Microfluidics
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Lecture – 19
Electrokinetics (Continued...)

Okay, so let us continue our discussion on in a dielectrophoresis for vertical trapping okay, for the dielectrophoretic force to be effective we said that we need to have a non-uniform electric field okay and we considered a rectangular channel configuration, where the gap between the you know; plates is much smaller than the width of the plate, so you can say that this is equivalent to a flow between 2 infinitely long parallel plates.

And we say that we had you know 2 asymmetric electrode configuration; 1 hemispherical electrode at the bottom of the channel and 1 flat electrode on the top of the channel, so we would be creating an non uniform electric field okay and the other requirement for dielectrophoresis is that the dielectric constant of the sphere has to be greater than that of the liquid, okay.

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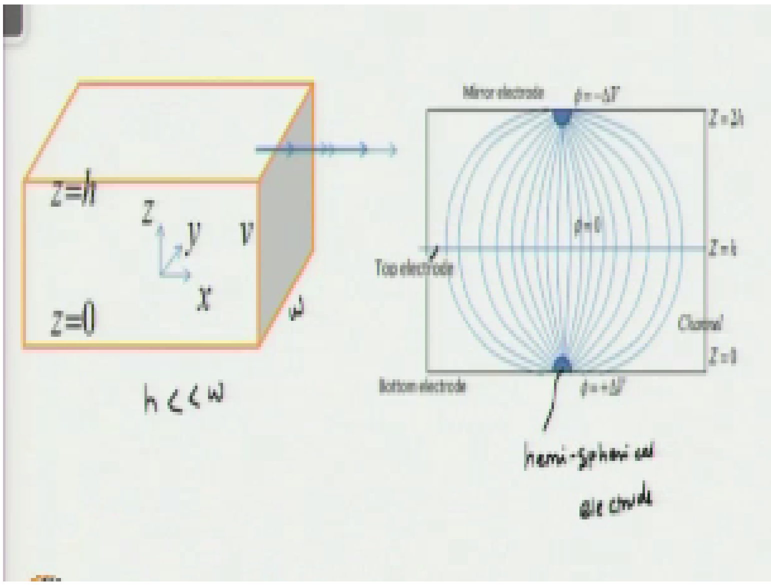

So, this is the case that we have been considering; we are considering a rectangular channel with height h and width w, okay and we said that the height is << w and as you can see on the right, this is the hemispherical electrode at the bottom of the channel okay. The hemispherical electrode and the top electrode; this is the top electrode which is flat okay.

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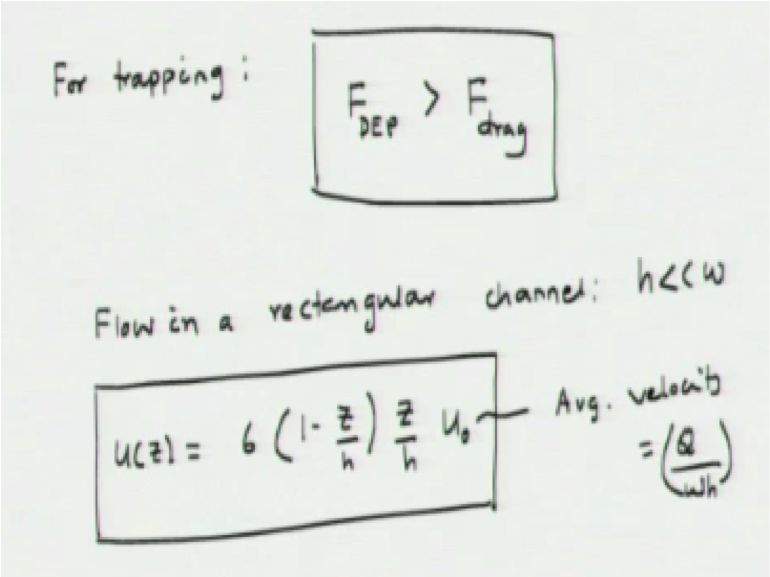

For trapping:

$$\boxed{F_{DEP} > F_{drag}}$$

Flow in a rectangular channel: h<<w

$$\boxed{u(z) = 6\left(1-\frac{z}{h}\right)\frac{z}{h}u_0}$$

Avg. velocity $= \left(\frac{Q}{wh}\right)$

So, we were saying that for the trapping for trapping; so there are basically 2 forces that are acting on the particle as we discussed; one is you know the dielectrophoretic force, which is acting towards the electrode and the other force is acting is the drag force, which is trying to carry the particle with the bulk fluid. So, you know for the trapping that the dielectrophoretic force F DEP has to be > the drag force okay.

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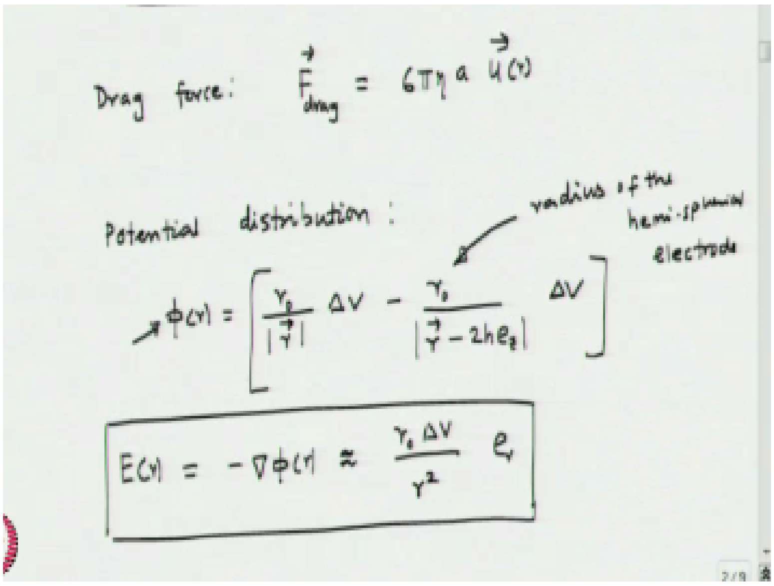

Drag force: $\vec{F}_{drag} = 6\pi\eta a \vec{u}(r)$

Potential distribution:

radius of the hemi-spherical electrode

$$\phi(r) = \left[\frac{r_0}{|\vec{r}|}\Delta V - \frac{r_0}{|\vec{r} - 2he_z|}\Delta V\right]$$

$$\boxed{E(r) = -\nabla\phi(r) \approx \frac{r_0\Delta V}{r^2}e_r}$$

So, we consider you know flow between in a rectangular channel; so we consider flow in a rectangular channel h < w and we say that you can write the velocity profile uz as 6 * 1 - z over h * z over h u0; u0 is the average velocity, which is flow rate divided by the cross sectional area, so u0 is the average velocity, which is Q over wh okay. So, the drag force you can write; the drag force F drag is 6pi ETA a * the velocity.

Let us say, this ur at any location r, okay this is also a vector; okay, these are all vectors. Now, we need to; if you look at here before we calculate the dielectrophoretic force, we need to calculate the electric field and to calculate the electric field; we need to know how the potential distribution is. To calculate the potential distribution in this case, since you have 1 hemispherical electrode, 1 flat electrode, we need to guess for a mirror electrode about the top plane okay.

So, we assume that there is another mirror electrode, which is sitting at the top okay. So, this is the symmetry plane for the mirror. So, if we do that, we can write down the potential distribution, so the phi r will be r0/ the position vector r * delta v – r0/ r - 2h, so that is along the z direction * delta v, so that is the expression for the potential, where r0 is the radius of the electrode; radius of the hemispherical electrode.

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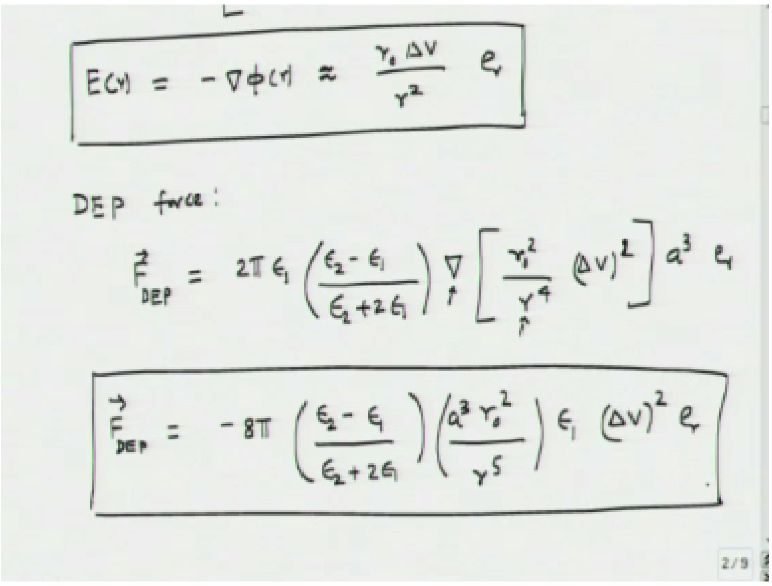

$$\boxed{E(r) = -\nabla\phi(r) \approx \frac{r_0 \Delta V}{r^2} e_r}$$

DEP force:

$$\vec{F}_{DEP} = 2\pi\epsilon_1 \left(\frac{\epsilon_2-\epsilon_1}{\epsilon_2+2\epsilon_1}\right) \nabla_{r} \left[\frac{r_0^2}{r^4} (\Delta V)^2\right] a^3 e_r$$

$$\boxed{\vec{F}_{DEP} = -8\pi \left(\frac{\epsilon_2-\epsilon_1}{\epsilon_2+2\epsilon_1}\right)\left(\frac{a^3 r_0^2}{r^5}\right) \epsilon_1 (\Delta V)^2 e_r}$$

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So, that is going to be the expression for the potential, so we can find out what is going to be the electric field, so Er is the gradient of phi r, which is; which can be approximated as r0 * delta v/ r square along the radial direction okay. So, that is the expression for the electric field if you differentiate potential with respect to r. Now, if you; we know the expression for the dielectrophoretic force.

So, the DEP force is F DEP = 2pi epsilon 1 * epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 * the gradient of the square of the electric field, so it will be r0 square/ r4 * delta v square, okay * delta v square. So, we can simplify this to write F DEP is = -8pi * epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 * a cube r0 square, here we are missing a cube here; a cube r0 square, r to the power 5 * epsilon 1 okay, * delta v square along radial direction; this is also along radial direction, okay.

So, since we are differentiating this with respect to r, we get a negative sign there and this becomes r to the power 5 okay. Now, we can see from this equation that the dielectrophoretic force varies as 1 over r to the power 5 okay, so the maximum DEP force would be realized when r is minimum and the minimum r is going to be the radius of the hemispherical electrode + 1/2 of the size of a particle okay.

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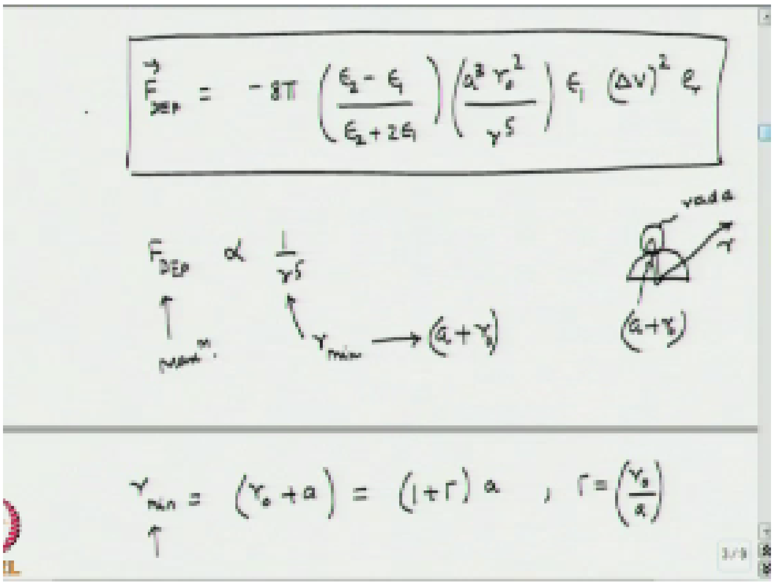

$$\boxed{\vec{F}_{DEP} = -8\pi\left(\frac{\epsilon_2-\epsilon_1}{\epsilon_2+2\epsilon_1}\right)\left(\frac{a^3 r_0^2}{r^5}\right)\epsilon_1(\Delta v)^2 e_r}$$

$$F_{DEP} \propto \frac{1}{r^5}$$

$$\underset{\text{max}^m}{\uparrow} \qquad r_{min} \rightarrow (a+r_0)$$

$$r_{min} = (r_0+a) = (1+\Gamma)a \quad , \quad \Gamma = \left(\frac{r_0}{a}\right)$$

So, that is going to be the minimum radial distance from the centre of the hemispherical electrode to the centre of the particular okay. So, what we see from here is that the dielectrophoretic force, where is as 1 over r to the power 5. Now, for the dielectrophoretic force to be maximum, r has to be minimum; so here, r minimum and the r minimum is; so we have a hemispherical electrode, okay.

Now, this is the centre of the hemispherical electrode and this is where we measure r, okay; r goes in this way and let us say we have a particle sitting here and which has you know radius a, so the minimum r is going to be this, okay and this is nothing but a + r. So, r minimum is going to be the radius of the particle + the radius of the hemispherical electrode, okay. So, you know; so the r minimum is r0, this is r0, not r, so this is r0 + a.

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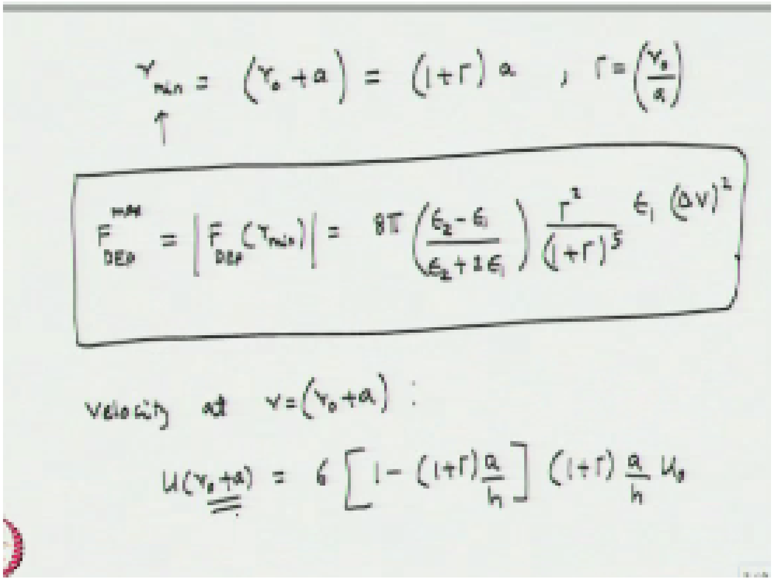

$$r_{min} = (r_0 + a) = (1+\gamma)\,a \quad , \quad \gamma = \left(\frac{r_0}{a}\right)$$

$\uparrow$

$$\boxed{F_{DEP}^{max} = \left|F_{DEP}(r_{min})\right| = 8\pi \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1}\right) \frac{\gamma^2}{(1+\gamma)^5}\, \epsilon_1 (\Delta V)^2}$$

velocity at $r = (r_0 + a)$:

$$\underline{\underline{u(r_0+a)}} = 6\left[1 - (1+\gamma)\frac{a}{h}\right](1+\gamma)\frac{a}{h}\,u_0$$

You can say that this is 1 + gamma * a, let us define gamma as r0 over a, so this is the ratio between the radius of the electrode to the radius of the particle. So, with that we can substitute this r minimum in the dielectrophoretic force equation here to find the maximum DEP force. So, the maximum DEP force; F DEP max can be F DEP r min, which is going to be 8pi epsilon 2 - epsilon 1/ epsilon 2 + 2epsilon 1 * gamma square/ 1 + gamma to the power 5 epsilon 1 * delta v square, so that is going to be the expression for the maximum DEP force.

So, we are actually taking the magnitude, so the negative sign is dropped off okay. Now, we are interested to see; you know what is the maximum velocity with which we can you know transport the sample, at the same time we are able to trap the particles from the sample, okay. So, you know the maximum DEP force is going to be when the r is going to be r0 + a okay, so we can find out what is going to be the drag force at that location, okay.

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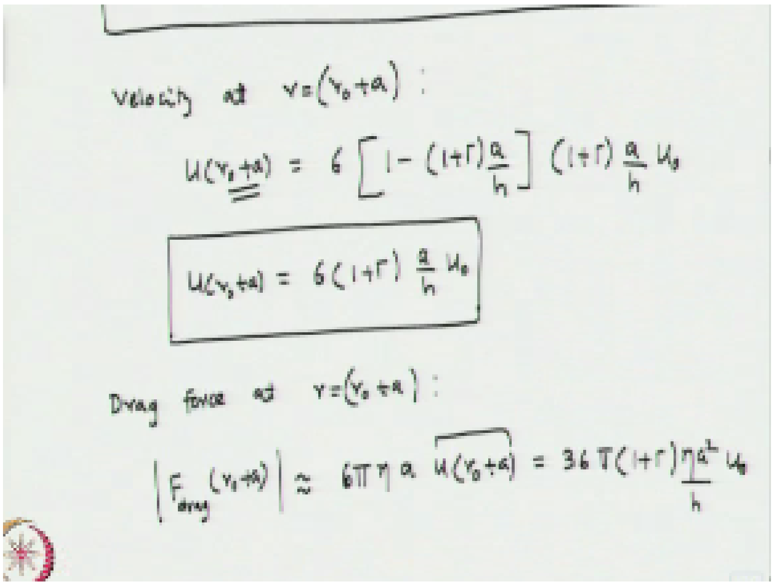

Velocity at $r = (r_0 + a)$:

$$\underline{\underline{u(r_0+a)}} = 6\left[1 - (1+\gamma)\frac{a}{h}\right](1+\gamma)\frac{a}{h}u_0$$

$$\boxed{u(r_0+a) = 6(1+\gamma)\frac{a}{h}u_0}$$

Drag force at $r = (r_0 + a)$:

$$\left|F_{drag}(r_0+a)\right| \approx 6\pi\eta a\,\overline{u(r_0+a)} = 36\pi(1+\gamma)\frac{\eta a^2}{h}u_0$$

So, first we find the velocity at r = r0 + a, so that is going to be so; u r0 + a is = 6 * 1 – 1 + gamma * a over h * 1+ gamma * a over h u0, okay, so that is the expression for the velocity if you substitute r as r0 + a, so this is what you would get. So, u r0 + a; you can simplify this to 6 * 1 + gamma * a over h * u0, okay right. So, from knowing the velocity, you can find, what is the drag at this location?

So, drag force at r = r0 + a is going to be F drag r0 + a, so finding the magnitude is going to be 6pi; so, 6pi ETA a and u r0 + a, so we have a radial expression for u r0 + a, so this is going to be 36 pi * 1 + gamma ETA a square over h * u0, okay. Now, for the trapping to be possible, the drag force has to be less than the dielectrophoretic force. So, in the limiting case let us say that F DEP = F trap, okay.

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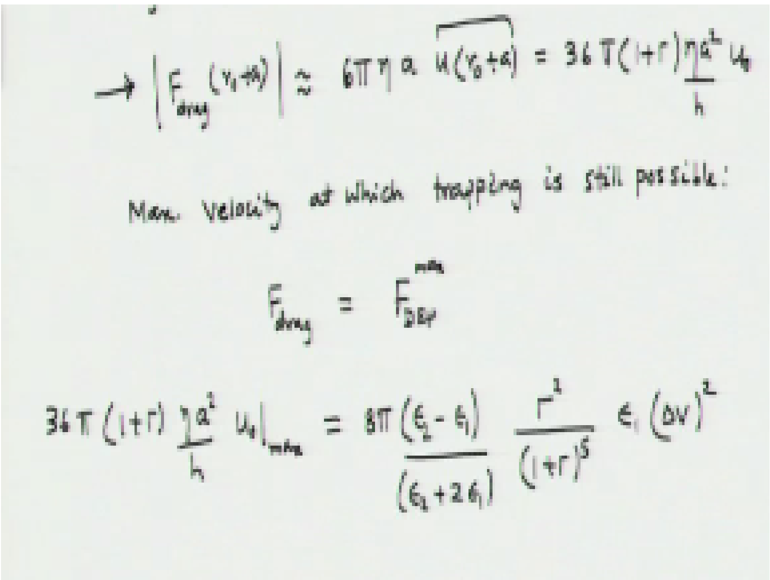

$$\rightarrow \left|F_{drag}(r_0+a)\right| \approx 6\pi\eta a\,\overline{u(r_0+a)} = 36\pi(1+\gamma)\frac{\eta a^2}{h}u_0$$

Max. velocity at which trapping is still possible:

$$F_{drag} = F_{DEP}^{max}$$

$$36\pi(1+\gamma)\frac{\eta a^2}{h}u_0\Big|_{max} = 8\pi\frac{(\epsilon_2-\epsilon_1)}{(\epsilon_2+2\epsilon_1)}\frac{\gamma^3}{(1+\gamma)^5}\epsilon_1(\Delta V)^2$$

So, you know the maximum velocity at which trapping is still possible, so F drag will be = F DEP max okay, so we can; we have an expression for the drag force where the DEP force is maximum and we have an expression for the maximum DEP force, so we can write the drag force is 36 * pi * 1 + gamma * ETA a square over h u0 that is maximum velocity that we are interest to find out is going to be 8pi epsilon 2 – epsilon 1/ epsilon 2 + 2epsilon 1 * gamma square/ 1 + gamma to the power 5 epsilon 1 delta v square.

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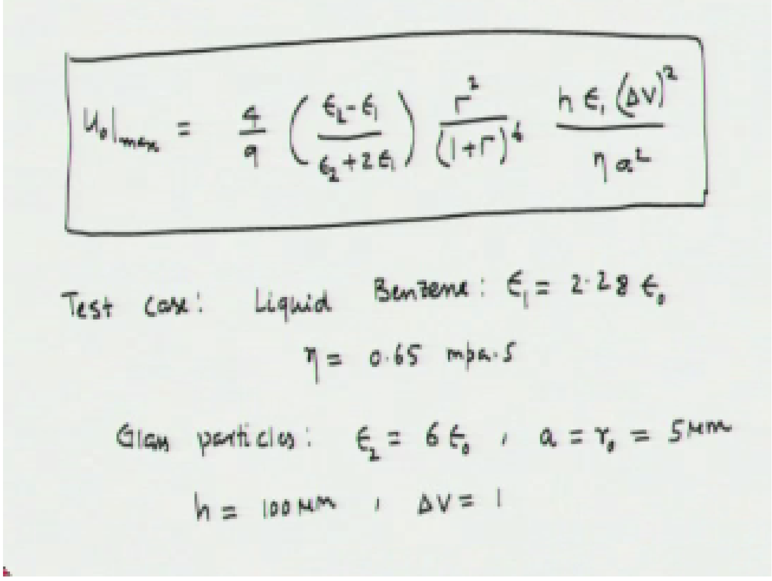

$$\boxed{u_0\big|_{max} = \frac{4}{9}\left(\frac{\epsilon_2-\epsilon_1}{\epsilon_2+2\epsilon_1}\right)\frac{\Gamma^2}{(1+\Gamma)^6}\,\frac{h\,\epsilon_1(\Delta V)^2}{\eta\, a^2}}$$

Test case: Liquid Benzene: $\epsilon_1 = 2.28\,\epsilon_0$

$\eta = 0.65$ mpa.s

Glass particles: $\epsilon_2 = 6\,\epsilon_0$, $a = r_0 = 5\mu m$

$h = 100\mu m$, $\Delta V = 1$

So, from here we can find out what is going to be the maximum velocity. So, u0 max is going to be 4/9 * epsilon 2 – epsilon 1 / epsilon 2 + 2epsilon 1 gamma square/ 1 + gamma 6 h epsilon 1 delta v square over ETA square, okay. So, that is going to be the maximum velocity of the particles or the samples at which trapping is still possible. Now, let us consider a test case; we consider a test case, where we have you know liquid benzene as the medium, okay.

And the properties are the dielectric constant epsilon 1 is the permittivity is 2.28 into permittivity of free space and the viscosity is 0.65 milli pascal second and let us say we have glass particles okay, so we have glass particles that we are trying to trap from liquid benzene, so glass particles whose permittivity is 6 times permittivity of free space and let us say the radius of the particle and the radius of the hemispherical electrode, they are the same is about 5 micron.

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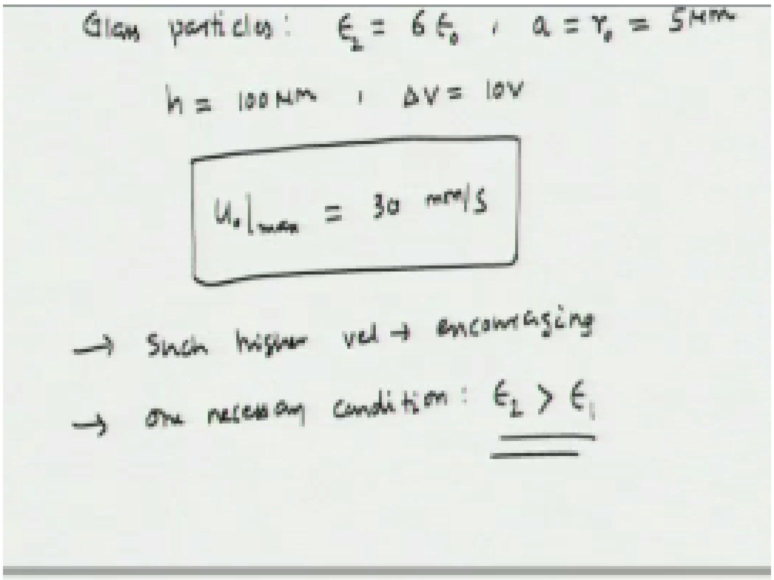

Glass particles: $\epsilon_2 = 6\epsilon_0$, $a = r_0 = 5 \mu m$

$h = 100 \mu m$, $\Delta V = 10 V$

$$\boxed{u_0\big|_{max} = 30 \ mm/s}$$

→ Such higher vel → encouraging

→ one necessary condition : $\underline{\underline{\epsilon_2 > \epsilon_1}}$

And let us say the channel height h is 100 micron and the applied voltage delta v is 10 volts, so under that condition, we can find out that the velocity u max is going to be about 30 millimetre per second okay. So, such a velocity is actually considerably high in micro channels, so which is very encouraging because the trapping is still possible at search higher velocities in a microfluidic environment okay.

So, you know but one necessary condition for you know; dielectrophoretic force is that always the dielectric constant of the particle has to be higher than that of the liquid okay, so such higher velocity is encouraging one necessary condition is that the dielectric constant of the permittivity of the medium has to be greater than permittivity of the liquid okay. So, now what we have done here is; we have taken an example of the electrode configuration.

But depending on the electrode configuration that we have for a specific problem, we can determine you know what is going to be the potential distribution, what is going to be the electric field and accordingly we can find out dielectrophoretic force and balance it with the drag force to calculate what could be the expression for the maximum velocity okay and from there for a given set of conditions, we will be able to find out what could be the flow rate at which we can drive the sample.

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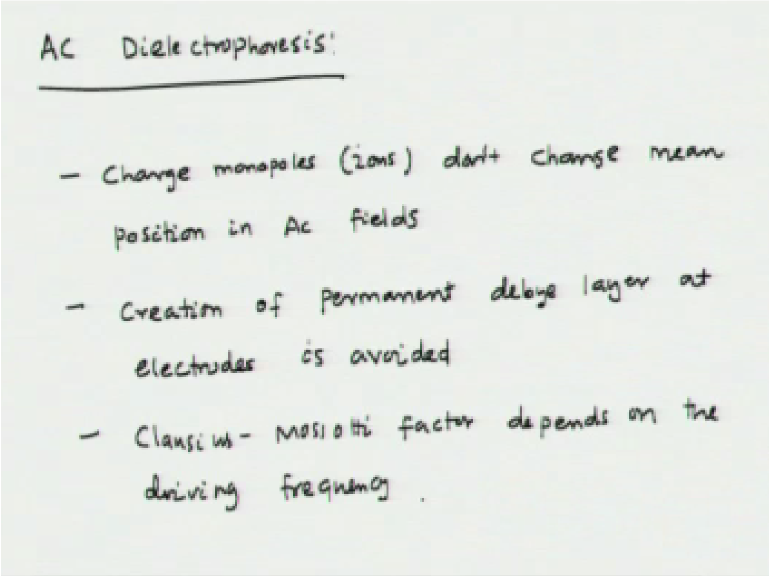

AC Dielectrophoresis:

- Charge monopoles (ions) don't change mean position in AC fields
- Creation of permanent debye layer at electrodes is avoided
- Clausius - Mosiotti factor depends on the driving frequency.

But still in achieving trapping of the particles present in the sample okay, so with that let us move on to talk about AC dielectrophoresis. So, what we have been looking at in the electrophoresis is DC dielectrophoresis okay however, the AC dielectrophoresis have certain advantages okay. The first advantage is that you know the charge monopoles for the ions will not in a move with respect to their original position in case of a AC dielectrophoresis.

Because the polarity of the electrode will change, so any drift of an ion in the first cycle will be compensated by the second cycle, so the ions in the liquid will not move relative to its mean position okay. So, the charge monolpoes or ions do not change mean position in AC fields okay. Now, the second advantage is that we talk when we talk about the electrodes that we use to achieve dielectrophoresis.

You know; we know that when you have a charged surface and we have a liquid that has non zero electrical conductivity will result in forming what is called the Debye layer, okay. So, in case of DC electrophoresis; dielectrophoresis, we have permanent Debye layer forming over the electrodes okay, so which is not a good sign for the dielectrophoretic force to continuously occur for a longer time okay.

And by going for AC dielectrophoresis, the permanent Debye layer formation and the electrodes is prevented okay because there is always a change in the polarity in each cycle, so there is no; not enough time for the Debye layer to be consistently there for a longer time, okay. So, what we have is the creation of permanent Debye layer at electrodes is avoided, okay.

The third advantage of the AC dielectrophoresis is that the Clausius mossotti in DC dielectrophoresis is depends on the dielectric constant of the liquid and the particle. In AC dielectrophoresis, the Clausius mossotti factor is also dependent on the driving frequency of the signal, so what did; this enables is that by manipulating the frequency of the signal, we would be; you can achieve in situ trapping or release okay.

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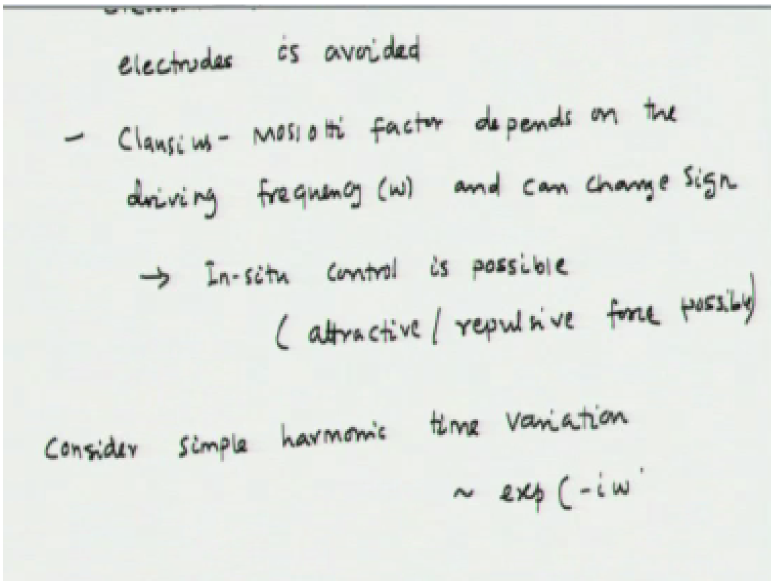

electrodes is avoided

- Clausius- Mossotti factor depends on the driving frequency (ω) and can change sign

→ In-situ control is possible
(attractive / repulsive force possible)

Consider simple harmonic time variation
$\sim \exp(-i\omega$

So, by controlling the driving frequency for a fixed set of particle and liquids, we can vary the sign of the dielectric force okay. So, selectively we can trap, we can release particle in situ okay, so that is possible in AC dielectrophoresis. So, in AC dielectrophoresis, the Clausius mossotti factor vector depends on the driving frequency okay. So, what this means is; so this is let us say driving frequency omega.

And the Clausius mossotti factor can change sign okay and hence the dielectrophoretic force can also change sign, so this enables in situ control is possible, so selectively attractive or repulsive force is possible; attractive or repulsive force possible, okay. Now, if you consider a simple harmonic variation; if you consider simple harmonic time variation, so something like you know exponential – i omega t, okay.

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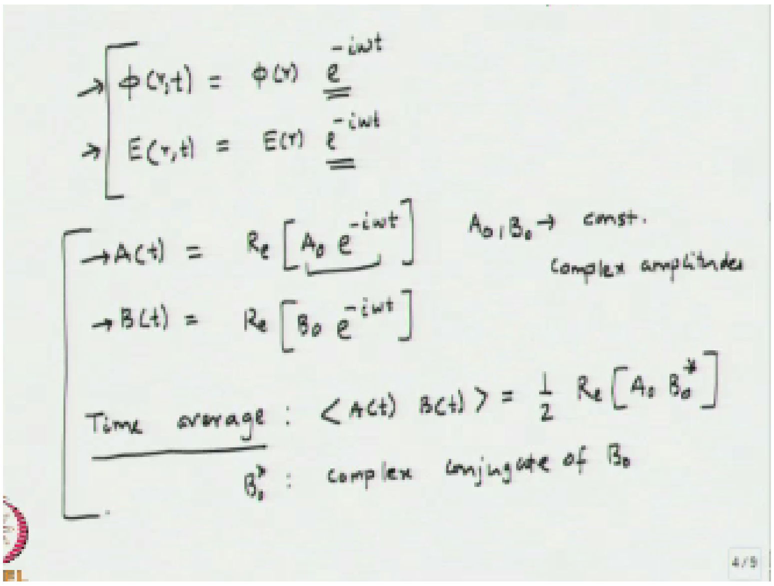

$\rightarrow \phi(r,t) = \phi(r)\, \underline{\underline{e^{-i\omega t}}}$

$\rightarrow E(r,t) = E(r)\, \underline{\underline{e^{-i\omega t}}}$

$\rightarrow A(t) = R_e\left[\underbrace{A_0\, e^{-i\omega t}}\right]$ $A_0, B_0 \rightarrow$ const. complex amplitudes

$\rightarrow B(t) = R_e\left[B_0\, e^{-i\omega t}\right]$

Time average : $\langle A(t)\, B(t) \rangle = \frac{1}{2} R_e\left[A_0 B_0^*\right]$

B_0^* : complex conjugate of B_0

You can express the potential phi rt as phi r * e to the power – i omega t and we can write the electric field as e to the power -i omega t, so these are; you know we are considering that the potential and the electric field following the simple harmonic time variation okay, as you can see there. Now, when we deal with simple harmonic time variation, we can have At; for example will be the real part of A0 e to the power –i omega t okay.

And we can define Bt as the real part of; so these are equivalent to or expression for potential and electric field, we are giving an example where; so this is the time dependent variation A, this is B; the real part of B0 e to the power -i omega t, where A0 and the B0 are called the constant complex amplitudes, okay. So, we can have time average of At and Bt, the time average of At and Bt is going to be 1/2 of the real part of A0 * B0 star, okay, where B0 star is the complex conjugate of B0.

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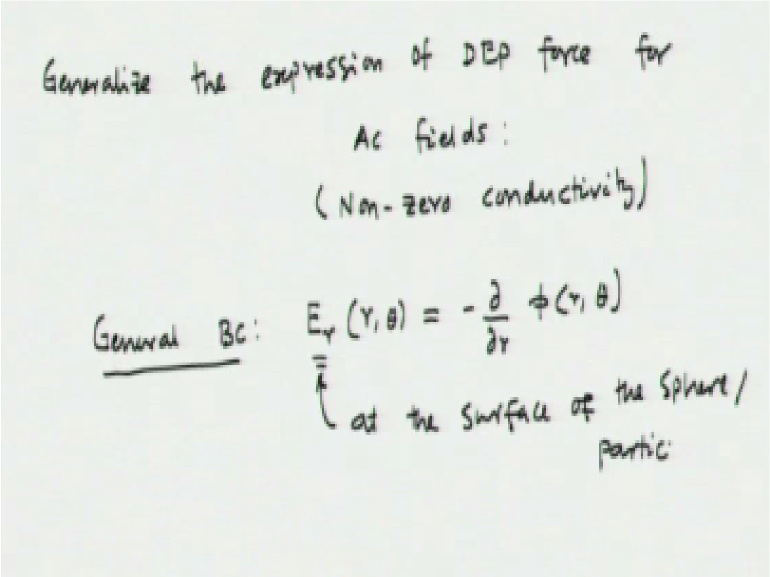

Generalize the expression of DEP force for

AC fields:

(Non-zero conductivity)

General BC: $E_r(r,\theta) = -\frac{\partial}{\partial r}\phi(r,\theta)$

at the surface of the sphere / partic

So, in AC dielectrophoresis since we will be talking about harmonic variation of the signal, we would be requiring these details while we talk about time average of the dielectrophoretic force okay. So, you require how we can do the time averaging of the dielectrophoretic force from these basics that we look at here, okay. Now, we can generalize the expression for DEP force; expression of DEP force for AC fields.

And here we say that the liquid has non zero electrical conductivity okay, so for that we would require the general boundary condition, which is nothing but that the electric field in the radial direction Er, r theta okay at a location r and along theta is - del over del r * phi r theta, so this is the simple definition of the electric field with respect to the potential, so which acts as the boundary condition in the radial direction okay.

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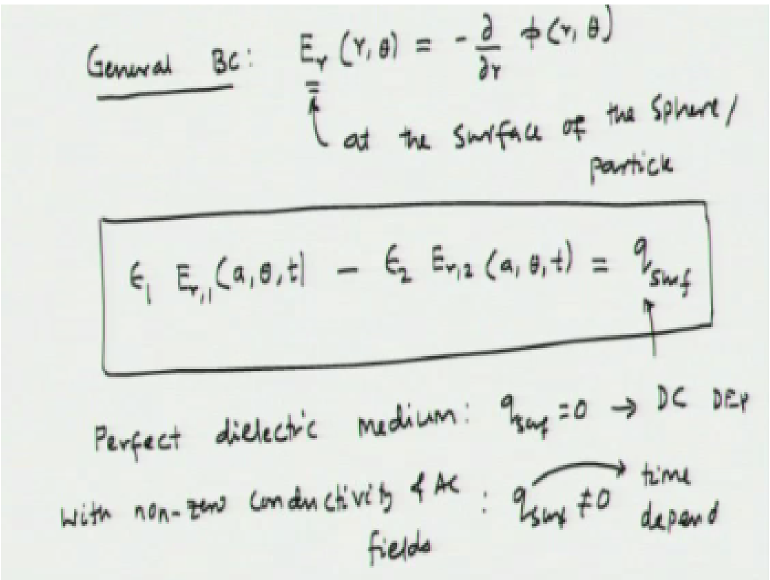

General BC: $E_r(r,\theta) = -\frac{\partial}{\partial r}\phi(r,\theta)$

at the surface of the sphere / particle

$$\boxed{\epsilon_1 E_{r,1}(a,\theta,t) - \epsilon_2 E_{r,2}(a,\theta,t) = q_{surf}}$$

Perfect dielectric medium: $q_{surf} = 0 \rightarrow$ DC DEP

With non-zero conductivity & AC fields: $q_{surf} \neq 0$ → time depend

And this boundary condition is at the surface of the sphere of the particle; surface of a sphere or particle, okay so we can write; you know if you remember we had written the boundary condition for DC case; epsilon 1 Er1 a theta, t, so in DC case, we did not have t, we had a and theta, which will be = epsilon 2 Er okay, so this will be - a theta t is = the charge density, okay. For DC case, we did not have the charge density.

So, epsilon 1 Er1 a theta was epsilon 2 Er2 a theta okay that was the boundary condition that we used in DC case. Now, here we have AC so time is coming into play and here also we are assuming that the liquid has non zero conductivity, so there is some charge density on the surface okay, which is taken into here. So, you know; if you have perfect dielectric; perfect dielectric medium, then the q surface is 0 and this was the case we considered in DC okay, DC dielectrophoresis.

And with non-zero conductivity; with non-zero conductivity and AC fields, the q surface is not = 0 and this is going to be time dependent, so the surface charge is going to vary with time; it is time dependent okay. So, this is the boundary condition we are going to use in AC dielectrophoresis, let us call this equation A, okay. Now, this time dependent charge q surface can be given by the charge conservation okay, by the Ohm's law.

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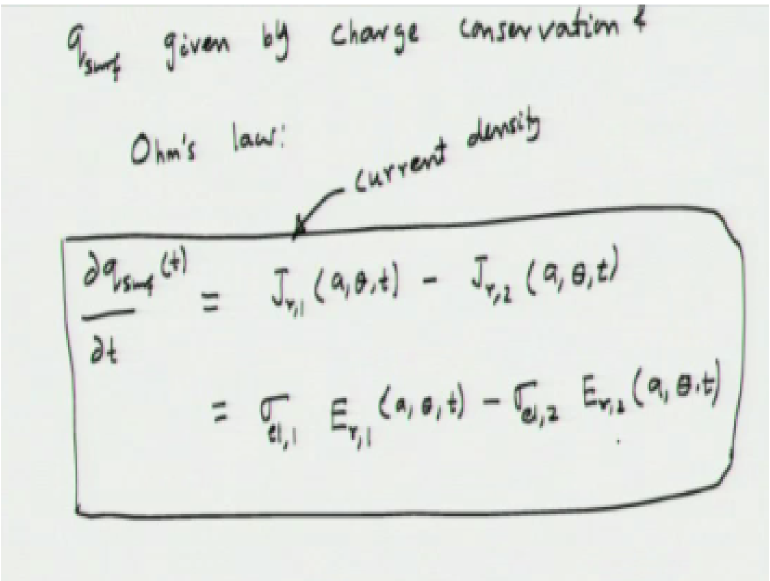

$q_{v,surf}$ given by charge conservation & Ohm's law:

current density

$$\boxed{\begin{aligned}\frac{\partial q_{v,surf}(t)}{\partial t} &= J_{r,1}(a,\theta,t) - J_{r,2}(a,\theta,t)\\ &= \sigma_{el,1}\, E_{r,1}(a,\theta,t) - \sigma_{el,2}\, E_{r,2}(a,\theta,t)\end{aligned}}$$

So, the q surface, which is the charge on the surface is given by charge conservation and Ohm's law okay, which is given by the del q surface, which is a function of time over del t is going to be jr1 a theta t – Jr2 a theta, so this J is nothing but current density, right. We have looked at this in electro hydrodynamics, this is nothing but current density okay and we can write current density in the form of electrical conductivity and electric fields.

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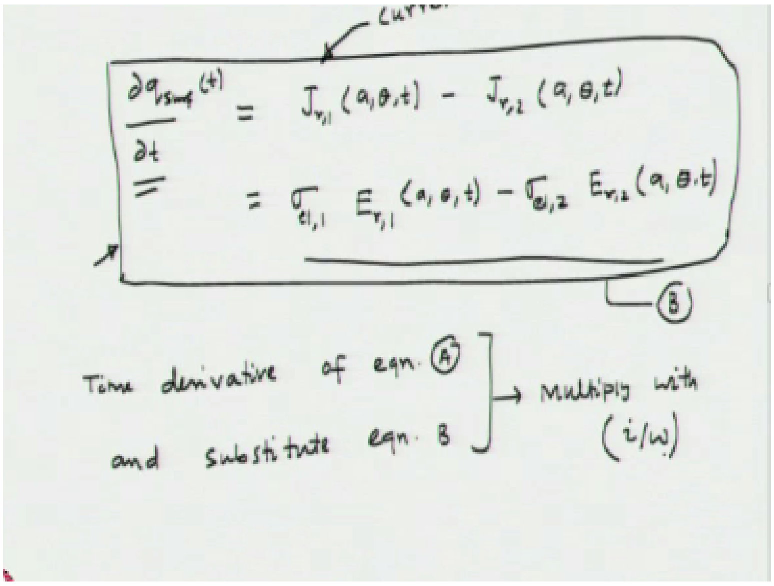

curr

$$\frac{\partial q_{surf}(t)}{\partial t} = J_{r,1}(a,\theta,t) - J_{r,2}(a,\theta,t)$$

$$= \sigma_{el,1} E_{r,1}(a,\theta,t) - \sigma_{el,2} E_{r,2}(a,\theta,t) \quad \text{Ⓑ}$$

Time derivative of eqn. Ⓐ and substitute eqn. B → Multiply with (i/ω)

So, if you do that we write electrical conductivity of 1 and electric field Er1, a theta t - electrical conductivity of 2, Er2 a theta t, okay so that is how the charge on the surface is going to vary with time and if you express that in terms of the electrical conductivity and electric field in the medium as well as the sphere or particle let us call this equation as equation B. Now, if we take time derivative of equation A, okay, you divide both side with d/dt; we take derivative D by DT on both sides.

And plug that in this equation okay, so you know; you take the time derivative here dq surface over dt, so we get time derivative of the left side and then in place of dq surface over dt, if you plug in this term on the right hand side, then this is what we will get, okay. So, we take time derivative of equation A and substitute in equation B, okay and after substituting we; so if you do that first and then multiply with i over omega okay.

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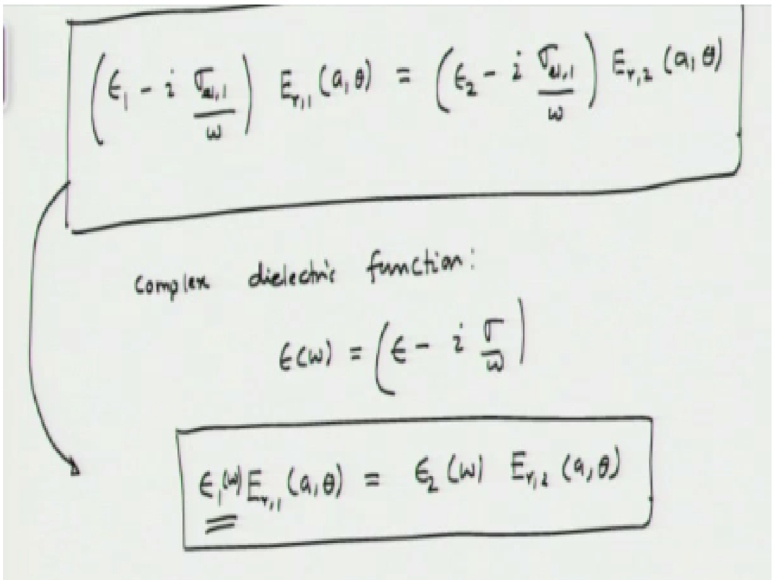

$$\boxed{\left(\epsilon_1 - i\frac{\sigma_{el,1}}{\omega}\right) E_{r,1}(a,\theta) = \left(\epsilon_2 - i\frac{\sigma_{el,1}}{\omega}\right) E_{r,2}(a,\theta)}$$

complex dielectric function:

$$\epsilon(\omega) = \left(\epsilon - i\frac{\sigma}{\omega}\right)$$

$$\boxed{\underline{\underline{\epsilon_1(\omega)}} E_{r,1}(a,\theta) = \epsilon_2(\omega)\, E_{r,2}(a,\theta)}$$

If you do that, we get this equation, we get epsilon 1 - i electrical conductivity 1/ the frequency * Er1 a theta will be = epsilon 2 - i sigma of 1/ w Er2 a theta, so this is the equation we get when you do the ever okay. So, here we can define a complex dielectric function; complex dielectric function, which is epsilon omega is going to be epsilon - i to sigma over omega. So, if we define that then this equation will be something like epsilon 1 * the electric field Er1 a theta okay, sorry; this will be epsilon 1 omega, will be = epsilon 2 omega r2 a theta.

So, this is this equation is similar to what we had seen for the DC case except that the dielectric constant is replaced by the complex conjugate of the dielectric constant, okay. So, we have a solution for the dielectric force for the DC case and if you modify the Clausius mossotti factor to include the complex dielectric constant, instead in place of the dielectric constant itself then we can get an expression for dielectrophoretic force for AC case, okay.

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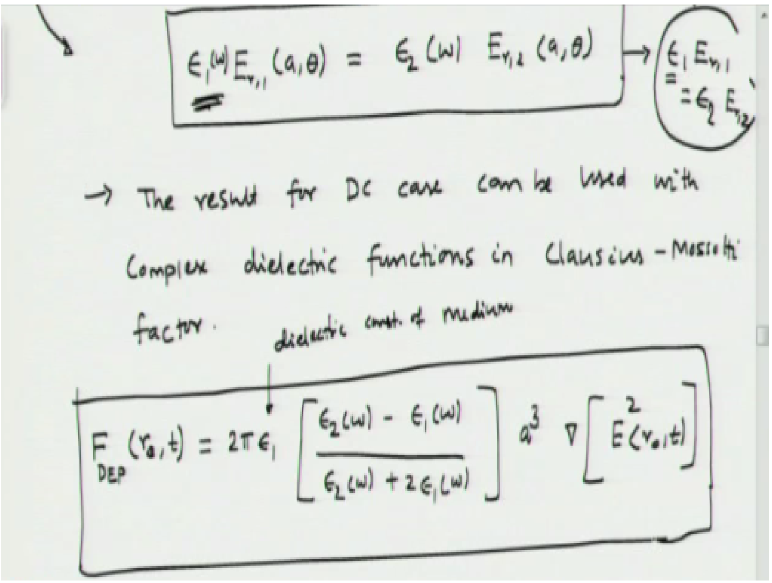

$$\boxed{\epsilon_1(\omega) E_{r,1}(a,\theta) = \epsilon_2(\omega)\ E_{r,2}(a,\theta)} \rightarrow \epsilon_1 E_{r,1} = \epsilon_2 E_{r,2}$$

→ The result for DC case can be used with complex dielectric functions in Clausius-Mossotti factor.

dielectric const. of medium

$$\boxed{F_{DEP}(r_0,t) = 2\pi\epsilon_1 \left[\frac{\epsilon_2(\omega) - \epsilon_1(\omega)}{\epsilon_2(\omega) + 2\epsilon_1(\omega)}\right] a^3\ \nabla\left[E^2(r_0,t)\right]}$$

So, what you do is; so, the result for DC case can be used with complex dielectric function in the Clausius mossotti factor, okay. So, this we can do because this equation is similar to our DC case, where we saw epsilon 1 Er1 = Epsilon 2 Er2, so this is what we saw in DC okay. So, the only difference is that this dielectric constant is replaced by the complex dielectric function okay, so we can write down the dielectrophoretic force for AC case, which is a function of let us say at ro t, okay.

This is going to be 2pi epsilon 1, so make it; please note that this dielectric constant is going to be the dielectric constant of fluid okay not the complex dielectric function, only the expressions in the Clausius mossotti factor becomes complex, so this will become epsilon 2 omega – epsilon 1 omega/ epsilon 2 omega + 2epsilon 1 omega * a cube * gradient of E ro t square okay, so this is the dielectric constant; constant of medium not complex dielectric function okay.

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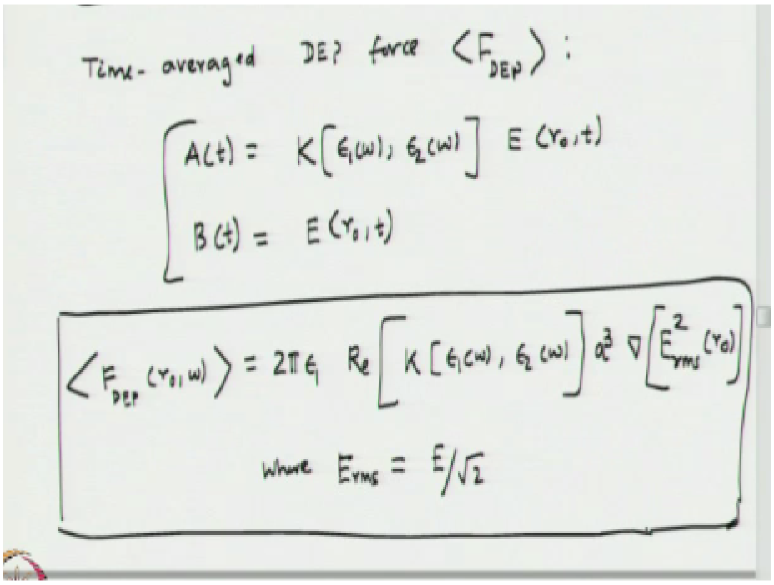

Time-averaged DEP force $\langle F_{DEP} \rangle$:

$$\begin{cases} A(t) = K[\epsilon_1(\omega), \epsilon_2(\omega)]\, E(r_0,t) \\ B(t) = E(r_0,t) \end{cases}$$

$$\boxed{\langle F_{DEP}(r_0,\omega) \rangle = 2\pi\epsilon_1 \, Re\left[K[\epsilon_1(\omega), \epsilon_2(\omega)]\right] a^3 \, \nabla\left[E^2_{rms}(r_0)\right]}$$

where $E_{rms} = E/\sqrt{2}$

So, this is going to be the expression for the dielectricphoretic force in a AC field, okay. Now, this is time dependent, so we have to find a time average dielectrophoretic force, so you can find the time averaged DEP force okay, which is F DEP. Now, here if you look at what we have seen earlier, okay we had At and Bt, so similarly here we define At as the Clausius mossotti factor K of epsilon 1 omega, epsilon 2 omega which is nothing but this term there okay * Er0 t.

And we have another term to define Bt is E of r0 t, so these are 2 functions okay from the dielectrophoretic force that you can extract that a time dependent okay, so we can average the dielectrophoretic force similar to the time average of a At, Bt, you can find F DEP r0 omega is going to be 2pi epsilon 1, the real part of the Clausius mossotti factor Omega, E2 Omega, real part of this term * a cube * the gradient of Erms square r0.

So, this is the root mean square electric field, where Erms is going to be epsilon over square root of 2, okay so this is going to the expression for the time average dielectrophoretic force okay in the AC mode. Now, you have to find that what is going to be the critical frequency about which this dielectric force is going to change its sign okay from positive, this may become negative.

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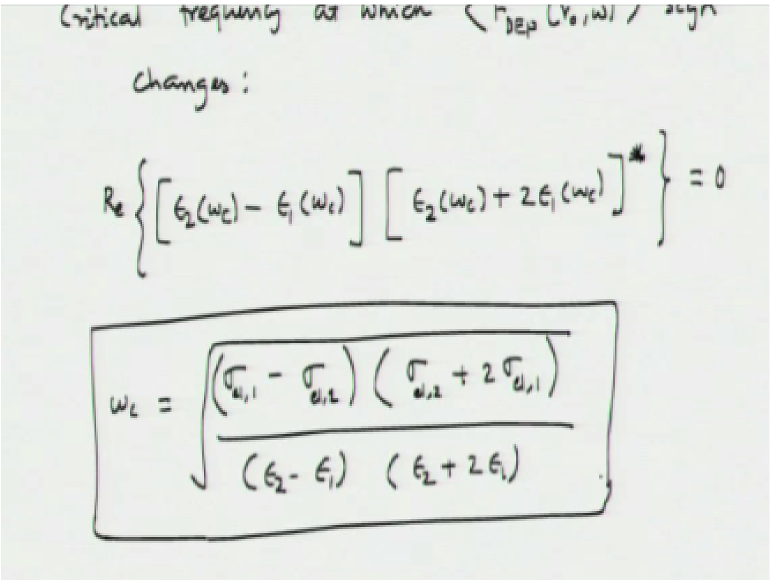

Critical frequency at which $\langle F_{DEP}(r_0,\omega)\rangle$ sign changes:

$$\mathrm{Re}\left\{\left[\epsilon_2(\omega_c)-\epsilon_1(\omega_c)\right]\left[\epsilon_2(\omega_c)+2\epsilon_1(\omega_c)\right]^*\right\}=0$$

$$\boxed{\omega_c=\sqrt{\frac{(\sigma_{el,1}-\sigma_{el,2})(\sigma_{el,2}+2\sigma_{el,1})}{(\epsilon_2-\epsilon_1)(\epsilon_2+2\epsilon_1)}}}$$

And as you know that the change of the sign is going to be decided by the value of the Clausius mossotti factor because that is the only term which has; which has the control over dielectrophoretic force okay. So, if you want to find the critical frequency at which the dielectrophoretic force sign changes, then we have to say that this part, so this; you know the real part of the Clausius mossotti factor will change between negative to positive okay.

And to find the critical value, let us equate that to 0 okay, so if you do that you can write the real epsilon 2 omega c omega c and we take to; this to the numerator + 2 E1 omega c star that means that is the complex conjugate is going to be 0. So, if that is 0, then we can find the critical frequency, okay. So, what we see here is that this part can control the sign of the dielectrophoretic force.

So, we say that this part can move from negative to positive okay, so since it moves from negative to positive at the critical value, this would be 0, okay so that is what we are doing here and if that is 0, we can find the critical frequency, which is going to be electrical conductivity1 - electrical conductivity 2 * electrical conductivity 2 + 2 electrical conductivity 1/ epsilon 2 – epsilon 1 * epsilon 2 + 2epsilon 1, square root of that.

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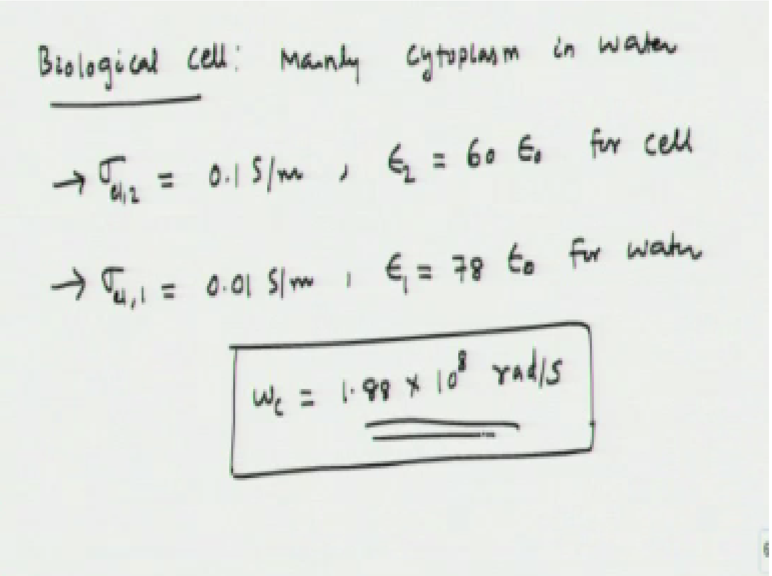

Biological cell: Mainly cytoplasm in water

$\rightarrow \sigma_{el,2} = 0.1\ S/m\ ,\ \epsilon_2 = 60\ \epsilon_0$ for cell

$\rightarrow \sigma_{el,1} = 0.01\ S/m\ ,\ \epsilon_1 = 78\ \epsilon_0$ for water

$$\boxed{\omega_c = 1.88 \times 10^8\ rad/s}$$

So, the critical frequency is going to depend on the electrical conductivity of the medium and the particle and the dielectric constant of the medium as well as the particle okay. So, the critical frequency at which the dielectric constant will change sign will depend on the electrical conductivity and the dielectric constant of the medium as well as the particle okay. Now, if we consider a biological cell and the biological cells have mainly cytoplasm in water, okay.

So, we have mainly cytoplasm in water, so the electrical conductivity of the medium is 0.01 siemen per meter and epsilon 2 is 60 times epsilon 0 and that is for the cell okay and for the medium, which is water, we have 0.01 siemen per meter and epsilon 1 is 78 times epsilon 0, this is for water. So, we have the particle and the medium. So, for this case if you find the critical frequency omega c turns out to be 1.88 * 10 to the power 8 radian per second okay.

So, that is typically the value of the critical frequency okay. What is interesting about AC at dielectrophoretic precious is that the Clausius mossotti factor is dependent on the frequency okay and that critical frequency is dependent on the property of the medium and the property of that object okay, so this can be used to; can be exploited in biology to separate living cells from dead cells and cancer cells from healthy cells.

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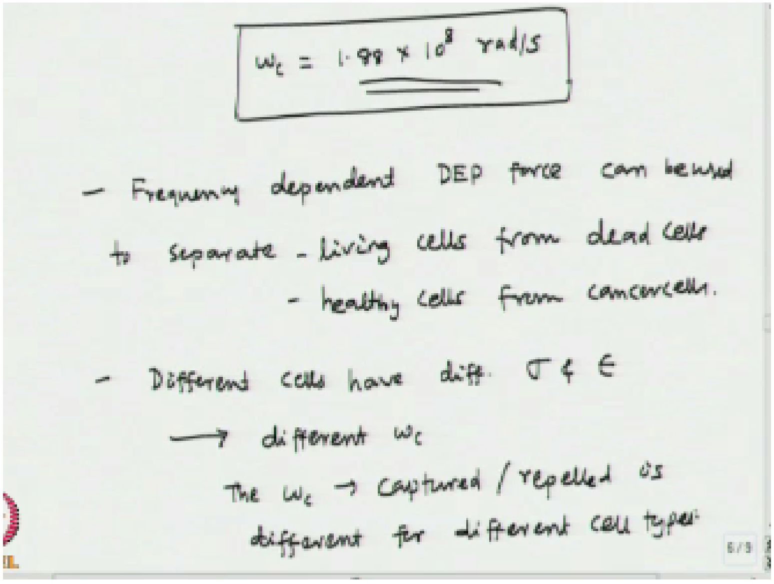

$$\boxed{\omega_c = 1.98 \times 10^8 \text{ rad/s}}$$

- Frequency dependent DEP force can be used to separate - living cells from dead cells
  - healthy cells from cancer cells.
- Different cells have diff. σ & ϵ

  $\longrightarrow$ different ω_c

  The $\omega_c \rightarrow$ captured / repelled is different for different cell types.

Because the electrical conductivity, the dielectric constant is different for healthy cells and diseased cells and they are different for living cells and dead cells okay. So, the critical frequency is going to be different for the different types of cells and this can be exploited to separate them okay. So, what we know from here is that the frequency dependent DEP force can be used to separate living cells from dead cells, okay and healthy cells from cancer cells okay.

And this is because the different cells of different sigma and epsilon and so this gives rise to different omega c, okay. So, the omega c at which they are captured or repelled is different for different cells; different cell types okay, so this has interesting applications in microfluidics.