qquad m = 1, 2, 3, \ldots$$

- gives positions of dark fringes
- **no** dark fringe for m=0

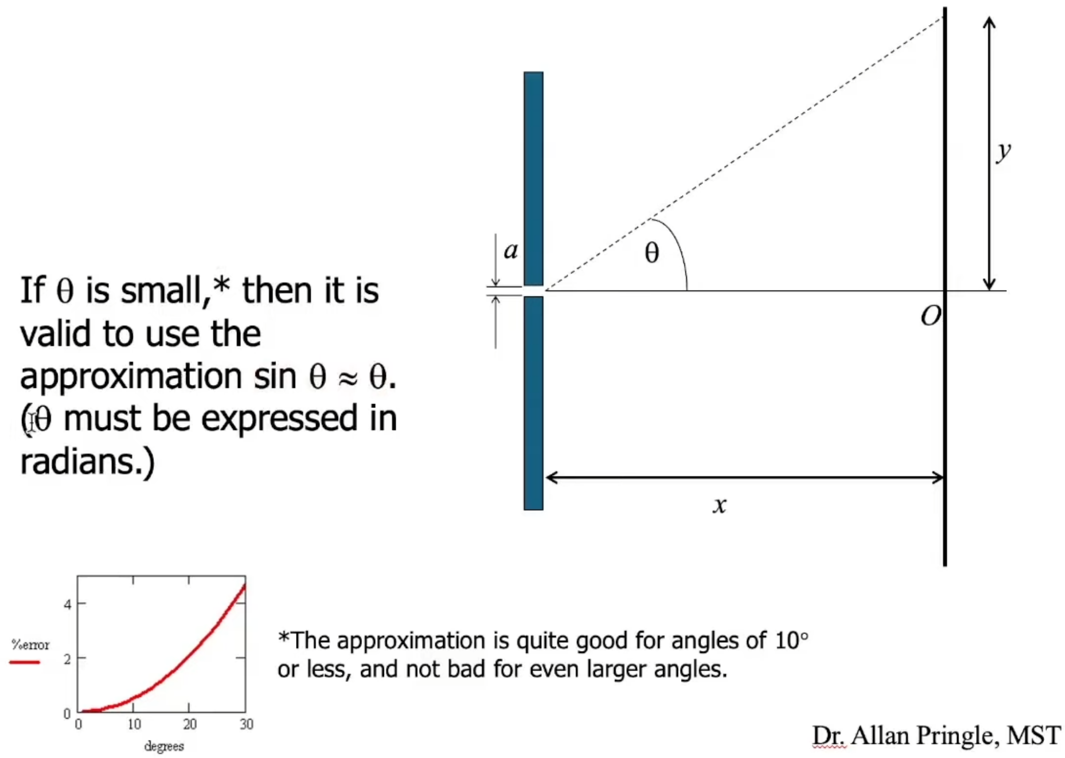


The bright fringes are approximately halfway in between.

Dr. Allan Pringle, MST

Now, when θ is equal to small, θ is equal to small. Then it is valid to use the expression, which is $sin\,\theta \approx \theta$. θ is in radians, remember. So, this θ is effectively small.

So if you plot the percentage error against the degrees, you will find that up to 10, this error is very minimal, but it is also about 4 to 5%. For an angle of 30 degrees. So even for large angles, this approximation, which is $sin\,\theta \approx \theta$, is not a bad approximation at all. So this is the primary thing that you want to know.



Now, in the single slit diffraction intensity, if we look at this, these are the general features of the distribution.

So this is the viewing screen. This is the screen that is at a distance of L from the slit. The slit has a width of a, and this is the angle. So you can see that when $sin\,\theta = 0$, which is this particular point, the central beam, you get the maxima. And most of the intensity is at the maxima, which is given by this.

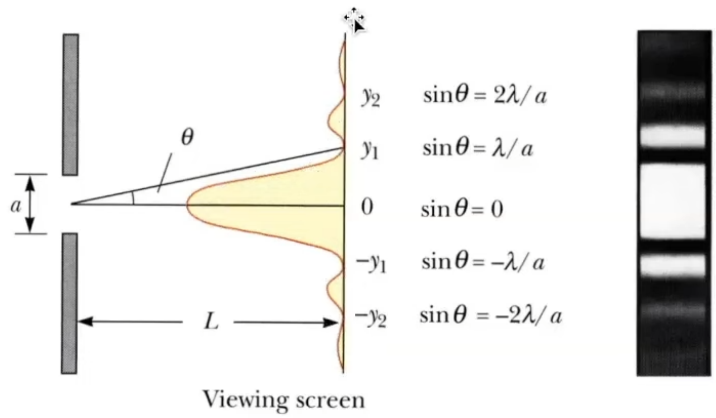
And it is twice the width of all the other maxima that you see. And progressively, the other fringes lie down. And these are basically the destructive fringe locations, as we have already seen. This was at sine theta equals lambda divided by a, which is $sin\,\theta = \frac{\lambda}{a}$.

This is something that we already did earlier. So these are the locations of the destructive fringes. Y = 0, which is basically this, is $sin\,\theta = 0$. Here we get the maximum. And these are the corresponding minima.

And the maxima, the central maximum, is actually the most intense. And it is twice the width of the secondary maximum. Secondary maxima are these. The tails of the wings, so to say, are the locations of the dark fringes that we can see over here; these are the locations where the brightness or the intensity is almost equal to zero.

Single Slit Diffraction Intensity

The general features of that distribution are shown below.



Most of the intensity is in the central maximum. It is twice the width of the other (secondary) maxima.

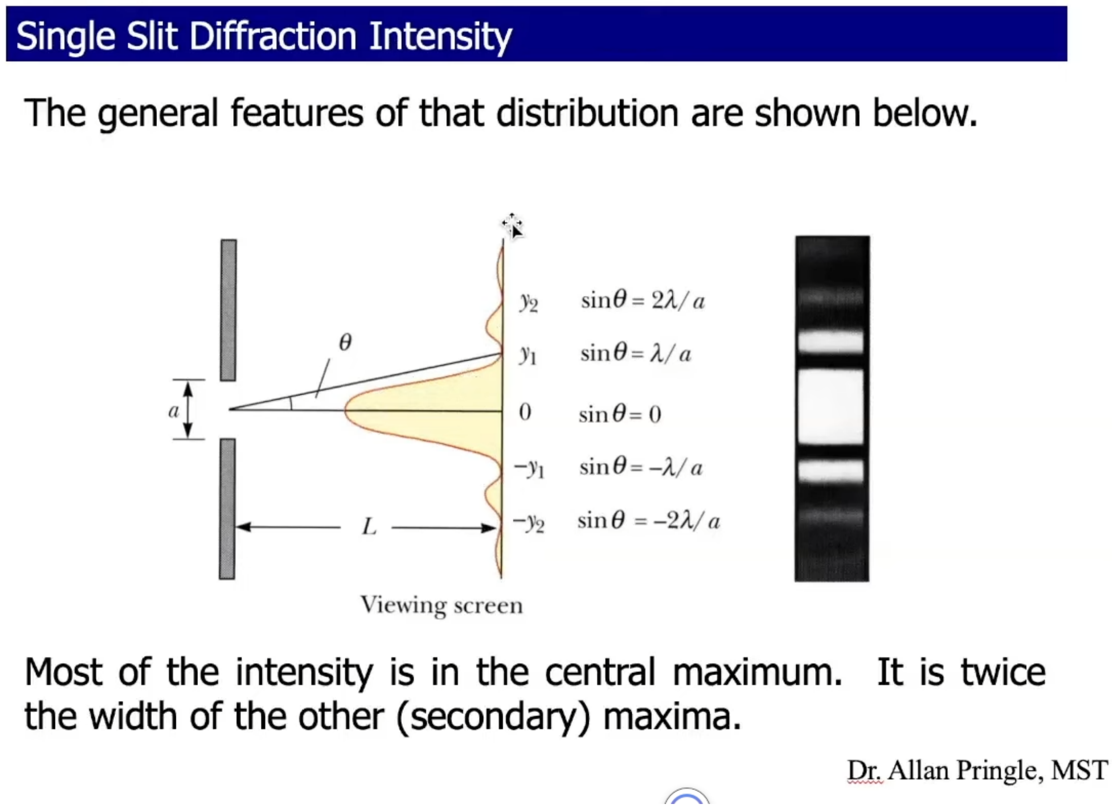
Dr. Allan Pringle, MST

So the starting equations for single slit intensity are, if you talk about beta, equal to 2 pi lambda into sine theta, 2 pi lambda A into sine theta, and I, which is the intensity, is the incoming intensity multiplied by this particular factor.

As you can see, $sin\,\theta = 0$. Okay, so that means β=0 . Okay, so in other words, this particular intensity goes to a maximum, all right? All right, so you can see that these are the maximum intensity, which happens at the center location, and the intensity drops progressively at higher values of your theta at different angles. So this angle, for example, will have a different intensity compared to that angle.

And those are all noted here. And those intensity profiles are given like this: So, as you can see from zero onwards for small angles, I mean, as you go on increasing the angle, this intensity compared to the central intensity will continue to decrease. This will continue to decrease. Okay, in a certain way. So that is the starting equation for a single slit intensity profile.

Okay, so this is what you get. This is how the profiles actually look. And these are the equations that correspondingly link the profiles. Okay, now that we have got it, we know what diffraction is all about, and the fact that diffraction through a slit will actually form these bright bands and these dark bands, and the central band will be quite intense, so to say.



Now, if we do the same thing, if we look from an imaging perspective, this is the light with an intensity and a wavelength of lambda, which is impinging on an aperture that has a size of a. Okay, and there is an image plane behind it.

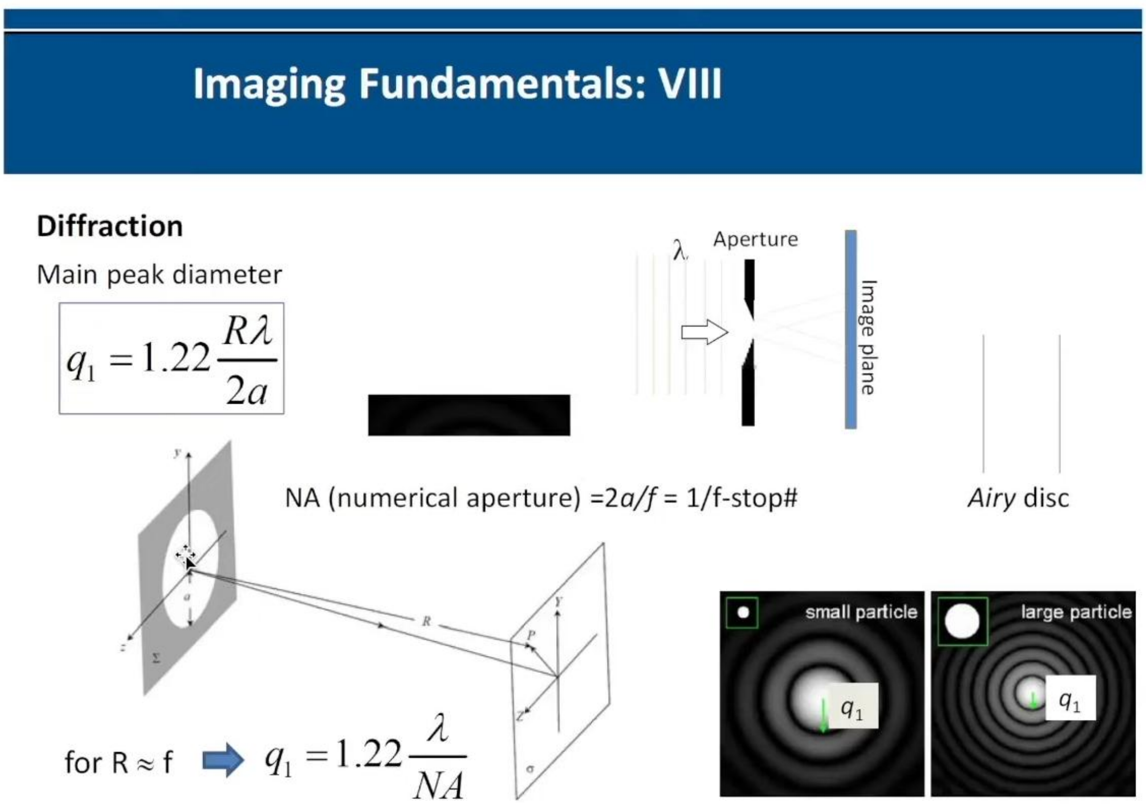
So the main peak diameter of the q_1 that you are going to record on the imaging plane is. So this can be an aperture; it can also be a small particle; it doesn't really matter. It can be in a particle as well. So what you can see is that it can be a slit, a particle, a hole, or whatever else you can think of. So the main peak diameter, which is q_1 that you see over here in this image, is what you look at in this image, okay?

$q_1 = 1 \cdot 22\frac{R\lambda}{a2}$. So what is $\frac{R\lambda}{a2}$? If R is equal to the focal length, then of course this becomes lambda by the numerical aperture. A numerical aperture is $\frac{2a}{f}$ or $\frac{1}{f-\#}$. λ is a wavelength, a is the size of the object, or in this case, the size of the hole, whatever you call it. R is basically the distance from the screen to the source. So this is the screen where you are putting in the imaging.

And if you place it at the focal length, this will be just the numerical aperture. This will be just lambda divided by the numerical aperture. If you put it at the focal length. If you put it at that particular focal length, So now you can see that you are not essentially imaging A; you are imaging something, which is q_1 in this particular case. And other things that you see, you see these interference bands in the same way.

So this is, for example, for a large particle; this is for a small particle. You see these interference bands, which are nothing but what is called an ARRI disk. This happens because of diffraction. So the diffraction leads to the formation of these Airy disks, which are formed around these particles. So this is an important thing that one should recognize in this particular context: if you are trying to image this object, what you are getting is something like a q_1.

which is not exactly the object, and you get these interference bands, which come directly as a consequence of this, because of the interference of cross-talking of these rays, one that we talked about, because of the destructive interference and constructive interference.



So now, if we look at a situation like this, where a particle diameter in the object plane, say, is dp. So this is basically the object plane, for example. So, as we said earlier, this can be your lens; it can be an object; it can be anything. So all these things are something that you should know.

So what happens is that there is a particle diameter, and the object plane is actually dp. This is, basically, a light sheet, for example. This is a flow direction. This is what people normally use in particle image velocimetry. The f number is the focal length divided by the aperture diameter, which we already know.

So this is the focal length of the lens, and then you are dividing it by the aperture diameter. For the diameter of the particle. So the particle diameter in the object plane, say dp, and the magnification is basically, you know $\frac{Z_0}{z_0}$. This is the magnification. From this information, the size of the particle image in the image plane is given by a particular expression.

In this particular expression, you see two interesting things. There is a term called ds squared, and then there is a term that includes the magnification of the actual particle diameter. So the $M_T^2 d_p^2$, which is the transverse magnification, which we already did. And so, plus the d_s^2, the whole square root is what is imaged on the image plane. Now, in the absence of d_s, recall that this M_T and d_p, the $d_i = M_T X d_p$.

In other words, the imaged particle diameter would be just the magnification factor multiplied by the actual particle diameter. So that would have been rather easy. You could have just divided one by the other, and you would have gotten the answer. That is what would have happened. But in this case, what you have is something even more significant; that is, you have this d_s^2, okay, plus this term which you are already familiar with.

So, what is this d_s^2? This ds squared is also called what we refer to as the diffraction-limited spot diameter, okay? MDTP is the geometric image diameter, so this is the diffraction-limited spot diameter, which is $ds = 2.4(1 + M_T)f^{\#}\lambda$. Now, this d_s, now you should recognize this factor that now enters into your equation, and this is a diffraction-limited spot. So you need to know this to actually calculate what the diameter of the imaged droplet will be and what the actual diameter is in order to find out what the actual diameter of the particle in question is, if that is the objective. Now, for a point object or a distant object where dp is almost equal to 0 and mt approaches 0, in that particular case, when the transverse magnification approaches 0 or it is a point object, this diameter has shrunk to 0; the d_p diameter has shrunk to 0, yet you still have the d_s.

That means no matter what lens you use or what kind of sophistication you use, after a certain point, your diameter doesn't decrease anymore. So whatever your diameter of the actual particle is, as it gets smaller and smaller, you get a response on the image screen. That is what you would expect. But after some time, you stop getting any response on the image screen at all because your diameter has now become equal to the diffraction-limited spot size. As a result, whatever you do, you still get a fixed diameter on your image plate.

You cannot reduce that. So that is a point of concern because this is what the diffraction-limited spot is. After a certain point, you cannot reduce the diameter of the object further. Just because you have the spot, which is no longer dependent on your original particle diameter at all, the imaging—whatever you are imaging—is no longer dependent only on your particle diameter, which is just given by your diffraction-limited spot. After some time, you'll still get a spot, but this spot does not correspond to any actual particle size; it is the diffraction-limited spot.

I think this part needs to be absolutely clear. So as we understand now, therefore, diffraction can happen when you actually have all objects slipped in the path of a light beam. And as a result of that, what happens is that you get these interference patterns. And as you know, there is a central maximum with progressively less bright minima. Less bright maxima, secondary maxima, and minima punctuated by this. So, that is a rather important thing that you should remember: this is what happens when you actually have diffraction.

Now, a couple of points to mention about diffraction are that it's basically the bending of light at the corner of an obstacle. And diffractions can also be of two types. The first type will be Fresnel diffraction, in which, if you recall, the distance from the aperture to the screen is finite. But it could also be Fraunhofer diffraction, where the distance of the screen from the aperture is basically infinite.

All right, so Fraunhofer and Fresnel. So these are the two types of diffraction that can happen over small objects and similar things. So this is an important thing to note about some small aspects of diffraction in general. Another feature of diffraction that you should note is that, fundamentally, the wavelength does not change in the diffracted wave. After diffraction, the frequency does not change, and the speed of the diffracted wave does not change, but the amplitude of the wave decreases. So these are the four points: the wavelength does not change, the frequency does not change, the speed of the diffracted wave does not change, and the amplitude of the wave decreases after the diffraction is over.

Also, one should note very important stuff over here because this is just like some fine print about diffraction, which you already know and we have covered quite a bit in this particular lecture. A shorter wavelength means that if the wavelength is shorter, the wave spreads after diffraction to a smaller area. If the wavelength is longer, that is, a longer wavelength, the wave spreads to a wider area for the same slit size or the same particle size. or the same obstacle size, so to say.

So shorter wavelengths after diffraction spread to a smaller area. Longer wavelengths after diffraction spread to a wider area. So, diffraction is also affected by the wavelength. The longer the wavelength, the greater the effect of diffraction. So this is an important thing that you should remember because we deal with lasers and light sources of different wavelengths to begin with. So this is an important part: the longer the wavelengths, the greater the effect of diffraction.

So now the opening comes. So if your opening is large, that is bigger than the opening, the wave spreads to a smaller area. The smaller the opening, the smaller the slip size; the wave spreads to a wider area. So, in other words, the diffraction is affected by the size of the openings. The smaller the size of the opening, the greater the effect of diffraction.

The smaller the size of the opening, the smaller it is. The smaller the obstacle, the greater the effect of diffraction. So that's why you cannot go down to a very small particle size. because of this. The effect of diffraction is greater.

Imaging Fundamentals: IX

Given:

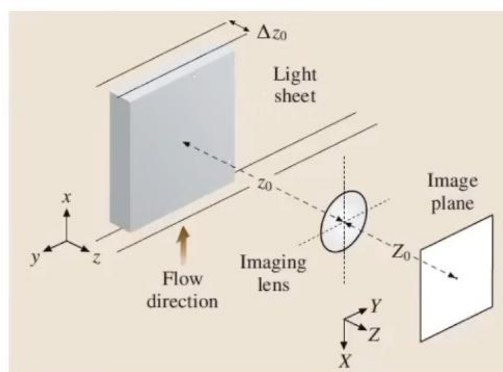
- Particle diameter in object plane d_p
- $f^{\#}$ (focal length divided by aperture diameter)
- Magnification $M_T = Z_0/z_0$

From this information the size of the particle image d_i is sought.

$$d_i \cong \left(d_s^2 + M_T^2 d_p^2\right)^{1/2}$$

$$\text{with } d_s = 2.44\left(1+M_T\right) f^{\#} \lambda$$

d_s is known as the *diffraction-limited spot diameter* and $M_T d_p$ is the *geometric image diameter*.



Point object or distant object:

$$d_p \to 0 \text{ or } M_T \to 0, \qquad d_i = d_s$$

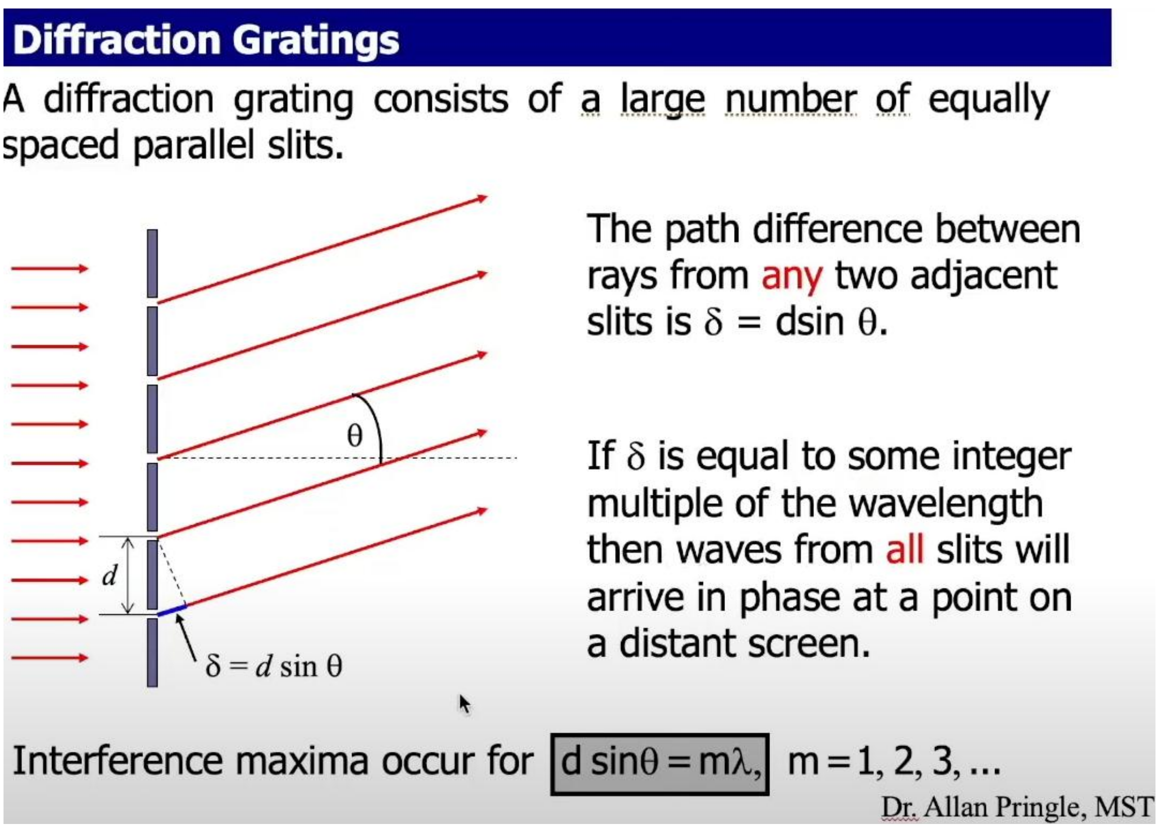
Diffraction limited!

This is called a diffraction grating. So diffraction gratings basically consist of a large number of equally spaced parallel slits, as you can see over here. So this is again a parallel beam of rays that is coming and impinging on this array of slits; these slits are separated by distances which are given as d, and they impinge on it, and therefore they are diffracted, or bending of light, at certain angles. The path difference between rays from any two adjacent slits, for example, these two points, is given by $\delta = dsin\theta$.

As you can understand. So this δ is equal to some integer number, an integer multiple of the wavelengths. Then all the waves from all the slits will arrive in phase at a point on the distant screen. So you understand this point very carefully. So this is a large number of slits: it can be 10,000 to 15,000 slits per inch, not per square inch.

So you have this ray of light that is coming and impinging. So the light that is going from here to here and the light that is going from here to here have a path difference, which is given as $\delta = dsin\theta$. Now, if this δ is equal to some integer multiple of the wavelength from all the slits, then waves from all the slits will arrive in phase at a point on a distant screen. Therefore, the interference maxima will occur for $m\lambda = dsin\theta$, where m is equal to one, two, or three. You understand, this is how you can fine-tune the grating.

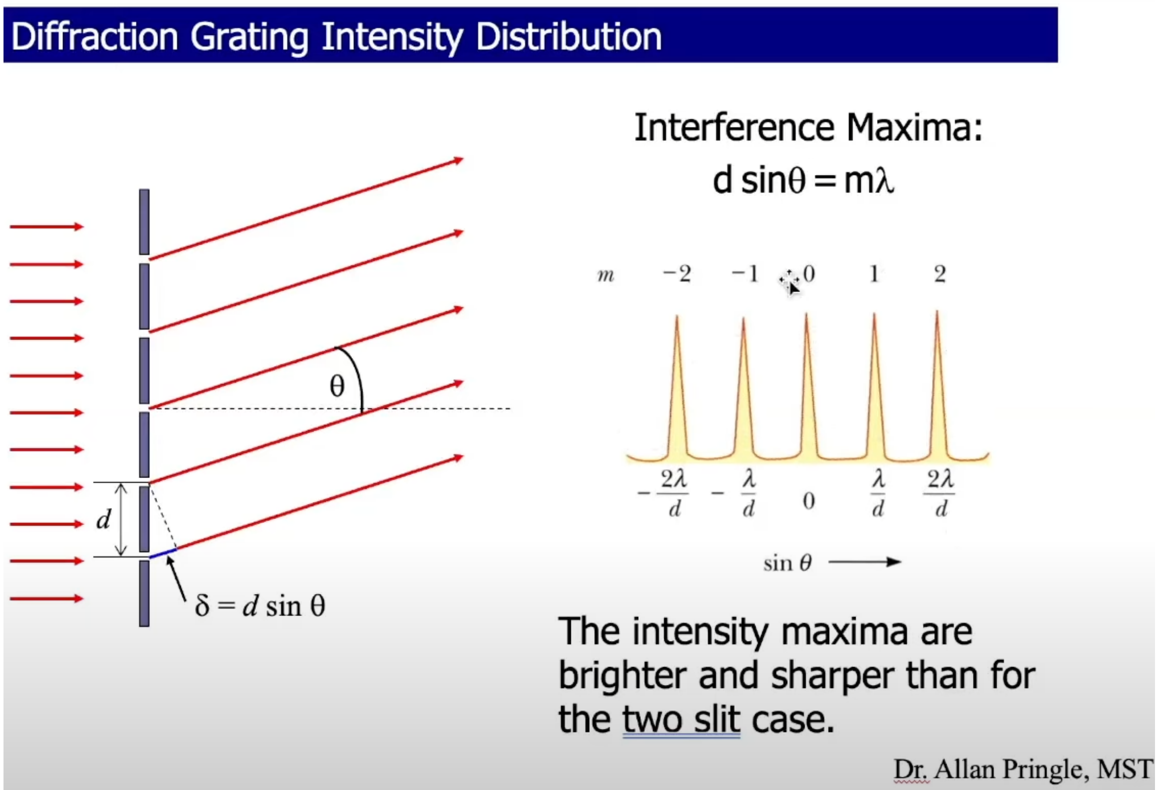
This is basically called diffraction grating.



All right. So the diffraction grating intensity distribution is given as $m\lambda = dsin\theta$. Assuming that this is an integer multiple, as we have already said, that this δ is an integer multiple, some integer multiple of the wavelength. Okay. So what happens here is that these are the intensity peaks.

So these intensity maxima are brighter and sharper than in the two-slit cases. So these are very sharp peaks that you get as your interference maxima. So this is how the diffraction grating is arranged. This is used for spectroscopic measurements very routinely. Used for many things, but this is something that you will see even in spectrometers; they have this diffraction grating, and we already talked about a little bit of how a diffraction grating need not be created with slits. They can also be created by, you know, other types of ways as well, but this is the most common thing.

These spacings are very small; as I said, 10,000 gratings per inch, which is a fairly large number, so to say.



Okay, so in the next class, we will look at optical aberrations now that we have completed diffraction. In the next class, we are going to do optical aberrations.

Optical Aberrations

Deviations from idealized conditions of Gaussian optics (paraxial descriptions) are known as aberrations. There are two classifications

Monochromatic Aberrations

- Spherical aberration
- Coma
- Astigmatism
- Petzval field curvature
- distortion

Makes image unclear

It is generally good practice to close the aperture of a lens by one or two *f*-stops, and to use a lens within its designed magnification range.

Deforms the image

Chromatic Aberrations