

# **Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer**

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**Week – 01**

**Lecture - 01**

**Introduction to Fluids – 1**

Okay, all right, so in these few talks, we are going to talk about basically fluid mechanics and give you a very bird's-eye view of it, so to speak. First, we need to know that for a measurement course, you need to understand why we need to take measurements. The reason we need to take measurements is as follows: we want to know how, for example, a fluid behaves. And once we know that, it can enable us to design new things, for example, a gas turbine combustor or even a microfluidic device—anything and everything. We can actually, if you know fluid mechanics and if you know what you can kind of measure, get this. So if you look at the fluid mechanics in general, there are, you know, what are the properties? So there will be the first material properties, right? So, material properties can be many things, such as mass, viscosity, thermal conductivity, which is  $K$ , and of course, molecular diffusivity.

These are all material properties, okay? And bulk modules and surface tensions. It is very important in the case of droplets, for example. And then, you know, the coefficient of thermal expansion, the index of refraction, which is  $N$ , and, you know, fluorescence, and, you know, specific heat under constant pressure, and so on. Similarly, you have the kinematic properties.

Kinematic properties. Okay, so the kinematic properties, like volume flow rate, displacement, acceleration, velocity, angular displacement, angular momentum, angular position, and vorticity, are very important, and so on. So there are many kinematic properties. Then, of course, you have dynamic properties. Dynamic properties.

Though we say properties, they are actually dependent on the flow. These are not something that you know a priori. So dynamic properties are force; then you have pressure; then you have stress; then you have torque. Then of course, so these are things that are applied onto the system. Then, of course, you have thermodynamic properties.

Thermodynamic properties look at it, so there are plenty of thermodynamic properties that you already know. Internal energy is one of them: internal energy, enthalpy, heat flux, work, energy, entropy, and temperature. So when you do fluid mechanics, you are

particularly interested in knowing parameters like velocity, and once you know velocity, you can find out things like acceleration and vorticity. You are interested in finding force, pressure, stress, and torque; these are things that you can measure, and similarly, you want to measure, for example, temperature. This is a very important quantity to which many diagnostic methods are dedicated for finding out the temperature.

Similarly, these are measured in different ways, not using the methods that we are going to talk about. We are going to focus on measuring these parameters, for example, and we are therefore going to use techniques like optics. Lasers are okay and stuff like that to measure these, so what we are trying to say is that the measurement techniques I have to put forth for doing those things are kind of, we would have point measurements, field measurements, and probe-based measurements. Okay. Probe-based.

## Material properties

mass, viscosity,  $k$ , molecular  
diffusivity, surface tension,  
 $n$ ,  $C_p$  . . .

## Kinematic properties ✓

Volume flow rate, displacement,

velocity,  $g$ , viscosity, . . .

## Dynamic properties

force, pressure, stress,

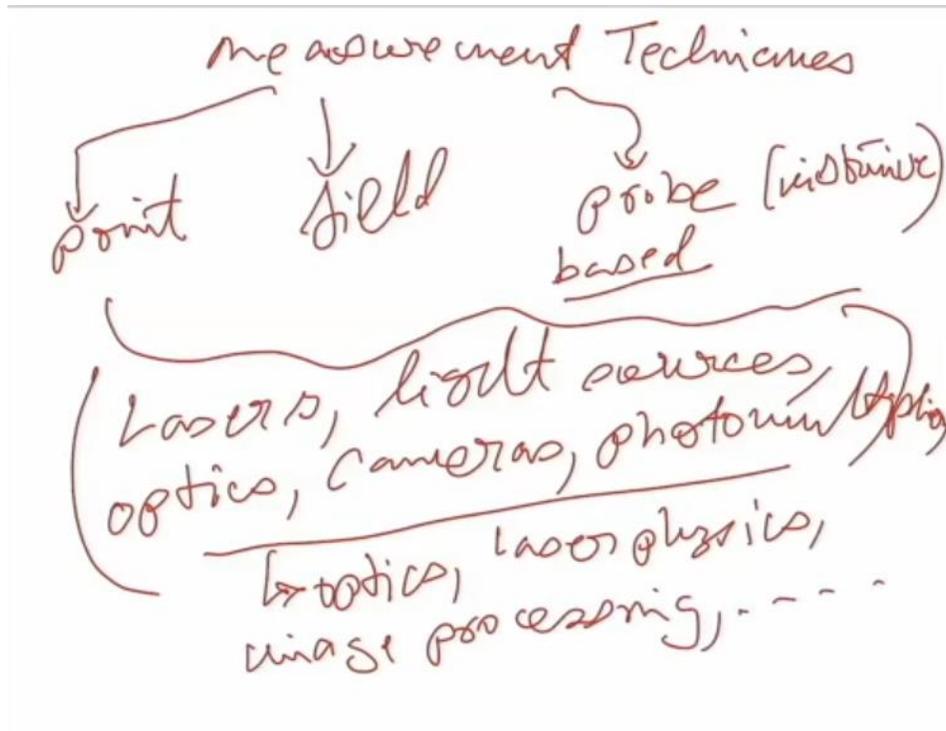
Torque

## Thermodynamic properties

internal energy, enthalpy,  
heat flux, work, energy,  
entropy, Temperature

These are intrusive. Okay. So these are the measurement techniques that we are going to use, and almost invariably, all of them will have lasers, you know, light sources. Okay.

Optics. And, you know, you will have cameras. Okay. Photomultipliers. And all this means you need to have a very good understanding of optics, okay, and a little bit of laser physics, okay, and of course image processing, and so on. Each specific technique has certain unique aspects attached to it.



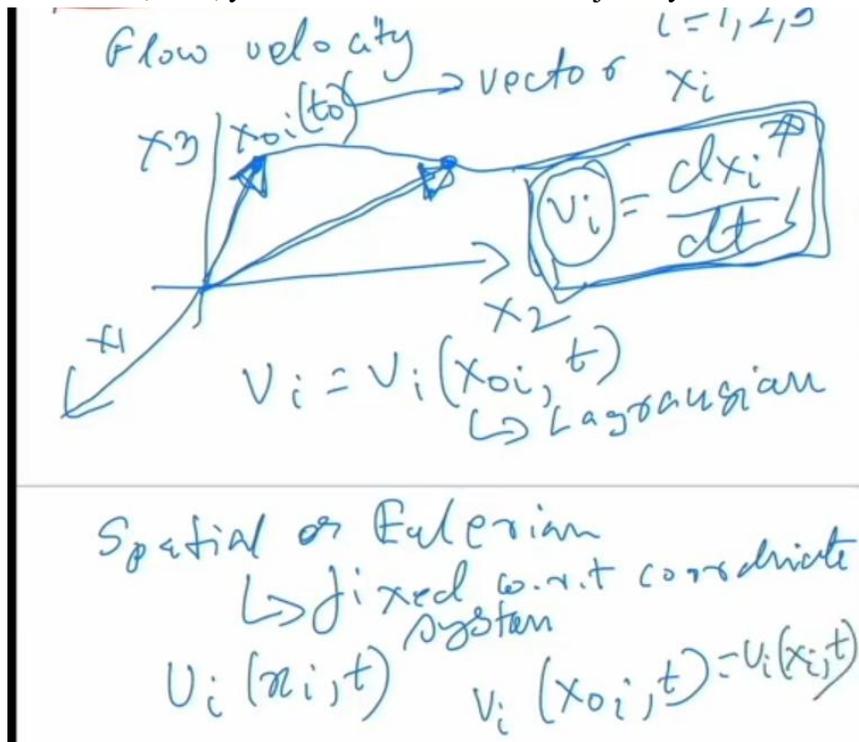
All right. So this is the kind of thing that we have. Now, let's do a quick recap or gain a quick understanding of the different flow velocities, because we know that it's a very important quantity to determine the flow velocity. And this, you know, is a vector. It basically has three directions. I am equal to one, two, three.

And then you have  $x_i$  in that direction. So you can also write it as  $xyz$ . It doesn't matter. So this is basically telling you how the particle behaves; for example, if you look at a typical Lagrangian. A situation like  $x_2$ , this is to say  $x_3$ , something like that.

And you have a particle trajectory like this. Initially, this vector, this position vector, was this. Then the position vector was this at some other points. So you have this change in the flow velocity at two different times:  $t_i$  and some later time. So basically, your velocity is given as  $\frac{dx_i}{dt}$ , which is essentially high school.

So this is the velocity in any particular direction. So you know the position vectors, and then you basically do a vector fraction. So one approach is to identify the initial coordinates of a fluid, namely the coordinates of the position of the fluid at the origin of time. So this is say at  $(x_{0i}, t_0)$  and then there is a final position. Which is at a later instant in time, right? So by doing this, you can find out the velocity, which is basically nothing but the slope of the path.

So this approach, okay, the velocity field can therefore be specified as a function of these two variables:  $V_i = V_i(x_{0i}, t_0)$ . This approach is called the material or the Lagrangian description of the motion. So this is Lagrangian; all right, this is the Lagrangian description of the motion. It basically involves tagging individual fluid elements, and this is not practical because, well, you have to. Calculate the trajectory of almost a thousand particles.



So it is much better to express the velocity field in terms of a fixed position with respect to the coordinate system. And of course, time, this approach is called the spatial or the Eulerian description. Spatial or Eulerian description is basically when you have a fixed coordinate system. So it was a fixed position. With respect to our coordinate system, the velocity, therefore, if I have to differentiate, is given as a function of  $(x_i, t)$ .

It is nevertheless understood that at all positions and for all times, the definition of flow velocity is unique. So, therefore,  $V_i(x_{0i}, t) = U_i(x_i, t)$ . According to the Lagrangian description, and this is basic fluid mechanics, in the Lagrangian description, the acceleration,  $a_i = \frac{DV_i}{Dt}$ . The rate of change of velocity in that particular direction, in an Eulerian description, when you go to an Eulerian or spatial description, is a little bit different; change the color as we write this. So here, the

$$a_i = \frac{\partial u_i}{\partial t} + U_1 \frac{\partial u_i}{\partial x_1} + U_2 \frac{\partial u_i}{\partial x_2} + U_3 \frac{\partial u_i}{\partial x_3}$$

So the first term on the right-hand side is called a local acceleration. And the remaining terms are basically called the convective acceleration.

Lagrangian description

acceln.  $\boxed{a_i = \frac{dv_i}{dt}}$

Eulerian description

$$a_i = \frac{\partial u_i}{\partial t} + \left( u_1 \frac{\partial u_i}{\partial x_1} + u_2 \frac{\partial u_i}{\partial x_2} + u_3 \frac{\partial u_i}{\partial x_3} \right)$$

$\hookrightarrow$  local acceln.       $\downarrow$  convective acceln.

Right, so local acceleration versus convective acceleration; the right-hand side of the equation—this entire thing, whatever you see on the right-hand side, is called the material derivative, or the substantial derivative of  $U_i$ , and it is usually designated as  $\frac{DU_i}{Dt}$ , substantial derivative.

So that includes the local acceleration and the convective acceleration, right? Similarly, fluid deformation, or deformation under stress, for example, press, is described by the rate of deformation tensor. Rate of deformation tensors. And that is given as, you know,  $e_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]$ . This is a cross derivative; remember, where  $i$  is equal to 1, 2, 3. So this is what we call the rate of strain or the rate of deformation.

So this is also something that you may know. Okay, so similarly, now that we know these two, this is important, you know, and the velocity description is given by a field, and the

rate of deformation tensor is given like that. Now, of course, just by doing this, you do not have everything at your disposal.

Fluid deformation under stress rate of deformation tensor

$$e_{ij} = \frac{1}{2} \left[ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right]$$

$i, j = 1, 2, 3$

You need an analytical description of the flow. What are the analytical descriptions of the flow? So the physical relationships between the flow variables are represented by analytical expressions, which constitute algebraic differential integrals.

It can also be integral differential equations. And there are certain empirical relationships that also go along with it. Okay, so they are based on the conservation principles. So, how many conservation principles are there? There is conservation of mass, conservation of momentum, and conservation of energy. OK, so when you are applying these principles to a closed fluid system, they can give rise to a set of integral relationships.

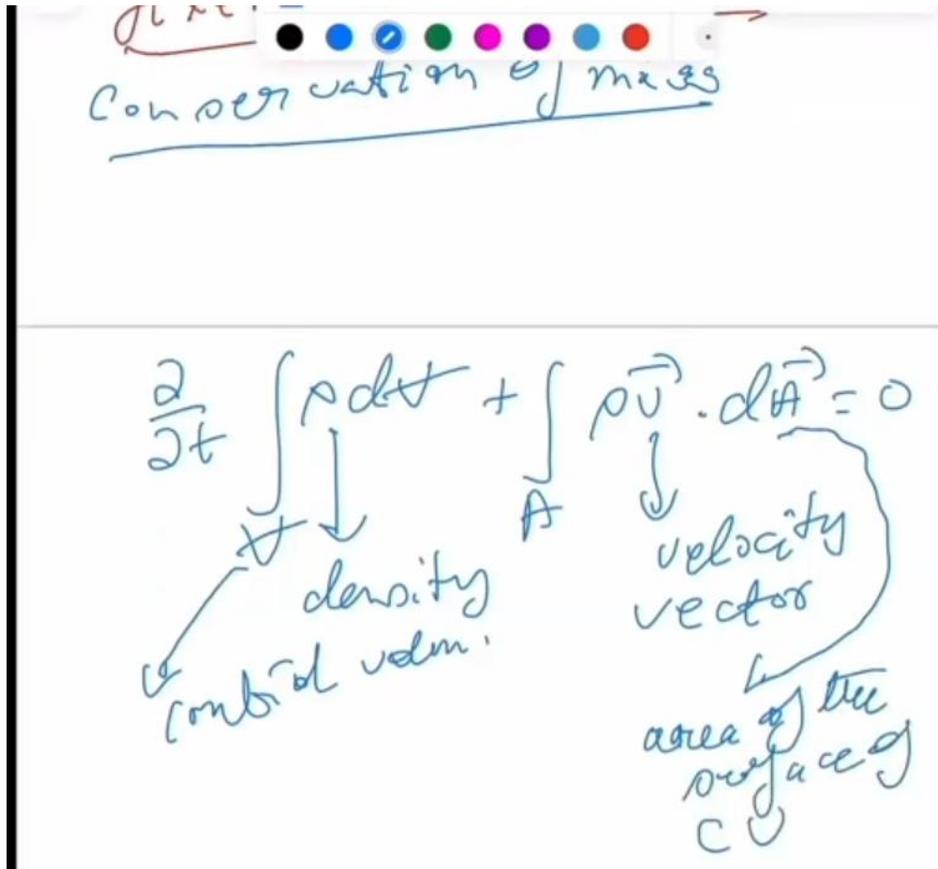
We also have differential relationships that we can represent as well. However, it is, you know, if you apply it to a fixed control volume, a fixed control volume. This is once again a part of a fluid, full-fledged dynamics course, but I'm just giving you a heads up. So the conservation of mass, for example, the continuity equation as we call it, is

$$\frac{\partial}{\partial t} \int_V \rho dV + \int_A \rho \vec{v} \cdot d\vec{A} = 0$$

This is the continuity equation. This is basically the density of the fluid.  $v$  is basically the velocity vector. And this is the control volume. Okay,  $A$  is the surface area of the control volume; the area of the surface of the control volume.

So this is all standard. And for a non-reacting multi-component flow, you can have similar equations expressing the species. And remember, in combustion dynamics or in

combustion diagnostics, that is also an important parameter. So this is the conservation of mass.



Similarly, you have a momentum conservation as well. The conservation of momentum is basically nothing but Newton's second law, so if it is given as, you know

$$\vec{F} = \frac{\partial}{\partial t} \int_V \rho \vec{v} dV + \int_A \vec{v} \rho \vec{v} \cdot d\vec{A}$$

F is the net force, the net external force. Because of Newton's law, remember, it is  $F = ma$ . F is the net external force acting on the control volume.

Control volume. So it is  $F = ma$ . Essentially, this  $F = ma$ . So that is the whole thing.

Momentum conservation  
(Newton's second law)

$$\vec{F} = \frac{\partial}{\partial t} \int_V \vec{v} \rho dV + \int_A \vec{v} \rho \vec{v} \cdot d\vec{A}$$

↳ net external force acting on the control volume

So then, of course, you have the energy, which is basically the first law of thermodynamics. So basically, it's the conservation of energy. So, conservation of energy, when you look at it, is like

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t} \int_V [u + \frac{1}{2}v^2 + gz] \rho dV + \int_A [u + \frac{1}{2}v^2 + gz + \int \frac{dP}{\rho}] \rho \vec{v} \cdot d\vec{A}$$

All right? So, Q is basically the rate of heat transfer. Transfer. Okay. And W dot is equal to what we call the mechanical power produced.

Power produced. Then there are shear stresses acting. This is not just mechanical power. Shear stresses are present, but normal stresses are not. Shear stresses are acting on the boundary.

Boundary. But not normal stress. Right then, of course, p is the pressure, z is the upward vertical axis, and g is the acceleration due to gravity. I think these things are kind of well-known.

## Conservation of Energy

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t} \int_V \left( u + \frac{1}{2} v^2 + gz \right) \rho dV + \int_A \left( u + \frac{p}{\rho} + \frac{1}{2} v^2 + gz \right) \rho \vec{v} \cdot d\vec{A}$$

$\dot{W}$ : work done by heat transfer modes  
 produced, shear stresses acting on boundaries  
 BUT NOT normal stresses.

Then you have the second law of thermodynamics, which is, if I write it properly,

$$\frac{\partial}{\partial t} \int_V \rho s dV + \int_A \rho \vec{v} \cdot d\vec{A} \geq \int_A \frac{1}{T} \left( \frac{\dot{Q}}{A} \right) dA$$

So  $s$  is specific entropy. And  $T$  is the temperature. Absolute temperature. The temperature and the absolute state. So by letting this control volume vanish towards a point, you convert these equations into differential forms, which you will see are.

→ second law of thermo

$$\frac{\partial}{\partial t} \int_V \rho s dV + \int_A \rho \vec{v} \cdot d\vec{A} \geq \int_A \frac{1}{T} \left( \frac{\dot{Q}}{A} \right) dA$$

specific entropy
Temperature

And of course, there are other conditions, such as if you have a solid boundary. This is like a wall, for example. Then you have the no penetration and no slip.

The no-slip and no-penetration. These are additional clauses that incorporate conditions, which basically tell you that the relative velocity normal to the contact surface actually vanishes. And no slip says that you cannot have relative velocity tangential to the contact surface. Now in two-phase flows, we have certain additional things that come into the picture, so if you have an interface like this, this is an interface, okay? So you have the  $\Delta P$  across the interface, okay? It is related to the surface tension. In three dimensions, this is basically the two radii of curvature, the principal radii of curvature. Across a plane interface, if this is a plane, then  $\Delta P$  goes to zero.

So, in a curved interface, when the interface is curved, then these two factors click. Sometimes they can be equal, as well. In that case, when they are equal, it is  $\frac{2\sigma}{R}$ . So in two-phase flows, this is a customary thing to do. And then you see that for a fluid, you know, for a flow near a wall, these are the other things that you do.

So there are other types of flows which basically, you know, require additional principles, like, for example, the law of chemical reaction for reactive flows. These are supplementary things that need to be added for reactive flows. And then there is magnetohydrodynamics. Dynamics for electrically conducting fluids in magnetic fields.

