Muffler Acoustics- Application to Automotive Exhaust Noise Control
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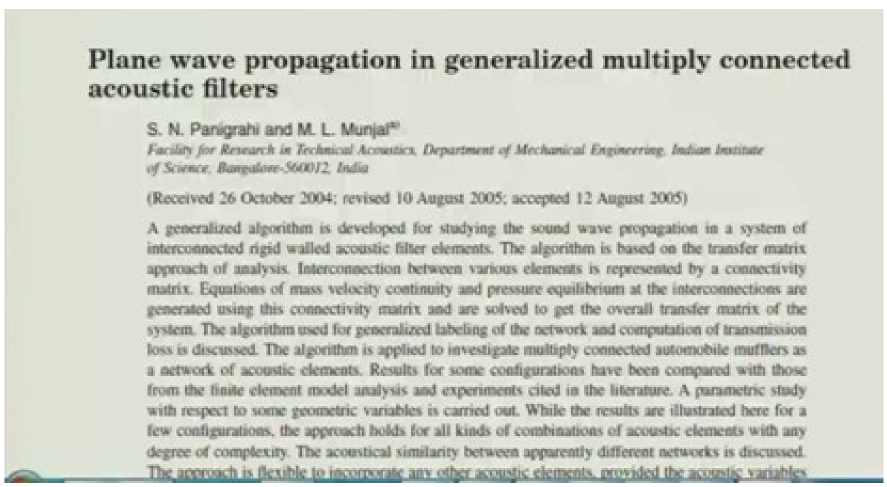
Lecture - 48
Impedance Matrix Characterization and Network Analysis of Perforated Element

Welcome to lecture 3 of week 10 of this Muffler Acoustics course, NPTEL course on muffler acoustics. So, last week I am sorry last class what we are discussing was that we will talk about much more complicated muffler configurations. So, let me just; let me just pull out; let me just pull out a nice paper that was published several years back on the multiply connected muffler thing.

So, just bear with me for a minute and I am going to pull out a paper for you in which the first time such multiply connected mufflers were kind of analyzed.


Plane wave propagation in generalized multiply connected acoustic filters

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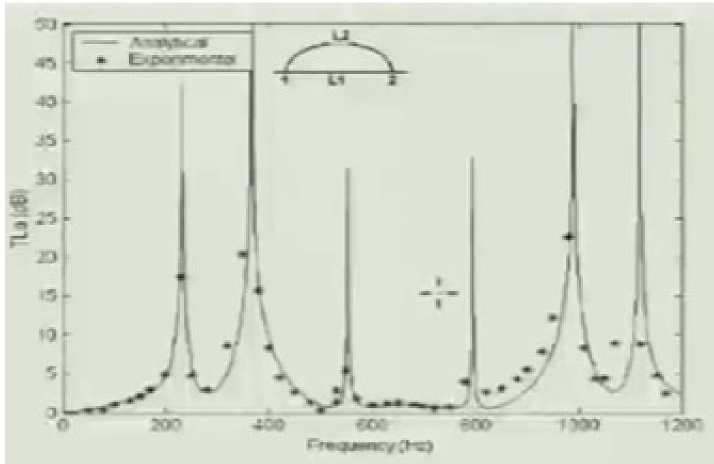
A generalized algorithm is developed for studying the sound wave propagation in a system of interconnected rigid walled acoustic filter elements. The algorithm is based on the transfer matrix approach of analysis. Interconnection between various elements is represented by a connectivity matrix. Equations of mass velocity continuity and pressure equilibrium at the interconnections are generated using this connectivity matrix and are solved to get the overall transfer matrix of the system. The algorithm used for generalized labeling of the network and computation of transmission loss is discussed. The algorithm is applied to investigate multiply connected automobile mufflers as a network of acoustic elements. Results for some configurations have been compared with those from the finite element model analysis and experiments cited in the literature. A parametric study with respect to some geometric variables is carried out. While the results are illustrated here for a few configurations, the approach holds for all kinds of combinations of acoustic elements with any degree of complexity. The acoustical similarity between apparently different networks is discussed. The approach is flexible to incorporate any other acoustic elements, provided the acoustic variables

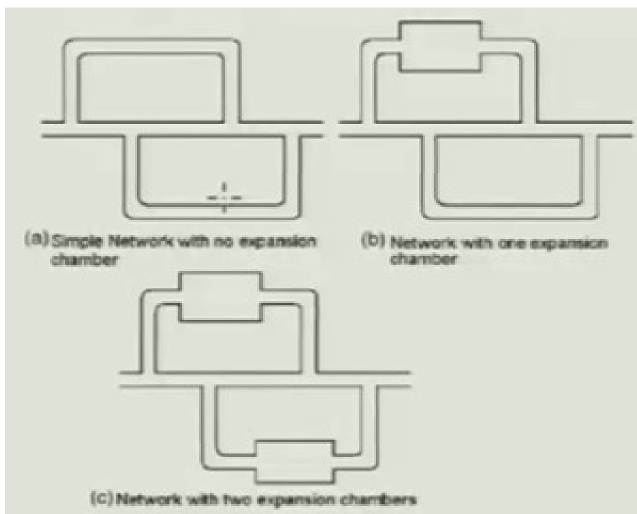
On plane wave propagation in generalized multiply connected acoustic filters or mufflers published several years back.

And then in this paper connectivity matrix you know to relate variables upstream to downstream variables was first talked about where waves can have multiple paths simultaneously.

So, there are number of important number of interesting kind of configuration.

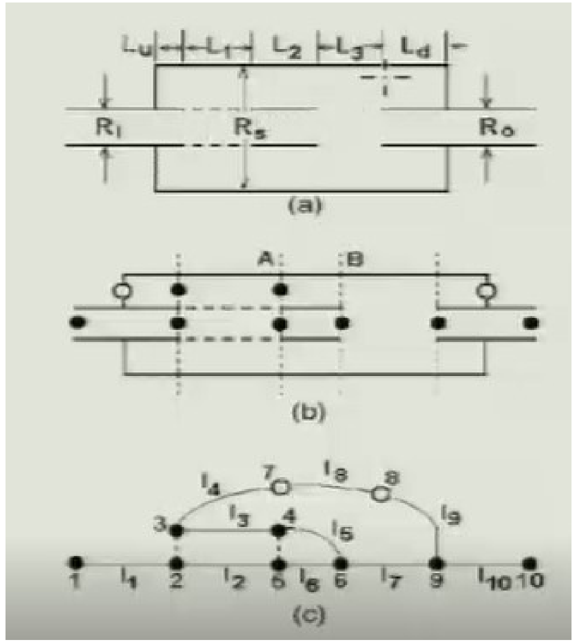


This is your Herschel Quincke tube the one that we just analyzed one of the most simplest multiply connected mufflers.

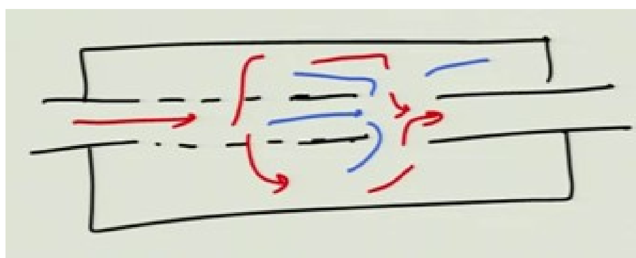


Then they have far more complicated ones; like for example, consider this one. You know a network connection with no expansion chamber in the middle, but you know they are multiple constructive or destructive interferences happed happening. And from this point onwards to this point we it is what we figure out the transmission loss and then you can have an expansion chamber here or an expansion two such expansion chambers and all that.

And you can have another such configuration we will probably discuss few such configurations in today's lecture like this is yet another such connection or probably yeah, this is the one I was mentioning.



So, you know this is let me go the slide and start talking about it.



So, you know drawing it in a rather simple manner, you know it is something like this, or maybe I need to modify the figure probably this is something like this. So, the reason this is multiply connected and cascading might not just simply help is because your waves can kind of go in different fashion. For example, it comes here, it can actually go in the annular cavity and propagate out propagate like this and again for the outlet it goes like this.

Another way is basically it straight it does not interact with this thing it just comes here and sees and see this is an annular cavity and this also the annular cavity and goes like this. So, there are multiple such paths that are happening. So, we will be analyzing a couple of such you know interesting configurations using MATLAB. So, for that end let us go to such figures.

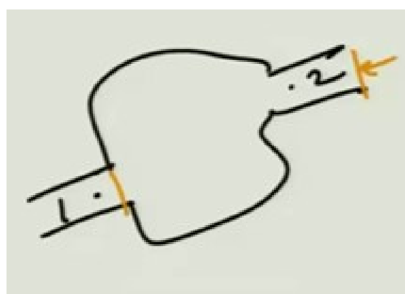
So, let us analyze this configuration where node numbering I will talk about that in a while. And, but before that let me introduce you for the first time to the concept of impedance matrix. So, let me use another slide; this is the first time in this course I am introducing the concept of an impedance matrix characterization.

Impedance Matrix

So, I eluded to this thing some time back, but basically what is happening is that instead of doing something

$$\begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix} = [T] \begin{Bmatrix} p_2 \\ V_2 \end{Bmatrix}$$

$$\begin{Bmatrix} p_1 \\ P_2 \end{Bmatrix} = [Z] \begin{Bmatrix} p_2 \\ V_2 \end{Bmatrix}$$



So, how do we what is the; what is the advantage like I was mentioning; you can have 3 or more elements with 3 or more ports they can be easily categorized or I am sorry characterized using an impedance matrix. So, suppose if we have you know this kind of a thing. So, we have port 1 here and port 2 here.

So, you know what we can do is basically have things like this here. So, this can be any element. So, what in order to characterize what we can do is that we can block this port and apply some kind of a piston excitation here and measure you know take. So, when you have v 2 0 just at the flush end what we,

$$\frac{p_1}{V_1} = Z_{11}$$

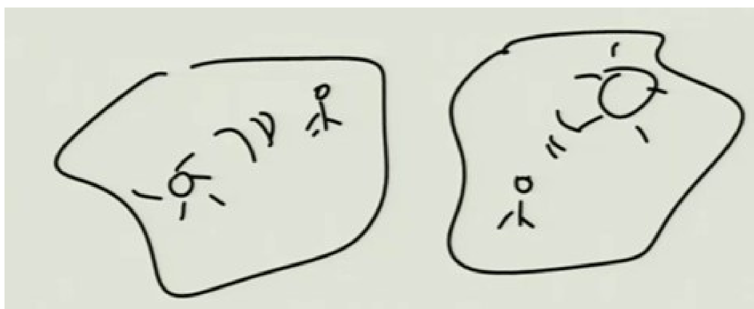
that is the parameter here. Similarly,

$$\frac{p_1}{V_2} = Z_{12}$$

Because v_2 is 0 mind you. So, this is Z_{11} times v_1 is plus Z_{12} times v_2, v_2 is 0. So, that is why we get this thing. So, just by measuring the pressure responses we can get stuff and then because then there is things like reciprocity acoustic reciprocity as.

So, this is you know probably you would have heard of this term also for the first time acoustic reciprocity.

Acoustic Reciprocity



So, what it means is that you know in a big room if you have a speaker that is radiating noise and a receiver a person. So, wherever response you are getting by putting the speaker here, the same thing will happen in the same identical room if you interchange the location of source and speaker. So, the same guy present here will respond will basically you know observe the same response if thus if the position of the source and speaker are interchanged.

The reason I bought up this was because if you were to block this port and apply a piston excitation somewhere here.

what I said now has to be $Z_{12} = Z_{21}$ and such systems which obey such which such a concept is called reciprocal system or reciprocal mufflers. So, you know such a thing does not quite exist in when you have your flow there is a flow direction.

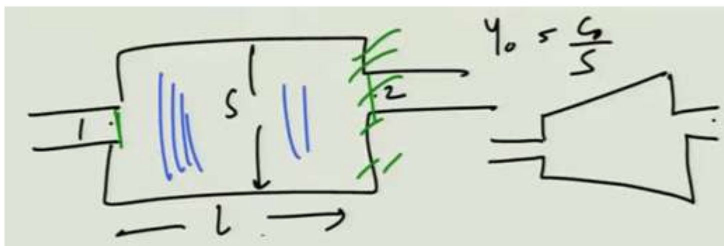
So, reciprocity is valid very much for a muffler without a flow just a filter, but when you have flow this thing does not necessarily is valid. So, this is just a comment that I wanted to make and

$$\frac{p_1}{V_1} = Z_{21}$$

$$\frac{p_2}{V_2} = Z_{21}$$

So, these are also your another parameters called impedance matrix parameters or Z matrix parameters just like transfer matrix is T matrix impedance matrix is Z matrix. So, what is a special about it?

Because now you know if you consider a simple expansion chamber like this a simple expansion chamber like this.



So, I drawn somewhere here, do not worry. So, this is the point 1 and point 2 just outside this is the length L. Remember our transfer matrix.

The transfer matrix.

$$\begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix} = \underset{[T]}{\begin{bmatrix} C & jY_0S \\ \frac{jS}{Y_0} & C \end{bmatrix}}$$

Now, impedance matrix will be p_1, p_2 is equal to $-jY_0$ times I am sort of running out of space. So, what I am going to do is I guess I have to rub this part.

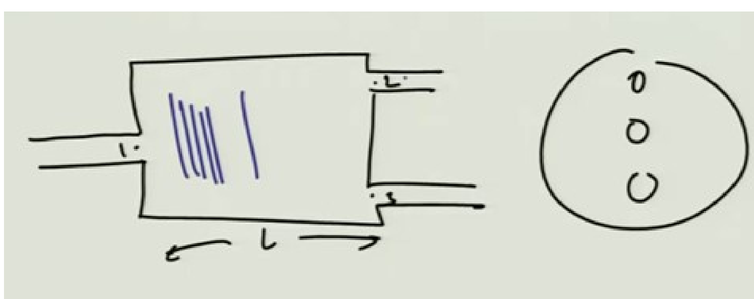
$$\begin{Bmatrix} p_1 \\ V_2 \end{Bmatrix} = -jY_0 \begin{bmatrix} Cotk_0L & coseck_0L \\ coseck_0L & Cotk_0L \end{bmatrix} \begin{Bmatrix} p_1 \\ V_2 \end{Bmatrix}$$

You know just starts from somewhere here. So, this is equal to $-jY_0$ is nothing but this is the cross section area of the chamber. So, y naught is c naught by s cot cosec and this is cosec cot and this is multiplied by v_1 and v_2 ok. So, p_1, p_2 is equal to $-jY_0$ times this is the matrix 2 cross 2.

So, this can be easily seen you know once you already have the relation a p propagation in a duct can be you know can be expressed as a into e to the power $-jk_0x$ plus b times e to the power $+jk_0x$ ok. This one is minus other is plus. So, you know then you apply your boundary find out the velocity expression, apply the rigid wall condition.

So, if you apply a piston that is flush bounded here and block it here and apply rigid wall condition over the entire face here and then its again it is a planar wave mind you. So, all these relation it is valid for a planar wave. So, this Z matrix is a counter part of the T matrix assuming plane wave propagation only right from the interface. So, when you have such a thing p by you know p by v will give you I am sorry p_1 by v_1 will give you $-jY_0$cot k_0L and p_2 by v_1 will give you cosec k_0L.

And this will be same because of reciprocity no mean flow and there will be cot k naught L also because it is a chamber. For a conical mufflers, if you have a thing like you know this if you have things like this kind of a thing then things would be sort of a bit different. This term would be different and reciprocity wise it will be the same they are off diagonal terms, but you know these are all certain things which we need to take care about.



But what I am trying to say is that suppose if we have a single or triple port system something like this. You know the length is L and the this is the circular cross section and that kind of a thing. So, you know this is 1, 2, 3; points 1 is just outside the chamber.

It is can be simply written in terms of inspection. So, port 1.

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix} = -jY_0 \begin{bmatrix} cotk_0L & \nearrow & cosc_0k_0^2 \\ cosc_0k_0L & cotk_0L & cotk_0L \\ cosc_0k_0L & cotk_0L & cotk_0L \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix}$$

So, now because of reciprocity p_1 by v_2 will also be the same.

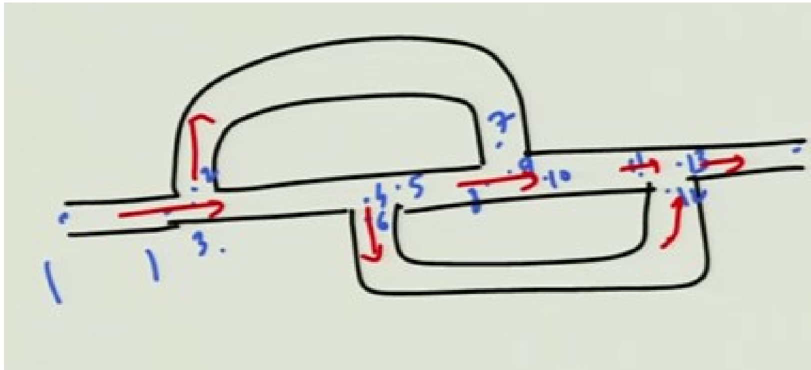
So, this will be the same, this will be $cosc_0k_0L$ same and here also will be same if you basically do this thing and same here $cosc_0k_0L$ and here will be $cotk_0L$ and $cotk_0L$. The idea is that there are by mere visualization of where the ports are located you can fill up the entries of the impedance matrix.

At least for the plane wave propagation for cases when you have ports located on the end faces. For on the side faces sides of faces one needs to do the algebra and work it out maybe in some of your assignment problems or may be in your end semester exam you can be asked to get the simple expressions.

So, there will be $-jY_0$ into $cotk_0L$ because this port is located here, there will be cosec here and cosec here. Now, for this port here you know this will be cosec, but you know p_2, I guess I made a mistake here I am sorry for that p_2 by v_2 is $cotk_0L$, but p_3 by v_2 is also $cotk_0L$ because it is on the same face, if it is on the opposite face then you have cosec.

Similarly, when you have excitation here p_1 by; p_1 by v_3 is cosec, but p_2 by v_3 is cot because it is the port is in the same face as the receiver port is in the same face as the source port somewhere here and this will be like this. So, depending upon where the ports are you can you know characterize it just by visualization and knowing some basic rule.

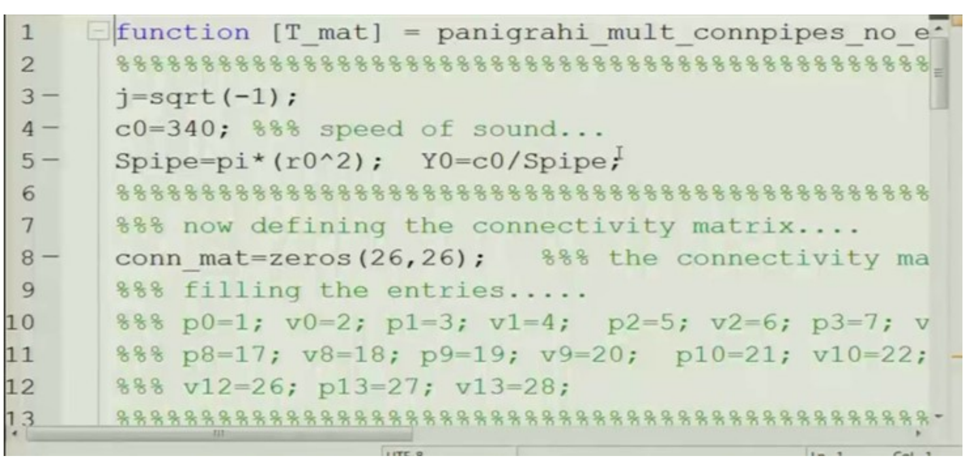
So, that is the advantage of your impedance matrix at any n cross n system can be worked out. Now, let us get back to the configurations that we discussed. We are doing you know this the configurations that was shown in the paper you know this one.



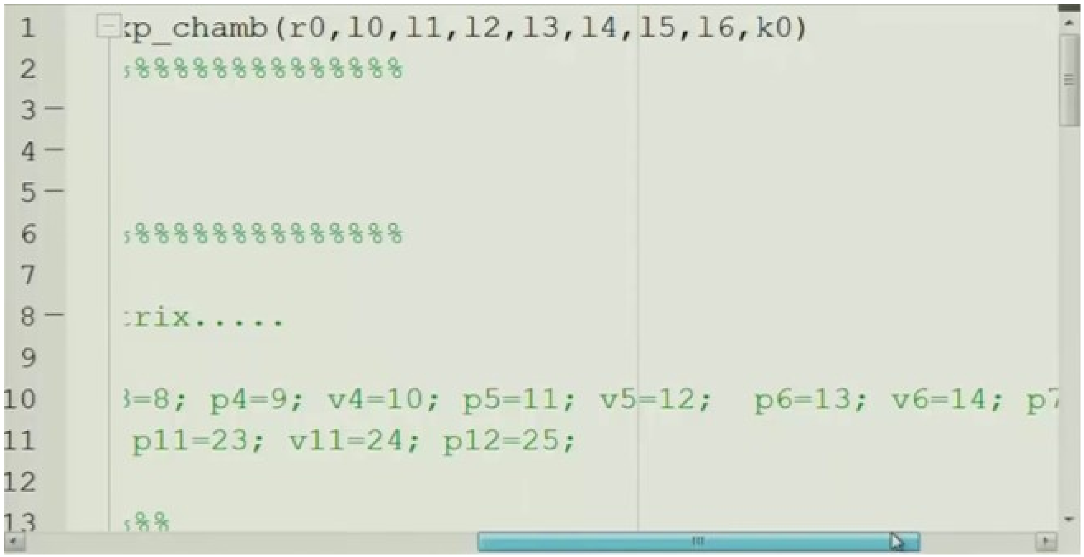
So, I can draw it again for you. So, this is something like this, this one was going like this and go like this and like this multiple such paths do exist. So, you know for such a system if we were to know number it like something like 1, 2, 3, 4 or probably 4 can be here just point here 5 and 6 and this guy can be 7, 8, 9, 10, 11, 12, 13.

So, what we want is basically relation between p_1 and v_1 and p_{13}, v_{13} or p_1, p_{13} related to v_1, v_{13} using an impedance matrix. Once we know impedance matrix we can convert it to transfer matrix very well and figure out the transmission loss. So, and you can also basically you know just like I was mentioning several weeks back just like we figure out the transmission loss in terms of the 4 pole parameters some of T_{12}, T_{12}, T_{22} and so on.

We you know we can also define transmission loss in terms of impedance matrix parameters you know in terms of Z_{11}, Z_{22}, Z_{12}, Z_{21} parameters or in terms of scattering matrix expressions you know we can easily do that, but what I would suggest is that we will kind of wait for that discussion to happen in a while. And right now, what we can do is that we can go to the MATLAB code and look at this configuration and get some transmission loss curves.



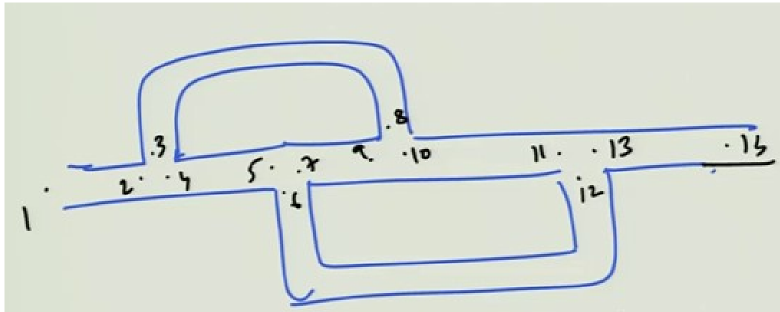
So, what we do now it is basically you know do a network analysis for such a configuration there are no expansion chamber they are just simple tubular elements then they still produce a lot of attenuation peaks and all that sort of a thing.



So, this is the thing. There are different lengths l naught, *l*1, *l*2, l r_0 is the radius which is assumed to be constant and *l*1, *l*2, *l*3 these are all lengths l1 to l6 which are probably not mentioned here this is just l say let us say this is *l*1 length, this is *l*2, *l*3, *l*4, *l*5 and maybe l 6 I guess.

So, you know 1, 2, 3, 4, 5 and maybe there is one more length *l*6. So, the idea is that idea is really that define the connectivity matrix, there are 14 such elements that I had drawn nodes I am sorry. So, 14 into 2 means and they can be actually more you can actually have more such elements somewhere here and here.

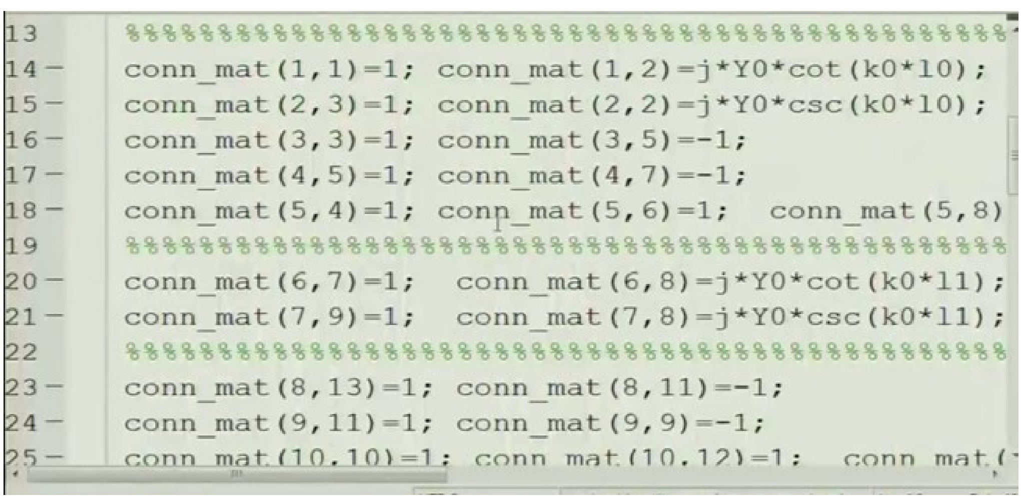
So, there can be more number of you know 16 actually you know maybe you can just renumber it. So, this can be 1, 2, 3, 4 and all that sort of the thing. So, something this is just the purpose of showing you how things can be done. So, I guess you have to renumber the some of the stuff maybe let us sort of do it. So, 16 to the 32, 32 number of variables because each node has two variables.



$14 \times 2 = 28$ variables;

So, p 1 is coming over here ok and this one was may go somewhere here well. So, if we were to number it like 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and then 12, 13 and 14 into 2. So, there are 28 variables because at each node there are 2 variables; pressure and velocity mass velocity. So, we want to express everything all the variables from p_1, p_2, p_3 till p_{13}, v_{13} in terms of p_{14} and v_{14}.

If we can do that then we can just take out those particular rows which represents the relation p_1, v_1 with p_{14}, v_{14} or probably p_1 and p_{14} in terms of v_1 and v_{14}. So, what has been done essentially in the MATLAB code? We can do the either way either which ways we can get the either the fully the impedance matrix or we can get the transfer matrix finally, we can because we can convert and do stuff because sound speed is 340 and some characteristic impedance of the pipe.



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(1,1)=1;  conn_mat(1,2)=j*Y0*cot(k0*10);
conn_mat(2,3)=1;  conn_mat(2,2)=j*Y0*csc(k0*10);
conn_mat(3,3)=1;  conn_mat(3,5)=-1;
conn_mat(4,5)=1;  conn_mat(4,7)=-1;
conn_mat(5,4)=1;  conn_mat(5,6)=1;   conn_mat(5,8)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(6,7)=1;   conn_mat(6,8)=j*Y0*cot(k0*11);
conn_mat(7,9)=1;   conn_mat(7,8)=j*Y0*csc(k0*11);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(8,13)=1;  conn_mat(8,11)=-1;
conn_mat(9,11)=1;  conn_mat(9,9)=-1;
conn_mat(10,10)=1;  conn_mat(10,12)=1;   conn_mat(
```

So, connectivity matrix there is a crucial part. Connectivity matrix is 26 cross 26 because like I mentioned we have 28 variables and we will probably be having equations 26 such equation. So, we can relate 26 variables in terms of the remaining 2 variables which are. So, we to uniquely determine or characterize the system.

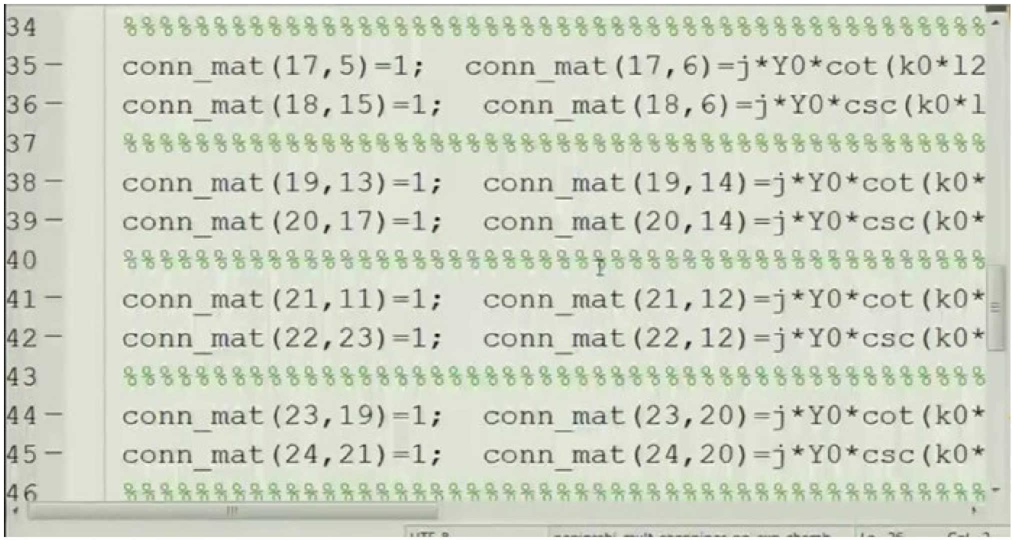
```
%%%%%
; conn_mat(1,4)= j*Y0*csc(k0*l0);   %%% 1
; conn_mat(2,4)= j*Y0*cot(k0*l0);   %%% 2
                                    %%% 3
                                    %%% 4
3)=1;                               %%% 5
%%%%%%%%%%%%
); conn_mat(6,10)= j*Y0*csc(k0*l1);   %%% 6
); conn_mat(7,10)= j*Y0*cot(k0*l1);   %%% 7
%%%%%%%%%%%%
                                      %%% 8
                                      %%% 9
t(10,14)=1;                           %%% 10
```

So, this one you know I am not going to go through the entire thing, but this is just I am just walking through the entire through the code in a brief manner. So, this is something like characterizing the tubular element upstream and this is the junction loss. So, they always occur in blocks ok. And then this is again your tubular element *l*1, *l*2.

```
conn_mat(10,10)=1; conn_mat(10,12)=1;   conn_mat(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(11,17)=1; conn_mat(11,19)=-1;
conn_mat(12,17)=1; conn_mat(12,15)=-1;
conn_mat(13,18)=1; conn_mat(13,16)=1;   conn_mat(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(14,21)=1; conn_mat(14,23)=-1;
conn_mat(15,21)=1; conn_mat(15,25)=-1;
conn_mat(16,22)=1; conn_mat(16,24)=1;   conn_mat(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(17,5)=1;   conn_mat(17,6)=j*Y0*cot(k0*l2
conn_mat(18,15)=1;   conn_mat(18,6)=j*Y0*csc(k0*l
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

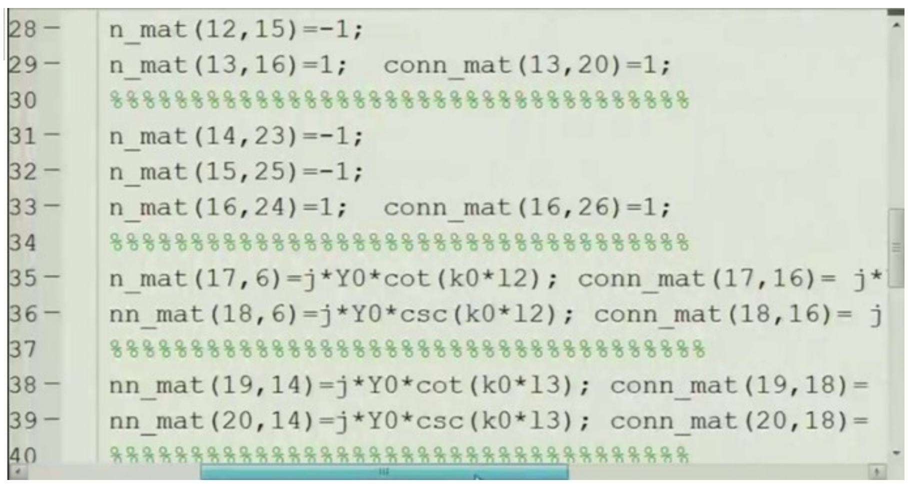
```
t(3,5)=-1;
t(4,7)=-1;
t(5,6)=1;   conn_mat(5,8)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
at(6,8)=j*Y0*cot(k0*l1); conn_mat(6,10)= j*Y0*cs
at(7,8)=j*Y0*csc(k0*l1); conn_mat(7,10)= j*Y0*co
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
at(8,11)=-1;
at(9,9)=-1;
mat(10,12)=1;   conn_mat(10,14)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mat(11,19)=-1;
mat(12,15)=-1;
```

And this is your; this is your this thing junction loss and similarly these are again your junction loss at a certain thing.



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(17,5)=1;   conn_mat(17,6)=j*Y0*cot(k0*l2
conn_mat(18,15)=1;   conn_mat(18,6)=j*Y0*csc(k0*l
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(19,13)=1;   conn_mat(19,14)=j*Y0*cot(k0*
conn_mat(20,17)=1;   conn_mat(20,14)=j*Y0*csc(k0*
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(21,11)=1;   conn_mat(21,12)=j*Y0*cot(k0*
conn_mat(22,23)=1;   conn_mat(22,12)=j*Y0*csc(k0*
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(23,19)=1;   conn_mat(23,20)=j*Y0*cot(k0*
conn_mat(24,21)=1;   conn_mat(24,20)=j*Y0*csc(k0*
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

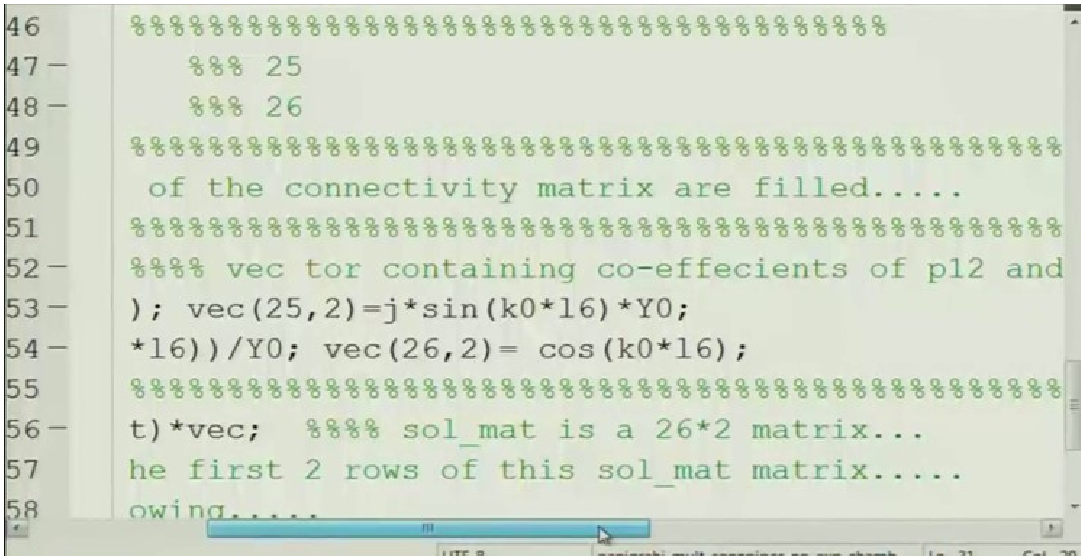
So, these are written by you know by a lot of bookkeeping and wherever you are getting



```
n_mat(12,15)=-1;
n_mat(13,16)=1;   conn_mat(13,20)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n_mat(14,23)=-1;
n_mat(15,25)=-1;
n_mat(16,24)=1;   conn_mat(16,26)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n_mat(17,6)=j*Y0*cot(k0*l2); conn_mat(17,16)= j*
nn_mat(18,6)=j*Y0*csc(k0*l2); conn_mat(18,16)= j
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
nn_mat(19,14)=j*Y0*cot(k0*l3); conn_mat(19,18)=
nn_mat(20,14)=j*Y0*csc(k0*l3); conn_mat(20,18)=
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

cot and cosec terms; that means, you should consider or understand that there is your wave propagation in the tube along those line. So, this is what it is. So, different so, you, so, the way connectivity matrix is written is quite unique.

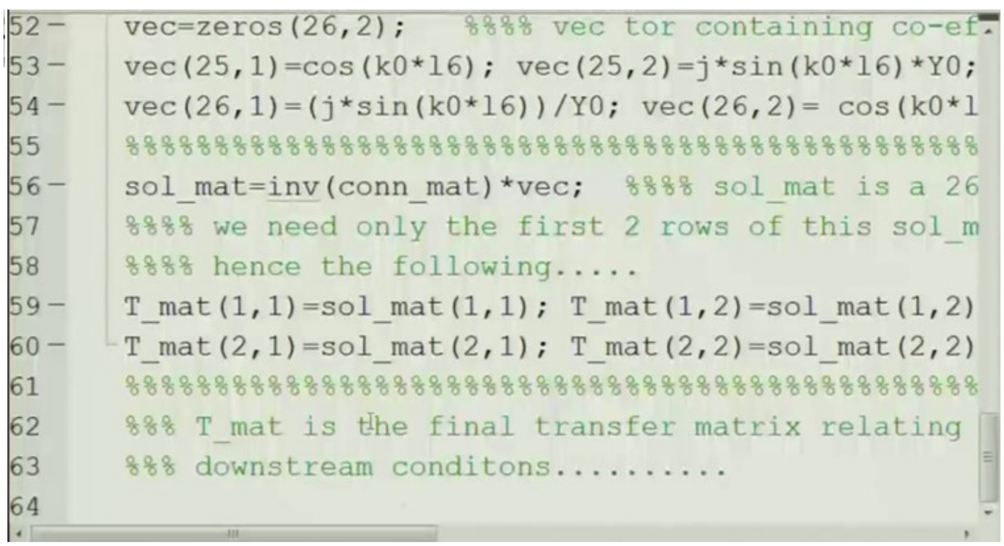
In the sense that in the first few rows you can consider the junction loss to be written first and then you know you should be you can get done with the junction loss and then talk about that propagation with an each element or vice versa or maybe have a jumble up jumbled up system in the sense that first junction loss are written and then your impedance matrix and so on so forth.



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   %%% 25
   %%% 26
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 of the connectivity matrix are filled.....
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% vec tor containing co-effecients of p12 and
); vec(25,2)=j*sin(k0*l6)*Y0;
*l6))/Y0; vec(26,2)= cos(k0*l6);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t)*vec;  %%%% sol_mat is a 26*2 matrix...
he first 2 rows of this sol_mat matrix.....
owing.....
```

So, finally, after writing all this, this is the final probably junction loss and or probably the propagation in the last downstream pipe and this is related to your some kind of a transfer matrix relation.

So, look well eventually what we get is a transfer matrix relation that is what we are heading towards. So, sol mat is inverse connectivity matrix.



```
vec=zeros(26,2);   %%%% vec tor containing co-ef
vec(25,1)=cos(k0*l6); vec(25,2)=j*sin(k0*l6)*Y0;
vec(26,1)=(j*sin(k0*l6))/Y0; vec(26,2)= cos(k0*l
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sol_mat=inv(conn_mat)*vec;  %%%% sol_mat is a 26
%%%% we need only the first 2 rows of this sol_m
%%%% hence the following.....
T_mat(1,1)=sol_mat(1,1); T_mat(1,2)=sol_mat(1,2)
T_mat(2,1)=sol_mat(2,1); T_mat(2,2)=sol_mat(2,2)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% T_mat is the final transfer matrix relating
%%% downstream conditons..........
```

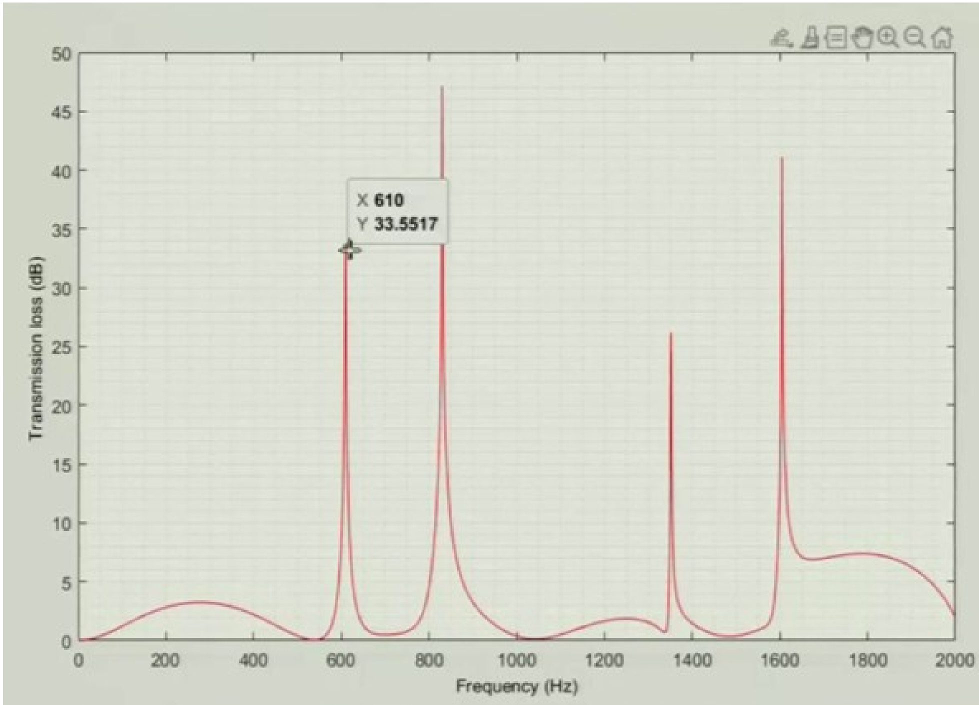
So, I did not purposely walk through the entire thing, because there is something based on the numbering you have followed and this numbering mind you is also not unique. Eventually you want relation between p_1 and p_{14} you can do it in whatever manner that suits you the numbering.

```
ining co-effecients of p12 and v12....
(k0*16)*Y0;
)= cos(k0*16);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
nat is a 26*2 matrix...
 this sol_mat matrix.....

ol_mat(1,2);
ol_mat(2,2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x relating the upstream conditions to
```

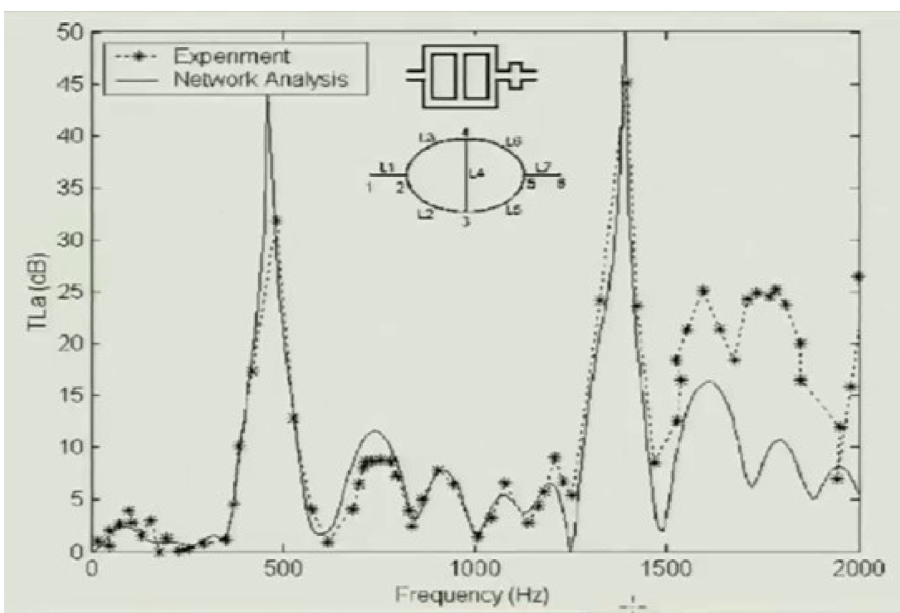
So and then there is a we need only the first two rows of this sol mat hence the following. So, T matrix is the final transfer matrix.

```
function [Tl]=transmissionloss(r0,l0,l1,l2,l3,l4,
c0=340;  %%%% speed of sound..should be same in a
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Y1=c0/(pi*(r0^2));
Y2=Y1;   %%%% impedances of the inlet/outlet pipe
[Tf]= panigrahi_mult_connpipes_no_exp_chamb(r0,l0
%%% computing Transfer matrix between inlet/outle
v=abs(     ((Y2/Y1)^0.5)*(Tf(1,1)+(Tf(1,2)/Y2)+(T
Tl=20*log10(v/2);
```

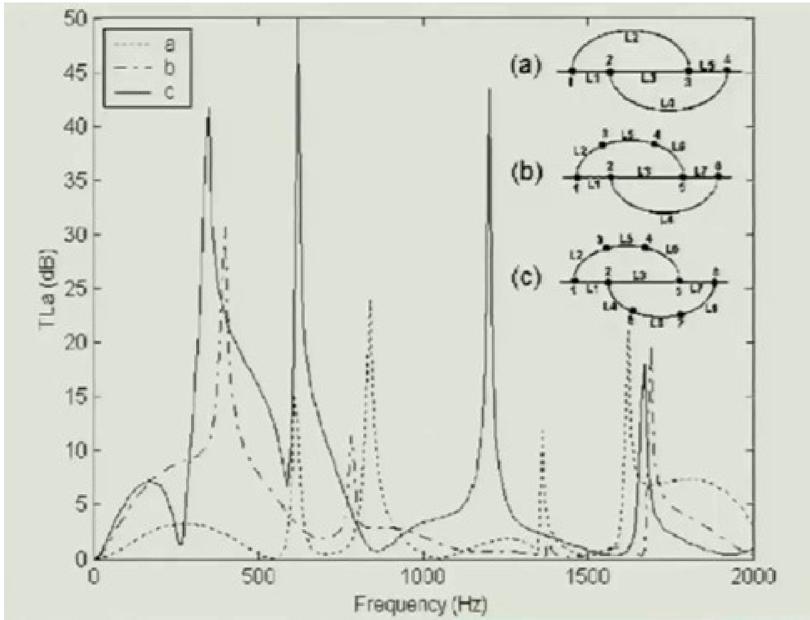
And then of course, remaining part remains the same you know you this guy calls the network system and finally, the transmission loss is plotted. So, this is already being computed. So, I will just show you one result for a given thing.



So, this was the result that we got where we have multiple such spikes, ok resonances that occur at this frequency and all these frequencies for a given length.



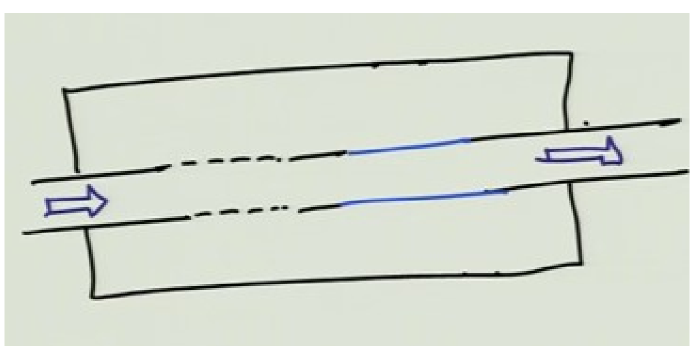
And let us carefully look at the graph. So, you know let me sort of like zoom it for you and a configuration a was this one.



And if you carefully look at the graphs here this one it goes like this and then you have peaks just about just above 600 or something and then below 1000 and this kind of a thing.

And that is precisely what you are kind of getting here. So, you know the idea is that we can actually represent things in terms of impedance matrix or transfer matrix it does not matter as long as we are you know doing a proper bookkeeping of terms. Another such configuration that I could talk about this was the one ok. So, we get this kind of a transmission loss graph and using a planar wave approach.

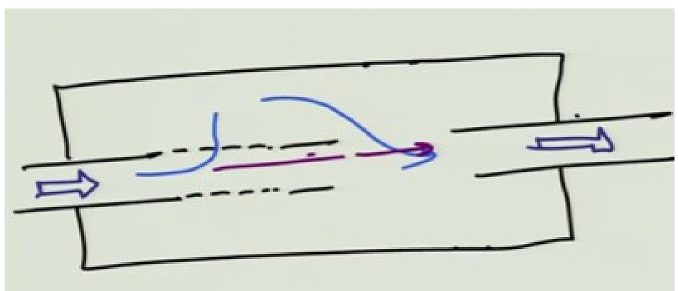
So, what we will do now as a final example for this particular lecture is that we will consider the configuration shown here. So, you know what we see here.



Let me just sort of redraw this thing in a better manner. We have an extended inlet here ok. So, there is a flow coming in here and it is going there. Now, you might be wondering how is it a multiply connected duct well because you know waves for example, if this pipe was something like fully joined something like this.

Then the waves would have actually had no way no other option, but to basically go through the straight through duct and; obviously, through interactions with the cavity here by means of a perforated thing ok. And then these cavities would have acted like a you know remember lecture 7 I am sorry week 7 or week 8 stuff where we did the extended tube resonator, but with the perforated bridge that is a concentrative resonator thing.

So, this would have exactly acted like a concentrative resonator if this thing was filled up or with or basically was connected with a pipe.

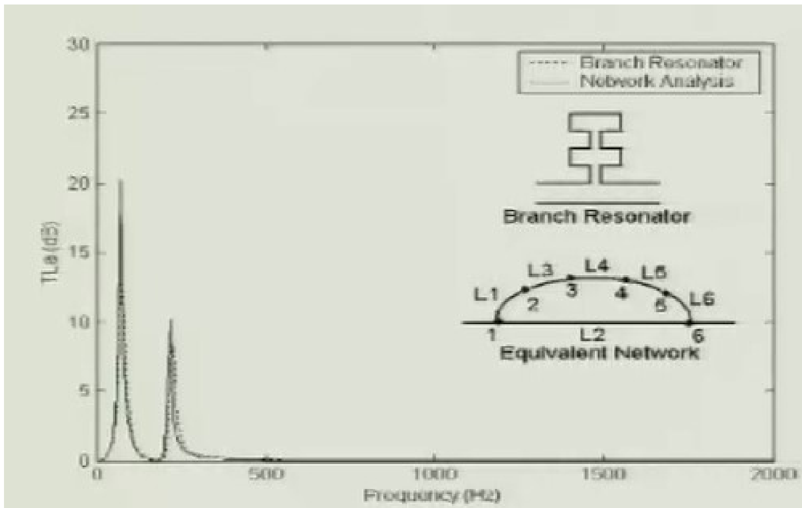


But since that is not the case what the waves can do is that they can interact with the cavity here through the perforated pipe and the waves can come back here and the other connection another wave another way the waves can propagate is basically straight coming through like this and interacting with the volume here some part of that gets reflected back and some part gets has a some kind of an interaction with the waves that already propagate from here.

So, this is one you know a rather unique or a simple example of how multiply connected path can be formed in a perforated tube muffler. Just by you know not just by leaving one of the ends open that is not connecting the inlet and outlet pipe. So, what going back to the paper by Panigrahi and Munjal published quite a few years back.

You know what they did was they basically used the network analysis or you know again the same thing equality of pressure at the nodes at the junctions and the some of the mass velocity is constant at a junction and then the individual transfer matrices for the different sections are known.

So, then you have this multiply connected mufflers and they analyze that using these junction loss in the plane wave propagation. So, plane wave propagation has been the method of analysis.



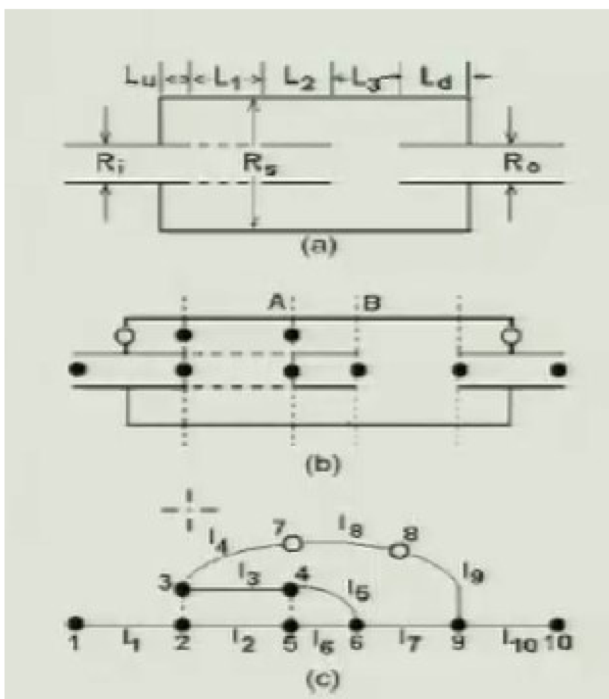
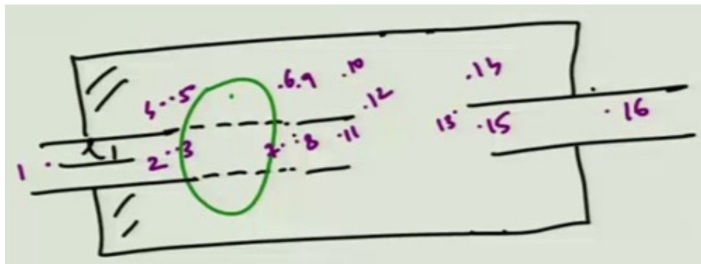


Figure 10: (a) Schematic of a multiply connected muffler. (b) Node marking scheme. (c) the branches have been numbered through the algorithm. $L_3 = L_4 = 0.1m$ $L_1 = 0.12m$, $L_2 = 0.12m$, $L_3 = 0.08m$, $R_1 = R_0 = 0.02m$, $R_x = 0.05m$.

So, what I am going to do is that very quickly I am going to go back to the configuration and start talking about perhaps how to analyze this very quickly.



So, let me number the node. So, we can number this as 1 ok and this node which is just here 2 and just the number which is here as 3, this is 4 just at the in the cavity and this is 5 ok and then this is 6, 7, 8, 9, 10, 11 and your 12 and then the entire section here is 13, 14, 15 and finally, at some point it is 16.

So, each remember our discussion in this thing each of the nodes has two variables pressure and velocity. So, how many of how many variables do we have? We have $16 \times 2 = 32$ variables, 16 out of which are pressures and 16 are mass velocities. So, now eventually we would like to obtain something like this, is not it?

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_2 \end{Bmatrix} = [T_{or}] \begin{Bmatrix} \tilde{p}_{16} \\ \tilde{V}_{16} \end{Bmatrix}$$

So, we have this sort of a thing ok. So, we relate p_1, v_1 to p_{16} and v_{16} using a transfer matrix representation, alright. So, how to do that? Basically again the same thing let me write down the constituent elements. So, you know we can do a kind of a network analysis.

So, I am just going to write down of the first few things and rest we can sort of follow data this is cot k naught. Let us say this length is something.

$$\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = -jY_0 \begin{bmatrix} \cot k_0\, l_1 & \cos\ e_c\, k_0\, l_1 \\ \cos\ e_c\, k_0\, l_1 & cot k_0\, l_1 \end{bmatrix}$$

So, this is for this part. Similarly, the annular cavity.

$$\frac{p_4}{V_4} = -jY_2\, cotk_0L_1; \quad Y_2 = \frac{4C_0}{\pi(D = d^2)}$$

So, you know like this we can also kind of define what is going to happen somewhere in here. We can what we can do is that, we can relate

$$\begin{Bmatrix} p_3 \\ V_3 \\ p_5 \\ V_5 \end{Bmatrix} = [T]_{4\times4} \begin{Bmatrix} p_7 \\ V_7 \\ p_6 \\ V_6 \end{Bmatrix}$$

What we had when we analyze the perforated two duct perforated elements relating the upstream variables to that in the downstream by means of a transfer matrix which is something like you know p7, v7 and here you have your p6, v6, is not it?

So, like this we can keep defining the transfer matrices of the kind instead individual elements and then apply junction loss. So, some of the junction loss which is you know fairly evident is that $V_2 + V_3 = 0$ and here we assume that the direction of the mass velocities is into the system looking into the systems and $V_4 + V_5 = 0$, $p_4 = p_5$ and $p_2 = p_3$ like this we can continue to analyze this.

So, let us go quickly to the MATLAB code which was written some time back by me and to analyze such a configuration which Panigrahi and Munjal analyzed long time back.

```
function [Tl]=transmissionloss(D,d,L1,l1,L2,l2,L3
c0=340;  %%%% speed of sound..should be same in a
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
S1=(pi/4)*(d^2);  Y1=c0/S1;
Y2=Y1;   %%%% impedances of the inlet/outlet pipe
[Tf] = connectivity_matrix(D,d,L1,l1,L2,l2,L3,k0,
%%% computing Transfer matrix between inlet/outle
v=abs(     ((Y2/Y1)^0.5)*(Tf(1,1)+(Tf(1,2)/Y2)+(T
Tl=20*log10(v/2);
```

```
function [T_mat] = connectivity_matrix(D,d,L1,l1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
j=sqrt(-1);
c0=340; %%% speed of sound...
S2=(pi/4)*( (D^2) - (d^2) );    Y2=c0/S2;   %%% ch
S1=(pi/4)*(d^2);                Y1=c0/S1;   %%% ch
S_cham=(pi/4)*(D^2);            Y_cham=c0/S_cham;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T= transfer_mat(D,d,l1,k0,t,dh,sigma); %%% trans
%%% now defining the connectivity matrix....
conn_mat=zeros(30,30);    %%% the connectivity ma
%%% filling the entries.....
%%% p0=1; v0=2; p1=3; v1=4;  p2=5; v2=6; p3=7; v
```

So, let us go to the connectivity matrix and. So, the this is the really the heart of the system heart of the code everything else is just a plotting thing or once you have got T mat to the overall transfer matrix this is just it is a regular stuff. So, I will not spend much time here sound speed and the annular volume S2 is the annular volume π / 4d square minus small d^2 and this is your area of the pipe.

```
c0=340; %%% speed of sound...
S2=(pi/4)*( (D^2) - (d^2) );    Y2=c0/S2;   %%% ch
S1=(pi/4)*(d^2);                Y1=c0/S1;   %%% ch
S_cham=(pi/4)*(D^2);            Y_cham=c0/S_cham;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T= transfer_mat(D,d,l1,k0,t,dh,sigma); %%% trans
%%% now defining the connectivity matrix....
conn_mat=zeros(30,30);    %%% the connectivity ma
%%% filling the entries.....
%%% p0=1; v0=2; p1=3; v1=4;  p2=5; v2=6; p3=7; v
%%% p8=17; v8=18; p9=19; v9=20;  p10=21; v10=22;
%%% v12=26; p13=27; v13=28;  p14=29; v14=30;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

And now this is the transfer matrix for the transfer matrix for the perforated section.

```
impedance   (annular region)
impedance    (inner pipe)
ristic impedance    (chamber)
%%
r the perforated section....



=10; p5=11; v5=12;  p6=13; v6=14; p7=15; v7=16;
24; p12=25;
```

So, this is something by now it should be very clear what is going on. So, this is your transfer matrix.

```
function [Tr] = transfer_mat(D,d,l1,k0,t,dh,sigm
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
j=sqrt(-1);
c0=340;
%%%%%%%%%%%%%%
Sc_ann=(pi/4)*( (D^2) - (d^2) );    %%%  (annular
Sc_pipe=(pi/4)*(d^2);                %%%  (inner
Sb_ann=Sc_ann;
Sb_pipe=Sc_pipe;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
zeta=(6*10^-3 + j*k0*( t + (0.75*dh) ) )/sigma;
A=zeros(4,4);  %%% A is the matrix to be integra
A(1,2)=-j*k0;
```

```
Sc_pipe=(pi/4)*(d^2);                %%%  (inner
Sb_ann=Sc_ann;
Sb_pipe=Sc_pipe;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
zeta=(6*10^-3 + j*k0*( t + (0.75*dh) ) )/sigma;
A=zeros(4,4);  %%% A is the matrix to be integra
A(1,2)=-j*k0;
A(2,1)=-( j*k0 + (4/(d*zeta)) );
A(2,3)=(4/(d*zeta));
A(3,4)=-j*k0;
A(4,1)=(4*d)/(zeta*(D^2 - d^2 ));
A(4,3)=-j*k0 -(4*d)/((D^2 - d^2)*zeta);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

So, basically you know you have two duct system and this is the perforated impedance for a stationary medium and then you have this sort of a thing.

```
                                 T(1,2)=T(1,2)*(c0/Sb
T(2,1)=T(2,1)*(Sc_pipe/c0); T(2,2)=T(2,2)*(Sc_pi
                                 T(3,2)=T(3,2)*(c0/Sb
T(4,1)=T(4,1)*(Sc_ann/c0);  T(4,2)=T(4,2)*(Sc_an
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Tr=inv(T);
Tr(:,2)=-Tr(:,2);
Tr(:,4)=-Tr(:,4);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  |Pb1|          |Tr11   Tr12     Tr13     Tr1
%%%  |Vb1|          |Tr21   Tr22     Tr23     Tr2
%%%  |Pb2| =        |Tr31   Tr32     Tr33     Tr3
%%%  |Vb2|          |Tr41   Tr42     Tr43     Tr4
```

```
Tr(:,2)=-Tr(:,2);
Tr(:,4)=-Tr(:,4);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  |Pb1|          |Tr11   Tr12     Tr13     Tr1
%%%  |Vb1|          |Tr21   Tr22     Tr23     Tr2
%%%  |Pb2| =        |Tr31   Tr32     Tr33     Tr3
%%%  |Vb2|          |Tr41   Tr42     Tr43     Tr4
%%% sections b and c are as in worked paper, sub
%%% innner or pipe region, while subscript 2 cor
%%% region....
```

So, we get this exponential basically relating downstream to upstream variable. Because at the end of the day well by some arrangement or something we kind of arrange that by taking some inverse into a suitable form and arrange it in such a manner the directions of the mass velocities are taken positive looking into the system. So, that is what this transfer matrix does, I will not go into too much detail.

But the end of the day the transfer matrix was basically what it does? It is it gets us this matrix p_3, v_3, p_5, v_5 related to p_7, v_7, p_6, v_6 where directions of p_7 and sorry v_7 and v_6 are looking into the system. Now, going back to the connectivity matrix once we get once we sort of get the transfer matrix of the perforated things, we define the 30 cross 30 connectivity matrix.

Because remember we have 32 variables and any 30 variables can be represented in terms of the balance 2 variables, pressure and velocity which we choose at the exit. We know because we have 30 equations, but we have 30, 32 unknown. So, any 30 equations can be represented as in the form of remaining 2 equation 2 variables which we choose to be the outlet variables.

```
S_cham=(pi/4)*(D^2);              Y_cham=c0/S_cham;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T= transfer_mat(D,d,l1,k0,t,dh,sigma); %%% trans
%%% now defining the connectivity matrix....
conn_mat=zeros(30,30);   %%% the connectivity ma
%%% filling the entries.....
%%% p0=1; v0=2; p1=3; v1=4;  p2=5; v2=6; p3=7; v
%%% p8=17; v8=18; p9=19; v9=20;  p10=21; v10=22;
%%% v12=26; p13=27; v13=28;  p14=29; v14=30;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  L2=L2+ 0.6*(d/2)*( 1- (d/D) ); L3= L3 + 0.6*(
%  l2=l2-(L2+L3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

So, and we can actually see why it is 30, you know why? Because, you see if you go back to your this thing you will get 2 equations for this plus 3, plus 4 equations. So, 2 let me keep writing down 2 plus 1 equation minus j k naught into k l something like that 1 plus 4 and then your 2 equations here 2 here and then 2 for this section 1, for this section and 2, again for this section, but we also have now junction loss.

So, junction loss would mean that you know we have one equation here and you know one equation here for the velocities in the pressure. So, you write here plus 3 and then you have similarly you know you have equality of pressure. So, you have another 3 equations. So, yeah and then you have another 3 equations here. Actually hang on I guess I just made a small mistake here. So, I made a small mistake here instead of 3 there will be 4 because 1 p you know you see v_2, v_3, v_4, v_5 and p_4.

So, here you have 4 such things and similarly plus 4 here sorry for that and then you have your you know v11 plus v10 plus v12 is 0 and p10 is equal to p11 and p11 is equal to p12. So, we have basically 3 such equations and you have also have 3 more equations here you know because of these things and. So, how many do we get?

$$2 + 1 + 4 + 2 + 2 + 2 + 1 + 2 + 4 + 4 + 3 + 3 = 30$$

So, we get 30 equations, but we have 32 variables. So, we can easily you know express 30 variables in terms of the balance at 2 variables. So, here is your detailed workout of the code I am sorry connectivity matrix is initialized with 0 entries everywhere and then you know you start from the first pipe.

```
%%% p0=1; v0=2; p1=3; v1=4;  p2=5; v2=6; p3=7; v
%%% p8=17; v8=18; p9=19; v9=20;  p10=21; v10=22;
%%% v12=26; p13=27; v13=28;  p14=29; v14=30;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  L2=L2+ 0.6*(d/2)*( 1- (d/D) ); L3= L3 + 0.6*(
%  l2=l2-(L2+L3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(1,1)=1; conn_mat(1,2)=j*Y1*cot(k0*L1);
conn_mat(2,3)=1; conn_mat(2,2)=j*Y1*csc(k0*L1);
conn_mat(3,3)=1; conn_mat(3,5)=-1;
conn_mat(4,4)=1; conn_mat(4,6)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(5,9)=1; conn_mat(5,10)=j*Y2*cot(k0*L1);
```

```
>4=9; v4=10; p5=11; v5=12;  p6=13; v6=14; p7=15;
23; v11=24; p12=25;



( 1- (d/D) );



nat(1,4)= j*Y1*csc(k0*L1);  %%% 1
nat(2,4)= j*Y1*cot(k0*L1);  %%% 2
                            %%% 3
                            %%% 4
%%%%
                            %%% 5
```

And you get you know this element now there is lot of bookkeeping I am not going to the entire code.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(1,1)=1; conn_mat(1,2)=j*Y1*cot(k0*L1);
conn_mat(2,3)=1; conn_mat(2,2)=j*Y1*csc(k0*L1);
conn_mat(3,3)=1; conn_mat(3,5)=-1;
conn_mat(4,4)=1; conn_mat(4,6)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(5,9)=1; conn_mat(5,10)=j*Y2*cot(k0*L1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(6,9)=1; conn_mat(6,7)=-1;
conn_mat(7,10)=1; conn_mat(7,8)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% now, relating connectivity matrix for the pe
conn_mat(8,5)=1;   conn_mat(8,13)=-T(1,1);   conn
```

But these are junction loss really for the you know for the first interaction of the pipe and the perforated thing.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mat(1,2)=j*Y1*cot(k0*L1);  conn_mat(1,4)= j*Y1*csc
mat(2,2)=j*Y1*csc(k0*L1);  conn_mat(2,4)= j*Y1*cot
mat(3,5)=-1;
mat(4,6)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mat(5,10)=j*Y2*cot(k0*L1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mat(6,7)=-1;
mat(7,8)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ctivity matrix for the perforated section..
mat(8,13)=-T(1,1);  conn_mat(8,14)= -T(1,2);
```

And then you have your annular cavity then your again the between the annular cavity and perforated pipe.

```
conn_mat(3,3)=1;  conn_mat(3,5)=-1;
conn_mat(4,4)=1;  conn_mat(4,6)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(5,9)=1;  conn_mat(5,10)=j*Y2*cot(k0*L1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(6,9)=1;  conn_mat(6,7)=-1;
conn_mat(7,10)=1;  conn_mat(7,8)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% now, relating connectivity matrix for the pe
conn_mat(8,5)=1;    conn_mat(8,13)=-T(1,1);   conn
conn_mat(9,6)=1;    conn_mat(9,13)=-T(2,1);   conn
conn_mat(10,7)=1;  conn_mat(10,13)=-T(3,1);  conn_
conn_mat(11,8)=1;  conn_mat(11,13)=-T(4,1);  conn
```

```
                                                %%% 3
                                                %%% 4
%%%%%%%%%%%%%%%%%
k0*L1);                                         %%% 5
%%%%%%%%%%%%%%%%%
                                                %%% 6
                                                %%% 7
%%%%%%%%%%%%%%%%%
 the perforated section..
;   conn_mat(8,14)= -T(1,2);    conn_mat(8,11)=-T(
;   conn_mat(9,14)= -T(2,2);    conn_mat(9,11)=-T(
; conn_mat(10,14)=-T(3,2);   conn_mat(10,11)=-T(
; conn_mat(11,14)=-T(4,2);   conn_mat(11,11)=-T(
```

Now relating the connectivity matrix for perforated section length all those things will happen.

```
conn_mat(8,5)=1;    conn_mat(8,13)=-T(1,1);   conn
conn_mat(9,6)=1;    conn_mat(9,13)=-T(2,1);   conn
conn_mat(10,7)=1; conn_mat(10,13)=-T(3,1); conn_
conn_mat(11,8)=1; conn_mat(11,13)=-T(4,1); conn_
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(12,11)=1; conn_mat(12,17)=-1;
conn_mat(13,12)=1; conn_mat(13,18)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(14,13)=1; conn_mat(14,15)=-1;
conn_mat(15,14)=1; conn_mat(15,16)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(16,17)=1; conn_mat(16,18)=j*Y2*cot(k0*L
conn_mat(17,19)=1; conn_mat(17,18)=j*Y2*csc(k0*L
```

So, it requires a little bit of practice in MATLAB coding as you would realize by now that you are going to get some MATLAB based assignment for final exam. The codes will be given to you, but you should be in a comfortable position to play around with some parameters and do something. So, you should also start doing your own coding and again these are all junction loss.

```
conn_mat(13,12)=1; conn_mat(13,18)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(14,13)=1; conn_mat(14,15)=-1;
conn_mat(15,14)=1; conn_mat(15,16)=1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(16,17)=1; conn_mat(16,18)=j*Y2*cot(k0*L
conn_mat(17,19)=1; conn_mat(17,18)=j*Y2*csc(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(18,15)=1; conn_mat(18,16)=j*Y1*cot(k0*L
conn_mat(19,21)=1; conn_mat(19,16)=j*Y1*csc(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(20,21)=1; conn_mat(20,19)=-1;
conn_mat(21,23)=1; conn_mat(21,21)=-1;
```

Wherever you have see 1 plus 1 minus 1 and all that these are all junction loss.

```
                                          %%% 13
%%%%
                                          %%% 14
                                          %%% 15
%%%%
ın_mat(16,20)= j*Y2*csc(k0*L2);   %%% 16
ın_mat(17,20)= j*Y2*cot(k0*L2);   %%% 17
%%%%
ın_mat(18,22)= j*Y1*csc(k0*L2);   %%% 18
ın_mat(19,22)= j*Y1*cot(k0*L2);   %%% 19
%%%
                                          %%% 20
                                          %%% 21
```

And then these are the equations relating the tubular elements cot cosec and then your these kind of things.

```
ın_mat(17,19)=1; conn_mat(17,18)=j*Y2*csc(k0*L2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ın_mat(18,15)=1; conn_mat(18,16)=j*Y1*cot(k0*L2);
ın_mat(19,21)=1; conn_mat(19,16)=j*Y1*csc(k0*L2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ın_mat(20,21)=1; conn_mat(20,19)=-1;
ın_mat(21,23)=1; conn_mat(21,21)=-1;
ın_mat(22,22)=1; conn_mat(22,20)=1;   conn_mat(22,
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ın_mat(23,23)=1; conn_mat(23,24)=j*Y_cham*cot(k0*
ın_mat(24,25)=1; conn_mat(24,24)=j*Y_cham*csc(k0*
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ın_mat(25,25)=1; conn_mat(25,29)=-1;
```

```
conn_mat(21,23)=1; conn_mat(21,21)=-1;
conn_mat(22,22)=1; conn_mat(22,20)=1;   conn_mat(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(23,23)=1; conn_mat(23,24)=j*Y_cham*cot(
conn_mat(24,25)=1; conn_mat(24,24)=j*Y_cham*csc(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(25,25)=1; conn_mat(25,29)=-1;
conn_mat(26,25)=1; conn_mat(26,27)=-1;
conn_mat(27,26)=1; conn_mat(27,28)=1;   conn_mat(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(28,27)=1; conn_mat(28,28)=j*Y2*cot(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(29,29)=1;     %%% 29
```

```
conn_mat(23,23)=1; conn_mat(23,24)=j*Y_cham*cot(
conn_mat(24,25)=1; conn_mat(24,24)=j*Y_cham*csc(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(25,25)=1; conn_mat(25,29)=-1;
conn_mat(26,25)=1; conn_mat(26,27)=-1;
conn_mat(27,26)=1; conn_mat(27,28)=1;   conn_mat(
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(28,27)=1; conn_mat(28,28)=j*Y2*cot(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(29,29)=1;     %%% 29
conn_mat(30,30)=1;     %%% 30
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% all the entries of the connectivity matrix a
```

```
*Y_cham*cot(k0*l2); conn_mat(23,26)= j*Y_cham*cs
*Y_cham*csc(k0*l2); conn_mat(24,26)= j*Y_cham*co
%%%%%%%%%%%%%%%%%%%%%
l;
l;
;   conn_mat(27,30)=1;
%%%%%%%%%%%%%%%%%%%%%
*Y2*cot(k0*L3);
%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ity matrix are filled....
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(28,27)=1; conn_mat(28,28)=j*Y2*cot(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
conn_mat(29,29)=1;     %%% 29
conn_mat(30,30)=1;     %%% 30
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% all the entries of the connectivity matrix a
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
vec=zeros(30,2);    %%%% vec tor containing co-ef
vec(29,1)=cos(k0*L3); vec(29,2)=j*sin(k0*L3)*Y1;
vec(30,1)=(j*sin(k0*L3))/Y1; vec(30,2)= cos(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sol_mat=inv(conn_mat)*vec;  %%%% sol_mat is a 30
```

And you know finally, we arrive at you know at the vector thing where you know vector matrix is a rectangular matrix 30 cross 2. Because 30 equations and 2 variable are your pressure and velocity data at the extreme thing at the extreme outlet pipe.

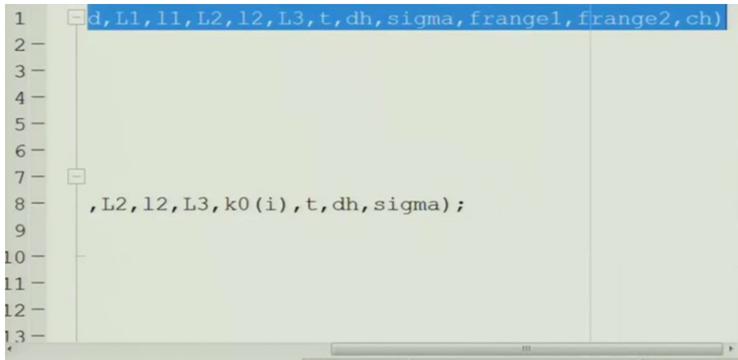
```
vec=zeros(30,2);   %%%% vec tor containing co-ef
vec(29,1)=cos(k0*L3); vec(29,2)=j*sin(k0*L3)*Y1;
vec(30,1)=(j*sin(k0*L3))/Y1; vec(30,2)= cos(k0*L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sol_mat=inv(conn_mat)*vec;  %%%% sol_mat is a 30
%%%% we need only the first 2 rows of this sol_m
%%%% hence the following.....
T_mat(1,1)=sol_mat(1,1); T_mat(1,2)=sol_mat(1,2)
T_mat(2,1)=sol_mat(2,1); T_mat(2,2)=sol_mat(2,2)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% T_mat is the final transfer matrix relating
%%% downstream conditons..........

```

And so, that this takes a simple form of cos and sin there should not be any problem. So, when you do solve inverse of connectivity matrix times vector it basically relates all the variables ranging from p1, v1, p2, v2 and so on; p15, v15 to p16 and v16 ok. So, it does that and then once you do that you get this transfer matrix ok.

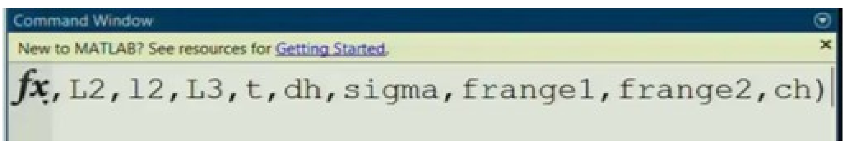
You get the transfer matrix and then because the choice of the variable was such that you know the p1 and v you know the rows 1 and 2 may corresponding to p1 and v1 pressure and mass velocities at the inlet pipe at the very extreme. So, we are just want we just want to extract that part from the sol mat, alright.

```
function [] =transmission_loss_plot(D,d,L1,l1,L2
f=frange1:1:frange2;
n1=size(f); n=n1(1,2);
c0=340;
k0=(2*pi*f)/c0;
Tl=zeros(n,1);
for i=1:n
      Tl(i)=transmissionloss(D,d,L1,l1,L2,l2,L3,
%       i
end
figure(1)
plot(f,Tl,ch)
axis([0,2000,0,70])
```

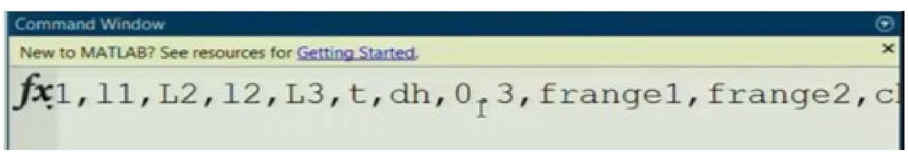
So, once we do that this function basically gets you the overall transfer matrix using which you compute the transmission loss and happily you know pass on the variable transmission loss value to the transmission loss plot.

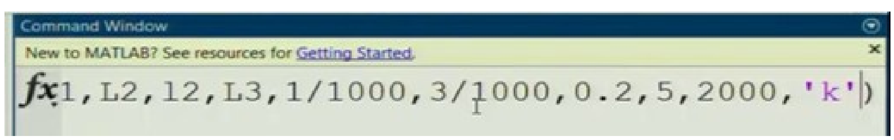


And what we can do is that we can sort of plot it straight away regarding this thing.

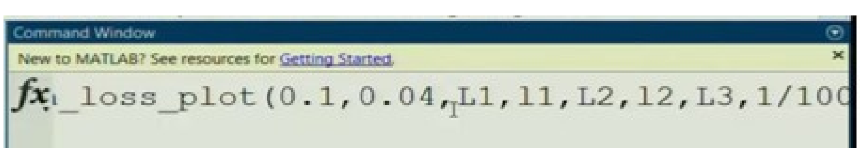


So, let me change the folder for you guys and otherwise it will show an error, it will show an error otherwise.

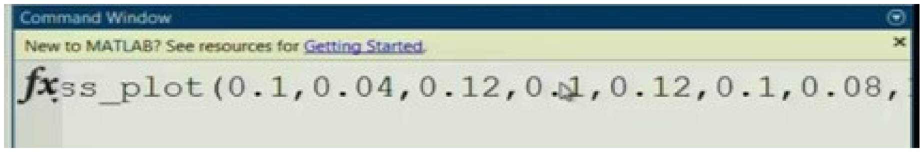




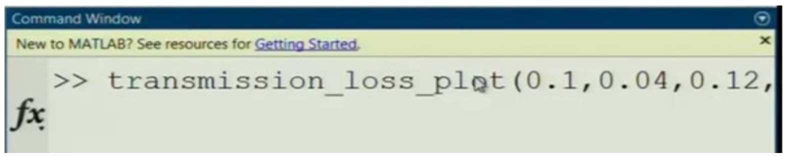
So, I am taking a nominal porosity say of 30 percent, maybe I can take you know who knows 20 percent whole diameter is 3 mm, thickness is 1 mm and this is 5 you can consider 5 and this is 2000, k and diameter can be 0.1, this can be 0.04.

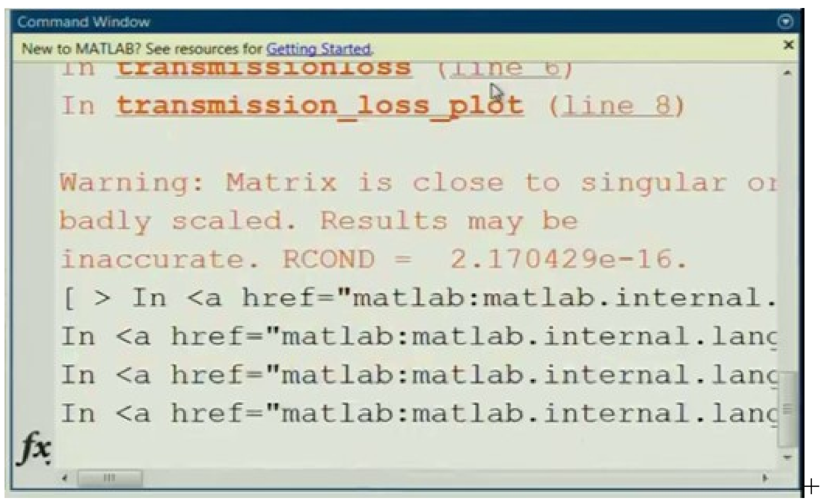


Now, what about L_1? You know let us go back to the paper and figure out what is L_1.

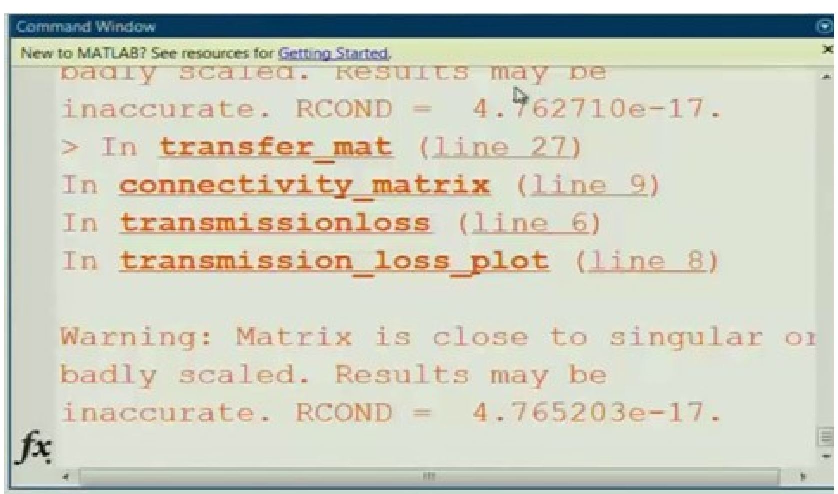


So, L_1 is your 0.12 and L_2 is also a 0.12, L_3 is 0.08 Lu, Ld is something different. So, what we will do is that we will put 0.12 and 0.12 and this guy is point I guess 08 ok 0.08. So, just hang on; yeah, so, 80 mm. So, and this can be your again be a 0.1 meters that is 100 centimeters where is it? And this is also 0.1.

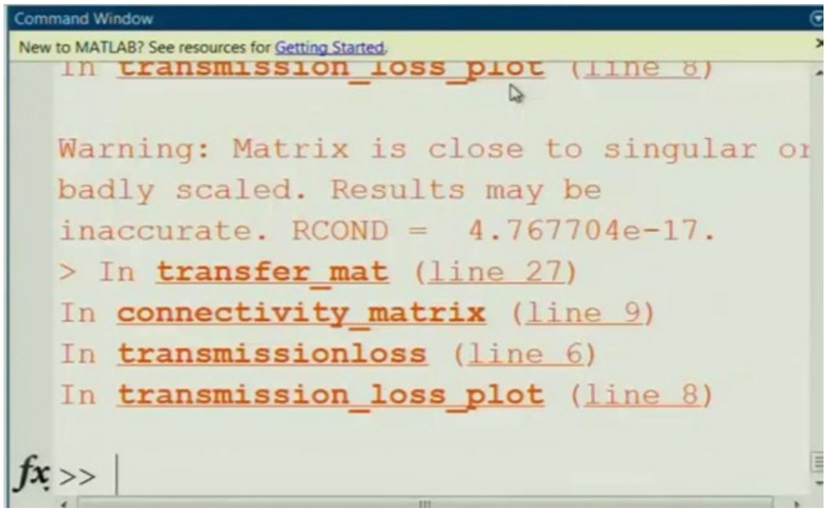




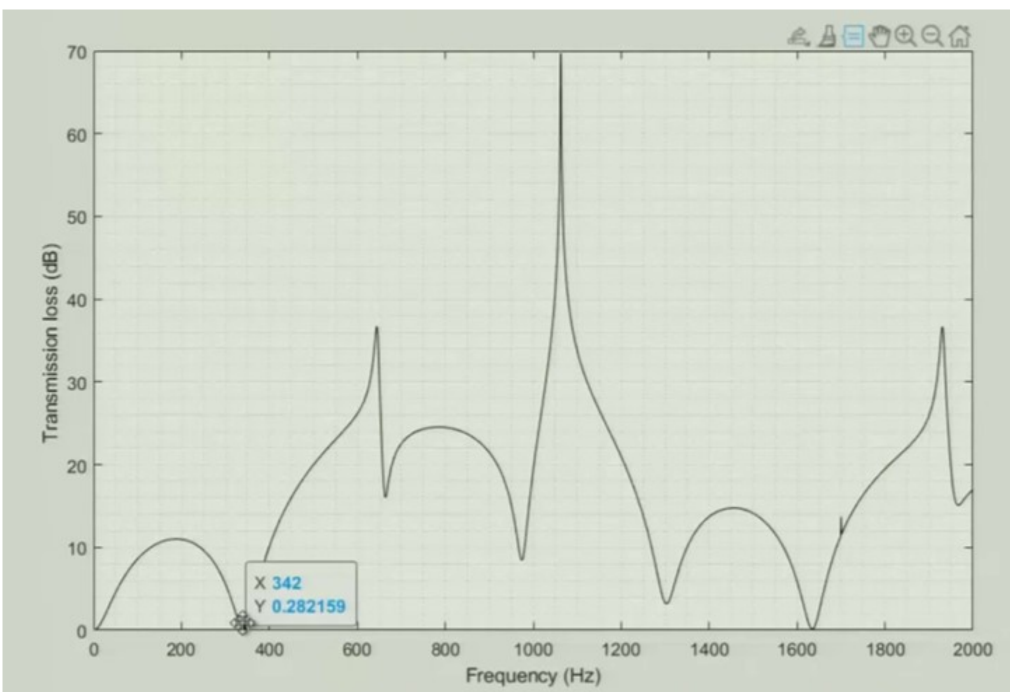
So, let us do the computation showing some things in here.



So, there might be some because of the exponential kind of a thing that because of the transfer matrix rendering the perforated thing, there might be some warnings.



But at the end of the day you get decent result and with small variation in the parameter you can verify how good or bad is it compared to the results obtained by Panigrahi and Munjal. So, you get you know these peaks here which are really kind of characteristic of your resonator peaks here as well as here.



Although what unfortunately it is not able to nullify the trough the resonance here that you are still getting. So, let us figure out what is the; what is this value it is I guess 342

Hertz and what is the chamber length in all? So, any idea what is the chamber length? So, this is your 0.1. So, this will become your 0.24 and 0.32 and 0.42, 0.52.

So, basically one can figure out what will be the resonance frequencies and you see they are not being the first actual resonance frequencies are not being able to nullify by the trough here. So, the idea is that well to just to demonstrate the multiply connected mufflers, but by playing around the sequence still tune the performance to nullify the trough here sitting somewhere here.

So, I guess I will stop here in this class and what I am going; this was an extended lecture, but what we are going to do in the next class is for the first time introduce concepts like a integrated transfer matrix approach. What it does is basically you know it can be used to analyze quite complicated muffler systems.

So, we will worry about the details in the next class using a MATLAB demonstration and some documentation that was prepared. So, till that time I say goodbye and I will see you very shortly.

Thanks.