Advanced Gas Dynamics
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Lecture – 37
Supersonic Flow past a 3D Cone at an angle of attack: Flow Visualization – II

So, let us continue from the last lecture right. So, we were basically looking at the conical flow at an angle of attack, right. And we you know it is been all talk, talk, talk. So, I you know it is to get a little confused. So, hopefully you know as we go along you will try to visualize things a little better. And I will you know go on those lines again today a little bit. So, today also we will there will be lot of talk, talk, talk and then you know we will go on to the equations and stuff. So, hopefully it should you know ring some bells by the end of it ok.

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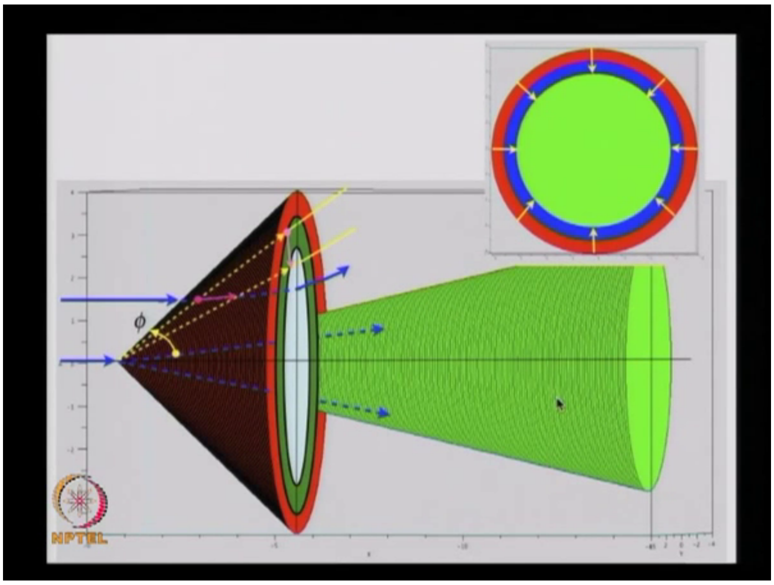

So, let us begin remind us as what we did. So, this was the axisymmetric case right. And so, if I had a streamline like that. And this is oh this is essentially showing is that it is going inside the shock structure, right; it is going inside the shock structure.

Which is the this red you know cone out here, right. And then I took another cone inside the shock structure which is shown by this dark green circle here right. So, this is a cone which you can think of that having the same vertex as this, but the cone angle is different for this say green cone. So, again so, the this streamline the way it goes in and it goes inside, and is this this streamline is basically traversing, right; into the cone so, it goes in and you know, if I were to look at the projection of this onto this plane or the x y plane, then I would see it crossing it somewhere over here.

So, the way we will look at that is that if I take say you know to a line like that. So, this point it is it is the point that did here just crossing. So, you can imagine that we actually have a cone, whose base looks like this green one. So, this point is actually crossing you know that particular cone, which is encased inside this red one. So, and if I look at that from this face so, that gets projected onto somewhere over here. So, then again if I have one more cone which is whose cone angle is again smaller than this green one, which is again you can basically see that we basically have 1 2 and 3. 3 and the 4th one is of course, this is our actual physical cone. So, this cone again is encased inside the green one, it has a yet smaller cone angle than the green one right. So, similarly here if I draw these 2 it is also passing through that. So, when so, it is basically what is happening is it is entering. First it enters through here through this shockwave. So, which is the red cone, right. And then it goes into another cone. It hits another cone of the shock, which is the green one. It goes further down and it hits the; this the cone which is in case which is projected at this blue base.

So, basically, I am looking at these 2 points. These 2 points, on the you know, these cones of shock that it is encountering. Say if I project this onto this face, they basically will project on to you know their basis. So, basically these 2 points, when the flow is moving from say the green cone into the blue cone. So, if I were to look at it from the x y axis, it would look like it is going from here to here. This something that we have done you know in the last lecture, just wanted to kind of you know remind ourselves. Now again this is a streamline which is basically moving through the shock, and it is going on the surface of the cone. So, these are straight streamlines, which are going on the surface of the cone. So, basically this for axisymmetric case, you have the cone structure like that you have the flow coming. And the flow equally diverges and goes pass the solid cone here.

So, this is what it look like. Now if we had to look at the this picture here, all right. If I were to look at this picture which is the if I am basically looking through the base of the cone. So, this green portion is the cone. The red is the you know, this shockwave. The blue is the slice that we were using we are not using the slice here. So, I guess I could have drawn this without it. So, you know just ignore the blue here if you have. So, if I want to look at this basically I will get several lines like this. You know, showing the traversing of the various streamline. So, it will look something like this if I were to look from here, through the base of it.

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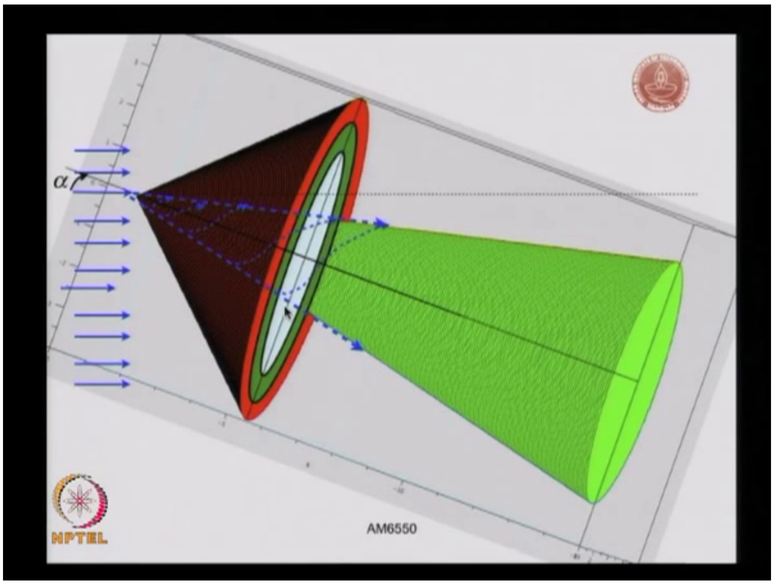

Now, let us go to the axisymmetric case. So, axisymmetric case we said. So, essentially this is with an angle of attack as you can see, this is the direction of the free stream, angle of attack now. So, we said there will be 2 streams, 2 streamlines one, right. At the top and one of the bottom.

And then the rest of the streamlines will basically behave like this. Let us discuss this a little bit more.

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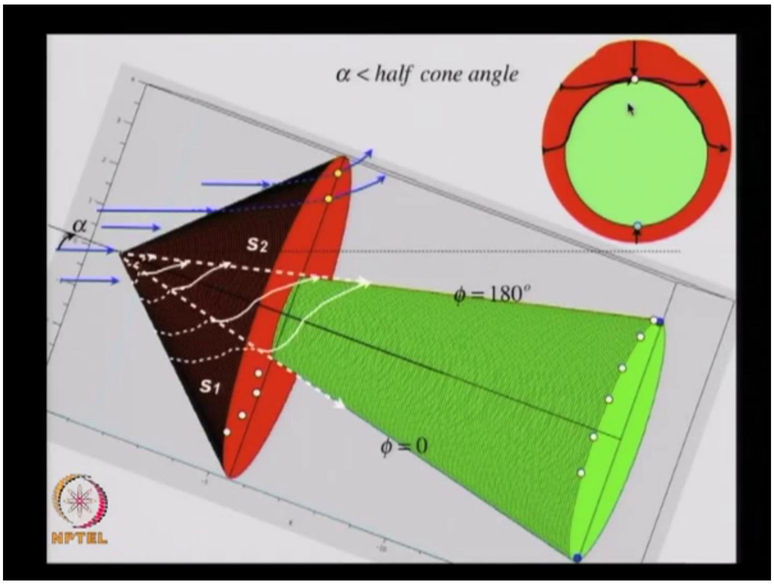

So, let us discuss this a little bit more. Now if what we see over here again. So, this is my shock structure, this is my shock, and this is my cone now. And you can see this is the angle of attack. Now let us consider basically 2 streamlines. One is you know at the bottom of the this streamline another is top of it. The reason I do that is because as we have I think we spoke about this briefly in the last lecture is that; because the another shock is a symmetric now, the shock is a symmetric about the you know this free stream axis, you know line. And what you can see is that the angle of the shock, the shockwave angle here is smaller than a shockwave angle here. So, the top part of this the top part of the shockwave is basically the weaker part of the shock, right. It is a weaker part of the shock.

And this here is the you know stronger part of the shock. Now let us consider this you know cone out here. So, we will. So, when we say the cone. So, this let us consider these you know the topmost if I consider 2 radial lines, how am I going to track the surface of this cone. So, let me consider 2 you know lines, 2 radial lines from say here. So, if I consider say 2 red lines from this you know cone vertex. So, a one radial line is basically phi is equal to 0, you rotate that and take it to phi is equal to 180 degrees. Let us we will come back to that now. So, we are looking at. So, this is say the topmost you know the cone the way the cone is projected here. So, this is the top version, to top most part of the solid cone this is the bottom most part of the solid cone, right. Now in this case 2 say for example, so, what we are trying to do here is essentially again, find out the projection and how the projection will look.

So, again we have a you know streamline like this. So, we have this streamline which again comes in goes inside into the shock structure, and you know reflects out. So, say it you know at this particular point, say it look you know we are looking at this particular point. So, if I were to if I look at the theory picture I see this streamline, if I were to look at just the projection of it which is if I were to just look at the circle red. It would boil down to one point over here, is not it? Similarly, if I have another in a another streamline here. So, and it is moving like this. So, basically again I would look at this particular point. So, say like we like we just did it for the axisymmetric case. So, if the flow is moving the streamline. So, if I take 2 points from the streamline. So, which is going from here to here. So, this will result in basically 2 points here on the red circle, which will give us the direction. So, again these are basically radially. So, the like the way we saw for the axisymmetric case.

So, when you see this. So, you will you can just imagine at this particular point an arrow which is pointing down. That will give us a direction in which the streamlines is moving. So, these are you know typical streamlines which we have just seen so far. Now let us consider. So, we let us consider this, a one radial line which goes here. And this is the phi here, and what is this S 1 now.

Now this S 1 is the; so, if when we have this flow, the free stream it comes from here. It encounters this part of the shock, at the bottom part of the shock. And so, you know we will have this this streamlines in this particular case. So, since here there is no, since there is no asymmetry; it is not going to like you know nicely spread out evenly spread out it is not going to do that. So, what we will have is that the flow here we will have basically 2 streamlines one going right down to the bottom most part of the cone which is shown by this you know white dotted line here.

And we will have one more at the top. Other than that, what we will have so, let us at this particular point let us say now since these 2 the bottom of the shock and the top of the shock are different because of the shock wave angle. So, this is the stronger part of the shock. So, at the end of the day what this means is that this has even my flow will encounter the shock across it. It will encounter say an entropy change of S 1. If we if we are encounter an ntropy as S 1 and if I so, if I for example, look at this you know radial line here. This is a radial line which is emanating from the vertex of the cone and going on the bottom most part or phi is equal to 0. It is like phi is equal to 0 line on the surface of the cone. So now, this so, if I am looking at the surface of the cone.

So, this projects onto the base of the cone as just this point, isn't it? If you were to look at this particular line from this face, if you just imagine that at this point just try looking at this from this face. So, this projects this white line here projects onto this green base at this blue point here. At this particular point. Similarly, we will draw the other you know radial line which is go which is basically going from the topmost part of the cone or the phi at the radial line at phi is equal to 180 degrees. And again, this encounter so, this here the change in the entropy is basically S 2. And again, if I project this line onto the base of the cone it we basically encounter this this it is basically projects as a point, which is this blue point. So now, that is all there is to it that is all the as far as the straight streamlines are concerned. That is, it address everything is curved.

What do I mean by that? So, like I said you have the flow which comes in and you have one streamline going right down the at phi is equal to 0 part of the cone you have one more going at phi is equal to 180 degrees. Now rest now what will happen is that there is this the lower part of the cone here, the lower part of the hock structure over here. So, that is facing basically the free stream, that is called the windward side. Again, you know using I think if you can think of a mountain and there is flow coming in it is a direct it is facing it directly. So, if you can think of that even when you think of that the flow will actually move past the mountain on to the other side. That is exactly what is going to happen here. So, you have this cone. So, the flow comes in enter. So, the shock reaches the bottom of the of the cone surface, and then curls around and goes up to the top. And that all these streamlines are say you know a curved.

So, if you look at this I draw this dotted line here, I draw. I have drawn this streamline here part of which is dotted another is solid. What this are? What this is means here is that this barf the curve which is the dotted one is going inside the shockwave, right. Is going inside the shockwave and then it goes from the bottom of the cone and curls around and goes to the top; when it tries to merge over here at this particular (Refer Time: 15:17) at this particular radial line. Now as we said that when it goes through this shock, it encounters a you know entropy of S 1. So, it will go from here and have an entropy n S 1, and then go and curl up and go here. Now before we go and try to understand all of that. Let us see how this would look if I project it onto the surface of the surface of the cone and the and the shock. So, if I were to look from here. So now, when I look at a 3D picture I see a curves, you know a curved streamline, you know going like this.

Now, if I look at this whole picture from this end. If I am looking into this green base, right. Through here then what would I see just pay attention to this. So, if I if just think of drawing several radial lines which intersect this streamline. So, where do you think that will project onto when we think of this base. So, just say look at the solid part of the streamline, which is on the surface of the cone. So now, various parts of this. So now, pay attention to this you know this this surface. So, the pay attention here if you see this. So, then these are various paths, various paths of this stream language project on to here. And you can see the top portion here the top portion here, well is merging with this line. So, the end of it will also project onto here which will be again this line this will be again this line. And this will actually merge with the blue a circle here.

So, this line should actually merge with this dotted line. So, I am just putting it on the side here to distinguish, but let us understand that this is actually the same point it should actually sit on top of this. So, that is how this the solid portion of this line projects onto the surface. What about this? Now this part will basically project on to hear. This read part of it, isn’t it? So, if I do that now look at this red concentrate on this red circle here. So, if I do that. So, various parts will look like this.

So, let us do that sort of again. So, we have say you know the streamlines, and we have these streamlines then we have this. If you look at this line which is going here now look how does this project now look at this blue the green base now. So, that is the S 1, and then. So, this is this is how it projects, again look at the top portion of the cone phi taken 180 degrees concentrate there we have a line like that which is S 2. And then this projects onto the green base as this blue line. And then we would have you know a streamline which is like that.

So, for the solid part of it again concentrated on the green base. So, you see various parts of it they will project as small points like this, and the end of it emerging with the phi is equal to 180-degree radial line. So, therefore, if you concentrate here, that also projects at the small point, right. And this part of it, again this part of it, it will project onto the red base let us do that again that is it. So now obviously, we will have several other you know streamlines. So, if I do that let us just look at this space here. So, so you have these particular streamlines. So, all of these all of these are going through the shockwave, and then the curl around from the bottom of the cone up to the top and the bottom is essentially the bottom is the windward side and the top is the leeward side. So, that how will this look you know, if I you know on the projected planes? So, let us look at that.

So, we will discuss this you know various points of it a; and a lot of the time the discussion will you know entail on a comparison with the axisymmetric case. And I do not know if that is the best way to understand this, but sometimes you can draw do draw some inferences. So, this is a shock structure here, and this is the cone the green one. Let us look at this now. Now this this line, what is this line? This is nothing but this streamline over here, if you look at this if you project this onto it. So, these points are the ones which are on the surface of the cone which is what is shown over here, and the remaining of it are in the shock which is over here. So, if you if you look at this in in in the projected plane, it will look something like this you know this particular streamline.

So, similarly you will have you know other streamlines of course, which we look like this and what we see that all of these streamlines; obviously, as you can see they are going through the top surface which is phi is equal to 180 degrees. So, they generate at various points, but they all converge at phi is equal to 180 degrees. So, and all of this point project at the same point, right. Same point here so, they are all going through this particular point. So, again so, these will be the you know like I said the these streamlines must be we will know. So, I am just drawing a couple here. So, you can see that we will have several you know streamlines, which will project on you know we can see the projection of that here. Now again if I look here where do these 2 points line these 2 points. So, the so, that this blue point actually comes here and this blue comes at the bottom over here right. So, if you look at this so now, what is the issue here? What is the problem? Or what is a not the problem are so on and so forth.

So now let us sort of discuss this a little bit now. I do not know if you are already seeing this. So, like we said yesterday a particular streamline, a particular streamline will have a single value of entropy. So, when the streamlines which are generating from here which are which are generating from the bottom or the windward side of the cone and moving up on the surface of the cone. So, this is encountering an entropy S 1. So, the all these streamlines out here you know, that you see have an entropy S 1 entropy change of S 1 as they go here. And these streamlines go ahead and converge on the top. Now this streamline also the I mean, this dotted line which is going along the surface. This also has a entropy S 1. Because it is moving in from the vertex and it is going along the surface of it.

So, it is on the windward side of the shock as well. So, this will also have an entropy S 1. So, on similar lines the line of the redder line and at the top of the cone at phi is equal to 180 degrees will also have an S 2. Now look at this, and I look at this over here. So, these streamlines these streamlines have an entropy of S 1. And when it comes and converges on this particular line. So, if you look at this blue point, and this white point now the blue point is a single point which is a same streamline sort of speak each has an entropy of S 2, but when these streamlines come and join these, they have a streamline and entropy of S 1, right. What does that mean that we have now when these are joining here, we have a single straight line which does not have a single value of entropy. It is actually multi valued.

So, you these streamlines when they come here they have S 1, and the streamline which is come which is here you know riding along the you know along the body of the cone has S 2. And they are all converging at phi is equal to 180 degrees. And they are all projecting onto this particular point here. So, we have several points there, if you look at just this point, you have several points there. You have several small such projected points, but all of them although they are the same location they do not have the same entropy right. So, these points have S 1 these points of S 2. So, clearly this is a singularity. So, this point is therefore, and then this line is there this line is therefore, called as vertical singularity, vertical singularity and I will probably go to the board in a while and then sort of write that out. So, this line is actually a line of vertical singularity, and it projects on to this point here.

So, you can see that this point on a on to the project plane here. Is it the point of vertical singularity? Also notice also notice here that the shape, and a shape of this projector plane is not any more concentric circles, like we did in the axisymmetric case, right. Isn’t it? Because the shock structure is you can see is no more, the shock structure is no more like symmetrically located about the cone if you look from here. So, that is why we have this little pout you know on the top for whatever. So now, this now all of this. So, all of this that we see here is basically when the angle of attack is less than half of the cone angle. So, this is the this is this angle, this angle is the half cone angle, and the angle of attack is less than the half cone angle. So, what we what we are seeing over here, what we see over here is this is the basically the case. So, we you know on the top portion on the top of the cone if you look at this. So, this is a projected plane etcetera we have a vertical singularity point over here.

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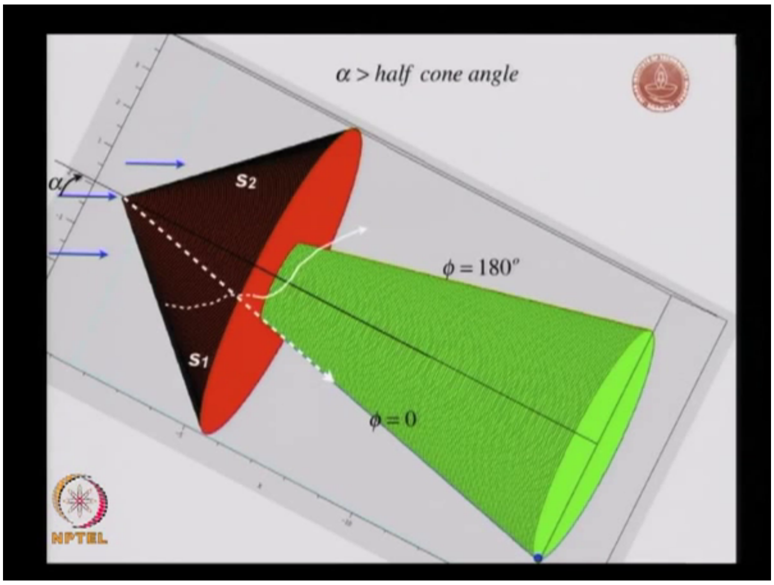

So, let us go to what happens if alpha? The angle of attack is greater than the half cone angle. What happens then? Right. So, we look at the same picture this alpha is just greater than the half cone angle it is the same picture. So, again we have to red line phi is 0 phi is 180 degrees and we have these streamlines. So, again the entropy change there is S 2 entropy change here is S 1 say and alright.

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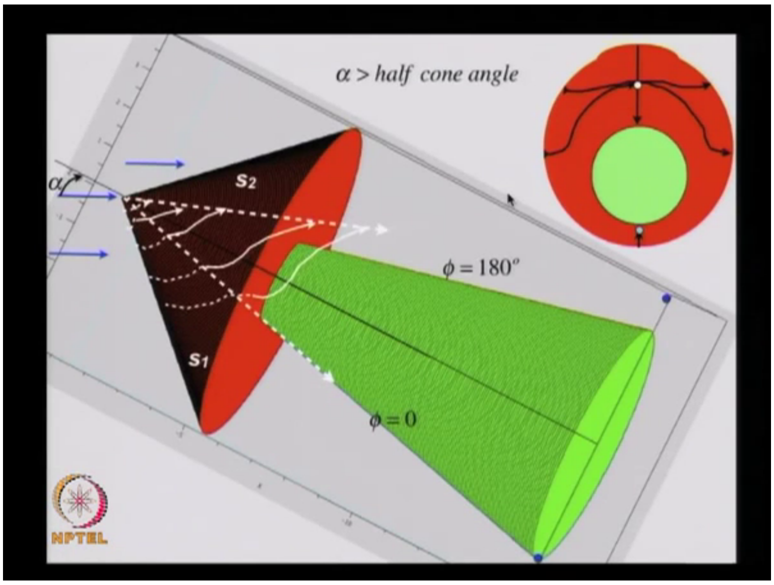

So now what we will look here? So, here if you this is S 2 and this red line again it projects on to it projects on to this point. Now let us look at one streamline. Now this streamline in this particular case it comes from here which is S 1 it is S 1 here, but it does not merge onto the just exactly the top of the cone. Because of a larger angle of attack, because of a larger angle of attack this does not merge exactly onto the line at the top of the cone. Basically, it does not merge onto the phi is equal to 180 degrees line radial line.

So, similarly we will have the other streamlines which again do not merge. But they do merge somewhere you know they seem to curl up and then they do merge somewhere, and if I draw a line through that if a line through that, I will see that this is they. So, there is merging onto a line which is not coinciding with phi is equal to 180 degrees. That is happening because the angle of attack is greater than half cone angle. And again, this straight line that I just drew is a radial line, we will have an entropy of S 2. So, this line in this particular case is also a point of is also a line of vertical singularity. So, compared to the previous case, in this particular case this line of vertical singularity has basically moved from the surface of this cone and above it, right. And now if I look at this picture and look at the base here. So, that projects on to so, if this particular thing it projects it does not project onto the surface here, but it is it is somewhere above it think of this, and if I extend this line. So, then I will be looking at from here. It is related somewhere over here on the flow field. So, it is now into the flow field.

So, therefore, if I look at this, if I you know look at this particular picture over here. So, again we see a you know sort of the I mean you know, this is a shockwave and you can see that it is even more skewed, or you know asymmetry is increased you know with the with the increased angle of attack about the cone. So, then here this is the streamline which is this one this is the streamline. This streamline is you know, above it is it is no more on inner sticking on to the surface of the console now the it is about. So, then again, all these streamlines pass through this. So, again we will have similarly you know all these streamlines which we learnt before. And therefore, we will still have this point here this point here, but the vertical singularity point will actually we lifted off the surface of the cone somewhere over here. So, if you can see this picture over here it is right.

So, this line is basically projected onto this point here, and that point if I look at it from here is actually at this location. So, unlike in the previous case for the alpha less than half cone angle, the vertical singularity point was here on the surface, right. He now it has lifted from here to here. So, what we can see from of course, both these pictures what I can we can see from both these pictures is that we have several streamlines, we have several streamlines which are you know converging at this particular point. So, there is a dense mesh or array or streamlines passing through this particular point. And so, which means what? You know, if I have a lot of lines passing through this. So, there is basically you know a large gradient of entropy, which is at this particular point going through this particular point. So, the entropy change is pretty large therefore, you know when you know the streamline will go near the surface of the cone because of this region.

Because we have so many streamlines converging at that point. So, this entails that we have a significant finite in a change in entropy, if we look at this. So, well that is something you know I want to discuss. Now I have there is something else here. For example, you know in this particular case v theta and v phi are finite. So, v theta and v phi are finite, and at a particular you know location anywhere in the flow field, we will have a certain velocity which is given by you know the component of velocity which will come from v theta and v phi. So, let us just sort of go to the board, and you know write out and see. So, anywhere in the flow field right.

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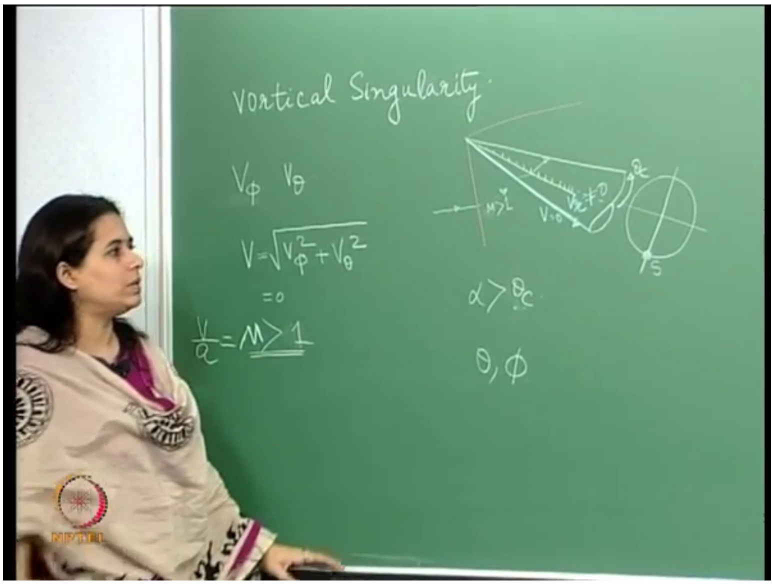

So, we will have say. I am not drawing any pictures here I think by this time you know where the v phi and v theta. So, the total velocity you can I will just think of this velocity component which is given by this. So, we have a velocity component which is given by this. Now if from the pictures that we just saw ok.

When we have the streamline, which is going you know, we have this cone say; this is my cone. And of we have the shockwave you know like that. So, we have these streamlines which come here and go, right. You know down here. So, write down here we have say this particular streamline. Now if you look at another streamline which comes from here, what will happen is it will enter the shockwave then reach the surface and then it will curl up. This is this will be a curve line, it will curl up. So, you can think that of course. So, it is curling up you know it is. So, it is changing it is phi it is changing it is theta. So, it has this streamline for example, has a definitely a finite you know velocity here, but look at this streamline, but look at this streamline. Now this streamline is not curling up, it is just going straight onto the surface of the cone, you know moving along it and it does not change it is theta does not change it is phi, isn’t it?

So, which means that the phi theta and v phi is 0, which means the velocity is 0. So, for this you know this this velocity here is 0. So, if you project it therefore, if the point that you are projecting onto the face. So, if we look at the projected point here, onto the cone surface as we were drawing if I look at this. So, this point projects onto here. This, so, we can therefore, in this particular case say this is a stagnation point, we can call this as stagnation point, but; however, if you if you look at this the velocity this streamlines still has a velocity in the radial direction. So, v r is finite, is not 0 v r is not 0, for this particular streamline, but v theta and v phi is 0. So, we can say it has the this is like a stagnation point based on this velocity, but truly there are really no stagnation points in a 3D conical flow, if you see from here.

So, for these of course, there are you know it is positive, it is it is finite. The velocity is finite, but for even for this radial line, the you know it is still has a finite v r, still has a finite v r. So, the I mean I did mean typically speaking or in a in a 3D conical flow there are no stagnation points, but you can think of this as a stagnation point over here. And this goes for you know all the cases that we have seen. And since we are talking about velocities, velocities over here. Now if this velocity of the fluid moving through of the you know the of these streamline moving, through the shock structure on to the cone etcetera. So, if this velocity is greater than the mach number, if this becomes super step actually this let us just say the corresponding mach here, the corresponding mach number this v. So, if the velocity of speed is a so, v by a so, the corresponding mach number is supersonic.

So, if we have supersonic, the curve velocity here if I have supersonic velocity here. So, I will end up having embedded shocks. So, say I for this particular streamline, I have a supersonic velocity which is say v r say for this mach is greater than 1. So, within the shock structure we can have you know, if we will consider several lines. So, you will have a you know series of shockwaves, we will have see it is a shock waves. And this sort of is a mach number. So now, this sort of a case will happen most of the times when for the alpha greater than half or say theta c, yeah. Actually, you know I can not do it here. So, this is my theta c half cone angle is here theta c. So, this this embedded shock thing, will happen most of the times when for a very high angle of attack. So, very higher angle of attack will end up in increasing the this velocity, and then if this becomes supersonic, we have this embedded shocks.

And so, like I said now this line let me just write it out. So, that you kind of you know understand what I am talking about. So, this straight line here is a line of what we say vertical singularity. So, if I had to write it out it is singularity. So, this is essentially vertical singularity these lines. One more discussion before we finally, stop this cone whatever we have started so far is, right. Circular, isn't it? We have a, right. Circular cone in the sense that if I have projected I essentially get a circle. And if you drop a perpendicular from the vertex here it makes 90 degrees with this, right. It is a red circular cone that is a, right. Circular cone it does not have to be like that always, it does not have to be like that always, you could have this this projection as an ellipse for that matter you can have that as an ellipse. And in that particular case, now if I do have an ellipse of that sort.

Now, even now if you do have that even for an axisymmetric case, when we have an elliptical projection of the cone, there will be 2 independent variables. You know, there will be theta and there will be phi. Unlike if you have a, right. Circular cone; where the phi is the phi does not like del phi does not matter v, phi is 0 for a, right circular cone for an axisymmetric case.

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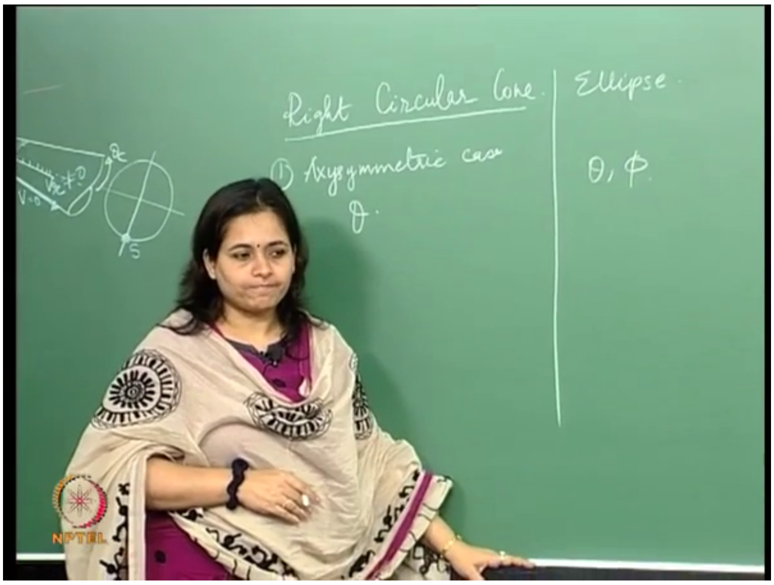

So, let us look at this. So, if I have a right circular cone I have a right circular cone, right and I have an ellipse. So, here for axisymmetric case, right. We have just one independent variable which is theta. But for the ellipse we still have theta and phi I think that is understandable, isn't it? I think it is easy to by the time if you look at the geometry try to look at it think of that as an ellipse instead of circle, I think this is kind of intuitive so. So, therefore, even for an axisymmetric case in the sense that the axis of the cone is in line with the free stream.

We still have a 3D flow when it comes to ellipse. One more thing before we sort of close with this. Now if you we have done basically an you know inviscid analysis over here right. So now, for this particular case; now if you come from here this is the right circular cone that we have been doing so far. So now, if we have the flow which comes here, that it curls up and it goes on onto the you know leeward side of the cone.

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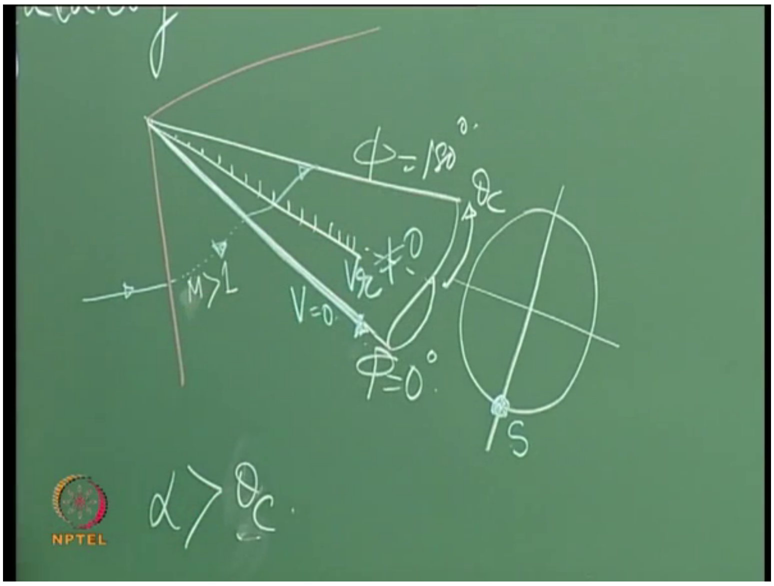

Let us call that this is 180 degrees, here phi is 0. Now from experiments when we have done numerically invested analysis here, now one experiments show that you know kind of match with our numerical findings. Because this portion of here is also in visit; however, when we come on this side of it, when we come to the leeward side of it, you know there might be flow separation at that you know, at this portion of the cone right.

So, therefore, here and inviscid analysis may not be sufficient to predict the properties or predict what is going on in around this region. So, I think that should be sort of all that regarding this. I think that that should conclude, and a what we will do now is essentially go and look at the equations. And see if we can you know make some headway into the way we you know found out a method you know, using the runge kutta method we solve the taylor method equation to you know solve for properties between the shockwave and the surface of the cone. Let us see if you can do something like that for a 3D case, and does that make it more complicated if. So, how much so on and so forth. So, we will deal with equations and stuff when we start on next lecture. So, this is a little bit on the you know, trying to visualize the 3D picture out here.

So, 3 dimensional and some of the issues which are associated with this. So, I think that will be all, and we will take it up from there, take it up from here next lecture.

Thank you.