Muffler Acoustics - Application to Automotive Exhaust Noise Control
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Lecture – 11
Sound Pressure Level, Intensity Level and Sound Power Level

Welcome to week 3 of our course on Muffler Acoustics. So, in this week,in the next two and half, three hours, we are going to talk about different things which will directly introduce us toacoustic filters.

INTENSITY and SOUND PRESSURE LEVEL

THEORY ACOUSTIC FILTERS

So, broadly we will be covering two topics in this week. First isdifferent terminologies that are used to measure sound pressure level like for example, its sound pressure level is measured in dB or dBA and intensity, how are they used, how are they related to quantifying the performance of acoustic filters so, theory part is what we are going to talk about in this week that is basically without; without mean flow.

So, like I have mentioned, all automobile mufflers have invariably mean flow because the exhaustgas is always slow towards the, towards theatmosphere.But before we consider the convective effects of mean flow, we will consider a much simpler case in which the medium is stationary that is themean velocity 0 and we will develop the basic theory of acousticwave reflection or transmissionin the absence of mean flow. So, we will see how this sort of basic elements constitutethe muffler analysis and we are going to elaborate on that.

So, let usfirst begin with some basic terminology that isbeing used.

<u>Sound pressure level</u>

So, we are following the terminology that p tilde or the perturbation sound pressure that is over and above theatmospheric or the ambient pressure so,both consider total pressure.

$$p_t = p_0 + \tilde{p} \rightarrow P_a$$

So, this particular thing is always what bothers us; this is what hits our eardrum and produces a sensation of sound. So, this obviously, has the unit of Pascaland our ear, the human ear canencompass a large range, largemagnitude of sound from very very faintest of noise to very incredibly loud noise like arocket roaring off.

So, we what we do is basically convert thePascal,

$$SPL = 10log_{10}\left|\frac{\tilde{p}}{\tilde{p}\,ref.}\right|^2 \; DB \qquad (1)$$

So, typically, here this is the absolute amplitude of the acoustic pressure disturbances in Pascal,

$$\tilde{p}\,ref = 20\,\mu Pa = 20\ \times 10^{-6}Pa$$

$$= 2 \times 10^{-5}Pa$$

So, that is the minimum threshold of noise that anyone can hear that is thehealthy human; human being can hear.

So, we always talk in terms of whenever you talk in terms ofwith the reference level, we are basically trying to; trying to basically quantify the noise produced with respect to this, the minimum sound that you can hear, the ratio of that. So, what we do is that this ratio can be a very small number of basically if it is much much less than this thing, then it is obviously, negative, but when $\tilde{p}$= p reference, then it is 1 and if it isvery large, it can be incredibly large number.

So, basically logarithm is required to properly account for the large variation in the magnitude that is possible in the variation of the sound, the sound pressure fluctuations so, we take obviously, the square of that. Now, generally this is, this has a unit typically called bell named after the scientist Graham Bell so, we put B, but then usually sound is in the decimal unit, we usually multiply this by 10 so, then this is called decibel. So,this is the expression for sound pressure level in decibel.

And thisequation can be simplified,

$$SPL = 20log_{10}\left|\frac{\tilde{p}_{RMS}}{\tilde{p}\ ref.}\right|^2\ dB \qquad (3)$$

So, this is in decibeland obviously, they are different in standard text and acoustics.

Andyou will find human ear is not equally responsive to different frequencies that is you tend to perceive 1000 Hertz frequencies, you basically human ear tends to respond to 1000 Hertz frequency in the mostin the; in the best possible manner, but low frequency is like less than 100 Hertz or 50 Hertz, the human ear does not respond to thatvery well in the sense that they are hard to perceive.

So, basically if you play them at the same level, it is perceive that 100 Hertz or 50 Hertz sound very low frequency noise isrelatively less efficiently heard compared to the 1000 Hertz that is why all your ambulances and signals, they are played close to 1000 Hertz if not exactly 1000 Hertz and again,at higher frequencies, we tend to hear themalright near about 1000 Hertz, but as we go up maybe about 8000, 10000 Hertz the again the same thing happens that we tend to hear them relatively less efficiently.

So, we have, the human ear basically is most responsive near 1000 to 3000 Hertz and beyond 20000 Hertz, a perfectperfectly healthy human being cannot hear sounds. Those sounds are called ultra-ultrasonics andfrequencies below 20 Hertz are called infrasonic.

Now, the reason that I have mentionedthese things is becausewe typicallyin view of the unequalreception of the human ear to different frequencies, weusually have a thing called A-filter, A-weighting. So, whatever sound we hear,unmuffled noise is a customary practice of course, toaccount forthe differentaccount for thehuman ear perception and add different weights. So, those are called A-weighting.

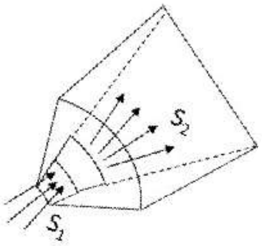
We right now, we are not going to go in that, but I just want to mention that herewhenever you account for the different frequencies that human ear listens to, you typically end up with thing called A-weighting and such a thing is calledA-weighted noise, but right now, we will just focus on the dB decibel level and not dBA.

Of course, the other scales also dBC orother ones, but that is what it is.Now, the other things also just like you have SPL, you also have intensity level. So, before talking about intensity level, let us formally quantify what is intensity.

So, now let us talk aboutdefinition of

Acoustic Intensity.

Now, basicallyspherical waves basically thewaves that propagate in three-dimensional space, they tend to; they tend to expand out. So, the same acoustic power that is imagine a sphere like this and if it is expanding out so, the area of the sphere becomes more. So, the same acoustic power spread over a sphere of a larger surface area.



$$\vec{I} = p(t)U_n(t)$$

So, if you have a look at this figure, if you consider a small slice of the sphere. So, this is let us say the cross-section areaS_1 and this is yourS_2. So, basically, what we do? If we go by the basic principle, intensityis nothing but basically power by

$$I = \frac{W}{A}$$

$$= \frac{F \times v}{A}$$

$$= \frac{pAv}{A}$$

So, here, $= pv$.

So, the instantaneous intensity then is yournothing butacoustic pressure, instantaneous acoustic pressure into the normal velocity that is the velocity; velocity of the particle is normal to the surface. In this case, this orangish line is normal to the surface. So, and here you have basically, when I talk about a bar, it means you need to, one needs to take the product of this thing and take the time average of the product.

$$I(t) = \tilde{p}(t)\tilde{U}_n(t)$$

$$I = \frac{1}{t_{avg}} \int_0^{t_{avg}} \tilde{p} U_n dt$$

Now, for non-periodic waves that is for random signal t;t average this tends to infinity, but for most signals is emitted by machines orother equipment's, they have some sort of periodicity. So, basically, we can talk about a certain period. So, that is t averages is finite.

$$t_{avg} \to \infty$$

There are special formulas for intensity that are developed for two somewhat overlapping class of problems; one is progressive waveor arbitrary wave form in lossless fluids and other others time harmonic waves which are need not be necessarily progressive and they can be in arbitrary fluids, but we will talk about air and we will talk about particularly about lossless fluids only.

So, this is the expression that we will use and here, this thing can be further simplified if we have a free space. So, if you remember, recall ourdiscussions in the 1st and the 2nd week, we sawthat,

$$\frac{p}{U_n} = \rho_0 C_0$$

$$\frac{p}{\rho_0 C_0} U_n$$

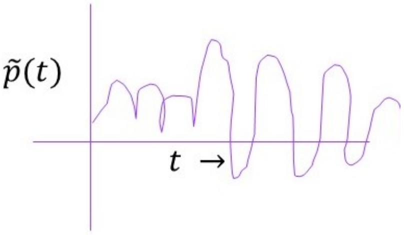
$$t_{avg} = T = \frac{2\pi}{w}$$

$$I(t) = \frac{1}{t_{avg}} \int_0^{t_{avg}} \frac{|\tilde{p}|^2}{\rho_0 C_0} dt = \frac{1}{\rho_0 C_0} \frac{1}{t_{avg}}_0 \int_0^{t_{avg}} |\tilde{p}|^2 \; dt$$

$$I = \frac{\tilde{p}_{RMS}^2}{\rho_0 C_0},$$

$$\tilde{p}_{RMS}^2 = \sqrt{\frac{1}{t_{avg}} \int_0^{t_{avg}} \tilde{p}^2 dt}$$

So, what one needs to do is that one if we know the time series of variation suppose if you havea signal the acoustic pressure signal and you using a microphone, microphones are used for measuring soundso, if you have certain variation like this so, once we know some time series, then we can write a algorithm, write a simple MATLAB code for example, or whatever programlanguage you use to figure out the RMS value and this can be used directly inthe equation (3) p RMSto give us the sound pressure level.



Now, talking about the intensity level, how do we go aboutmaking use of the intensity to that. So, just like we have defined sound pressure level, intensity level,

$$IL = 10\ log_{10}\left|\frac{I}{I_{ref}}\right|$$

$$I = \frac{\tilde{p}^2_{RMS}}{\rho_0 C_0}; I_{ref} = 10^{-12}\frac{w}{M^2}$$

So, for now, we will just put this as I reference andsimplify this expression.

So, intensity level

$$IL = 10log_{10}\frac{\tilde{p}^2_{RMS}}{\rho_0 C_0 I_{ref}} = 10\left\{\log\left(\frac{\tilde{p}_{RMS}}{p_{ref}}\right)^2\frac{p^2_{ref}}{\rho_0 C_0 I_{ref}}\right\}$$

Let us go to the another slide,

$$IL = 10log_{10}\left|\frac{\tilde{p}_{RMS}}{I_{ref}}\right|^2 + 10\log_{10}\left|\frac{\tilde{p}^2_{ref}}{\rho_0 C_0 I_{ref}}\right|$$

$$= 20log_{10}\left|\frac{\tilde{p}_{RMS}}{\tilde{p}_{ref}}\right| + log_{10}\left|\frac{(2 \times 10^{-5})}{414 \times 10^{-12}}\right|$$

$$IL = SPL - 0.16 \simeq SPL$$

$$(dB)(dB)$$

Now, what this yet another thing called soundpower level, not to be confused with SPL, this is calledPWL and lot of machines that you buy for example, air conditioners, refrigerators and other electrical appliances used in householdor even inindustrial setup, the manufacturer often has to specify theacoustic power that is radiated by the machine in dB or dBA.

$$(PWL)$$

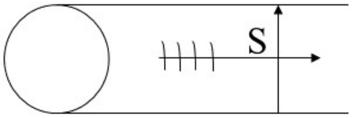
$$W_{ref} = 10^{-12}w$$

$$PWL = 10log_{10}\left|\frac{w}{w_{REF}}\right|$$

$$w = I.s$$

Now, basically, sound pressure level is a measure of the total acoustic energy per unit time emitted by the source. It cannot be converted to sound pressure level until additional information is provided. Specially, like the directionality of the sound, distance from the source and so on, but nevertheless, a useful expression for the sound pressure radiatedwhich will be; which will bevisually particularly useful for evaluating the performance of mufflers is basicallyintensity into S.

So, suppose if you have a ductand you have some progressive waves going in certain direction so, intensity into surface area so, suppose this S is thecross-section area so, intensity into surface area is the acoustic power.



So, we just sawa while back thatintensity,

$$W = \frac{\tilde{p}_{RMS}^2}{\rho_0 C_0} .S$$

So, we will see in due course that acoustic pressure as a function

$$\tilde{p}(x) = \overrightarrow{Ae^{-jk_0 x}}$$

we are neglecting the waves that are coming from the opposite direction. We are considering only waves that go like this in this direction and not allowing waves to come. So, it is a progressive wave.

$$\tilde{p}_{RMS} = \frac{A}{\sqrt{2}}$$

Now, the RMS thing would beA by root 2. So, typicallyyou would get things like this. So,we will eventually see the acoustic power carried by a progressive wave.

$$W = \frac{|A|^2}{2\rho_0 C_0} .S$$

we see how these definitions are useful tothese expressions are useful for evaluatingthe performance of mufflers.

There are few derivations of course, that I thought it is little important to talk about now. So, generalized expression forimpedance is p byU or U n whatever you are comfortable with normalized impedance and this isreal part,

$$Z = \frac{\tilde{p}}{\tilde{U}} = Re\ (z) + j|_m(z)\tilde{p} = z\tilde{U}$$

$$= |Z|e^{j\theta}$$

So, intensity then becomesomegaby2 pi and if you were talking about a period,

$$I = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} Re\ (\tilde{p})Re(U)dt$$

$$T = \frac{2\pi}{\omega}\ 2\pi f\ T = 2\pi \quad => \quad \omega T = 2\pi \quad => T = \frac{2\pi}{\omega}$$

So, in the expressions that we have at our disposal basically, these ones t averagewould become T, this would be equal to 2 pi byomega or omega is the angular frequency in Hertz. So, nowlet us write down quickly what it means for us. It means pressure can be thought of pressure is nothing butimpedance times U so, it is we talked about real part of this. So, real part of pressureinto real part ofvelocity into dtbecause these signals can be put in the complex form so, this again would become 2π/ω.

So, we need to go to the next slide andformally define,

$$I = \frac{\omega}{2\pi}\int_{0}^{\frac{2\pi}{\omega}} |Z||U|^2 \cos(\omega t + \beta + \theta)\cos(\omega t + \beta)dt$$

So, once we do the algebra work out, these particular nasty looking expressions andwhat are we going to get so, most likely after a lot of simplifications, we probably would get this as

$$= \frac{1}{2}|Z|cos\theta|U|^2$$

$$Zcos\theta = Re\ (Z);\ \frac{|U|^2}{2} = \widetilde{U}_{RMS}^2$$

We obtain Iequals real part of Z because this is something that we are taking togetherand this thing we are clapping at here, we get,

$$I = Re\ (Z)U_{RMS}^2$$

So,clearly the physical significance is that if we keep the material, the propagation material constant like air and if we have increased particle velocity that is the oscillations are going on at a with a larger amplitude, larger U RMS, then obviously, the acoustic intensity will increase many a fold and this will have consequences on the hearing that we do.

So, yet another form, this is similar to youryou know electrical circuits, we will probably talk about that in a much greater detail, but for now, let me present to youyet another expression for the intensity which,

$$I = \frac{p_{RMS}^2}{|Z|}\cos\theta = \frac{1}{2}|\rho||U|cos\theta$$

$$= \frac{|P|^2}{2}$$

How did we end up getting this? So, if we recall this one,U also can be written as p / Z right. So, we can put these expressions here to simplify further. So, once we do that, we can get real part of pressure into real part of pressure that is p RMS square and you will get in the denominator Z cos theta and if you simplify this thing further, we are probably going to end upwith this expression.

And this can be furtherwritten as half ofmagnitude of P magnitude of U into cos thetawhere theta is the angle between the acoustic pressure and the particle velocity. So, if they are in phase, then cos theta is 1 so, you just have half of magnitude of p and U that is basically in the far field acoustic pressure and acoustic velocity are in phase, you typically end up withthings likethis thing where rho naught c naught is sort of absorbed in this constant p.

Then, you can keep on presenting different expressions for this, but in context of duct, duct work on propagation throughone-dimensional ducts, we typically we will see that we will end up with a progressive wave of amplitude A, the acoustic power that is carried by wave in this direction is given by,

$$w = \frac{|A|^2}{2\rho_0 C_0} S$$

Similarly, the waves that propagate in the opposite direction their magnitude is B and the acoustic power carried by them in this direction isB square by2 rho naught c naught S.So, we will worry about all those things in the next lecture. So, for now, we will stop here and talk aboutacoustic filters that is somefundamental elements used in mufflers in the next lectures.

Thanks.