Engineering Drawing and Computer Graphics
Prof. Rajaram Lakkaraju
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Module - 02
Lecture – 11
Conic Sections – III

Hello all, welcome to our NPTEL online certification courses on Engineering Drawing and Computer Graphics. I am Rajaram Lakkaraju from Mechanical Engineering IIT Kharagpur. We are in module number 2, lecture number 11, especially focusing on Conic Sections.

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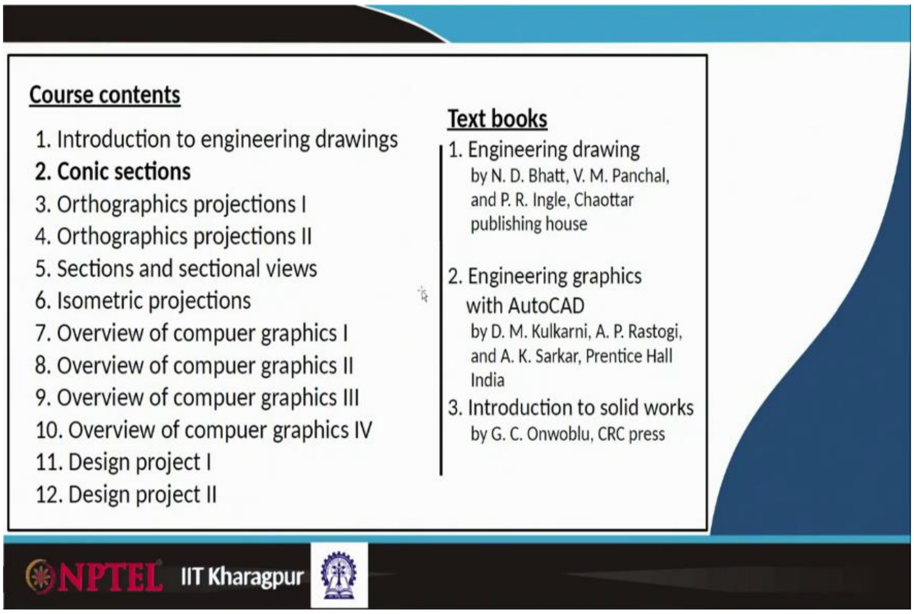

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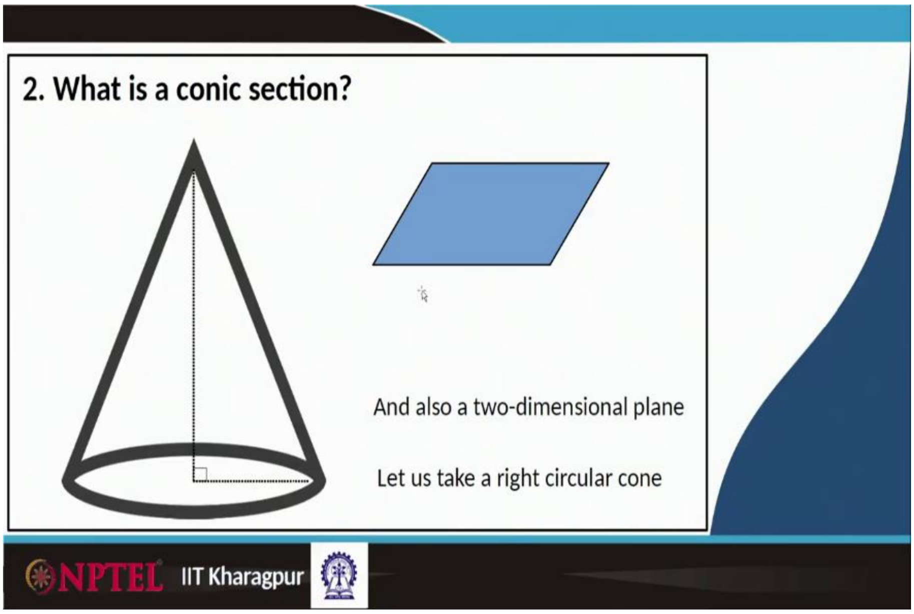

So, first of all, let us look at what is a conic section? Let us take a right circular cone; right circular cone, if we are dropping from apex a line, it makes a perpendicular line to the base. Let us draw that.

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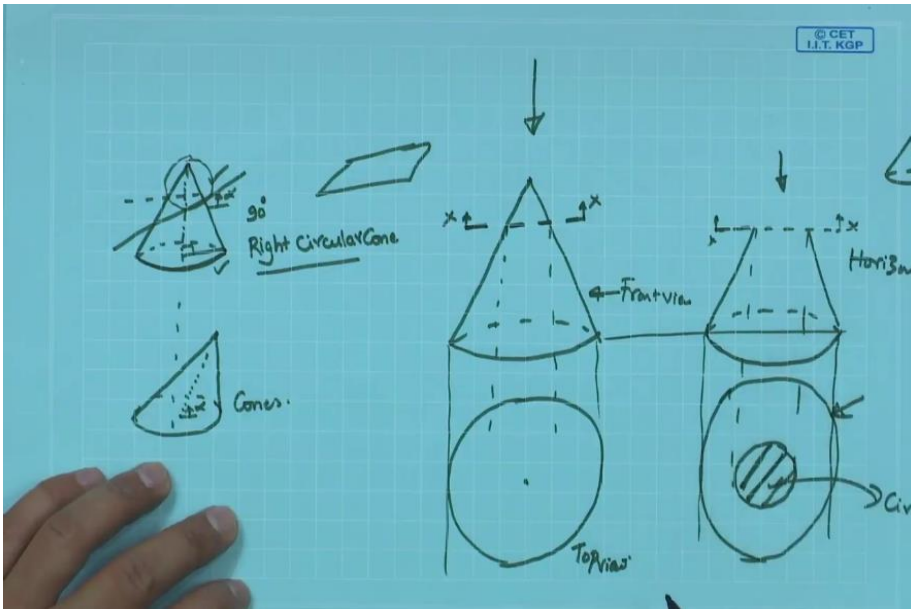

So, if I am taking a cone, so, usually in drawing practice, anything behind the object which is not straightforwardly visible, we show it by dashed lines. So, this is a standard convention in engineering drawing.

This line, if we are looking from the frontal view of a cone, this is visible; and this one may not be visible if it is opaque kind of object. If it is transparent, it might be visible; in that case, we will show it by dashed lines. So, any invisible kind of things and the object is present, we show it by dashed lines, and this is the apex.

From the apex, if we are dropping a perpendicular, and usually centre lines we represent by dash-dot convention. So, this one with the base it makes a perpendicular angle; that means it is 90 degrees, such kind of cones are called right circular cones. In certain cases cones might be non-right circular; that means let us draw a circle, your cone base might be in that way.

So, this one if from apex all the way to the centre of the circle, if we are joining that may not be 90 degrees; it makes an angle alpha, such kind of cones are called right circular cones, and these are generally named as cones. Let us take a plane, is more like taking a knife passing through cone; we can cut the cone perhaps at that section. So, slicing the cone in the horizontal direction, one can make it.

Similarly, one can slice this cone somewhere away from that horizontal location; that means with an inclination angle alpha, that also we can really make a slice. In both cases, if we are removing the top part, we will get a different kind of curves.

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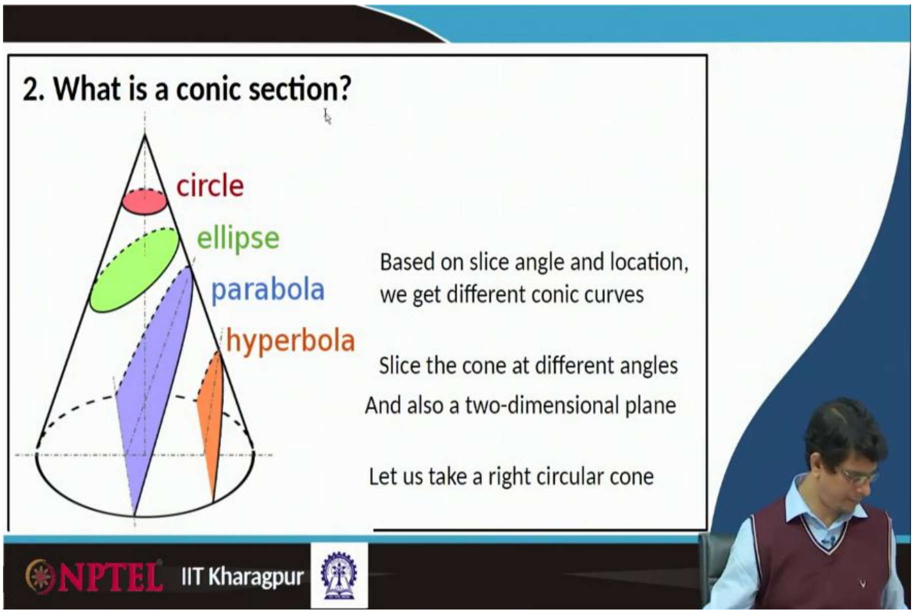

For example, for the same cone the right circular cone; if we are making a cut section, usually cut sections are shown.

If we are making something like slicing or whatever, we show it by long dashed thick, long and thick dashed lines; that indicates a perhaps there might be a section. Usually, it ends, we show it something like a mark x-x; that means it is a kind of plane which we are going to slice it, cut the section and show that view.

So, if we are looking at a view all the way from the top; a cone without any cut sections, perhaps it might look like a circle. This kind of views what we are calling, from the top we are trying to look at this is what we call top view. We will learn more about these views in later part of the sections and this point, a tip we will see it is something like that. This is what we call front view; from the front side, we are looking at it, and if we are looking a cone from the top, it might look like a circle.

So, let us take a section x x, slice it; remove this top portion, so that if we remove that top portion, perhaps this part we have removed it by section x x and the remaining part perhaps kept it there.

So, if we are looking from top view for this cone again, this part we see it like a circle. So, cut section, so usually, we show it by hash line, and this is non-cut section; but the material is present, so again a larger circle we will see.

So, this entire circular base; one circle we will see and the cut section, we removed the portion, so through hash lines, we see it as a circle. So, any horizontal plane for a right circular cone if we are slicing it, we will see it as circle here, this is the circle.

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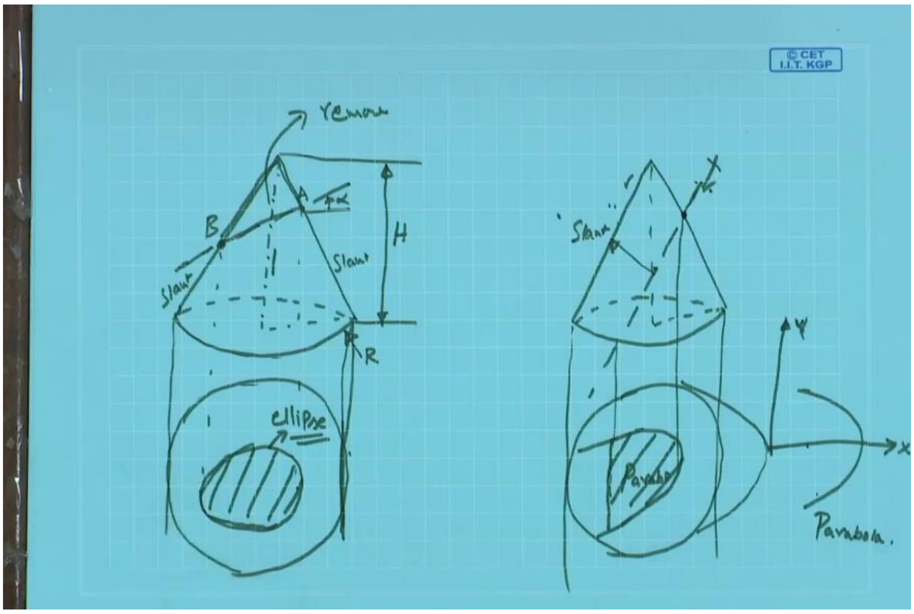

So, let us look at an inclined section. Again for a right circular cone; we take another cut section, this is the centerline, this is the radius.

Let us call this one this; our convention is in drawing we will show it exterior to that object, let us call H and this one radius or diameter we can show some radius.

Now if we are taking an inclined section making an angle alpha, remove this top portion, remove it and visualize this part. So, this bottom portion looks like a circle in the top view. So, this circle will be bounded by these lines and this point matches there and this point matches there; this is on the top side, this is on the bottom side.

So, if we are looking at it a circle, we will see something like in that way. This is an elongated circle which we usually call ellipse, having a major axis and minor axis. So, any inclined section, but it has to cut on both the sides let us call A point, B point; on both sides of this cone if it is going to cut the slant once. So, this is one of the slants, one other one; around the entire circumference, we have a slant.

So, on both sides of the slant, if it is going to intersect here and here; then the top portion if we are going to remove it, the remaining leftover portion we see it like an ellipse. So, let us look at the other inclined section, for the same right circular cone. Now in this case parallel to one of this slant, this is the slant edge; parallel to this slant, if we are making a cut section.

So, we have to move parallel to that and perhaps make a cut section. So, this line, this line if it is parallel; the projected one this part in a circle we see as a parabola.

So, we are removing this. So, up to this portion, we see it as a parabola. This is the way we construct a parabola. So, if our coordinate system is on the inclined plane like x-axis normal to that y-axis, flip this x-axis, y-axis; then we will see something like a parabola in that way, x-axis, y-axis, it depends on where exactly we are translating.

If we are focusing centred around that, if this is the origin of the coordinate axis; then our parabola passes through this point. If the origin is somewhere located, then we will see parabola in that way. The first point what we have to remember is, from slant if we are going in a parallel direction, make a cut section, remove; the top portion the leftover portion on the coordinate axis of x and y we see it as a parabola.

If we are picking other inclined section, we see something named hyperbolic curves. So, based on the slice angle and location, we get different conic curves; that is a reason we call, from the cone. These are constructors, so we call them as conic curves. On the sections are generated from the cone by slicing it; so such kind of sections are called conic sections, and these are very powerful curves in engineering applications.

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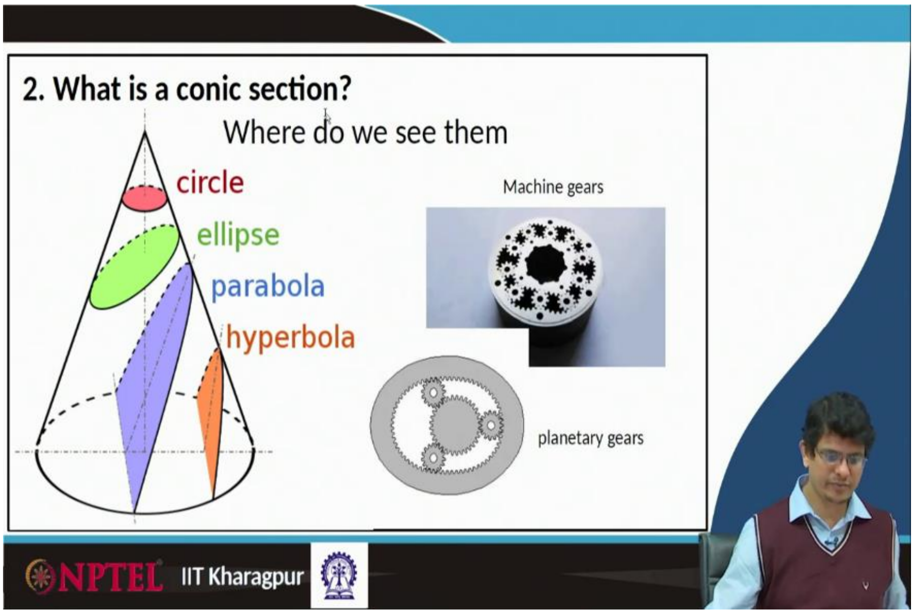

For example, let us look at any machine gear or mechanical gears; in automobiles, perhaps for the cycle, motor vehicles, even pulley shaft kind of things, the circles are quite common features. So, the circle we will get it by slicing it horizontally to a right circular cone. Ellipse is quite common for planetary gears and off centred kind of transmission kind of systems.

So, here let us look at that planetary gear. So, here if the driving motion is made by this circular gear if it is centred that; we will be in a position to transfer this power to this gear and that gear and finally through this planetary gear kind of arrangement also.

So, this planetary gear sometimes comes here, sometimes goes up through major, minor axis and transmit the power to the required locations; this is the way we get a power transmission from this circular, elliptical kind of gears.

And to design anything or perhaps to send it for production line, constructing ellipse inside of that constructing gears is the most important thing; for that purpose, we are learning about this how to construct an ellipse or circle.

Mathematically also we can represent it by x square by a square plus y square by b square with constant number or 1; that way geometrically we can construct, but when machining and other things are happening, the mechanical tools has to construct this ellipse on a metal place.

So, there we do not straightaway use this pen, paper kind of thing; the mechanical arrangement has to be in such a way that, finally it construct an ellipse. For example, if we are taking your mini drafter, one part of the mini drafter is fixed on one side and once you tighten this scale; wherever you move this mechanical drafter, it always construct vertical and horizontal lines.

So, for that purpose, we are using a four-bar mechanism to construct this kind of vertical, horizontal lines. So, your mechanical device though we are not drawing constructed horizontal-vertical lines; but the bars linkages move in such a way that it always construct these horizontal-vertical lines.

Similarly, when you are machining these tools; if one requires this kind of ellipse is, there should be a standard protocol to construct these ellipse and circles, and that is the thing what we are going to learn for this conic sections.

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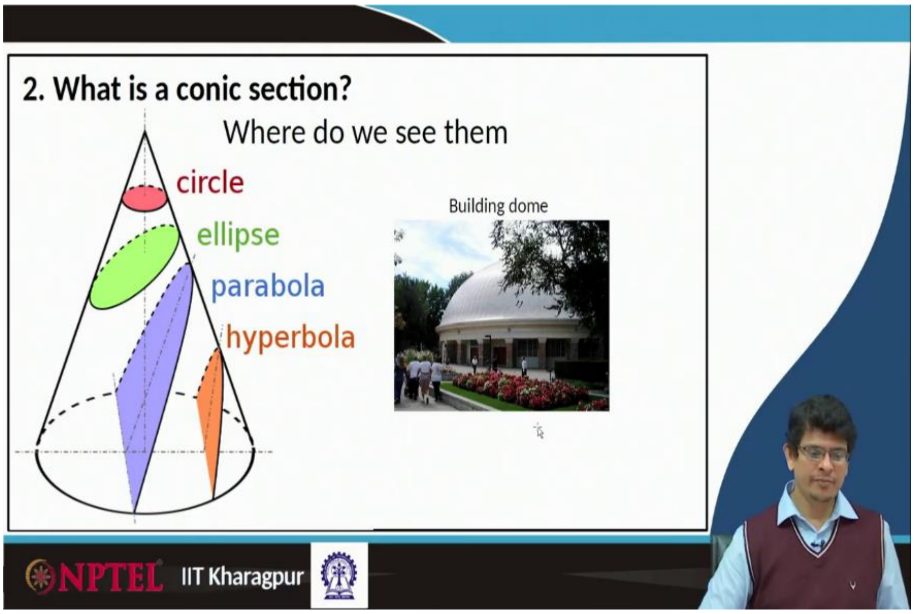

The variety of applications, for example, like building domes; here there is a picture, a construction, top of that it might be a circle, it might be elliptical kind of curve. So, if we are looking at on the exterior of the surface, it might be making an ellipse. Why it is an ellipse? Because here the major axis and the minor axis are of different lengths, that is a reason it forms an ellipse.

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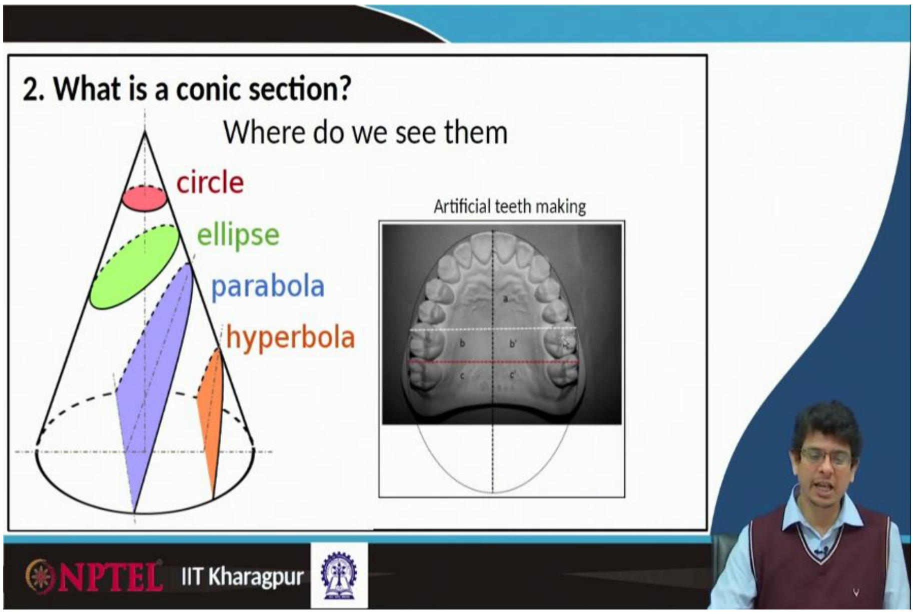

For a dentist, if he wants to make these teeth, artificial implants and so on. First of all, he has to really design where exactly these teeth location, tooth location supposed to be there and which form it this tooth has to be arranged.

For that purpose, usually our human mouth it is in the elliptical format of the different major axis, minor axis; so for an ellipse. So, here ellipse is shown by these dashed lines; the major axis is this longest one, and the minor axis is this shortest one. At different locations normal to that major axis, teeth will be arranged.

So, a dentist has to arrange, first of all, construct either through software or geometric construction; locate what should be the teeth and where exactly it has to be located. So, for that purpose, first of all, one has to construct an ellipse, this is the idealistic case.

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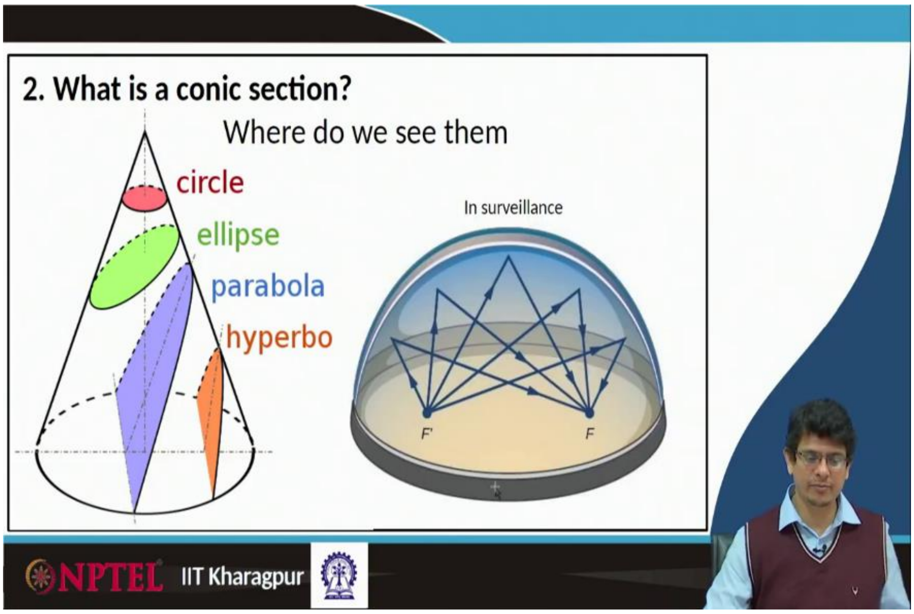

Similarly, in surveillance like cams, CCTV cams, spy cams; the light rays are perhaps from the object the reflected rays comes, finally focused it either one point or two points.

So, all these light rays come hit the surface; internal reflections happen; finally, they will be focused at one or two points on this line. If we are looking at this, perhaps this curve forms an ellipse. So, it has always two foci centres; F and F prime.

So, all these rays internally reflected, finally converge at these points. Or if we have light sources at two locations, they will be emanating in radial directions; but all these things internally reflected, finally focus two points F and F prime. For surveillance, cameras also we require this elliptical kind of sections.

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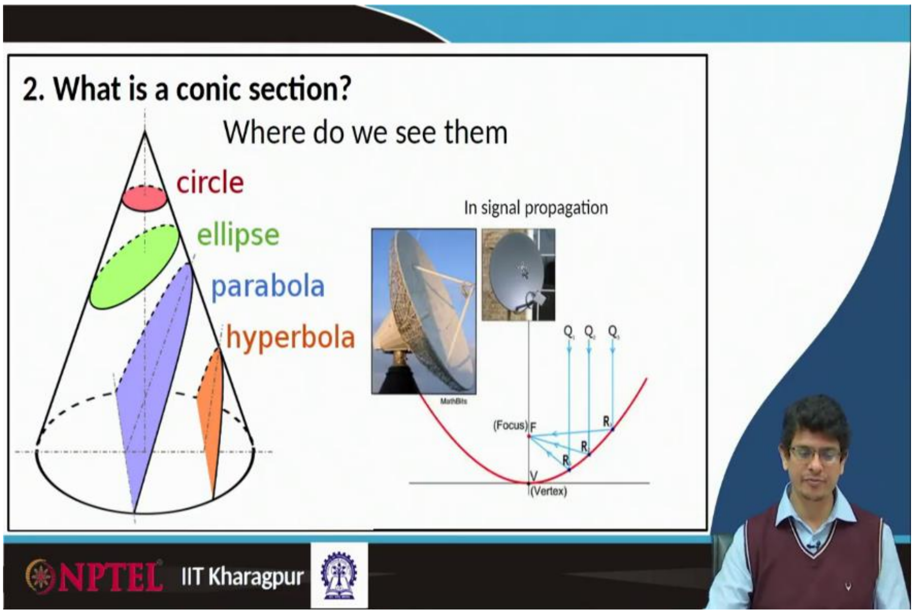

Let us look at the first signal processing. For signal processing, either these satellites sending or from extraterrestrial waves are continuously coming; they might be parallel beams which strike this parabolic kind of dish antennas mirrors, goes finally in reflux and converge to one point. That strengthens the signal; from there, you collect it, pass it for data analysis or for the TV.

So, for example, here let us look at our typical dish antenna for this communication. These entire points will be focused at one point. So, the parallel beams from satellites coming, they will reflect touch this parabolic mirror, reflux at different locations; from there finally, converged to a point focal, focus or foci point and this point what we call vortex for a parabola. And once this is converged is amplifiers modulated and finally send it for signal processing.

So, our parabolas are majorly useful for this kind of signal processing applications.

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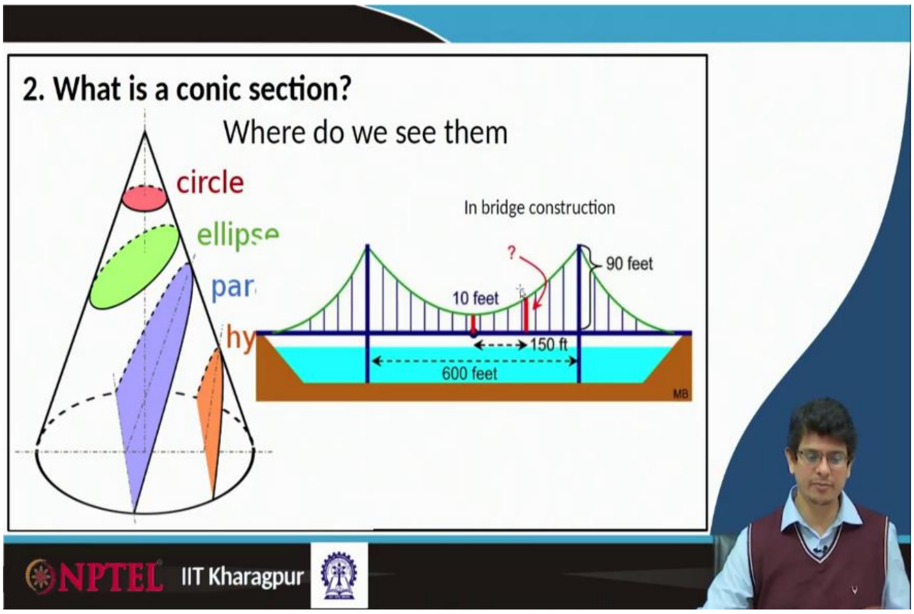

Similarly, if it is suspension bridge; so here this is the river which is coming in the streamwise direction and above which there is a bridge. Usually, the suspension bridge, the load of the cable has to be distributed; so there will be mass or pillars, and there will be more like a cantilever kind of suspension bridges will be there.

And these will be connected by ropes, the entire weight will be suspended on these wires, and typically these wires will be of parabolic kind of path. So, for bridge construction also constructing these parabolic curves and determining what should be this rope length from one point to other point is very much required; unless we construct the parabola, we will not be in a position to know how much length it is supposed to have for the load distribution.

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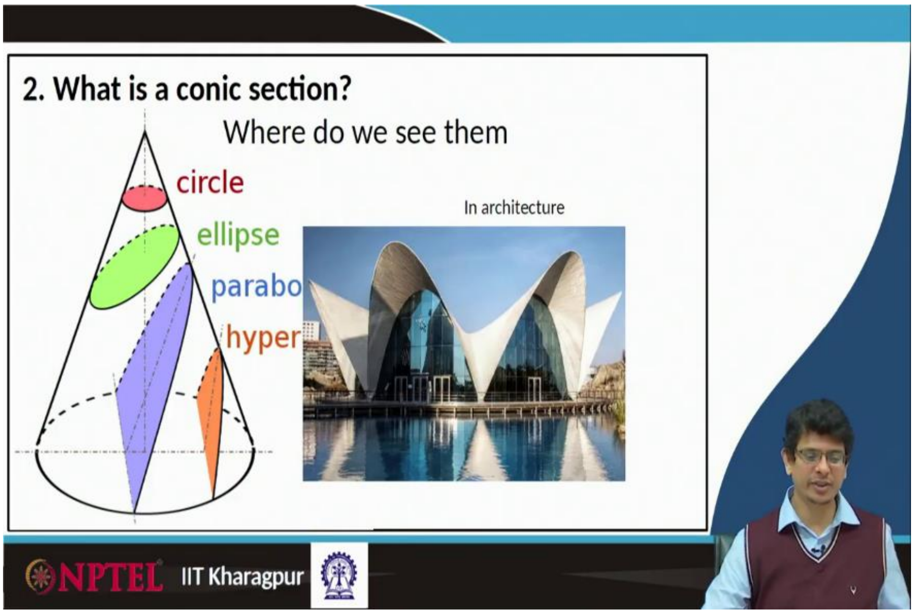

Similarly, in architecture also this parabolas gives aesthetic aspects in nice appeal. So, here for these domes, we see that kind of parabolic constructions. These parabolas sometimes might be inclined on the vertical direction also; it may not be the y is equal to x square kind of parabolas or x is equal to y square kind of parabolas, but with off-centred and tilted kind of parabolas also we will come across.

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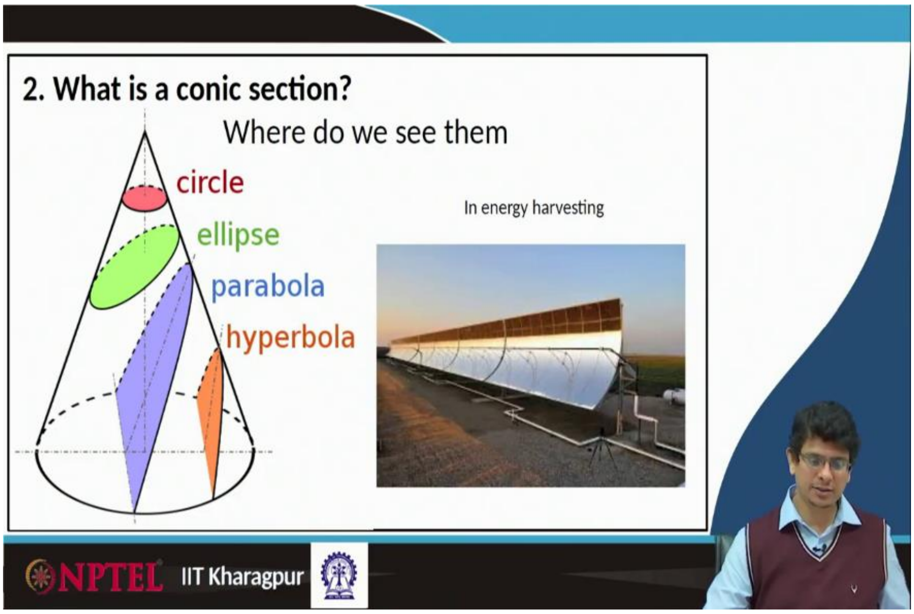

Similarly, for energy harvesting, these days green energy people are aiming for; for that purpose, we have to collect this solar power, heat the panels or converge through solid-state kind of devices, all this photo photonic energy will be converted and finally transmitted.

Or perhaps for solar heating applications, you collect these solar beams; focus on one point through which water will be passed, so that temperature will be increased and finally utilized for solar heating of water. For that purpose also people go with parabolic kind of mirrors.

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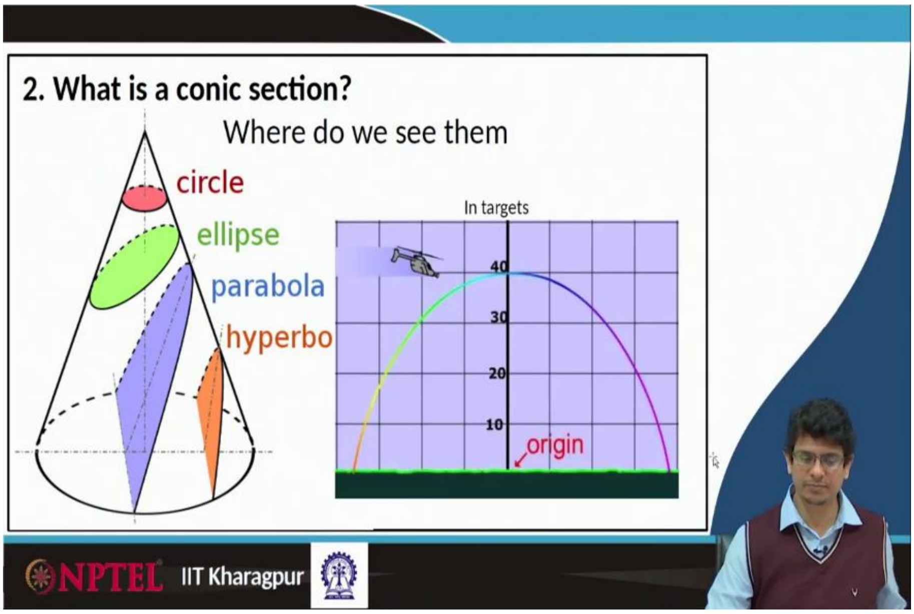

Similarly, to target specified application, for example, there is a helicopter which is releasing a bomb; it usually goes in parabolic format, it might be bombed or perhaps in calamities supplying food products to flooded zones. With uniform velocity, if this helicopter is moving; if it releases a packet, it comes in parabolic format, and finally this is the origin.

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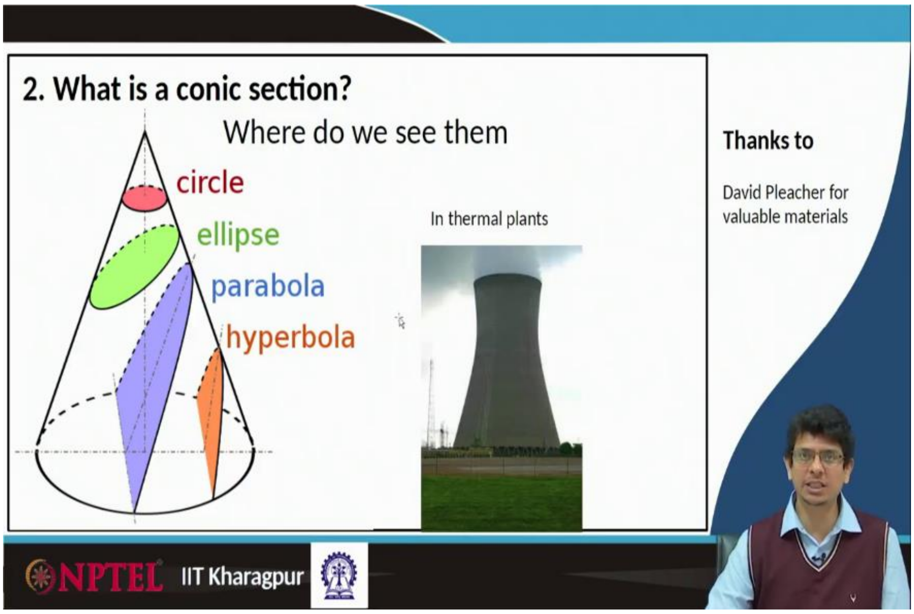

Hyperbola has special applications for cooling towers and draft towers. Here near thermal plants, whatever this steam which drives the turbine blades, again has to be condensed. To condense that steam or to cool that, we will have chilling towers and chilling ponds; and hot water will be dripped from this hyperbolic kind of towers, the hot water comes down all the way and coolant or air goes all the way up. So, a draft tube will be constructed, for that purpose, usually, hyperbolas are very useful.

So, this is one kind of cooling tower utilized for thermal plants. And the exterior of the curve represents part of the hyperbola. So, this is also hyperbola; this one is also hyperbola; the middle line usually we call this directrix lines, which we will learn later.

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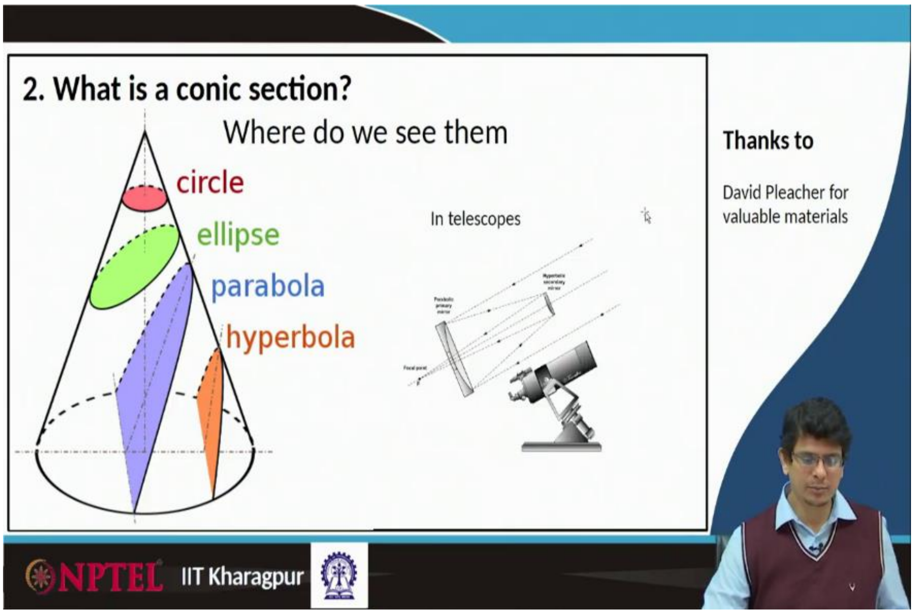

Similarly, for telescopic applications and understanding about a plan is a combination of parabolic mirrors and also hyperbolic mirrors will be used. This is a telescope, on one side you will have a parabola; so all these rays come heats that, focus to one point and from that one point hyperbolic mirror will be used, again it will be focused at some other point so that it will be amplified or enlarged image one will be in a position to get. So, occasionally they use these parabola hyperbola combinations also.

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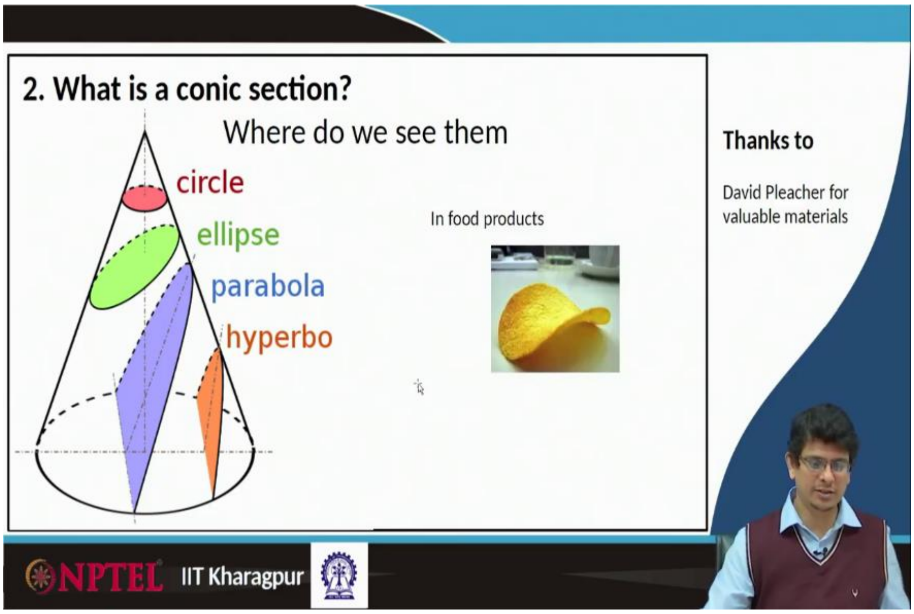

Even for products food products like chips, especially the brand name we call Pringles; the shape of that curve forms typically a hyperbolic curve.

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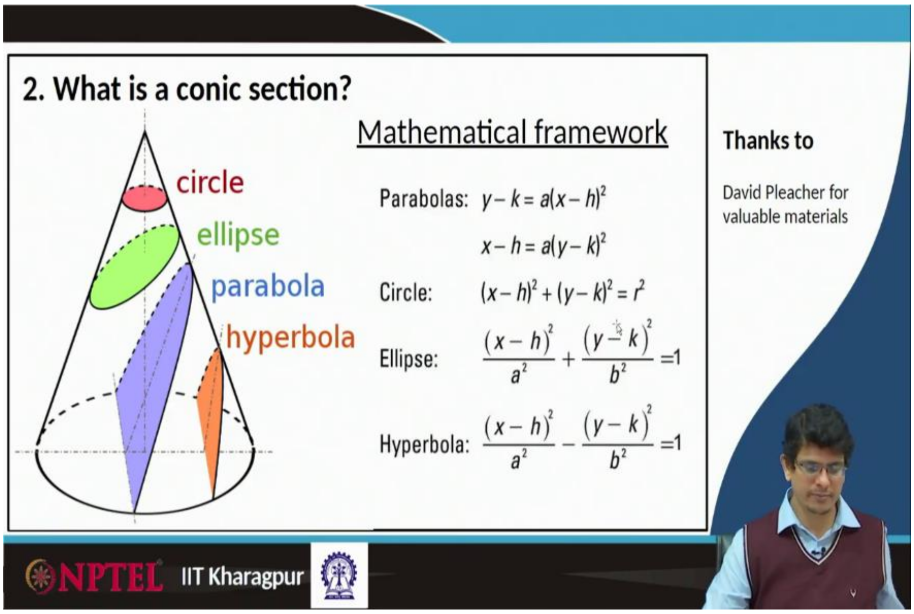

If we are summarizing at the mathematical level; a circle have a functional form based on where exactly centre is located, if it is located at h and k location.

For example, this is the coordinate axis, somewhere x and h coordinates.

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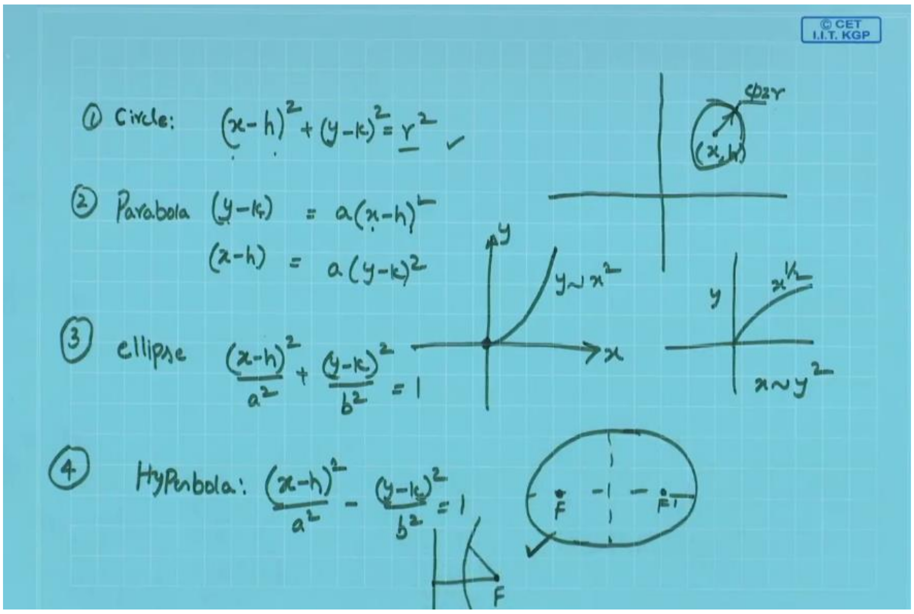

If a circle has to be drawn, it forms

$$(x-h)^2 + (y-k)^2 = r^2$$

If it is a parabola, if it is in the vertical direction, is satisfies

$$(y-k) = a \times (x-h)^2$$

So, if it passed through this origin, and that equation is:

$$y = x^2$$

Sometimes there are parabolas which go,

$$y = \sqrt{x} \qquad or, \qquad x = y^2$$

The third one ellipse, the mathematical definition is satisfied

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

The fourth one is a hyperbola, which satisfies

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

We will see the typical construction of this hyperbola ah ellipse, hyperbola; hyperbola always be having two directrices. So, we will see, understand the relation between the focal point to the directrix and the distance from this focal point to any point on that curve in the next class.

Thank you very much.