Advanced Gas Dynamics
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Lecture - 24
Subsonic Flow over a Wavy wall

So, continuing with the wavy wall problem. So, we were supposed to find out the surface velocity potential, ok.

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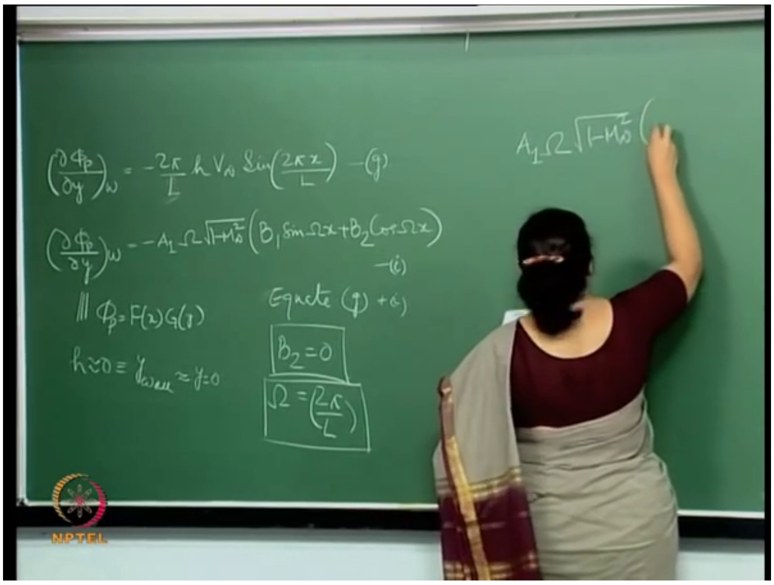

So what, given the equation for the contour of the wavy wall. So, what we found out we are basically these two relationships, I think it read like that, right. So, this is what we gone I think we call this as equation g. Now this is something we got were using the physical contour equation of the wall itself which was given to us right, you got that. And then we use the while solving for the separate solving for the governing equation we use a method of separation. And then using that so numerically we got again this relationship again at the wall, we also got this to be, right. And we said this equation as i. So this equation was I think I said this is where we kind of stopped in the last lecture.

So, now what is left to do is basically equate you know this expression and this expression. So, essentially this is also, right. And this is the equation we got because we said that our solution is basically right; we because we said this equation resultant for the fact because we said phi p was, right. And then we got expressions for Fx and Gy and we were using boundary conditions right. Now we were using boundary conditions so this is essentially our Fx out here and using boundary condition which is at the wall.

Now, since here we said that this is going to be small perturbation. So, which means that h is very small and which means that y at the wall is nearly y is equal to 0. So, this is where so we got these 2 equations. Now if we have to equate these two, so you can therefore, these two can be equal only of course we have got basically you can see if we got a sin term here and a cos term here. So, for if g, if we equate I say; so if we equate these two so this is going to be possible only a if the; we can do this only if this is possible. So, if this is true and of course also; so essentially.

And also here if because in there is no cos term on this side, so this B 2 of course needs to go to 0. And if you see this in here this omega term also needs to be 2 pi by L. So, there you go. So we found something for; these are the two relationships right. So, therefore if you had to write this, so let us say write this now with B 2 equals 0 and omega been equal to 2 pi by L.

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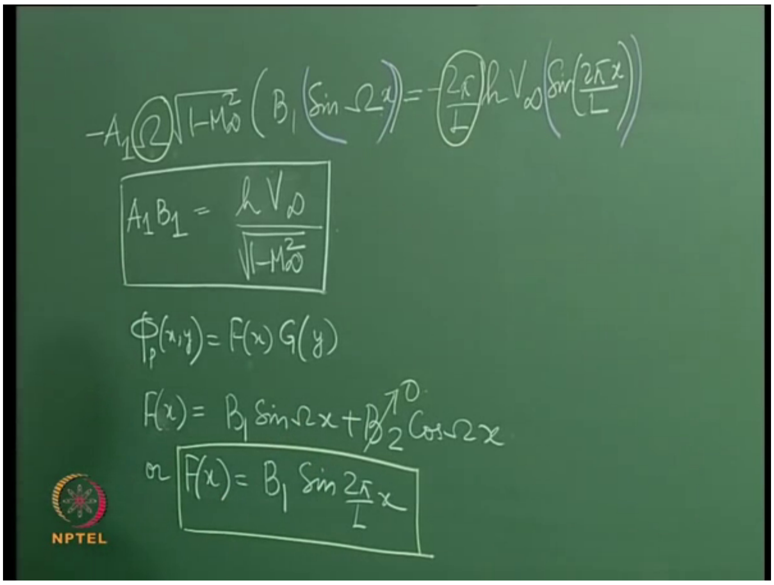

So, if I do that so therefore what we get is this, is equal to this right. Therefore, what we get is A1 B1 right. Now A1 B1, sorry I missed on the negative sign here. So, what we have here is that A1 B1 is equal to 2 pi by L so 2 pi by L, so that goes out; so omega is equal to 2 pi by L or so that cancels out. So, what we get here is h V infinity.

So, this is what we get from here. So, let us sort of look at this one more time. So here now this omega is equal to 2 pi by L which we just found out right, so therefore they cancel out. And again this term sin omega x and this is also the same so they cancel out. So, therefore we are basically left with A1 into B 1 right is equal to h into V infinity here and we take this term here. So, therefore, A1 B 1 is equal to this.

Now what we had from earlier like it was this right and our equations; so therefore. Now Fx has now become; Fx essentially so Fx becomes with B2 equal to 0 let me write this again. So, we had written Fx to be, right. So, here now this Fx now becomes; omega is 2 pi by L right. So, Fx becomes this right, because we found out that B 2 is 0.

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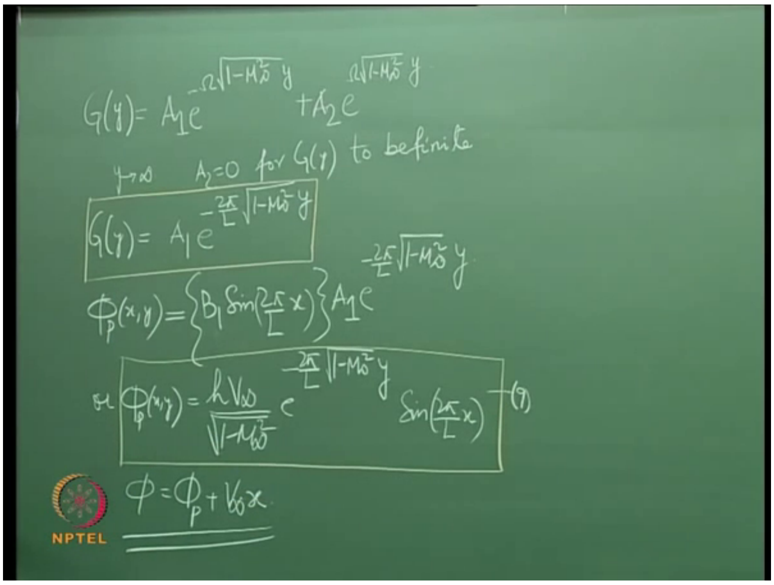

So, again let us go here. So, G x sorry Gy was given by this and we also said that because at y tends to infinity to for this two remain finite. So, this was the condition that we use so that Gy therefore, becomes 2 pi by L again for that 2 1 minus y ok.

So, again this is my expression for Gy right. So, let us therefore now right, which we get as into this is my Fx and Gy which is A1 E to the power into 1 right. So we can basically now what we know here is that this A1 B 1 we got an expression for that. So, A1 B 1 is h into V infinity by this, so we could actually use that they are right. So we could write as A1 B 1 is h right that then we have the exponential term right. So, then we have say let me write this like each of the power minus 2 pi by L this right. Therefore, this is my perturbation potential right, so then this becomes my perturbation potential. So, I think I have called this as equation 9, so let us keep this is equation 9.

So, now that we have got this our original problem was that we need to find a surface velocity potential right and we said that we were going to divide our fraction that into the free stream and a perturbation potential right. So, basically we said that this was equal to right, so this was the perturbation potential and this was the free stream. So, now we have got an expression for this in here. So, to this if we add this then therefore, we get the total velocity total surface velocity potential this is what we get right, ok.

So, now the next task was that now having found this, you have got the first job out of the way. The next job was to find out a surface pressure coefficient, we also needed to figure out what the surface pressure coefficient would look like. So, let see what that means, now what I am going to start with what you mean by the surface pressure coefficient.

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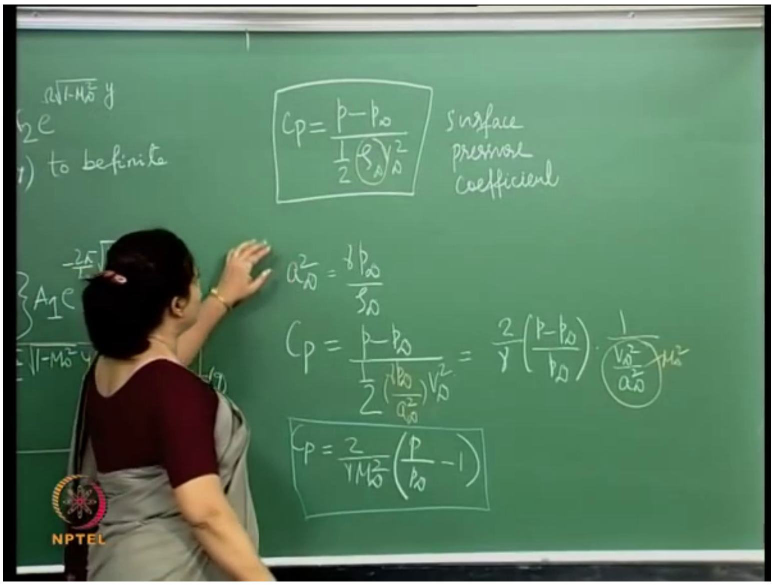

So, we will call that basically as it is pressure coefficient sort of write that as Cp right and you essentially say. So, this is the dynamic pressure and this is the static pressure difference, so this is our coefficient of surface pressure coefficient ok.

So, now we need to get an expression for this over the wavy wall. So, let see what how we will do that or if you can do that from here or not. So, let us look at this and use this is we need an expression for this essentially, this is my surface pressure coefficient. Now let us use this right. So, if I do that, then what we get over here if you see right. So what I am going to do is have this rho infinity right, I am going to write in terms of right.

So, therefore you can pretty well see from here so therefore, this can be written as what? This can be written as essentially written as 2 by gamma into p minus p infinity by p infinity right, the same thing I can write this way. So, you can see here this makes some sense to us right. So, this I can write as the Mach number. Therefore I can write Cp to be 1 by M into. So, therefore I have now basically transformed this equation into this introducing the Mach number right. So, we have the Cp over here which is now written in this form.

Now essentially what we seeing here that you know this sort of solving a problem like this, involves a lot of mathematics right; it is mathematically pretty exhaustive. So, now having done this now total enthalpy, here is constant. So, say I take any point in the flow field, so basically what I will have is right so this is it.

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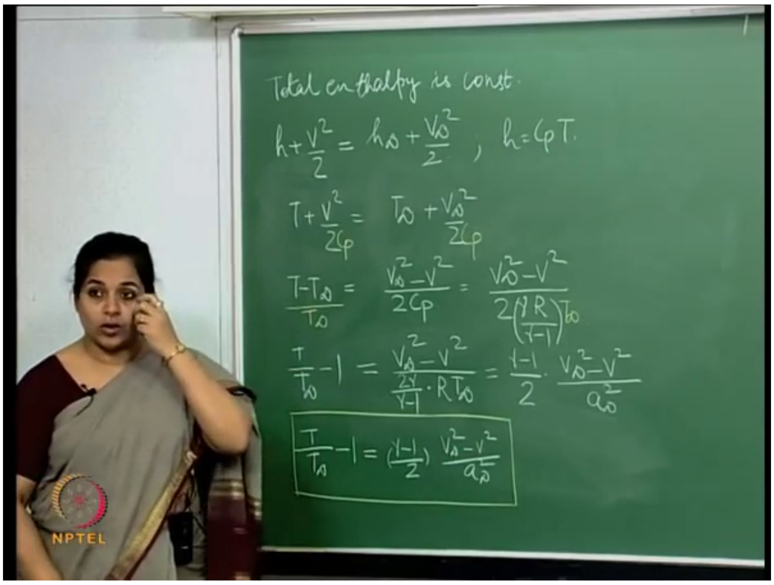

Now, the reasons I write this is because total enthalpy is constant, now for a clerically perfect gas we also know that this is equal to we know this right. So, if you know this then let us write this out. So, for this equation becomes right so if I do this here and then so let us sort of I will you know not skip steps here maybe. So, then what we do over here let us divide by Cp right. So, what we get here is dividing by Cp throughout, so if I divide by Cp throughout what I get is this right, this is what I get and again what we get over here is T minus T infinity is equal to 2 into Cp right, now this Cp again can be written in terms of the gas constant and gamma.

So, therefore this again I can write as this is my Cp here if I write it in this term right. So, if I do that so therefore again I divided by T infinity right then this equals to right. I divided by T infinity throughout so if I divide by T infinity throughout this is what I get right. So, you can see this gamma RT infinity is equal to what V infinity squared right speed of sound. So, what we get so therefore we can write this as say this gamma RT 1 is equal to a infinity square right.

So, now if you look at this term here, so therefore what we essentially are looking at. So, the let us clear out this. So, what we are looking at is this into so this is what we are looking at this one here.

Now, let us see what this V looks like right this is the local velocity right and this is the free stream now as we have said earlier.

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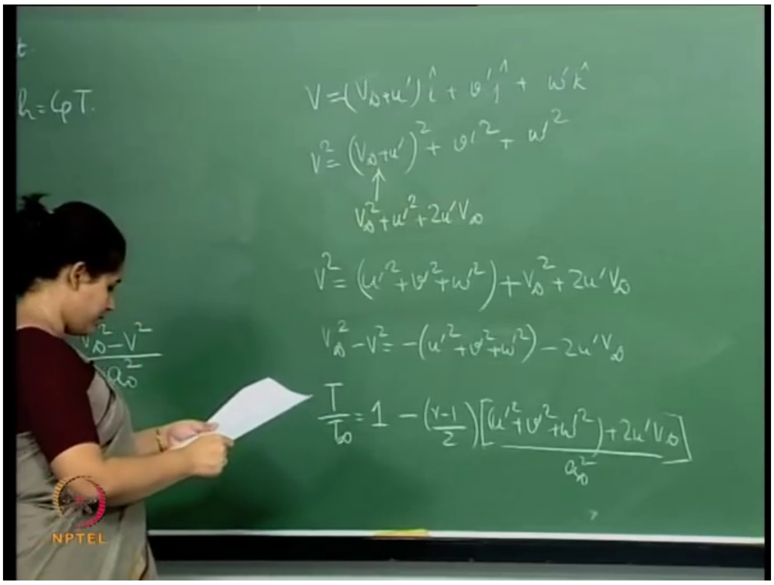

Now, U is V is what? V is the free stream plus the perturbation potential right plus the perturbation velocities basically ok.

So, this is my local velocity isn't it? So, therefore when we say V square right when we say V squared it is equal to what? It is basically equal to this right. So, if I do this what I get from here so what I get from here is essentially. So, then this term becomes right; so therefore, I can write this whole V square term as; so therefore V square becomes this right. So, total V square term is this, now what we have over here is T in this particular expression is V infinity squared minus V square. So, V infinity square minus V square then becomes right this is what we get. Therefore our this term here T by T infinity right, so T by T infinity then becomes I take the 1 to the right hand side 1 and so this becomes the whole thing is say negative right and by a infinity square this whole thing right.

So, this is what we get using the as T by T infinity hopefully now you have saying where we are going. So, let is kind of just stuff a little while we see what we were doing this, so much Mach involved sometimes kind of get model is where we are going right. So, we have to find out the surface pressure coefficient. So, we started from this right and then we if for this density term here, from we introduce the speed of sound in the free stream we did that and use and consequently we were able to introduce this Mach number. So, what we were able to get from these heaps, what we were able to do here is write this term in this fashion introducing the Mach number.

Now, having done that we went back and said that the total enthalpy is constant. So, we started with this equation right and what we have done here is essentially work with this enthalpy constant, this relationship and got an expression by for T by T infinity right we just did the Mach out here and got T by T infinity to be in this form.

So, clearly what we will do now what we need out there however, Cp is p by p infinity we have got T by T infinity. So, we will basically use the isentropic relationship right. So, having done that so what we will now use these isentropic relationship right. So, we have from isentropic.

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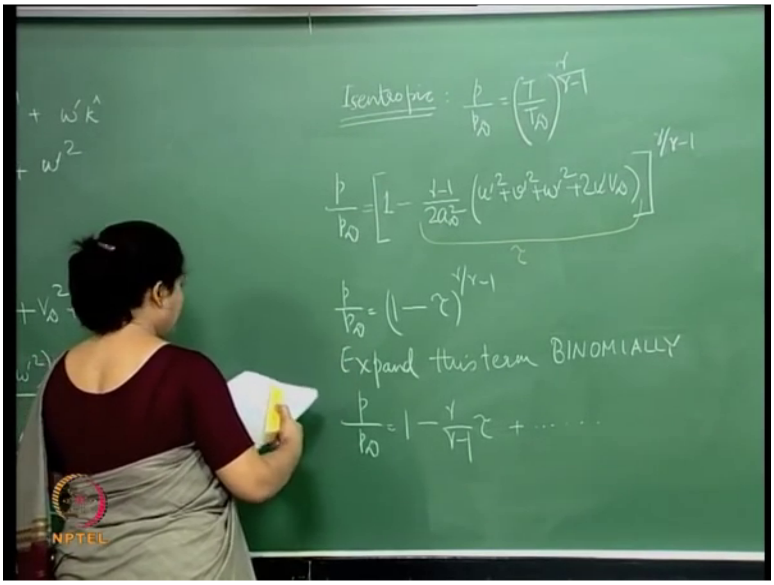

So that means that this is what we get from isentropic. So, we actually have a relationship for p by p infinity which is related to T by T infinity, T by T infinity you can see we have written here in terms of just the velocities right. We have written here just in terms of velocities this helps us in this present case why because, for our present case we have basically found out the surface velocity potential.

Now, from the if that surface velocity potential is known to us and we can get all the velocities. So, in here you can see we have the perturbation velocities and the free stream all right. So, this term is basically known to us so therefore, if I let us write p by p infinity. Now, therefore p by p infinity so we can write as say 1 minus right this raised to this power, now let us say that let us call say this entire term let us say we will call this term as tau. So, then let us call that is tau and this is what we get right now. If we have this now what we will do out here is expand this binomially, if I expand this binomially so we will expand this binomially if I do that what do I get? So then what I get is as so and so forth. So, this is what how we will get we neglecting the higher order terms ok.

Now, if I do this so therefore, basically we just introduced this term gamma minus gamma by gamma minus 1 into this tau. So, what that will do so if you go over there. So, what that will essentially do here, so now let us go ahead and do the remaining math of this ok.

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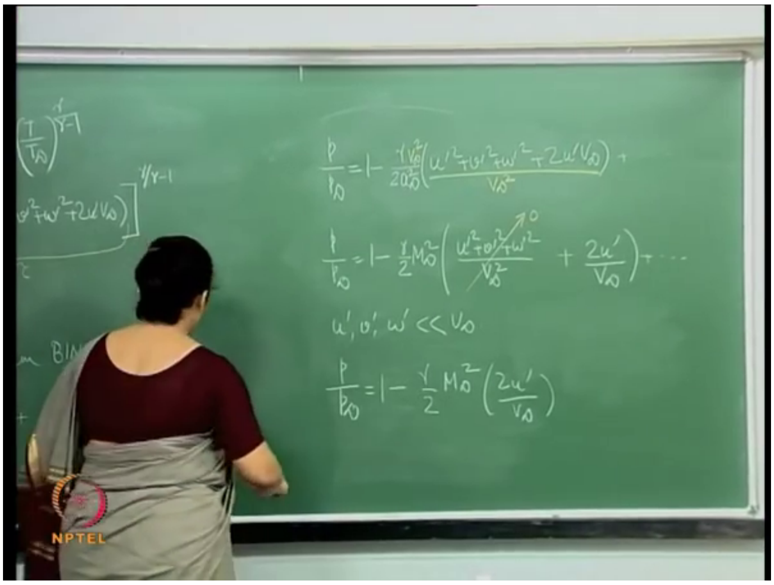

So, what that does is essentially that p by p infinity is therefore than equal to. So, I will just straight away right this if you get a gap multiply this term by gamma by gamma minus 1. So, what we get is this right and then this way this is essentially so we have other terms over here ok.

Now, we will do a very simple thing, when what we will do here is we will take multiply this by V infinity. So, if I do that so essentially what I will do is multiply this whole thing by this was a to the power square. So, we will multiply this by V infinity square right, if I do that then what do I get. So, therefore what I get here is right.

So, this is where I introduce my Mach number so and here what I get is this is what we get right, this is what we get now at this point of time let us introduce this small perturbation theory right. So, small perturbation theory which means right, so therefore the squares etc. So, if I use a small perturbation theory so I can basically say that we can ignore this term. So, what we are left out essentially is this. So, if I am left with this, then what happens to my Cp, let us see what the relationship that we get for Cp.

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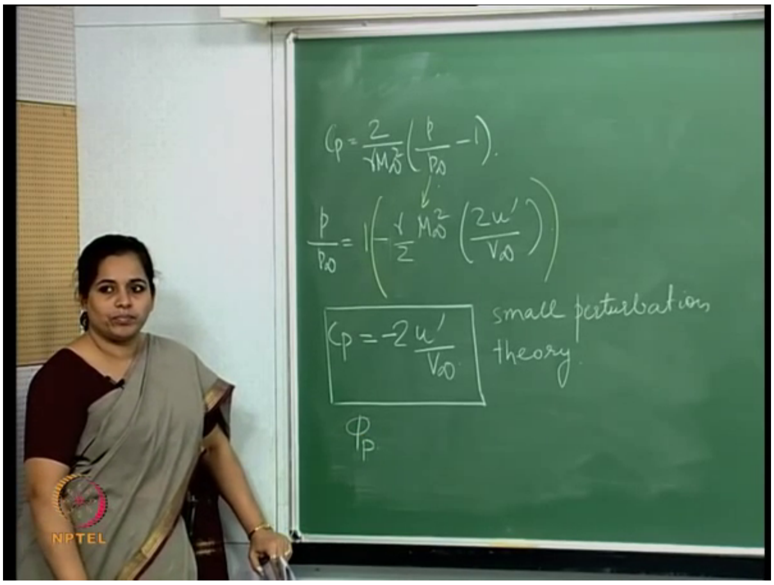

So, now Cp was essentially this was my Cp expression and what I have got now is that p by p infinity is 1 minus gamma by 2 into M infinity square right into this is what we get right. So, therefore p by p infinity minus 1 so therefore, p by p infinity minus 1 is basically this term. So, what we get here so p by p infinity actually is just this term, so therefore Cp is nothing but this right and this is from we have a negative. So, we have a negative sign also so minus 1; so p by p infinity minus 1 is actually negative here. So, let is say we include that is wrong. So, essentially this right so p by p infinity minus 1. So, if I do that so this is my Cp assumes a small perturbation theory ok.

So, now that we have got this now we know the expression for we can find out u dash because now we know we have an expression for the perturbation potential. So, therefore u dash is basically Del phi p Del x right. Therefore this is the small from using the small perturbation theory therefore, the Cp depends only on the u component of the perturbation potential, but it depends on the or just the u component of the perturbation velocity ok.

Now, let us see if we can draw some more inferences from this, it is write out this Cp and see what we get. Now, since we have phi p and expression for the velocity potential let us see and let us plot that over the wavy wall and see what we get.

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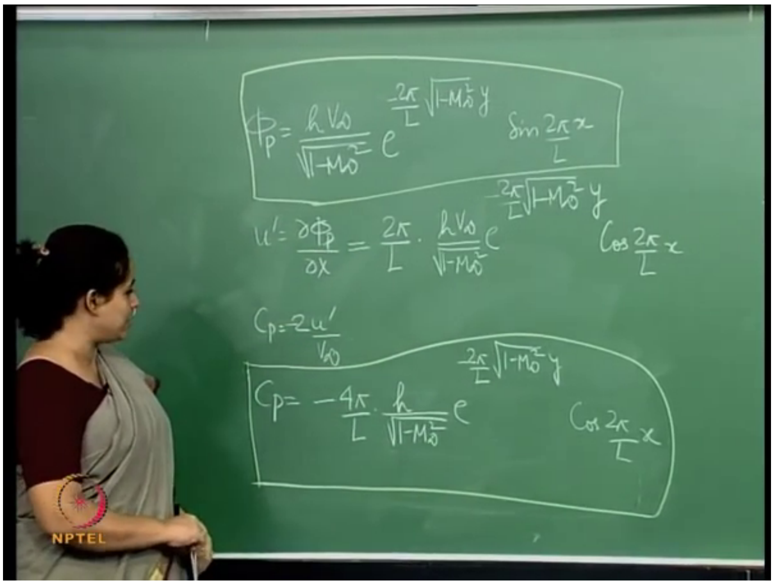

So, let us see what did we get for phi p, so this was our phi p was this right. So, exponential that so this was our phi p, so what we need here u dash. So, u dash is nothing but right this is what it is. So, what shall we get over here? So, this from using this expression for phi p what we get is point right. So, this is our u dash using this perturbation velocity potential so then our Cp coefficient is 2 into this right. So, Cp is 2 into u dash by this so which means that Cp, therefore Cp is equal to minus 4 pi L into h by 1 minus we divided by V infinity. So, that cancels out and e to the power into so now I have an expression for Cp as well, so we have an expression for the velocity potential.

So, this is the perturbation velocity potential and add to that V infinity x and you get surface velocity potential and we also have a expression for the coefficient of pressure right. Now let us see how this looks like. So, this is essentially giving us a feel for the velocities right, velocities this is giving us a feel for the pressures.

So, let us see you know physically what this actually means. So, if I have if I look at this now so let us write out one more term in here. So, this is my perturbation potential. This is the coefficient of a pressure potential, a coefficient of pressure expression and this is what was given to us right, so this is the wall is not it. So, this is essentially the actual equation of the contour of the wall itself ok now.

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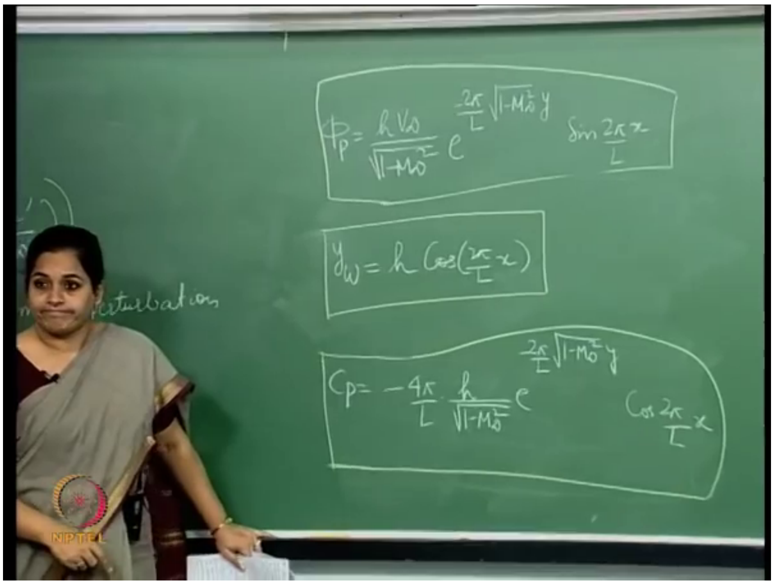

So, what you can see is that in here the perturbation potential, perturbation potential is depending on this sin 2 pi x by L, whereas you have the coefficient of pressure which is a cos 2 pi x by L right.

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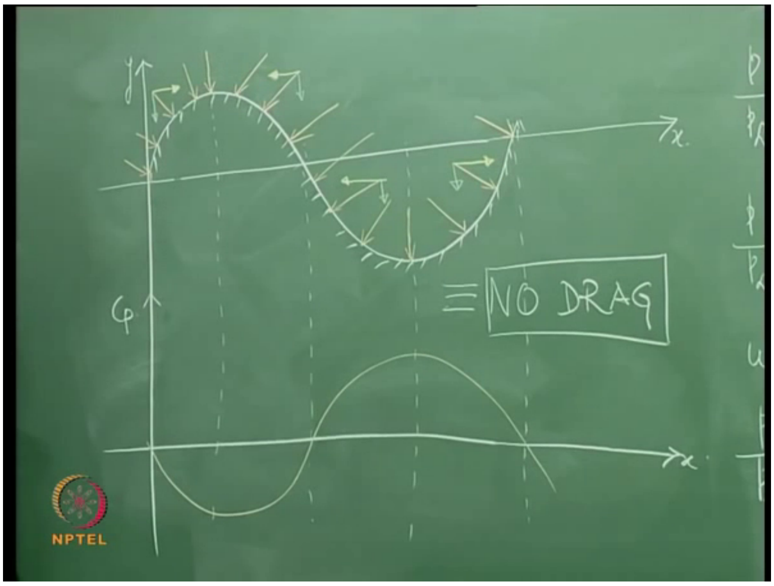

Let us plot this out and see what we get for 1 case; we will actually plot the wall and the other case we will plot the Cp over it. So, you can see that is unlike phi p and Cp the wall and Cp actually vary according to have the same cosine variation, they have the same cosine variation; However, they have a 180 lab lag right. So, let see what that means, so say we have this wall out here, so say we have this wall out here, so this is the wall ok.

Now, what we have here. So now we have, now will now look at the Cp; however is 180 degree phase lag. So, which means if I had to draw the Cp on this curve, on this plot with respect to this I would get is essentially this right. So, this would be my Cp and this is how it would look with respect to the wall itself ok.

Now, what is this mean now what this means out here is that is a pressure is acting on the wall. So, I am going to exaggerate this and draw it like this. So, this is exactly how the pressure looks like, it will act normal to the surface which is what I am doing right. So, this is how the pressure is acting on the surface right. So, essentially this is and this is the plot ok.

Now let us look at one thing you know we will consider say to these you know pressure vectors like this, if I consider this and let me resolve this. So say I am considering exactly there is 2 pressure vectors here, when I say pressure vector just meaning these arrow of a here which denoting the direction of the pressure right.

So, what I am drawing here trying to show here is I am taking exactly to immediately opposite exactly opposite these you know pressure vectors and what I am just resolving this in the x and y direction, if you look at that so similarly say a we have take this right. So, then we have right so this is what we get. So, if you have this you know what do you see what do you know understand from this if you feel and if you do that for every other you know for all the pressure vectors for the entire pressure diagram out here on this wall, what will we see do you see that do you see that the horizontal components these horizontal components cancel each other out right.

So, we essentially have in this case as you seen we have this pressure distribution right. We have this kind of a pressure distribution which is say a 180 degree phase lag compared with respect to the given wall and so then what we see from here is that it is a symmetric distribution right, it is a symmetric distribution as a result of which the horizontal components cancel out right.

So, if this is so if the horizontal components of the pressure cancel out what does that mean physically is that there is right, there is no drag that is what it means and this again do you see this is free if physically possible. So, therefore you know that creates what we know as a d Alembert's paradox ok.

So, what we can you know in the end what we should be a sync here is that let us say we use what we have done, here is we have used a linearized theory what we have done here is written a essentially.

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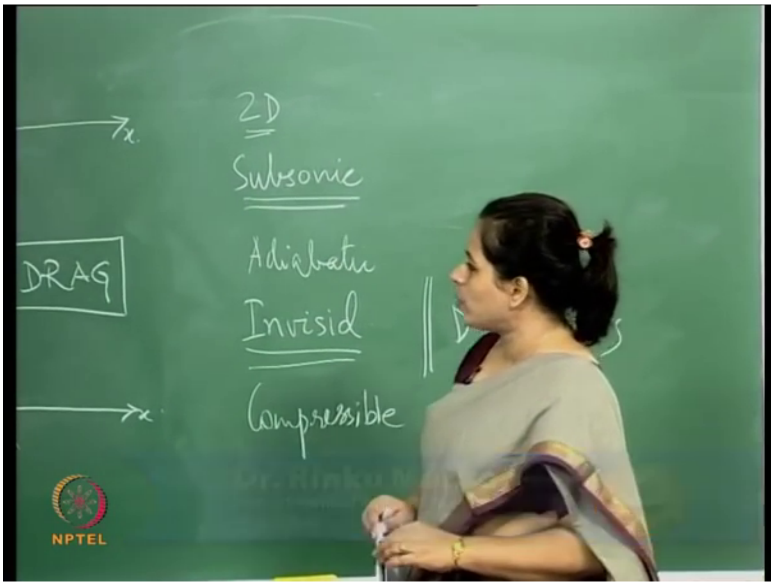

So, this now let us see what this means to us. So, what we have studied here is 2D, we have said subsonic then well adiabatic of course, we have used the velocity potential right. So, therefore this was not and but this was compressible flow ok.

So, in this case so this sort of a problem, when we took the wavy wall we took a 2D problem we said it for subsonic it was adiabatic. So, Invisid you took an Invisid we did an inverse a solution for this and we were able to solve equations etc. What we find however, is that what we get here is that all the horizontal components cancel out of the pressure, so that means there is no drag there is no drag in the x direction there is no pressure component. So there is no drag so this is what is typically known as the Alembert's paradox ok.

Now, also now what we will do here is also look at this pressure here and you can see that the coefficient of pressure varies according to x and y, we will just study this a little more and then we will do the entire problem again for a supersonic case. So, we will continue this.

Thank you.