Advanced Gas Dynamics
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Lecture - 14
Unsteady Shock Waves: The Shock Tube

So, we have been doing shocks and now we have done expansion fans right. So, what we will do today so far do today actually is the theory of the shock tube. Now for that what we will study is the unsteady motion of the shocks. And what we have studied so far is standing normal shocks or standing or say stationary oblique shocks right.

What we will do today is, what if these were not standing if these were moving. So, basically unsteady motion. So, let us see now what we have sort of studied so far is this. So, you have say, an object like that.

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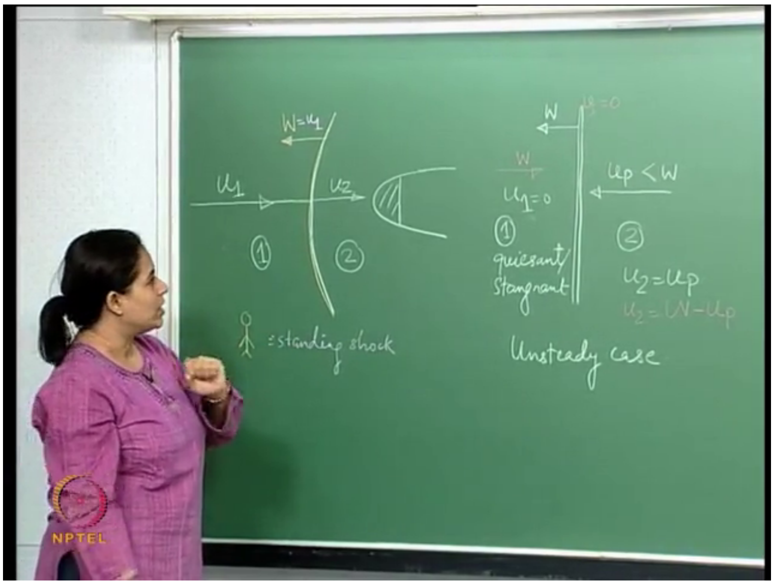

Say this is a an obstruction, right. You have flow say coming in, and say some velocity say u 1 it intersects a certain say shockwave.

So, let us say this is the normal shockwave over here, right. And then it continues to flow in this region say as u 2, right. Now let us say actually the, so what? So, someone is standing say over here. So, standing someone standing over here is actually seeing a stationary shockwave. Now just turn think about this. Now say this shockwave actually was moving this is the shockwave, right. This is actually moving into the flow say with a velocity which is W, right?

So, someone is standing here. So, therefore, someone is standing here. So, in that case if this is moving with the velocity W which is exactly equal to u 1, right. Then the relative velocity between this streamline, and the shock is 0. So, therefore, a and observer standing over here is actually going to see is what we say as a standing shock, right. What he is going to see is a standing shock right. So, this is what we have done so far.

Now, let us do something like this. Now let us say that this u 1 is actually 0. So, let us just say that we have. So, essentially this is my region 1, right. And this is region 2. So, let us come here, and see if we have say a normal shock, say like this. So, this is region 1 and this is region 2, right. And here u 1 is equal to 0, and this shock is actually moving into this region. This shock is moving into this into region 1 with a velocity of w, right. Now and this region here is quiescent, right or stagnant

So, basically you have region 1, with their where the velocity is 0, and you have these you have the shockwave moving into it with a velocity W. And this causes mass motion of region 2. So, let us just say the velocity of that in here is; say up this is usually say less than W. So, in this case this is my u 2, let us just call that as up.

So now if you basically. So, this is the now, this is actually an unsteady case. This is actually the unsteady case. So now, let us try and understand this with an example problem, right. That is, you actually have a flow field like this. So, essentially if you were to numerically you know do this, it would we will see using the problem; that it would probably mean just superimposing. So, if you superimpose this flow field, right with a velocity W. If you superimpose this flow field with a velocity W, then what will happen is; this u 1 will then become W. This becomes the velocity of the velocity of the shock then becomes 0.

So, what we see is here, and u 2 actually then becomes W minus up, right. And then that becomes similar to this standing shock, in that case it becomes standing it becomes very similar to the standing shock, and then we should be able to use the normal shock tables, and the corresponding isentropic tables right.

Now, let us do a problem, and try and understand a little more about this. So, say we have a problem here. So, let me do the diagram.

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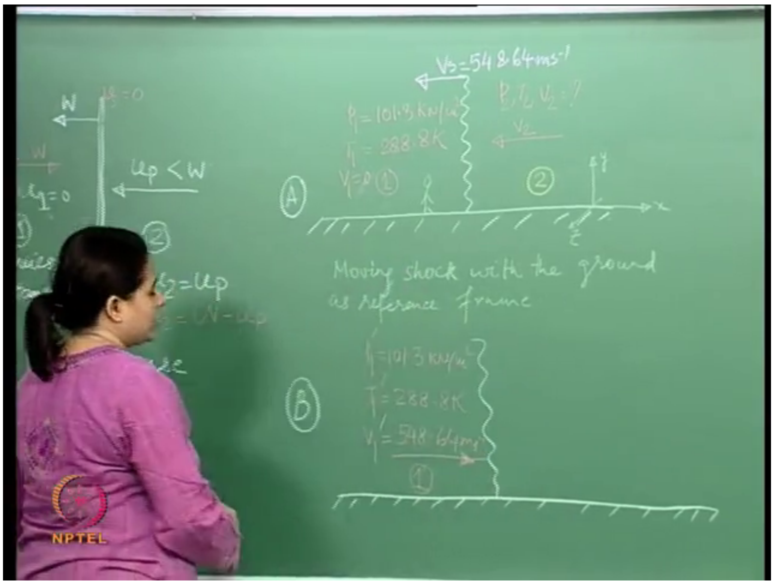

So, I have, right and I have an unsteady shock like that. And this is moving into the flow field, and the velocity of this shock is equal to that, right. And say this is region 1 ok. So, let us say this is region 1, and this is region 2, and let us say. So, the flow properties here are given. So, p 1 is, right then the temperature is, right. And the velocity is 0.

So, what we need to find out is that we have a quiescent region 1 into which a shockwave is propagating with the speed of 548.64 meters per second. So, that will induce motion in this region too. So, what we need to find out right. So, v 2 is essentially, this is v 2. So, this is let us say this is the case a. So, this is case A this is case A. So, this is, so now the point is that this here my reference frame is somewhere over here.

This is my reference frame. I have a reference frame like that. And say I am standing somewhere over here. In this, that particular reference frame. So, what I see in here is exactly this picture right. So, this is a moving shock, right. With the ground as reference frame. Now how do we transform this? Like we said how do we transform this to the steady state case. So, what we will do there. So, we can do that in 2 2 in 2 ways.

Now, one is we will superimpose this entire flow field, right. With the velocity vs equal to 548.64 meters per second to the right. So, in that case. So, let us do that. So, for the steady state case. So, what we will do is we have, this is case B. So, what we will do? So, this is my region 1. So, I have, let us call this dash. T 1 dash is that and now V 1 dash is equal to 548.64 meters per second and in here the flow is in this direction. The shock is not moving anymore, because v as net velocity of the shock is 0. So, velocity of the shock in this case is 0.

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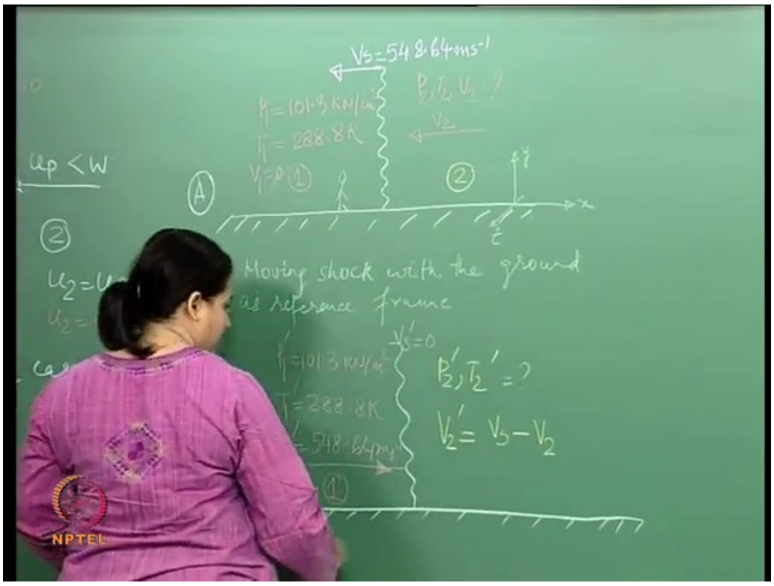

So, what we now? So, therefore so, p 2 dash T 2 dash is equal to what? Now v 2 dash is of course, like we said before. So, we have basically vs minus v 2, right. Or we will just write out the value. Let us just write out the value.

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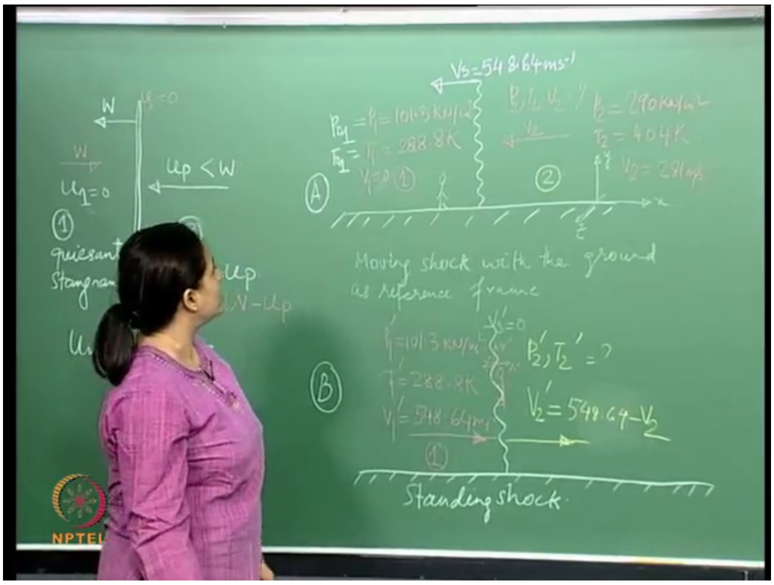

So, which is 548.64 minus 3 2 right.

So, like we said here that you know when the shock moves into the quiescent flow, it with a same velocity W. It induces flow in this region 2, we will call that as u p. So, u 2 and this; obviously, the velocity of this is less than that of the shock. So, when I superimpose this flow with the velocity W in the opposite direction. So, therefore, basically the flow will start doing in the other direction. So, in this case v 2 dash is this. And this is moving in this way.

So now what I have over here is essentially the steady state case. So, we have a flow which is coming in at velocity of 548.64 meters per second, and it is moving beyond the shock at a velocity v 2 dash. And we have a standing shock. And you could also look at this in other way. What we have basically done is moved the reference frame here to here.

So now we have a reference frame over here, or you can say this person I was standing here with on the ground with respect to this reference frame. So now, I am actually riding this wave, I am actually riding on this wave, and taking a look at things going on. So now, basically I have moved my reference frame from here to here. So, let us say that us dash. So, that is essentially the difference between the in terms of calculating this. So now, let us just look at the property. So, what we need to find out is p 2 T 2 v 2, these are the these are the parameters that we need to find. And let us see what we will do.

So, let us look at this picture over here. So, what we will do here is this. So, first things first.

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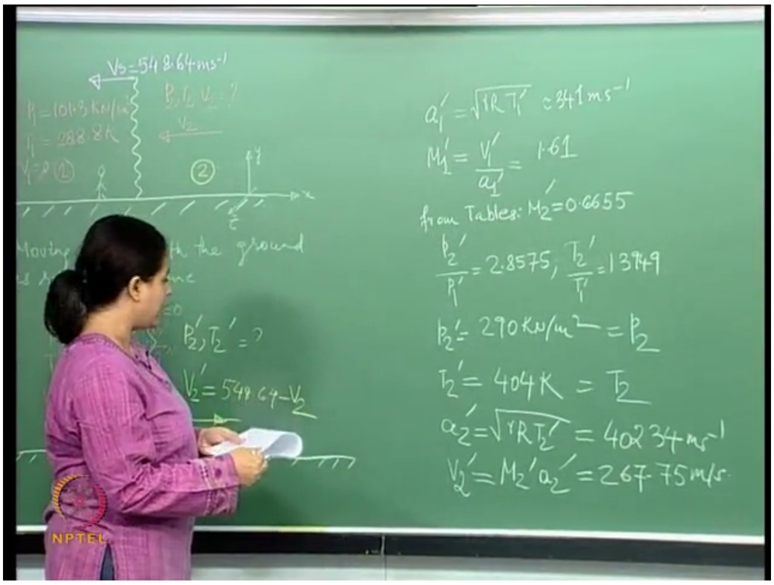

Now, a one dash. So, this we are going to use this table, right. We are going to use look at this picture out here; which is the standing shock. And this is this is the standing shock. And so, let us say this is; so, this is the standing shock. So, what we will do is now calculate the velocity which is T 1 dash right. So, that is around right.

So, we the these value are given. So, the static pressures and the temperatures, now these were given. So, we have this right. So, then corresponding to this we have M 1 dash which is right. So, V 1 dash is again 548.64, right. And a one is what we calculate it is what I get from here is; so, as you can see we have a mildly supersonic flow impinging on this shockwave ok.

So now corresponding to this now corresponding to this, mach number from the normal shock tables, right. From the tables for corresponding to this mach number we get M 2 dash which is point that, we get and we get this. So, therefore, from here if we get this then we need to find we could find out what is p 2 dash right. So, p 2 dash with respect to that is basically P 1 dash is available right. So, we get p 2 dash to be equal to, right. Now T 2 dash is equal to around.

So, this is what we get. Now clearly for obvious reasons now and this these this is the pressure and temperature, right. Now this is equal to right. So, basically with you know respect to the we will basically follow the steps which we have done for normal shocks before. So, once we do this we get the corresponding ratios, and then we get p 2 dash and T 2 dash right. So now, these pressures now p 2 dash and T 2 dash is the same as p 2 and T 2 think about why, right. Why should these change or should these not change.

Now, all we are doing from here to here is changing the reference frame. We are just changing the reference frame, now because of that, so now, the region here; however, you know the properties as such we will see the static pressures or temperatures do not change. So, therefore, this what whatever we have calculated here is the same as p 2 and T 2. So, that. So, whatever we calculate here in terms of pressure and temperature is the same as and we answered the case for the for this region.

So, again now a 2 dash, right. We get this as around 402 point this therefore, now we can calculate v 2 dash, right. We can calculate v 2 dash which is equal to M 2 dash a 2 dash and this is around 267-point meters per second. So, therefore, now we can calculate v 2. Now we can calculate v 2 from here. So, from here we can get basically, let us just say a v 2 so, therefore, again.

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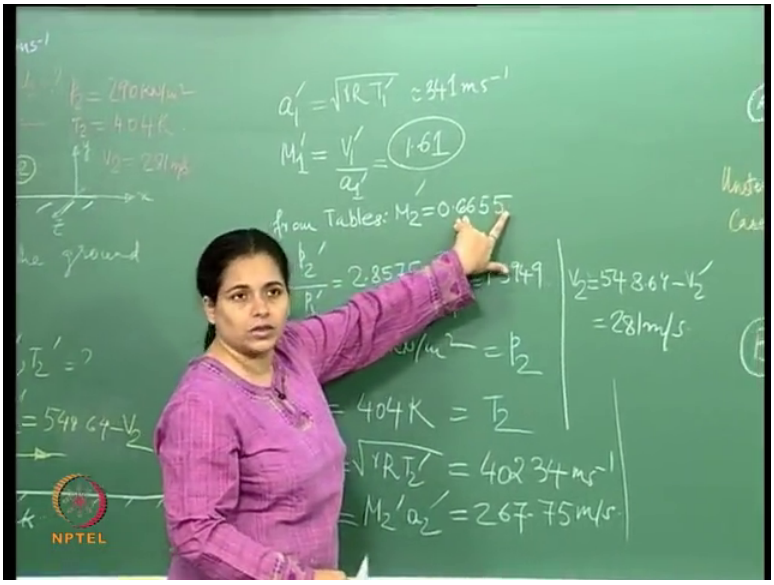

So now v 2 is therefore, 548.64 minus v 2 dash, right. And that comes out to be 281 meters per second. So, we get the v 2. So, such is very not an issue here. So, let us just write this down.

So, p 2 is 290 kilo per meter square, then T 2 is 404 kelvin, right. And v 2 is 281 meters per second.

So, p 2 T 2 and v 2. So, we are basically able to calculate this now. So, like we see that the static temperatures and temperature and pressure is pretty much the same for the unsteady case as well as the steady case. That is what we have seen so far. Now let us go ahead and calculate the stagnation conditions. Stagnation conditions for both these cases, let us do this and see what we get.

So now say for you know for the unsteady case. So, for the case for this for the unsteady case.

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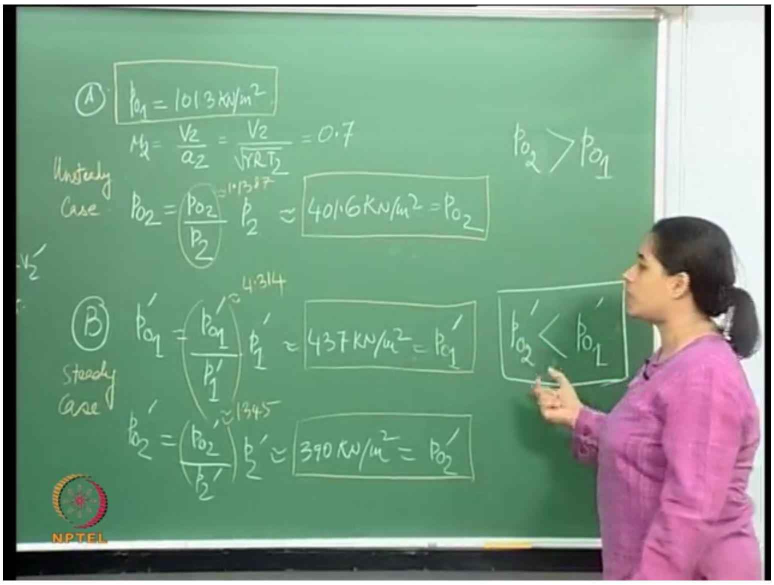

So now, for the unsteady case this is quiescent, isn’t it? So, therefore, I can say that this P 1 is equal to the stagnant pressure as well. Similarly, this is also T naught 1. So, this is for the unsteady case. So, therefore, for the unsteady case p naught 1 is equal to, so then M 2 right. So, in here. So, this becomes. So, v 2 is something we know and this is gamma R T 2.

So, as you can see that now we found out T 2. So, vt 2 is known, right. V 2 is also known. So, if we do that we get a corresponding mach number which is around 0.7, which is around 0.7 right. So, therefore, now p naught 2. So, what we are doing here is the stagnation pressure in region 1 is p naught 1. What we try to calculate now is the stagnation pressure in region 2. So, p naught 2 we can write as p naught 2 by p 2, right into p 2.

Now, what we will do is we will go to the isentropic tables, and corresponding to M 2 equal to 0.7 subsonic, and we will calculate we will get this value p naught 2 by p 2, right. And what we get we get this this is around 1.1387. You can you can check that. So, and p 2, p 2 is something that we have calculated. P 2 is what we already have static condition 290, 290. So, this comes out to be around 402 actually.

So, what you see over here is that the stagnation. So, let us just mark this. So, stagnation p naught 1 is this, right. And this is p naught 2. So, p naught 2 is this, and this is for the unsteady case. So, this is for the unsteady case. Now let us go to the steady case now. So, let us go to the steady case right. So, this is my due to the change of reference frame, but this is my standing shock definition right.

So now here for this in this case. So, p naught 1 dash, right. Now that is equal to, right. Now the mach number M 1 dash right. So, the mach number M 1 dash is what we calculated earlier. So, which is 1.61 now with respect. So, for this mach number again we go to the isentropic tables, right. And we calculate these ratio. Now this comes out to be around say 4.314, and what we get here P 1 dash is known to us essentially. So, what we get around here is 437 kilo newton per meter square, right.

So, essentially this is p naught 1 dash. Now the next thing is p naught 2 dash again similarly. So, p naught 2 dash by p 2 dash into p 2 dash. Again, we go to the isentropic tables, and M 2 dash was point you know 0.6655. So, around that value this graph comes up to be 1.345 again this is from the isentropic tables. So, corresponding to M 2 dash we get this ratio, and then this comes up to be right. So, this is my p naught 2 dash. So, this is my; so, now let us take a step back and see what we are getting. Now what we will see let us let us sort of write this down. So, for so this is for the steady case. So, what we see over here for the unsteady case is that stagnation pressure behind the shock, right. Is greater than the stagnation pressure in front of the shock. And what we see over here for the steady state case that the stagnation pressure behind the shock is less than the stagnation pressure in front of the shock.

Now, this is expected is not it this is what we have seen before. So, this is what we have you know seen before, but what we see is that the nature of the stagnation conditions is different, for the steady case and the unsteady case. And this is happening because of the change in reference frame right. So, this is the difference if you if you see if you if you look at the stagnation pressures. So now, let us go back let us go back again and calculate these stagnation temperatures, and see how those look like. So, again say for the unsteady case which is a case A right.

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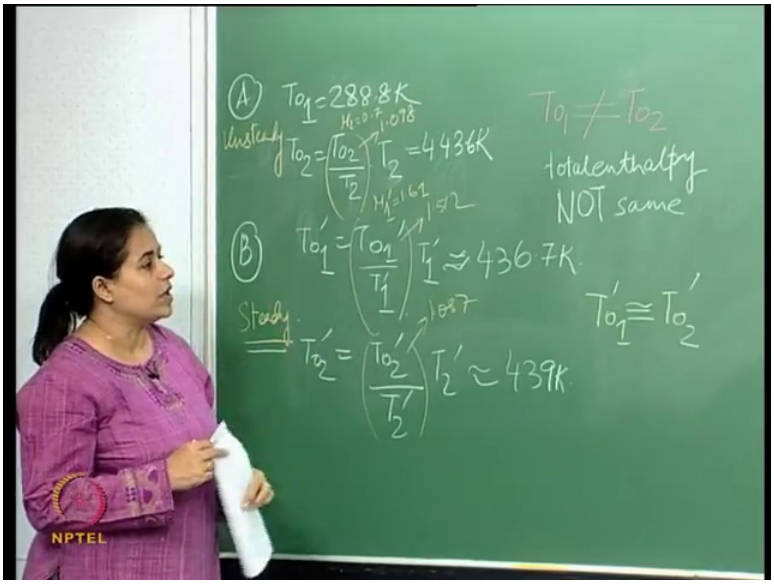

So, if we calculate stagnation temperatures again, T naught 1 is basically given. So, T naught 2 is again T naught 2 by T 1 into T 1 sorry, this is right. So, we get this and again this comes up to be, so we have this here.

So, this comes up to be (Refer Time: 28:10). So, we had a mach number of 0.7. So, M 2 is 0.7. So, for M 2 is equal to 0.7 this comes out to be around 1.098. So, again we go to the isentropic tables, we have calculated the M 2, you know just you know when we calculate the pressure stagnant pressures.

So, based on that from the isentropic from the isentropic tables, we can get this is 1.08 and this comes out to be around 443.6 kelvin, we get this. Now again for the. So, this is for the unsteady case, right. Now from the steady case. So, this is for the unsteady case. So now, for the steady case. So, again here, so for M 1 dash which was 1.61 right. So, for M 1 dash which is 1.61, this becomes t this ratio is around 1.512 right.

So, then I get this t is around 436.7 kelvin. So, I am I have kind of really approximated this. So, you can get you can really do better calculations than me and you know get more correct values. I am just basically trying to drive home certain points. So, then we do this. So, similarly this is what we have been doing so far. So, so again this; so, this value this value comes out to be around 1.0 and 2. So, again this comes out to be around 439 kelvin like I said there are a lot of approximations. So, essentially what we see here, what we see here is that that for this is for this steady case. Now there is a very stock revelation here. What we see over here is that for the unsteady case T naught 1 is not equal to T naught 2, right is not equal to 2, but for the steady case.

So, which essentially means and a total enthalpy. So, stagnant temperature is here. So, which essentially means that the total enthalpy is not constant across an unsteady shockwave. Whereas, for a steady case it is. So, for a here total enthalpy not same. Total enthalpy is not same across an unsteady shockwave. Whereas, total enthalpy is conserved across a steady shockwave. So, that is what we have seen from this example. So, essentially what we see is that the static pressures and temperatures they do not change. So, we can just sort of superimpose the velocity, and then you know look up the table as we did for any normal shockwave; however, the stagnant pressures and temperatures they are not same, because total enthalpy is different because of this because of this changing reference frame.

So, alright I think we will go ahead now and into the shock tube; which is what we wanted to which is which is what is I want to start today. Now so now, a shock tube is actually a practical application of shockwaves, and expansion of fans etcetera, it is used widely. So, we are going to kind of briefly study the theory of that. So, what we saw here in this particular case that we have a flow in a steady case what we basically see there is a flow coming in right. So, you could now just think of you know the shockwave which is moving exactly the same velocity here, this is also moving velocity here.

So, this is if you put the reference frame over here with respect to this. So, this becomes stationary and this is the flow in front is moving this way with the velocity V 1 and then it is moving with v 2 dash. Now there is a start difference here though, that what we are actually seeing is that we have a quiescent flow stagnant flow and a shockwave is moving into it, right. That is causing the flow in this region to there is a mass movement of the you know mass in here in region 2, which moves starts moving with the velocity v 2.

Now, we could do this. So, essentially what we can do which is the shock tube right. So, we can implement this say in a, yes. So, it for a for a shock tube. So, essentially what we will have is they right.

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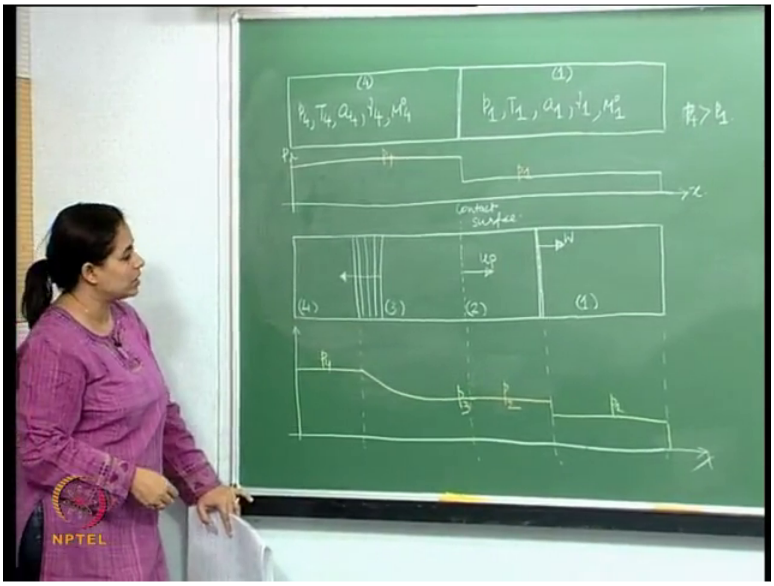

So, we have a tube we have a duct set, and this the there are 2 chambers in it which are separated by a diaphragm. This is separated by a diaphragm, let us call this session as 4. Let us call this section as 1.

Now, this section has a pressures of p 4 t 4. So, essentially it could be different gases they do not have to be the same gas. So, I put the gamma as 4. So, P 1 T 1 and they also have different molecular weights. So, essentially it could be m is the molecular weight. So, let us just say molecular weight the differential is from mach number. So, essentially this is this is the system that we have. So, we have a tube, we have we can have the same gas or 2 different gases, right. A and there are do these 2 chambers here that which are separated by a diaphragm. And then essentially this p 4 is greater than P 1.

So, in here p 4 is greater than P 1. Now before I start this let us go back to this problem one more time. Now what you can see here is for a standing shock, the way we look at all the properties. The way we developed all our relationships. That we said the property changes, or the properties behind the shock etcetera or the strength of the shock you know weak or strong basically is a function of the incoming mach number, right. That is what we did; however, if I look at the unsteady case I cannot do that I mean this is quiescent. There is really no mach number property out here. So, in this particular case the change of properties is basically governed by the ratio of the pressures. The pressure ratios between the 2 regions. So, in because if you see over here, I can you know we have not talked about a shock shockwave if at all, but even if there is a shock say now, but for some reason by some way we are able to create a shock over here. Now that is basically now that is a shock which we will start moving. And then it will cause changes in properties.

So now the shock the way it will move and whether it will be strong or weak and which way to move which way it will move to the, right. Or the left basically will depend on to this pressure ratio. If p 4 and p 1. So, for unsteady cases. So, the pressure ratio is what is dominant. So, if we have this. So, let us say so, if I am going to draw this. So, if I say let me draw this. So, essentially what I have over here is this.

So, let us just say this is x, right and this is pressure. So, therefore, this is p 4 and this is p 1. So, this is it. Now what we will do is we will break this diaphragm, we will break this mechanically or using some you know passing electricity or voltage. So, we will break this diaphragm, then what happens? So, let us come here if; so, what I have done now is I have broken this diaphragm. So, what I will have in here is that there is a shockwave, there is a shockwave, which starts moving into this quiescent flow, and I am going to call the velocity as 2. So, this is region 1 again, right. And I have an expansion fan; which starts moving into this region. Now this is again the region 4.

So, let us look at this. So, I have broken the diaphragm, what that will do is it will create a shockwave which starts moving to the right and let us say the velocity of that is W. And it is also going to cause an expansion fan to move into region 4, and yes to region 4. Now this so, before that now what this is going to then cause is cause the region. So, this in here u 1 of course, is 0 as we know now again this is going to cause you know as we as we said mass motion of the fluid of the fluid behind it. So, say this will stop moving say a velocity u p. And so, again let us call this region as 2, and this region as 3.

Now, this is actually this is similar to the slip lines that we spoke about, right. When there was interaction of shocks. So, this is similar to that and there is a discontinuous entropy change across this line. So, across this body so, we go across this say a slip line. So, this we can we can basically say as contact surface. Let us call this as say contact surface. So, let us just draw the so, how does the pressures look like? So, what we have over here is that in here again we will have. So, this is the. So, let us say. So, let us divide these regions. So, we have this, this in here, right. As we have done before so, let us this this region is say p 4. So, this is p 4 and this is P 1.

So now in here. So, as we have done earlier, across this constant contact surface, right. The enthalpy is discontinuous, but the pressures are same. So, in here the pressures are going to be same. Now will p 2 be greater than or less than P 1. Let us go back here. We calculated p 2 to be larger than P 1, remember this problem that we did. So, this is the this the this is this is for the unsteady case, and we found out that when you have an unsteady flow like this. Then the pressure behind the shock is the pressure in front of the shock is actually more than that behind it. So, therefore, p 2 is going to be larger. So, therefore, in here. So, let us just say p 2 is here in this region right. So, this is p 2. Now for. So, I am just draw this right. So, let us just say this is my p 2. So, this is p 2. Now for this region now for this the for the for the third region, here the pressure is the same. So, let that pressure be same over here, but this is an expansion fan. So, it will gradually increase the pressure up to p 4.

So, this is basically. So, at this point. At this point it is p 3. So, here p 3 3 is equal to p 2 and then it increases up to p 4 the pressure here. So, this is essentially our shockwave. So now, if I were to if I were to just look at say let us look at just the shockwave part of it first. So, essentially what we have now in this case, right. Is a an unsteady shockwave which is propagating into region 1, right? And we have a an expansion fan which is propagating into region 4.

So, what we are going to first look at is this right in front of the properties in these 2 regions over here. So, similar to what we have done in the problem so far. So, if we do that now the governing equations, right. The governing equations let us just for the for the steady state case right. So, the for the steady state case we had right.

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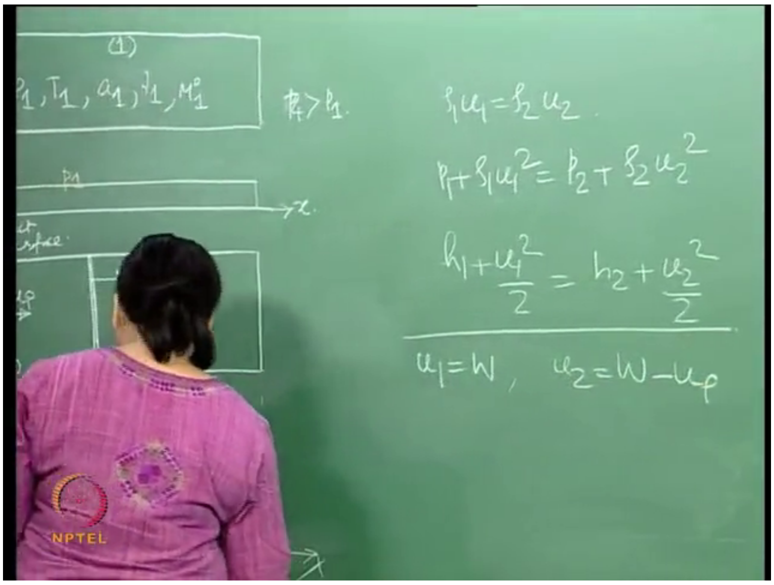

And so, this is the continuity momentum and energy equations right.

Now, as we did in the problem, what we basically what we will do out here is now these velocities you are new to by this time now you understand, this is basically with respect to the wave, right. This is where. So, the reference frame is basically, if you are sitting on the wave or for a steady state case for, so the shock is not moving it is stagnant. So, we just have to keep that in mind when we look at this picture over here. So, in this case our W 1 for our unsteady case here. So, u 1 is essentially W right. So, again that is what we will do it will impose the whole impose the impose a velocity W in exactly in in this direction right. So, that this becomes stagnant. So, our W 1 then therefore, becomes our u 1 therefore, becomes W and u 2 becomes W minus up right.

So, if you are if we are basically going to now we are just going to consider the turning shockwave. So, in this case like we did in the previous case, we will impose a velocity which is W exactly in the opposite direction. So, initially this one u 1 is 0.

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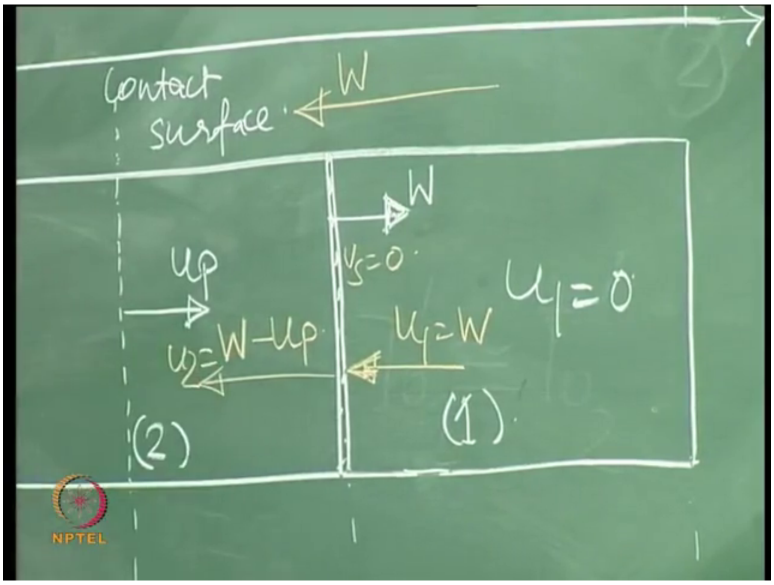

Now when we impose the velocity, then what happens? So, if I impose a velocity like this, which is W, then u 1. So, new u 1 is equal to W, which is in this direction, right. This new W is 0 velocity of the shock now becomes 0, and this velocity then becomes W minus up. So now, then this is exactly similar to a standing shock. So, the velocity in front which is u 1 is W and velocity, beyond that this is your u 2 is equal to W 1's u p. So, that is just a reference change. And so, what we do is put these values in here we just put these values in here. And let me sort of write that down. So, what we get in terms of how the properties look like right.

So, what we get in here is this.

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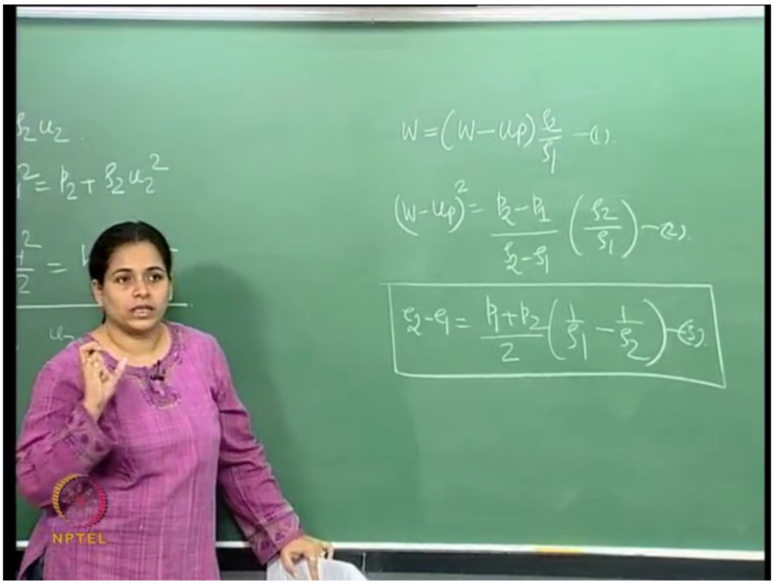

So, W is W minus u p. So, we get that then again, we get then we have. So, this essentially your governing equations reduce to this. Now also now what you see from here is that this is the exact same (Refer Time: 49:37) equation for a standing shock. And which should be the same because in (Refer Time: 49:43) equation will basically relating all the thermodynamic variables often in front of the shock, and we are not really relating that to the mach numbers right.

So now so, let us just write down 2 more relationships; so now, for a calorically perfect gas. So, for a calorically for the calorically perfect gas right.

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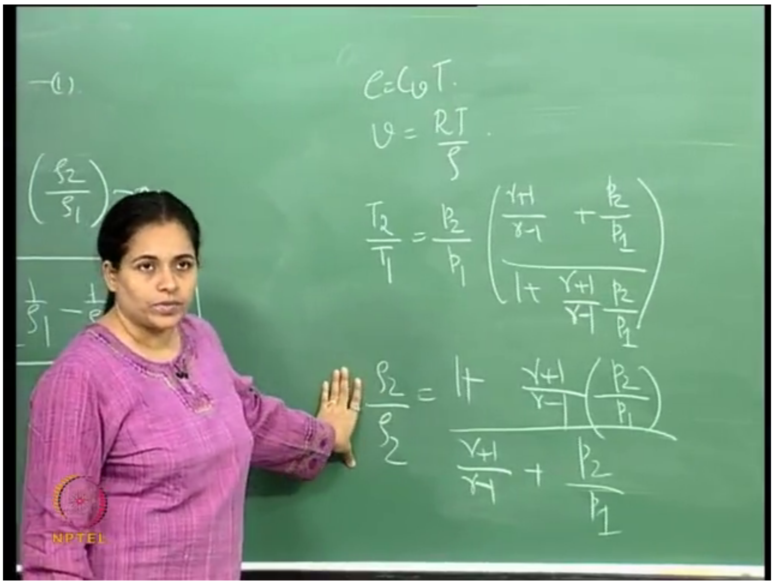

And so, using that you use using that into this equation we will get these 2 relationships, right. Now we will get these 2 relationships.

And so, what we are basically what you can see over here in this the way I am writing this that the change in properties are essentially being related to the pressure difference across the right. So, you can see the change in temperatures is basically in terms of the pressure ratios in front and behind the shock. So, that is the same for the density change as well. We will continue from this in the next lecture.

Thanks.