Lecture - 08
Micro-Scale Fluid Mechanics

Okay let us talk about 2 phase Poiseuille flow. Here we talk about 2 fluids between 2 infinitely long parallel plates phase and the 2 fluids have different viscosities. Okay. So you know, we would achieve an interface between the 2 fluids which is flat so the young Laplace or pressure drop would not exist, okay. And we would have the interface between the 2 place at some height extra, okay. So with that let us look at the sketch.

(Refer Slide Time: 00:49)
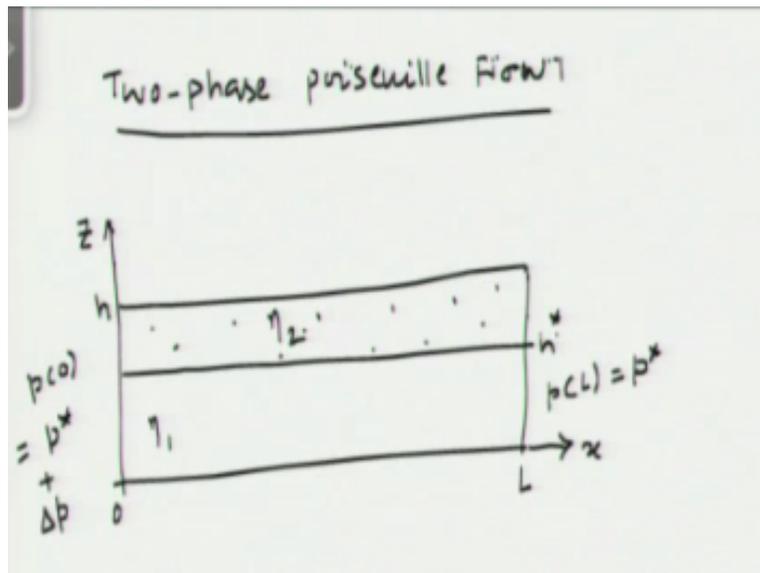
So we talk about 2-phase Poiseuille flow. So here we talk about 2 parallel plates. Let us say this is x and this is z, this height is h and we have let us say this is the interface okay at h* okay. So here we have the p(0) at the inlet, it will be p* + delta p and here we have p(l) will be = p* this is l. Okay. So this fluid has viscosity it have eta 1 and this fluid has a different viscosity this is eta 2. Okay.

(Refer Slide Time: 02:18)

- Two fluids: η_1 & η_2

- Flat interface at $z = h^*$

⌐— Young Laplace Pr. $= 0$

→ Pr. field is similar to single phase Poiseuille flow

$$p(x) = p^* + \left(1 - \frac{x}{L}\right)\Delta p$$

So here, now we have 2 fluids viscosity eta 1 and eta 2 and we have a flat interface at z= h*. And since the interface is flat a Young Laplace Pressure is going to vanish so this is 0. So the Pressure field is similar to similar to single phase Poiseuille flow that means it varies linearly okay. So p(x) = p* + 1-x over L*delta p. So that is how pressure is going to vary between inlet and outlet.

(Refer Slide Time: 03:44)

Bottom Layer: $0 < z < h^*$

Top layer: $h^* < z < h$

$$u(z) = u_1(z) \quad 0 < z < h^*$$
$$= u_2(z) \quad h^* < z < h$$

No-Slip BC: $u_1(0) = 0$, $u_2(h) = 0$

So you know as I said that 2 fluid layers for the bottom layer—bottom layer is between 0 < z < h* and the top fluid layer is between h* < z < h, okay. So you know in the 2 domains the velocities we can write, so u(z) = u1(z) when 0 is < z < h* and it is u2(z) when h* < z < h. Okay. So we can satisfy the No-Slip boundary condition, a No-ship boundary conditions are u2 at--

sorry we are in this case we are taking 1 and 2 so u1 at z=0 is going to be 0 and u2 (h)=0 is going to be 0. So those are No-Slip conditions.

Now we can generalize expression for the velocity that we have derived for single phase fluid between 2 infinite parallel further plates and you can apply it to this situation for 2 different fluids, okay. So if you do that you know for a single phase—

(Refer Slide Time: 05:50)

No-Slip BC: $u_1(0) = 0$, $u_2(h) = 0$

Single phase fluid between infinite parallel plates:

$$u(z) = \frac{\Delta p}{2\eta L} (h-z) z$$

Generalize:

$$u_1(z) = \frac{\Delta p}{2\eta_1 L} (h_1-z) z$$

$$u_2(z) = \frac{\Delta p}{2\eta_2 L} (h-z)(z-h_2)$$

For single phase fluid between infinite parallel plates we know the velocity expression so u(z) is going to delta p over 2 eta L*H-z*z. So this satisfies the boundary conditions at the top and bottom all assuming there is a single phase fluid. Okay. So this case if you generalize what we would get is we can write u1(z) = delta p over 2 eta 1 L, so the delta p and L they are the same for both the fluids okay*h1-z*z. okay.

So this will satisfy the boundary condition e1 at 0=0, okay. So for the velocity u2 you can write u2(z) is going to delta p over 2 eta 2 L*H-z*z-h2. So this will also satisfy the boundary condition u2 h=0 okay. So now you can see here there are 2 constant we have introduced h1 and h2, they are constant. And this constant could be determined using the boundary condition.

So now in addition to this No-Slip boundary conditions we have additional boundary conditions coming at the interface, okay at the interface between the 2 fluids, we would have you know

additional boundary conditions coming in. The one boundary condition at the interface would be that the velocity of the 2 fluids will be equal at the interface and the second boundary condition is that the shear stress will be equal, okay.

So eta 1*the velocity gradient will be eta 2*velocity gradient for the other fluid, okay. So if you do not satisfy the shear stress continuity boundary condition then there will be on physical forces that might be present at the interface, okay.

(Refer Slide Time: 08:50)

So we would have boundary conditions at the interface so one would be the velocity u1 (h*) = u2 (h*), okay. So this is velocity continuity boundary condition and the other one is shear stress continuity, so eta 1*del u1 over del z = eta 2*del u2 over del z and this will be evaluated at h* is at h*. So using these 2 boundary conditions we can you know, we can apply these 2 boundary conditions to this equation here okay, these equations for the velocity field. And if you do that what you will get is we will evaluate the 2 constants.

(Refer Slide Time: 09:59)

$$h_1 = h + h_2$$

$$h_2 = \left[\frac{\left(\frac{\eta_1}{\eta_2} - 1 \right)\left(1 - \frac{h^*}{h} \right)}{\frac{\eta_1}{\eta_2}\left(1 - \frac{h^*}{h} \right) + \frac{h^*}{h}} \right] h^*$$

So we can find that h1 will be = h+h2 where h2 will be given by this expression, eta 1/eta 2-1*1-h*/h*/eta 1 or eta 2*1-h* over h* sorry +h*/h*h*. So we get an expression for 2 constants h1 and h2 which can be used in the expression for the velocity here, okay. So you would have the expression for the u1(z) as well as u2(2) okay. So if you plot that if you plot all the velocity would look like you know here, it would be increasing parabolically and there will be discontinuity in the velocity profile where the gradients are going to be different.

So the slope of the profile is going to change here, okay. So you know, these are the velocity profile and you can see that there is a change in the slope that is occurring at the interface between the 2 fluids. And this change in slope is because of the equal shear stress requirement and given that the viscosities are different, okay.

So you know, 2 fluids in my micro channels has practically importance in applications for example in case of you know flow cytometer where we need to focus a sample fluid using a seed fluid and there our goal would be to bring down the width of the sample stream to a size equal to the size of the particles that we want to detect so the particles would move single file.

So in that case you know we were talking about a case similar to what we discussed, would be having 2 different fluids different viscosities and we can perform analysis similar to what we just did to you know determine at a particular flow rate what would be the width of the sample

stream, okay. So with that let us move and discuss time dependent flow where the velocity will be a function of time.

We will consider a simple case, case of a quick flow where you know a time=0 both the place will be stationary and will have a fluid attached in between the plates and for time t > 0 one of the plate would start moving. And when it starts moving how the velocity of the fluid is going to change with time, and we know at steady stress in quick flow the velocity profile will be linear but we are interested in the time domain where the flow is still a you know in the transient range between 0 to sum time t before it gets fully developed, okay.

(Refer Slide Time: 13:59)
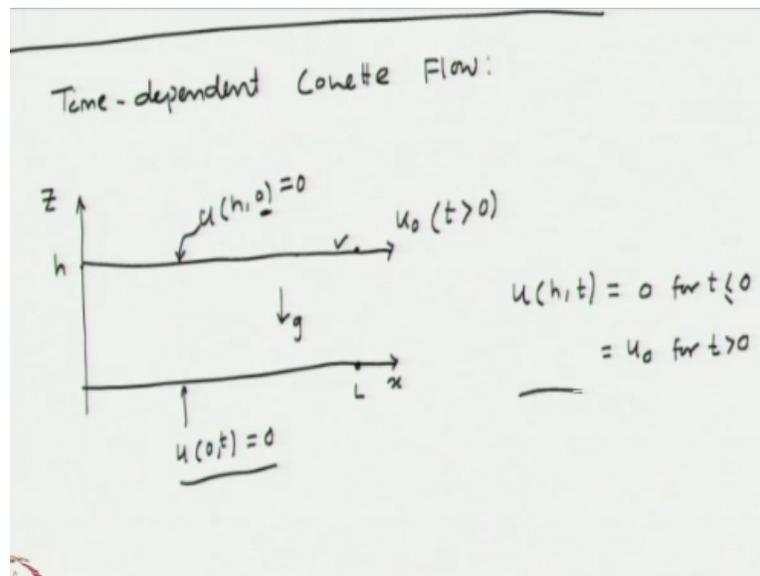
So let us talk about Time-dependent Couette flow. So here we have one of the plates, let us say this is the bottom plate this is x direction and this is z direction and this is the second plate both are same length, let us call it L and this is moves are a velocity u(0) for t > 0 and this is u(h,0) 0 this is No-Slip boundary condition, it will always be satisfied on the top wall is the same case on the bottom wall u(0,t) will be 0. Okay.

So this is the time t=0 this is the stationary but at some finite time this is going to move. So this is only valid at time t=0 but this plate is stationary so at all time this is going to satisfy the No-Slip boundary condition. We consider g in this direction and this is h is the separation distance

between the plates. So we can write that u(h,t) so the velocity of upper plate here = 0 for t < 0 and it is going to 0 or t > 0. So that is the situation we are talking about.

(Refer Slide Time: 16:20)
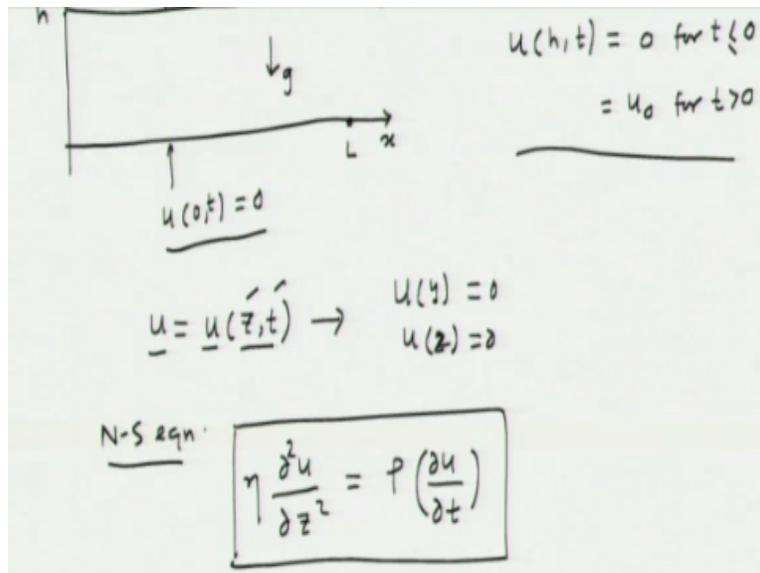
So and here similar to the Poiseuille flow we can u as a function of z and time. So what it means is that velocity along y direction and velocity along sorry velocity along z direction there 0 and we have only x velocity which is a function z and t. So we can write the Navier-Stokes equation for this simplified case, here you can write del square u which is a function of z and t over del z square*eta that is the viscosity will be = Rho*del u over del t so that is the unsteady term.

(Refer Slide Time: 17:16)
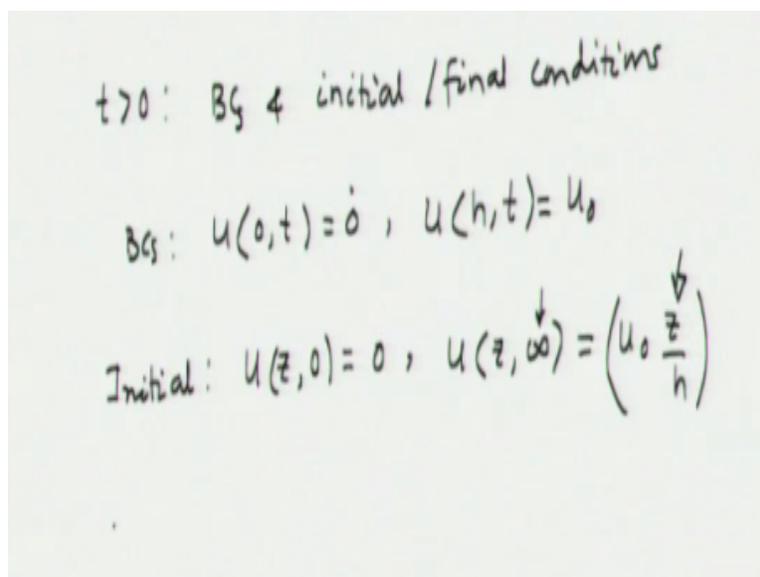
So this is the equation which is relevant here. So this is the unsteady term, this is the viscous term so for T > 0 we can write the boundary conditions and the initial as well as final conditions, final condition will be the steady state conditions, so the boundary conditions can be written as u (0,t) will be 0 and u(h,t) will be u0. So the velocity of the top wall is going to be u0, so that is the velocity of the top wall.

And the bottom wall is addressed so this is going to be 0, the initial condition is going to be u(z) time=0 is going to be 0 and the final condition when time is infinite u (z, infinite) is going to be u0*z over h, okay, so that is how when time = infinite the velocity is going to vary in this fashion, okay. So linear, linearly varying with z, okay.

So this equation is a linear partial differential equation, linear second order partial differential equation which can be solved by Eigen expansion. So here in the velocity term we have, velocity has 2 variables, one is z and the other is time, okay. So by Eigen expansion we would be able to separate, you know, the velocity field which is a function of z and t to 2 different variables, okay, 2 different functions with single variable.

We will have u and z and time t and t, okay. So if you do that—
(Refer Slide Time: 19:37)

So this is a linear partial differential equation second order that we are talking about, okay, this is the second order linear partial differential equation. And to solve that we would use method of Eigen expansion where u(z,t) will be expressed as u and z and t and t, okay, so it will be the velocity field which is depended on 2 different variables will be converted as the product of 2 variables with 2 functions with single variable, z and t separately.

So if you do that we can write the solution u(z,t) in terms of 4ier expansion we can write this u(z, infinite), so this is the steady state solution, okay, this is the steady state -n, Cn*u and z, t and t, okay, which you can also write as U0*z over h. This is the steady state solution, okay what we see here, right, - summation n Cn un(z) Tn and t, okay, so this is the, our expected solution using the again expansion, okay.

So, you know, this is the steady solution. What it means that the expansion here, the same here, it will vanish when t = infinite. So these terms, the expansion term will be = 0 whereas T tends to infinite then only we will have only these solutions. Okay.

(Refer Slide Time: 22:25)
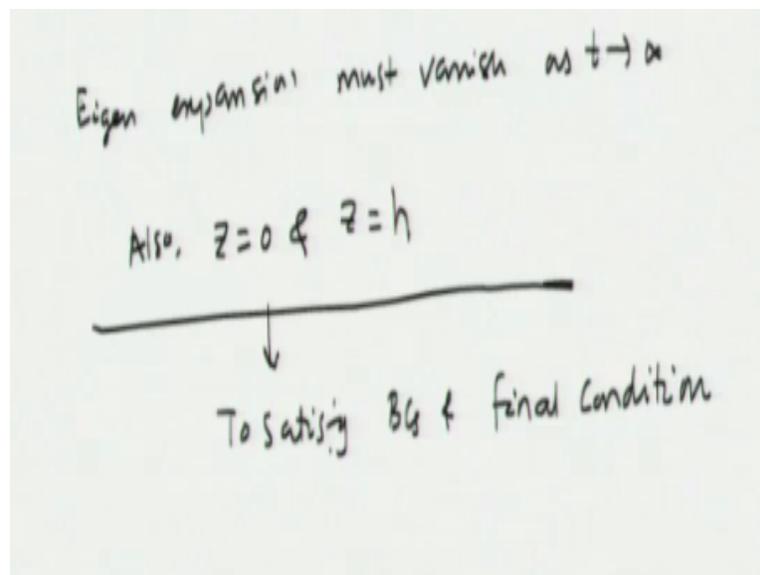
So you know, so again function Eigen function, Eigen expansion must vanish as t tends to infinite and also when you are talking about z=0, okay, when you are talking about z=0 this function should be 0, okay, and when we are talking about z=h, okay, this function is going to be

the time T=0 is going to be 0 and time T > 0 is going to be u0. Okay. So this expansion is going to vanish also for time T= 0 so z will be 0 so this term will be 0, okay.

And the for z=h this term is going to be u0, okay. So the expansion term must also vanish for z=0 and z=h so then only the boundary conditions and the final conditions will be valid. To satisfy boundary conditions and final conditions that means steady state conditions, okay. So now if we substitute this-- the Eigen expansion in this equation here what we get is this.

(Refer Slide Time: 24:12)

We get an expression Tn(t)*del square Un(z) over del z square will be =, so here you would have nu, the eta over Rho so that will be 1 over nu, kinematic viscosity, 1 over nu*Un(z)*del Tn (t) over del t, okay. S we can also write this as, let us separate the variables divided by, sorry, del square U Un(z) over del z square/ Un(z), okay, will be = 1 over nu*del Tn over del T/Tn(t). Okay.

So here if we look at this expression the left hand side is a function which is depended on z, the right hand side is a function of t, and these 2 expressions can be equal only if these 2 are = constant, okay. So we can, they will be equal only if they are equal to constant and we call this constant as lambda n, okay.

(Refer Slide Time: 25:52)

$$\frac{1}{U_n(z)} \frac{\partial^2 U_n(z)}{\partial z^2} = -\lambda_n = \frac{1}{\nu} \frac{\partial T_n(t)}{\partial t} \frac{1}{T_n(t)}$$

Solutions:

$$U_n(z) \sim \sin(\sqrt{\lambda_n}\, z)$$
$$T_n(t) \sim e^{-\lambda_n t}$$

So Lambda n – lambda n is a constant, okay. So in that case we would have 1 over Un(z)*del square Un(z) over del z square = -lambda n and it will also be = 1 over nu del Tn(t) over del t* 1 over Tn(t), okay. So, you know, this can be solved, right, this can be solved easily we you can find the solution, and the solution can be Un(z) is going to look like the sin of square root of lambda n*z and the Tn(t) is going to look like e to the power –lambda n*t.

Because this is, you know, first order and this is second order so this is how then equation, the solution are for Un(z) and Tn(t) will look like.

(Refer Slide Time: 27:24)

Solutions:

$$U_n(z) \sim \sin(\sqrt{\lambda_n}\, z)$$
$$T_n(t) \sim e^{-\lambda_n t} \checkmark$$

$$\lambda_n ? \rightarrow T_n(\infty) = 0 \rightarrow \lambda_n \rightarrow \text{must be} +ve$$

$$\boxed{\lambda_n > 0}$$

$$U_n(0) = U_n(h) = 0 \quad (\text{No-slip})$$

Now if you want to determine lambda n, how do we determine lambda n? We have to use the conditions Tn infinite is going to be 0. So when time tends to infinite this Tn term is going to vanish. So if that is going to vanish from here you can see that lambda n has to be positive, okay, lambda n must be positive, okay. So this is something we are talking that about steady stage.

So in that case if Tn infinite will be 0 as T tends to infinity this is only possible when lambda n is positive. So lambda line must be positive. So lambda n is > 0, right. Now we have additional conditions. We have Un(0) = Un(h) which is 0, okay and this is No-Slip condition, right, even h, right. So from there if you look at this equation here, what do we see here?

(Refer Slide Time: 28:53)

$$\lambda_n \,? \;\rightarrow\; T_n(\infty) = 0 \;\rightarrow\; \lambda_n \rightarrow \text{ must be } +ve$$

$$\boxed{\lambda_n > 0}$$

$$u_n(0) = u_n(h) = 0 \quad (\text{No-slip}) \rightarrow$$

$$\boxed{\sqrt{\lambda_n} = \left(\frac{n\pi}{h}\right)} \qquad n = 1, 2, 3 \cdots n.$$

We see that lambda n, square root of lambda line must be n pi over h, okay. So only if square root of lambda n will be N pi over h, this term will go to 0 satisfying this condition here, okay. So, you know n here is 1 2 3, okay up to n.

(Refer Slide Time: 29:26)

$$\boxed{\sqrt{\lambda_n} = \left(\frac{n\pi}{h}\right)} \qquad n = 1, 2, 3 \cdots n$$

eigen expansion const C_n ?

$$u(z, 0) = \underline{0} \Rightarrow T_n(0) = 1$$

$$\sum_{n=1}^{\infty} C_n \sin\left(n\pi \frac{z}{h}\right) = u(z, \infty) = \left(u_0 \frac{z}{h}\right)$$

$$\Rightarrow C_n =$$

Now to determine, now we have determined lambda n, now let us try to determine what is this expansion constant is going to look like? Okay. So the Eigen expansion constant, Cn to be determined and to do that you use u(z,0) is going to be 0, right, when time=0, this is 0 so Tn(0) is going to be 1. Okay. So Tn (0) will be 1, right. So Tn(0) is this expression, right, here Tn(0) will be 1. Why Tn(0) will be 1?

Because when T=0 this term is going to be 1, right. So Tn(0) will be 1 when T=0, so what we get from here we get n=1 to infinite, Cn sin n pi z over h, okay, this term here, Cn* even z, this is becoming 1, Tn(t) is becoming 1, so you are talking about this term n=1 to infinity. This term is going to be our, nothing but U(z, infinity), okay. Because this is 0, so if this is 0, this you can take it to the other side of the equal sign. So this is U (z, infinity) or U0* z over h, okay. This is what you will get. Now from here you can get Cn and substitute in the velocity field equation.

(Refer Slide Time: 31:44)

$$u(z,t) = u_o \left[\frac{z}{h} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (-1)^{n+1} \sin\left(\frac{n\pi z}{h}\right) \times \right.$$

$$\left. \exp\left(-n^2 \pi^2 \frac{\nu}{h^2} t\right) \right]$$

$$n = 1$$

$$u(z,t) = u_o \left[\frac{z}{h} - \frac{2}{\pi} \sin\left(\frac{\pi z}{h}\right) \exp\left(-\pi^2 \frac{\nu}{h^2} t\right) \right]$$

And if you do that you can get Cn here and if you substitute in the velocity equation you get z, u (z,t) = u0* z over h–2/ pi, summation n=1 to infinity, 1 over n-1 to the power n+1*sin n pi z over h* there is an exponential term which will be multiplied here, this is exponential –n square pi²*Mu over h²*t. Okay. So this is going to be the expression for the velocity field when we are talking about transient state, okay.

And as you see here this is an exponential term which decays quickly as n increases, okay. And the best value of N for which this will have some quantity is n=1 because we said that n varies from 1 to n here, okay. So we can say that for n=1, the value can be the best value, okay and as n increases it is going to decay very fast. That is what we see from this equation.

So if we put =1, then you can write u (z,t)=u0* z over H – 2/pi*sin pi z over h* exponential –pi square*Mu over H²* t, okay. This is the expression when n=1, for N>1 this value again will be very negligible, okay. So we just evaluate for n=1. So here if you look at this term we can define.
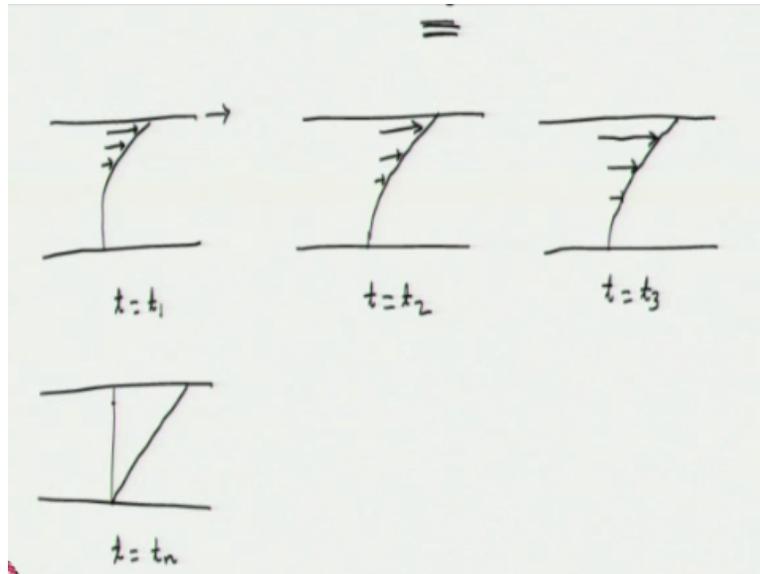**(Refer Slide Time: 34:41)**

$$\tau = \frac{h^2}{\pi^2 \nu}$$

As $t \uparrow \Rightarrow t = N\tau \rightarrow$ Steady state

$t = t_1$ $\qquad$ $t = t_2$ $\qquad$ $t = t_3$
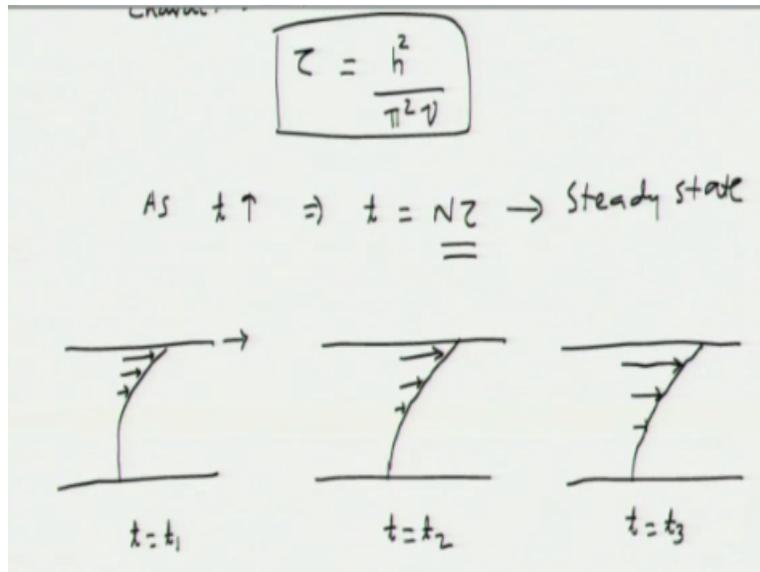
A characteristic time scale, let us call it Tau that will be h square/pi square Mu, okay. So as T increases and it becomes, T is you know let us say some n times Tau then we get steady state, okay. So what it means is that as the characteristic time constant is small we obtain the steady state very quickly. So if you are talking about a quad flow situation, where the height between the 2 place is small or the kinematic viscosity is very high, the steady state will be attained very quickly, okay.

So if you look at the profile, how it going to vary with time, when you draw various time steps, so let us say at time=t1, it is going to start something like this, okay, so this plate is moving right. So it will move something like this and time=t2, this is going to bend little further and this is time t2 and this is time=t3, right.

(Refer Slide Time: 36:41)

And when we have T=Tn, large time stamp which is depending on the characteristic constant we would have a flat velocity profile, okay.

(Refer Slide Time: 37:08)
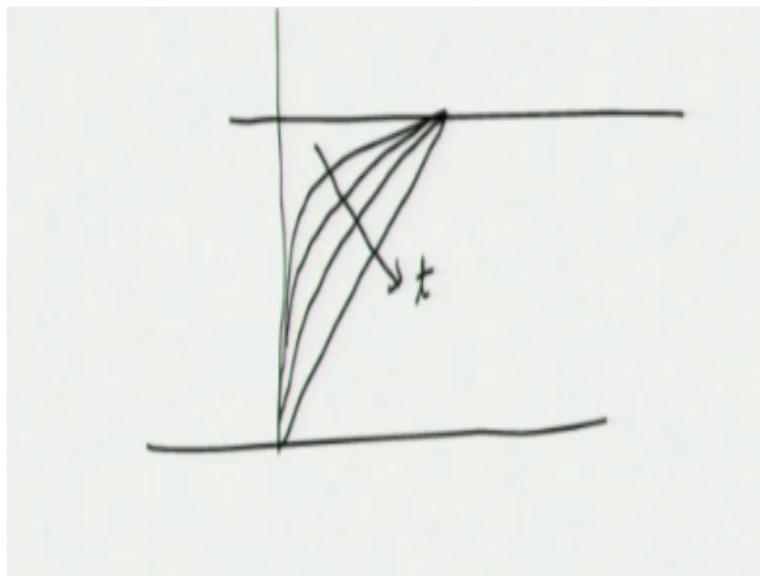
So if you superimpose all these on a single plot, this is how it is going to look like. What will happen is initially it is going to be something like this, then it will be like this and like this and the finally it is going to be linear, okay, so this is as time increases, okay. With that let us move on and talk about slip flow in case of micro channel flows, so we will consider quad flow situation where we will talk about the liquid, sorry the fluid to be gas, okay.

We know that in case of liquid flows, if the length scale is > 10 nanometer then we can apply the Navier-Stokes equation with No-Slip boundary condition, okay. But if the fluid is gas then if the notional number is < 0.001, then we are able to use Navier-Stokes equation with No-slip boundary condition and if the notional number is between 0.001 and 0.1 then we can still use Navier-Stokes boundary condition but with slip, okay.
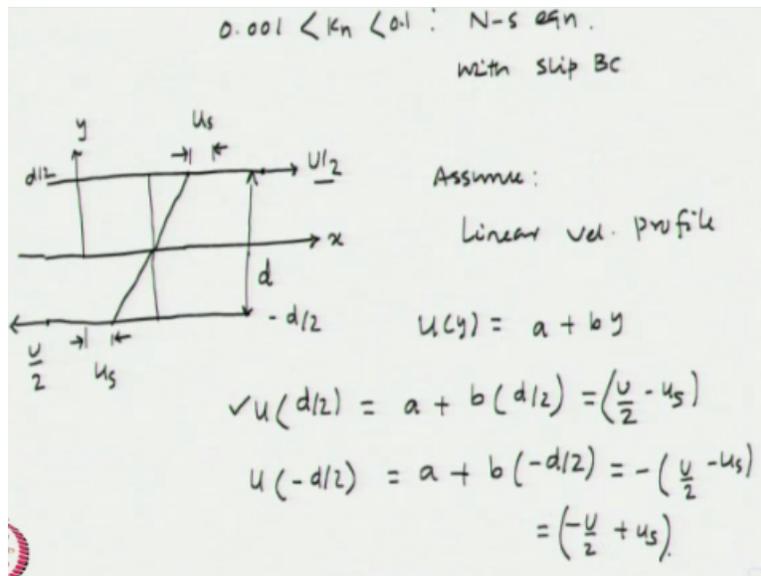
So we will discuss gas flows in a micro channel in case of quad flow if the fluid is gas and notional number in the range 0.001 to 0.1, how we can obtain an expression for the velocity profile.

(Refer Slide Time: 38:58)
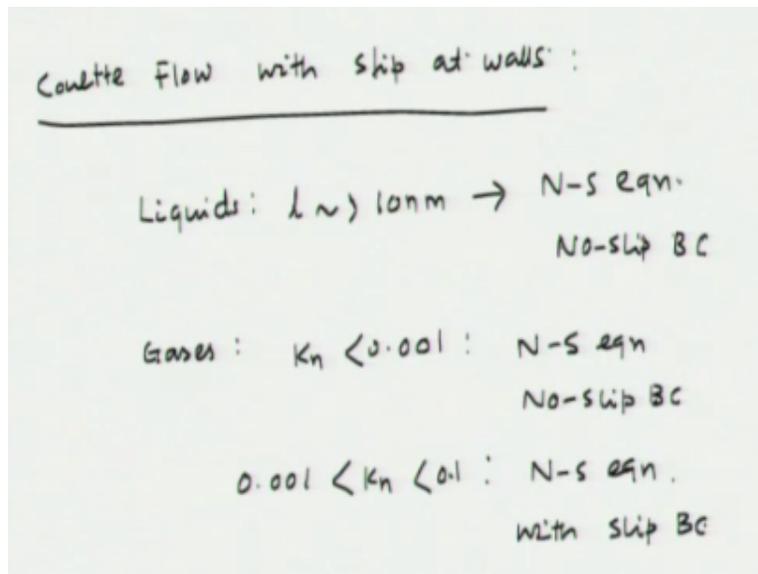
We will talk about quad flow with slip at walls, okay. Right, so we know that for liquids if the length scale is of the order or > 10 nanometer then we can use Navier-Stokes equation with No-Slip boundary condition, okay. Now for gases if notion number is < 0.001 we can use Navier-Stokes equation and No-Slip boundary condition. If notion number is between 0.001 to 0.1, we can use Navier-Stokes equation with slip boundary condition, okay.

(Refer Slide Time: 40:35)

At the top of the figure:

$$0.001 < K_n < 0.1 : \text{N-S eqn.}$$
$$\text{with slip BC}$$

Assume:

Linear vel. profile

$$u(y) = a + by$$

$$\checkmark u(d/2) = a + b(d/2) = \left(\frac{U}{2} - u_s\right)$$

$$u(-d/2) = a + b(-d/2) = -\left(\frac{U}{2} - u_s\right)$$
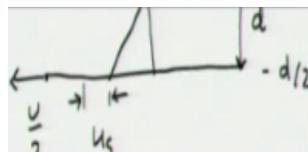$$= \left(-\frac{U}{2} + u_s\right)$$

So we consider here 2 parallel plates or quad flow, so this is one plate, this is another plate and we say this is the middle line between the plates and this is let us call it Y, this is X, this is moving at velocity U/2 in this direction and this plate is moving at velocity u/2 in this direction. So the relative velocity is U and so, D is the separation distance. The separation distance between the plate is D, so this is at D/2 and this is at (-)D/2, okay.

So we assume that the velocity profile to be linear, linear velocity profile which you know from the liquid flows. So if you do that you can write UI will be A + BY, okay. So U at D/2 is going to be A + B X D/2, okay, that is the upper wall we are talking about and at the lower wall, U-D/2 = A+B X (-) D/2, okay. And let us say this is the profile nature and here we say there is a slip velocity of US and here also there is a slip which is US.

So the plate is moving at U/2 but the fluid is moving at a lower velocity as compared to the plate because there is a slip and so it is moving at U/2-US, right. So this is U/2 – US and for the bottom plate the fluid is moving at U/2 in the negative X direction and the fluid is moving at a lesser velocity US, so this is going to be minus because it is going in negative X direction U/2 – US, okay so this is (-) U/2 + US, okay.

(Refer Slide Time: 43:48)

$$u(y) = a + by$$

$$\checkmark \quad u(d/2) = a + b(d/2) = \left(\frac{U}{2} - u_s\right) \quad —①$$

$$u(-d/2) = a + b(-d/2) = -\left(\frac{U}{2} - u_s\right)$$

$$= \left(-\frac{U}{2} + u_s\right) \quad —②$$

$$\boxed{a = 0} \qquad \boxed{b = \left(\frac{U - 2u_s}{d}\right) y}$$

So now if you look at equation 1 and equation 2, if you solve these 2 equations we can see that A=0. We can clearly see that the constant A is going to be 0 and B, so A and B are 2 unknowns in these 2 equations. So B can be determined as U-2US/B x Y, okay. This is going to be B.

(Refer Slide Time: 44:24)

Maxwell's slip model:

$$\dot{u}_s = \left(\frac{2-\sigma}{\sigma}\right) \lambda \left.\frac{\partial u}{\partial y}\right|_{y = \pm d/2}$$

$$= \left(\frac{2-\sigma}{\sigma}\right) \lambda \left(\frac{U - 2\dot{u}_s}{d}\right) \rightarrow$$

$$\boxed{u_s = \left(\frac{2-\sigma}{\sigma}\right)\left(\frac{\lambda}{d}\right)\left[\frac{U}{1 + \left(\frac{2-\sigma}{\sigma}\right)\frac{2\lambda}{d}}\right]}$$

So now we can use the Maxwell's slip model. So using Maxwell's slip model we know the expression for the slip velocity at the wall which is a function of the momentum accommodation coefficient, lambda and the velocity gradient, okay. So you can write that US is equal to 2-sigma divided by sigma, sigma is the momentum accommodation coefficient*Lambda*DelU / DelY at Y= ± T/2, okay.

So you can write this as 2 - sigma / sigma x lambda. Now from this equation, you know A and B, so if you substitute in that equation you have an expression for the velocity profile which is basically B x Y, right. B x Y because A is 0. So you know, sorry B is going to be this, so this is velocity is BY, so U is going to be U-2US / D x Y, okay. So you can append the velocity gradient which is going to be U-2US/D, okay.

So US is 2 – sigma / sigma*lambda over d *, so from this equation you can write US, so here is US and US. So you can write US to be U, whereby 1+ 2 (-) sigma / sigma x 2 lambda / D. So that is the expression for the slip velocity, okay.

(Refer Slide Time: 47:01)

Now you can determine, no, here you can check as lambda by T to 0, your slip velocity US is tending to 0 and which is the case when notion number is very, very small, okay. So the slip velocity is not existing. Now from the expression for velocity, we can obtain the expression for CS stress. So CS stress can be written as Tau = Mu delU / delY at Y = ± D/2.

So you can write this as Mu x U – 2US / D which is the velocity gradient and so that can be written as Mu x U / 1+ 2 - sigma / sigma x 2 lambda / D, okay. So now knowing the CS stress we can calculate the skin friction coefficient. The skin friction coefficient which is CF can be calculated.

(Refer Slide Time: 48:46)

$$\rightarrow C_f = \frac{\tau}{(\rho U^2/2)} = \left(\frac{\mu}{\rho U_d}\right) \frac{2}{\left[1 + \left(\frac{2-\sigma}{\sigma}\right)\frac{2\lambda}{d}\right]}$$

$$\Rightarrow \boxed{C_f M_a = \frac{M_a}{R_e}\left[\frac{2}{1 + \left(\frac{2-\sigma}{\sigma}\right)2 K_n}\right]}$$

$$\text{If } K_n \rightarrow 0 \quad : \quad \boxed{C_f = \frac{2}{R_e}}$$

So you can write CF, CF = Tau, we have an expression for the CS stress Tau, so Tau / row U² / 2, okay. So you can write this as Mu / row UD x 2 / 1 + 2 - sigma / sigma x 2 lambda / D, okay. Just substitute for U there, okay. From there you can write CF x Mac number is going to be Mac number / Re which is Reynolds number x 2 / 1 + 2 – Mu / Mu x 2 x notion number, okay. So this expression can be used to predict or to calculate momentum coefficient, okay.

So if you do an experiment where you can determine the Mac number, Reynolds number, notion number, other parameters, skin friction coefficient, you can use this formula to predict the value of the momentum accommodation coefficient sigma, okay. So here if you look at this expression here, if notion number goes to 0, okay, so lambda / D, notion number goes to 0 the what we see here, we see that the skin friction coefficient is going to be 2 / Reynolds number.

And this is valid for liquid flows in channels where the skin friction coefficient is 2/Reynolds number, okay.

(Refer Slide Time: 51:07)

$$K_n \to \infty \quad (\text{free mol. regime})$$

$$\frac{M_a}{R_e} = \sqrt{\frac{2}{\pi K} K_n}$$

$$C_f = \sqrt{\frac{2}{\pi K}} \left[\frac{1}{\left(\frac{2-\sigma}{\sigma}\right)} \right]$$

Now if notion number 10 is to infinity, then we are talking about free molecular regime, right. Free molecular regime and in that case if you put notion number infinity here, okay, you have to rearrange the term so that you can consider the regime notion number 10 to infinity, if you do that you will get CF will be equal to and you have to use Mac number over Reynolds number is equal to 2/pi K square root*notion number that we have seen, right, Mac number, Reynolds number and notion number can be related.

So here K is specific hit ratio. In that case you can show that CF is going to be 2/pi K square root *1/2-sigma over sigma, okay. So this is the limit when notion number is very high. Notion number is infinite. So let us stop here.