Muffler Acoustics - Application to Automotive Exhaust Noise Control

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Lecture - 58
MATLAB Demonstration for Transmission Loss Calculations

Welcome back to our NPTEL course on Muffler Acoustics. So, this is lecture 58. Probably the this is the 3rd lecture of this week and what we are going to do today is going to have a sort of an extended session on MATLAB demonstration of the analytical mode matching method to evaluate the transmission loss performance of extended inlet and outlet muffler configuration and also in the process do a limiting case of a concentric expansion chamber.

So, you know that was a very basic element that we studied in the first; in the first in the very first few lectures probably from the lecture 4 onwards. So, all that why you know that time we studied analysis and found out nice clean analytical expressions. And then with understanding that you know at higher frequencies the 3 dimensional effects are important and then we deferred our discussion to the 3D effects until this time until this week.

So, what we are going to do is that for the first time we will consider analytical mode matching that is you know the full 3D field is considered the full modal expansion is considered in the ports as well as in the main chamber.

And then using these mode expansion we will you know enforce the conditions of continuity of pressure and velocity, what we discussed in just in the last lecture last couple of lectures combined 56 and 57 and using the equations that we set up. So, we are going to; we are going to do the MATLAB demonstration.

So, before we go to the MATLAB demonstration, let me very quickly go; let me just very quickly walk you through the finally, the mathematical formulation that we covered in the last class.

$$\begin{bmatrix} \quad & & \quad \end{bmatrix}\begin{Bmatrix}\{A_n^-\}\\ \{B_n^-\}\\ \{C_n^+\}\\ \{C_n^-\}\\ \{D_n^+\}\\ \{E_n^+\}\end{Bmatrix} = \begin{bmatrix} \quad \end{bmatrix}\{A_0^+\}$$

This was the formulation that we did you know let me just go back to the few slides.

$$\begin{array}{l|l l}
A_0^+ & A_0^- \neq 0 & A_n^- \,\Big|_{n=0,n_1} \; q \\
A_1^+ = 0 & A_1^- \neq 0 & B_n^- \,\Big|_{n=0,n_2} \; q \\
A_2^+ = 0 & A_2^- \neq 0 & \tilde{C}_n^+ \,\Big|_{n=0,n_3} \; q \\
\vdots & & C_n^- \,\Big|_{n=0,n_3} \; q \\
\vdots & & \\
A_n^+ = 0 & & D_n^+ \,\Big|_{n=0,n_2} \; q \\
E_n^+ = 0 & are & E_n^+ \,\Big|_{n=0,n_1} \; q
\end{array}$$

6q

$$(\nabla^2 + \mathrm{k}_0^2)\,\tilde{p} = 0 \qquad j = \sqrt{-1}$$

$$\tilde{p}_n(r,\ z) = \sum_{n=0}^{\infty} \left(\vec{A}_n^+ e^{-jk_nz} + A_n^- e^{-jk_nz}\right)\psi_n(r)$$

$$A_n^+ \text{ and } A_n^- \text{ are the model}$$

$$\psi_n^A = J_0\left(\frac{\alpha_n}{k_0} r\right)$$

$$k_n = \sqrt{k_0^2 - \left(\frac{\alpha_n}{l_0}\right)^2}$$

So, you know that is we started off with the modal summation formulation in the ducts. And you know there was we clearly mentioned that you know because of the concentric nature of the location of the ports only the axis symmetric modes the they will be excited or those are the ones that are important to be considered. The circumferential mode of the cross modes are not necessary.

So, we will consider only the radial mode. So, the axis symmetric modes and that is why we have written this formulation there is no dependence on θ that is the angular dependence there is only dependence on radius R ok. And we get this kind of a thing where n of course is the n is a modal particular nth radial mode or n plus 1th radial mode and this is clearly Bessel function of order 0.

$$k_n = \sqrt{-1}\ \sqrt{\left(\frac{\alpha_n}{k_1}\right)^2 - k_0^2} = j\sqrt{(-)}$$

$$A_n^+\, e^{-j} j\sqrt{\left(\frac{\alpha_n}{k_1}\right)^2 - k_0^2}\ z = A_n^+\, e^{(-)z}$$

$$A_n^+\, e^{-j} j\sqrt{(-)}\ z = A_n^-\, e^{(-)z}$$

$$\tilde{p}_E\,(l,z) = \sum_{n=0}^{\infty}\left(E^+ e^{-jk_n z} + E^- e^{-jk_n z}\right)\psi_n^E\,(l)$$

$$\psi_n^E\,(l) = J_n\left(\frac{\alpha_n l}{l_0}\right)$$

$$k_n^2 = \sqrt{k_0^2 - \left(\frac{\alpha_n}{r_0}\right)^2}$$

So, we have this formulation in the duct A and you know similar things will happen in the outlet duct.

$$\tilde{p}_C\,(r,z) = \sum_{n=0,1,2}^{\infty}\left(C_n^+ e^{-jk_n^2 z} + C_n^- e^{-jk^2{}_n\, z}\right)\psi_n^E\,(r)$$

$$\psi_n^C\,(r) = J_0\left(\frac{\alpha_n}{k_0} r\right)$$

$$\left(\frac{k_0}{l_0}\right) = \begin{matrix} 3\ to\ 6 \\ \\ 9\ to\ 36 \end{matrix}$$

$$k_n^C = \sqrt{k_0^2 - \left(\frac{\alpha_n}{l_0}\right)^2}$$

$$\tilde{p}_B\,(r,z) = \sum_{n=0,1,2}^{\infty} \left(B_n^+ e^{-jk_n^B z} + B_n^- e^{-jk^B{}_n\,z}\right) \psi_n^B\,(r)$$

$$\left\{D_n^+ e^{-jk_n^D z} + D_n^- e^{-jk^D{}_n\,z}\right\} \psi_n^D\,(r)$$

$$k_n = \sqrt{k_0^2 \left(\frac{\alpha_n}{k_0}\right)^2}$$

And in the chamber we had this formulation in the annular cavity we had this.

$$k_n^B \sqrt{k_0^2 - \left(\frac{B_n}{k_0}\right)^2}, \qquad where$$

$$\psi_n^\beta = J_0\left(\frac{\beta_n r}{k_0}\right) - \frac{J_1\,(\beta_n)}{N_1\,(\beta_0)}\, N_0\left(\frac{\beta_n l}{k_0}\right)$$

$$J_1\left(\beta_0 \frac{l_0}{k_0}\right) - \frac{J_1\,(\beta_n)}{N_1\,(\beta_n)}\, N_1\left(\frac{\beta_n l_0}{k_0}\right) = 0$$

$$\tilde{p}_A\big|_{z=0} = \tilde{p}_C\big|_{z=0}$$

$$\tilde{p}_{\boldsymbol{B}}\big|_{z=0} = \tilde{p}_C\big|_{z=0}$$

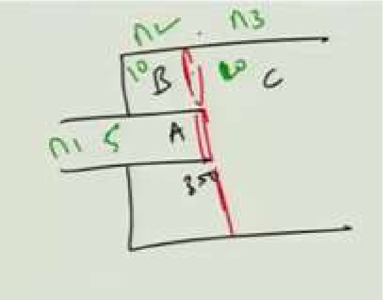
$$\widetilde{U}_A\Big|_{z=0} = \widetilde{U}_C\Big|_{z=0}$$

$$\widetilde{U}_B\Big|_{z=0} = \widetilde{U}_C\Big|_{z=0}$$

$$(\psi)[\psi] \quad \iint_{ds} J_0\left(\frac{\alpha_n}{k_0} r\right) J_0\left(\frac{\alpha_s}{r_0} r\right) r\; dl\; d\theta$$

$$C_n^{+} e^{-jk_n^C} l_3 \quad \rightarrow \tilde{C}_n^{+}$$

$$-jj\sqrt{\left(\frac{\alpha_n}{l_1}\right)^2 - k_0^2} \quad \tilde{C}_n^{-}\ C(-)$$



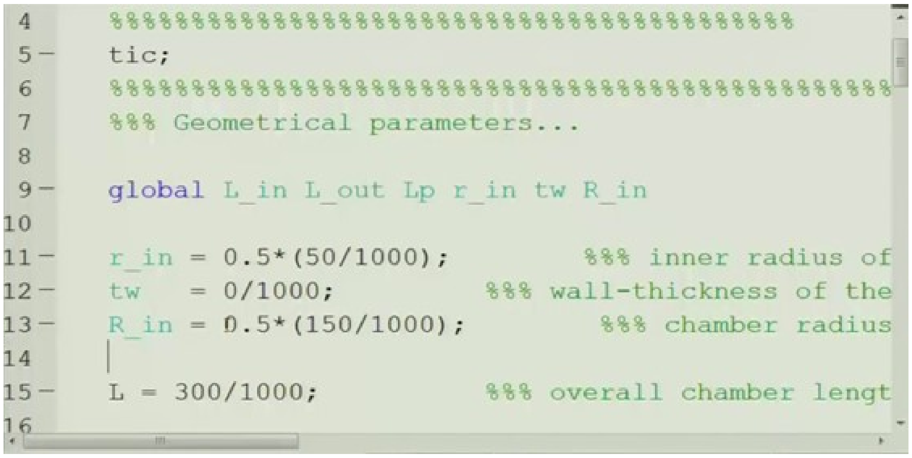
And all these things were they were taken from Selamets paper published almost more than 20 years back now and very well cited paper. And then that kind of after that paper there were lots of such similar papers on analysis of extended inlet and outlet systems.

So, what we basically do is that, we enforce continuity of pressure p A is equal to p C over this part as VC and p A I am sorry p B is equal to p C at the annular part and then velocities are also matched and then we you know multiply make exploit the mode orthogonality to set up a system of equations you know you can refer to the last lecture.

And finally what we eventually you know want to do is find out the reflected model coefficients An i and then the model coefficient

$$\left[\begin{array}{c|c|c|c|c} \quad & \quad & \quad & \quad & \quad \\ \hline \quad & & & & \\ \hline \quad & & & & \\ \hline \quad & & & & \\ \hline \quad & & & & \\ \hline \quad & & & & \end{array}\right] \begin{Bmatrix} \{A_n^-\} \\ \{B_n^-\} \\ \{C_n^+\} \\ \{C_n^-\} \\ \{D_n^+\} \\ \{E_n^+\} \end{Bmatrix} = \begin{bmatrix} \qquad \\ \\ \\ \\ \\ \end{bmatrix} \{A_0^+\}$$

All these things in terms of the incident wave amplitude A_0 t at the extended inlet pipe ok. So, we would we would like to do that and. So, all these equations were set up and they were chunks of this matrices. So, we will you know we will soon see; we will soon see how it is going to happen.



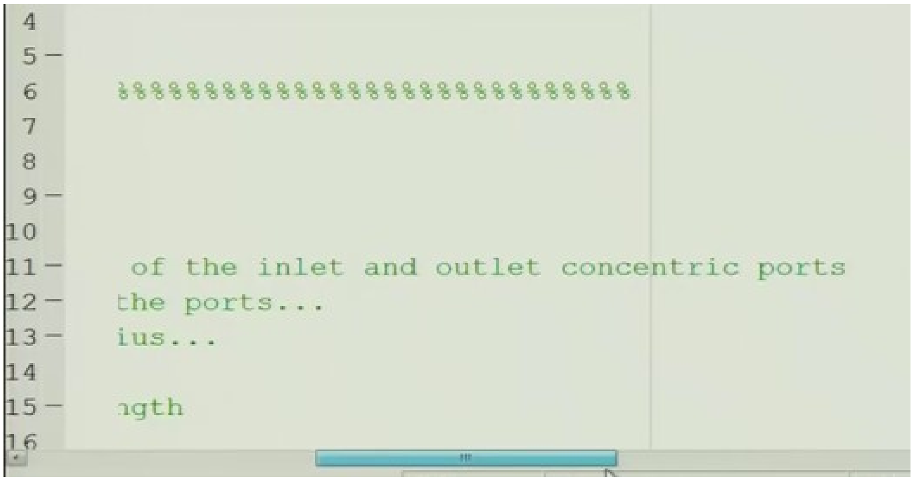
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tic;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);        %%% inner radius of
tw   = 0/1000;          %%% wall-thickness of the
R_in = 0.5*(150/1000);        %%% chamber radius

L = 300/1000;          %%% overall chamber lengt
```

For that let me just walk you through the MATLAB code to begin with. So, what we do? This is the main code in which you can change a number of parameters; just it is as simple as changing a few parameters like r underscore in tw R underscore in.



```


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



 of the inlet and outlet concentric ports
the ports...
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ngth
```
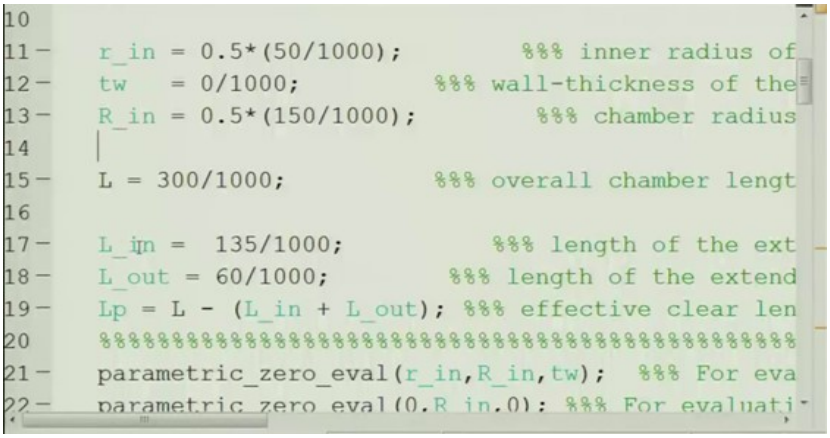

So, this is basically the inner radius or the inlet and outlet pipe. Note that there is another important thing that I simultaneously did in the MATLAB code. I incorporate the effect of wall thickness which has not been considered in Selamets work. And in previous papers by Chaitanya and Munjal in applied acoustics almost about 10 years back, they did consider the wall thickness effect, but then they considered it using the full 3D formulation using some commercial package.

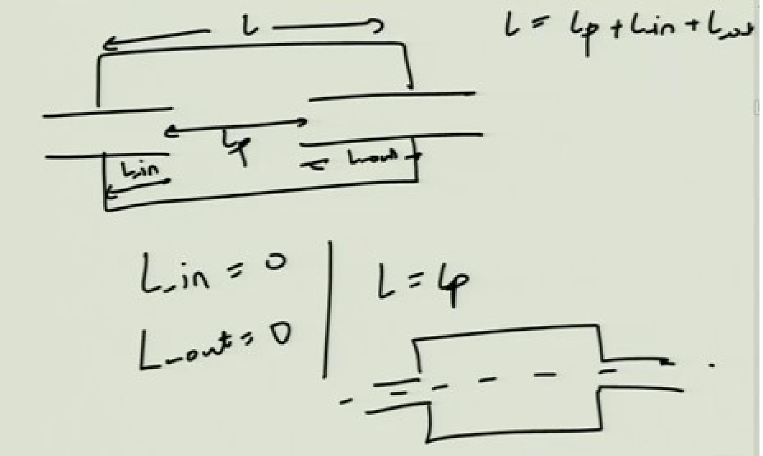
So, this was the first time that we are considering analytical mode matching incorporating the thickness of a pipe thickness ok. Right now I have just set it to 0. So, that we have a feel of what is going to happen I mean this is just a initial case. So, we will gradually put it here some nonzero value ranging from you know say 1 mm to 2 mm maybe up till until about 5 mm you can have really thick pipes.



```
r_in = 0.5*(50/1000);         %%% inner radius of
tw   = 0/1000;          %%% wall-thickness of the
R_in = 0.5*(150/1000);        %%% chamber radius

L = 300/1000;           %%% overall chamber lengt

L_in =  135/1000;           %%% length of the ext
L_out = 60/1000;         %%% length of the extend
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);  %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati
```

So, even this small thickness of the pipe can dramatically affect the end correction and it can influence the overall performance. Right now it has been set to 0.

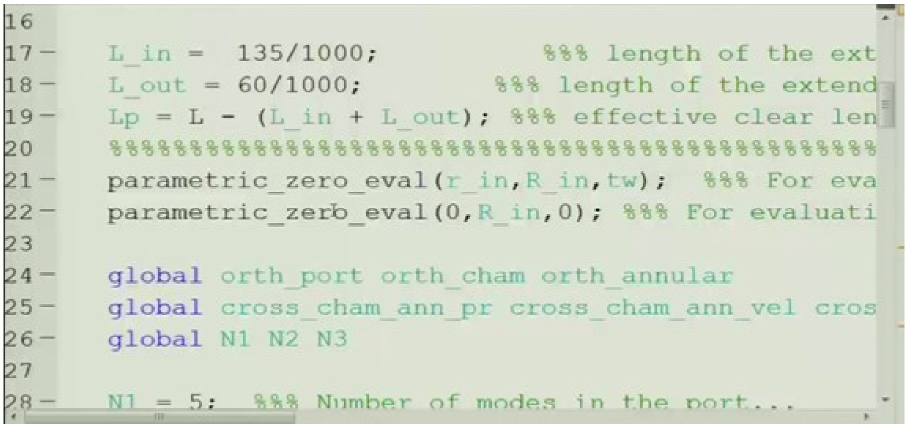
So, let us; so you know go through these parameters the these values later. But let us understand the symbolic conventions L is the length of the chamber, L underscore in is the length of the extensions at the inlet, L underscore out is the length of the extension at the outlet and this is the end to end length that is Lp that is the end.



So, if you recall our discussion. So, this was our; so what I really mean is that if this was the formulation, you know let us say L_n is the geometric length. L underscore out is this length. And from here to here it is L_P ok this is L_P. So, this end to end length or clearance length you can talk about; obviously under the limiting case when Li this is 0 and this is also 0 ok.

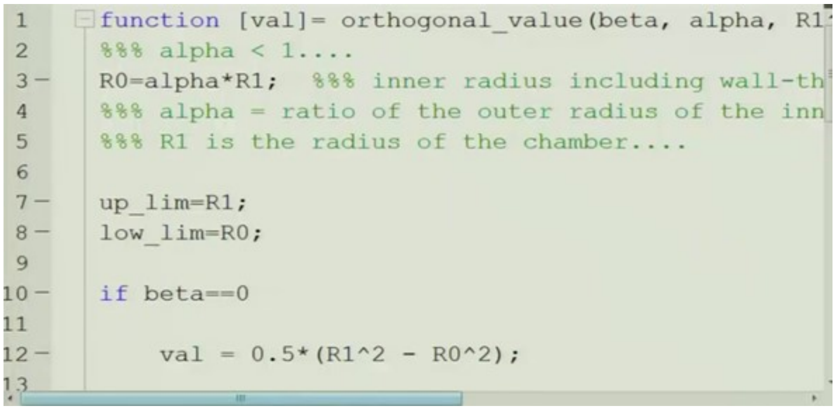
Then you will get and this is L. So, L is equal to L_P plus this out. So, under this limiting condition L is equal to L_P that is overall chamber length and we will get a concentric expansion chamber the simplest of all muffler element. And we will, but we can do 3D analysis without resorting to going to any commercial software at least for such a configuration ok.

This is the advantage of using this mode matching technique. And very quickly we can do that there is; obviously, no finite element is involved here. So, there is no need of meshing or things like that. We can exactly evaluate in terms of the Bessel function ok, we can surely do that.

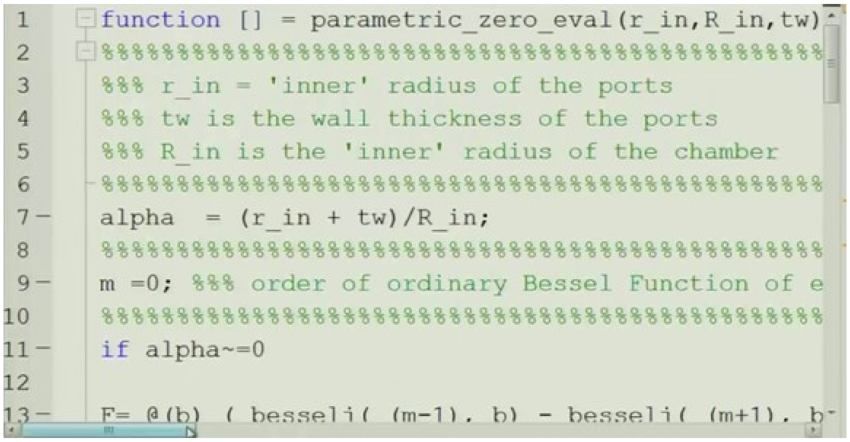


```

    L_in =  135/1000;          %%% length of the ext
    L_out = 60/1000;         %%% length of the extend
    Lp = L - (L_in + L_out); %%% effective clear len
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    parametric_zero_eval(r_in,R_in,tw);   %%% For eva
    parametric_zerb_eval(0,R_in,0); %%% For evaluati

    global orth_port orth_cham orth_annular
    global cross_cham_ann_pr cross_cham_ann_vel cros
    global N1 N2 N3

    N1 = 5;  %%% Number of modes in the port...
```

Now, let us go to the MATLAB code very quickly and this parameter is something that I explained. So, we evaluate the zeros or resonance frequencies of the circular chamber or the annular cavity using a parametric this file.

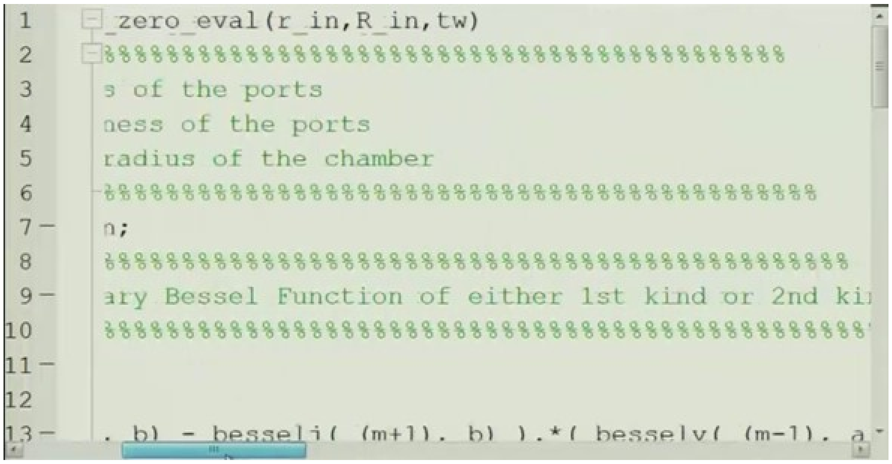


```
function [val]= orthogonal_value(beta, alpha, R1
%%% alpha < 1....
R0=alpha*R1;  %%% inner radius including wall-th
%%% alpha = ratio of the outer radius of the inn
%%% R1 is the radius of the chamber....

up_lim=R1;
low_lim=R0;

if beta==0

    val = 0.5*(R1^2 - R0^2);
```



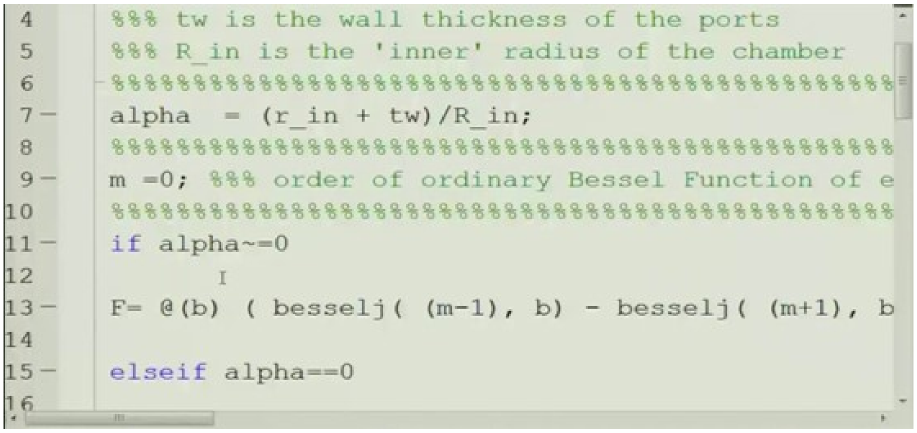
```
function [] = parametric_zero_eval(r_in,R_in,tw)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% r_in = 'inner' radius of the ports
%%% tw is the wall thickness of the ports
%%% R_in is the 'inner' radius of the chamber
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpha  = (r_in + tw)/R_in;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m =0; %%% order of ordinary Bessel Function of e
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if alpha~=0

F= @(b) ( besseli( (m-1), b) - besseli( (m+1), b
```

Which is somewhere which is basically sets the order of Bessel function to 0.

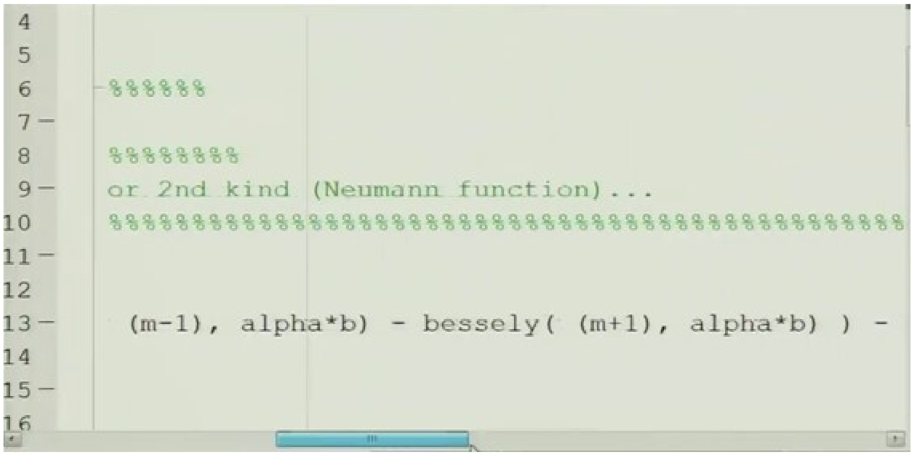


```
_zero_eval(r_in,R_in,tw)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
s of the ports
ness of the ports
radius of the chamber
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
n;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ary Bessel Function of either 1st kind or 2nd ki
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


, b) - besseli( (m+1), b) ).*( bessely( (m-1), a
```

Takes in a few arguments r in R capital R underscore in wall thickness and calculates alpha, the effective the incorporating the wall thickness effective ratio of the inner pipe diameter to port diameter to the inner diameter of the other chamber ok.

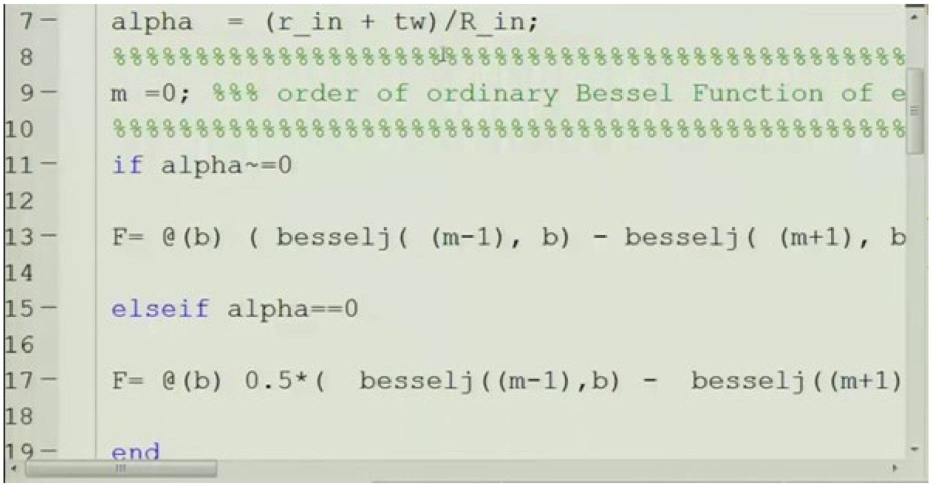


```
%%% tw is the wall thickness of the ports
%%% R_in is the 'inner' radius of the chamber
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpha  = (r_in + tw)/R_in;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m =0; %%% order of ordinary Bessel Function of e
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if alpha~=0

F= @(b) ( besselj( (m-1), b) - besselj( (m+1), b

elseif alpha==0

```



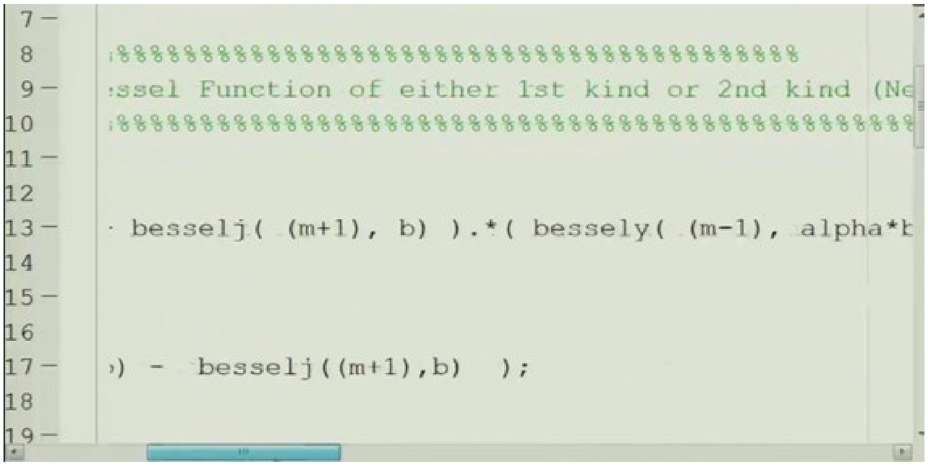
```



%%%%%%

%%%%%%%%
or 2nd kind (Neumann function)...
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


 (m-1), alpha*b) - bessely( (m+1), alpha*b) ) -



```

And then if alpha is not 0, which is the case of annular cavity. We you know the this function is evaluated which is nothing, but you know incorporating rigid wall condition on the inner pipe as well as the chamber inner surface of the chamber.



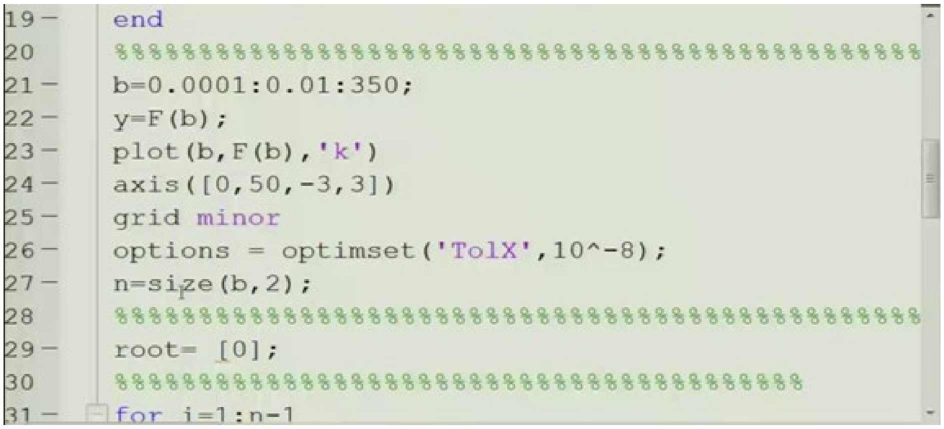
```
alpha  = (r_in + tw)/R_in;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m =0; %%% order of ordinary Bessel Function of e
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if alpha~=0

F= @(b) ( besselj( (m-1), b) - besselj( (m+1), b

elseif alpha==0

F= @(b) 0.5*(  besselj((m-1),b) -  besselj((m+1)

end
```

So, this is the function that is defined. And if alpha is 0 that is if you are considering you know r underscore; basically we are considering the modes of the full cavity or the full chamber.



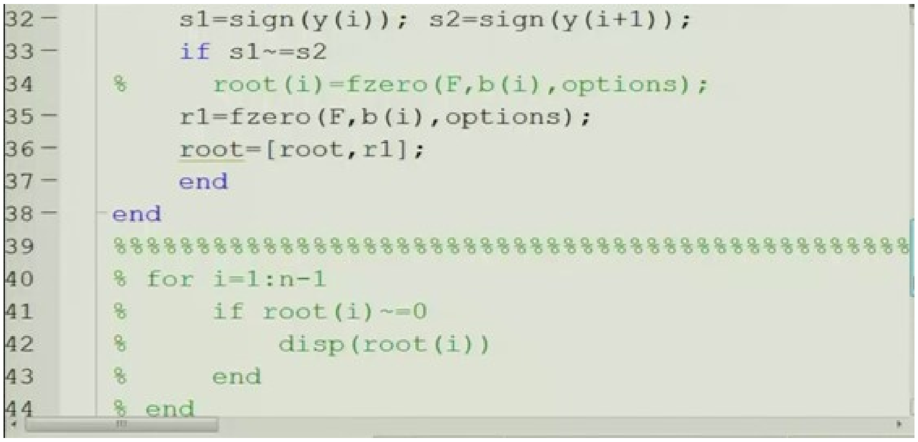
```
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
:ssel Function of either 1st kind or 2nd kind (Ne
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



· besselj( (m+1), b) ).*( bessely( (m-1), alpha*b



:) -  besselj((m+1),b)  );

```

Then this function can be defined and this function can also be used to get the resonance frequencies of the port, that is inlet and outlet port.



```
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
b=0.0001:0.01:350;
y=F(b);
plot(b,F(b),'k')
axis([0,50,-3,3])
grid minor
options = optimset('TolX',10^-8);
n=size(b,2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
root= [0];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i=1:n-1
```

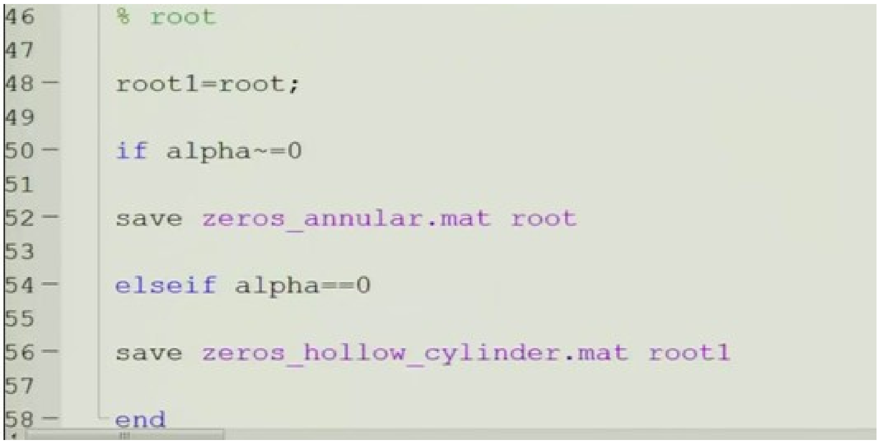
So, and then you know using route bisection method we what we do is basically we plot we define these functions and then we plot it. This plots the function.



```
    s1=sign(y(i)); s2=sign(y(i+1));
    if s1~=s2
%     root(i)=fzero(F,b(i),options);
    r1=fzero(F,b(i),options);
    root=[root,r1];
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% for i=1:n-1
%     if root(i)~=0
%         disp(root(i))
%     end
% end
```

And then using root bracketing method you know it looks for a change of sign. So, these are the some of the codes that you know you can write on your own also, root bracketing method that is look for a continuous function wherever the sign changes you know the and one of the roots must be located at least that interval s1 and s2 must have at least 1 root and we ensure that these intervals are very small and none of the roots are missed or things like that.

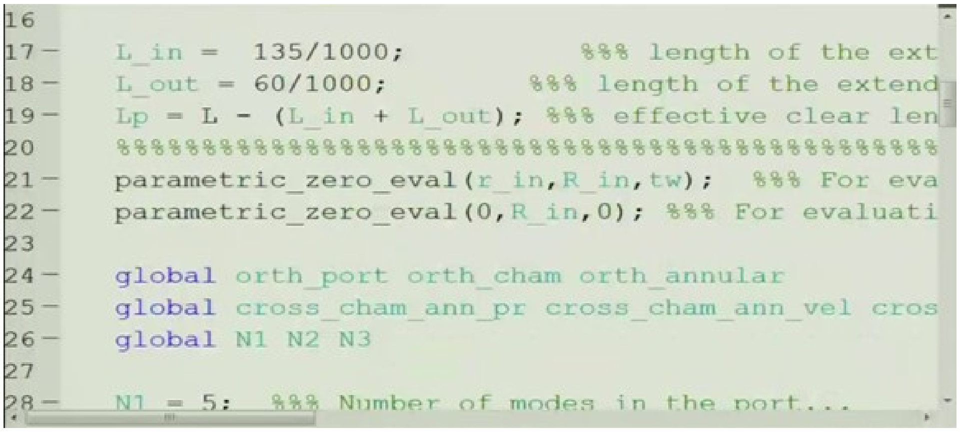
And then we evaluate the zero using the f zero method which is you know function given in MATLAB f zero. We could also you do you know evaluate this using a root bisection method, but I used root f zero.



```
% root

root1=root;

if alpha~=0

save zeros_annular.mat root

elseif alpha==0

save zeros_hollow_cylinder.mat root1

end
```

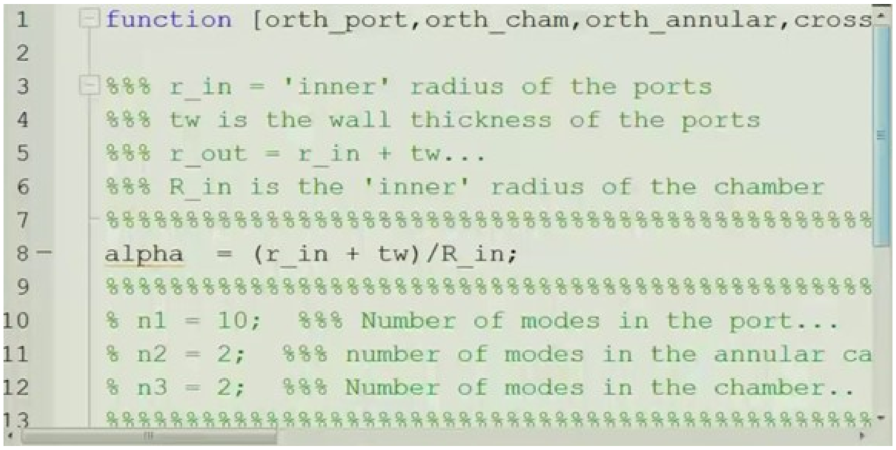
And figured out that all zeros are correctly ordered and then its basically saves it; obviously, the first 0 is 0 only that is a planar wave mode in both annular cavity and this thing. So, especially the evaluation of the anything pertaining to the annular cavity where Bessel functional was second kind Neumann function was also involved. Because that function goes to infinity singularity problem at 0.

So, there you have to be careful with the zero-2 mode or the plane wave mode. You can just directly consider a plane wave rather than evaluating it using Bessel function that is only exception So, if alpha is not 0, then you consider zeros of annular cavity otherwise this, this thing. So, this it is stored in this file that you get a series of zeros that resonance frequencies.



```
    L_in =  135/1000;          %%% length of the ext
    L_out = 60/1000;         %%% length of the extend
    Lp = L - (L_in + L_out); %%% effective clear len
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    parametric_zero_eval(r_in,R_in,tw);   %%% For eva
    parametric_zero_eval(0,R_in,0); %%% For evaluati

    global orth_port orth_cham orth_annular
    global cross_cham_ann_pr cross_cham_ann_vel cros
    global N1 N2 N3

    N1 = 5;  %%% Number of modes in the port...
```

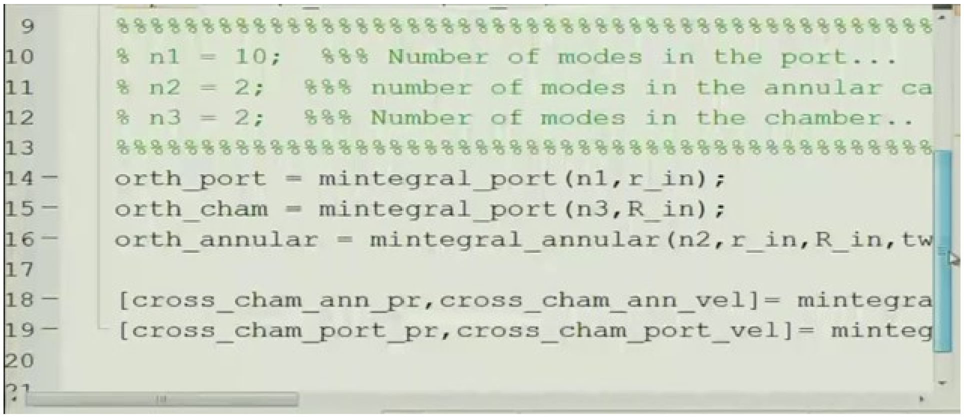
Once you do that it evaluates the parametric zero and then we need to compute the number of integrals, you know in the last class I mentioned about the number of integrals like you know integral of Bessel function in the chamber you know Bessel function in the chamber r square. So, this is what the orthogonal file looks like.



```
function [orth_port,orth_cham,orth_annular,cross

%%% r_in = 'inner' radius of the ports
%%% tw is the wall thickness of the ports
%%% r_out = r_in + tw...
%%% R_in is the 'inner' radius of the chamber
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpha  = (r_in + tw)/R_in;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% n1 = 10;  %%% Number of modes in the port...
% n2 = 2;  %%% number of modes in the annular ca
% n3 = 2;  %%% Number of modes in the chamber..
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

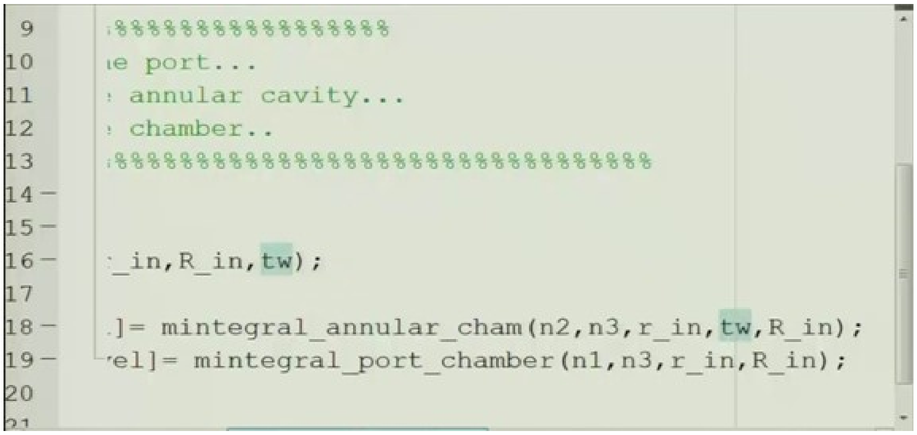
```
s_cham_port_vel] = integrals(r_in,R_in,tw,n1,n2,
```

This is the integrals file which basically evaluates all the pertinent integrals.



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% n1 = 10;  %%% Number of modes in the port...
% n2 = 2;  %%% number of modes in the annular ca
% n3 = 2;  %%% Number of modes in the chamber..
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
orth_port = mintegral_port(n1,r_in);
orth_cham = mintegral_port(n3,R_in);
orth_annular = mintegral_annular(n2,r_in,R_in,tw

[cross_cham_ann_pr,cross_cham_ann_vel]= mintegra
[cross_cham_port_pr,cross_cham_port_vel]= minteg
```

So, this m integral port is basically the file that calculates the orthogonal values, which is which is basically obtained by multiplying the Bessel functions or the mode shape function with the square of the mode shape functions and weigh and with the weighing factor being the radius and integrate it over 0 to r underscore in or R underscore capital R underscore in depending on whether we have a port or a chamber.

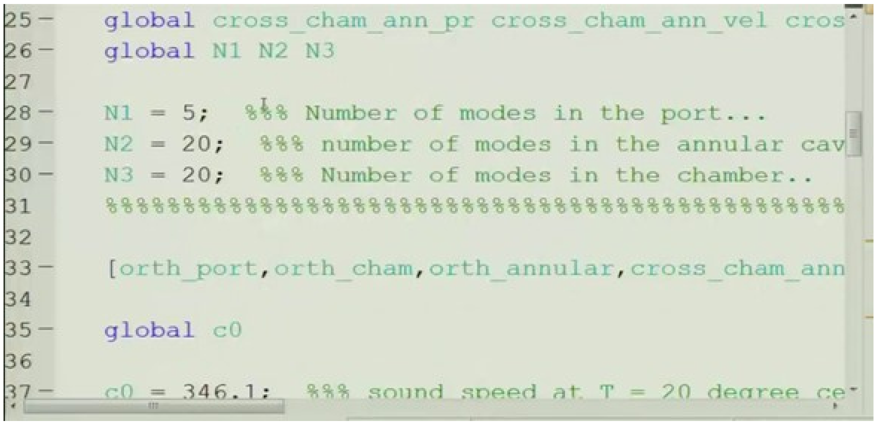


```
%%%%%%%%%%%%%%%%%
e port...
 annular cavity...
 chamber..
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


_in,R_in,tw);

]= mintegral_annular_cham(n2,n3,r_in,tw,R_in);
el]= mintegral_port_chamber(n1,n3,r_in,R_in);
```

And for the annular cavity go we go now from basically from R underscore in plus tw that is the incorporating the thickness that is a trick we employ. That you know let me just; let me just explain this point a bit more carefully. So, what we do is basically we the effective the incorporating the thickness of the pipe the effective radius of the annular cavity in a radius will become r underscore in plus tw.

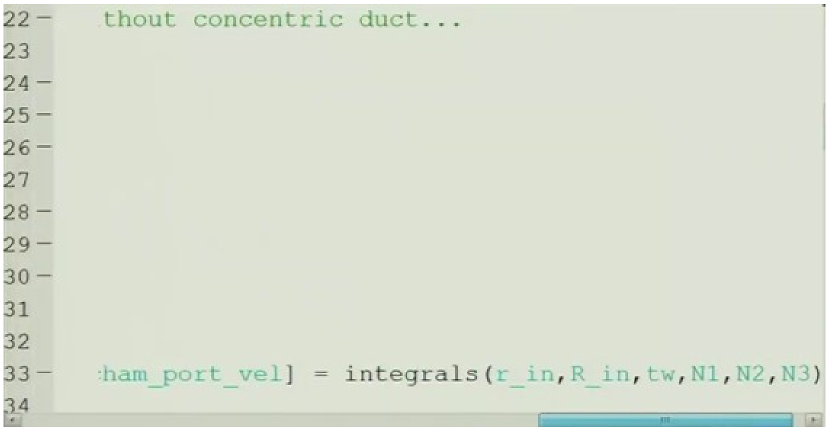
And the outer radius will be the radius of the chamber that is R underscore in. So, the limits of integration for such this thing will be small r underscore in plus tw and the upper limit will be capital R underscore in. So, this is how we calculate orthogonal values and the cross modes will also be there as we discussed in the last couple of lectures. So, this is what is calculated.

Note that these all these values are frequency independent; so we just need to compute it once before starting the frequency loop, that is very important if you want to optimize the runtime of your code.



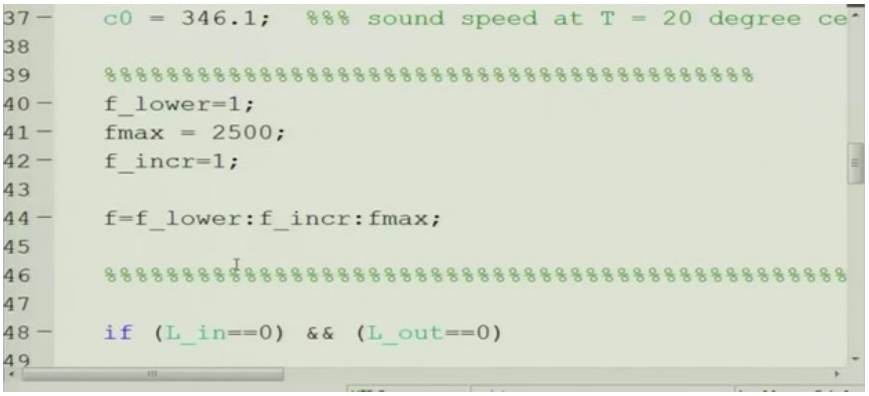
```
global cross_cham_ann_pr cross_cham_ann_vel cros
global N1 N2 N3

N1 = 5;   %%% Number of modes in the port...
N2 = 20;   %%% number of modes in the annular cav
N3 = 20;   %%% Number of modes in the chamber..
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[orth_port,orth_cham,orth_annular,cross_cham_ann

global c0

c0 = 346.1;   %%% sound speed at T = 20 degree ce
```

Otherwise, if you calculate it every time it is going to take a lot of time especially if you have large number of frequency steps. So, this is like the overarching file run code and here we have to also specify the number of modes in the port that in the annular cavity and the one in the chamber. So, once it and then these variables are declared global, because you know the scope of this thing is also there in the other files.



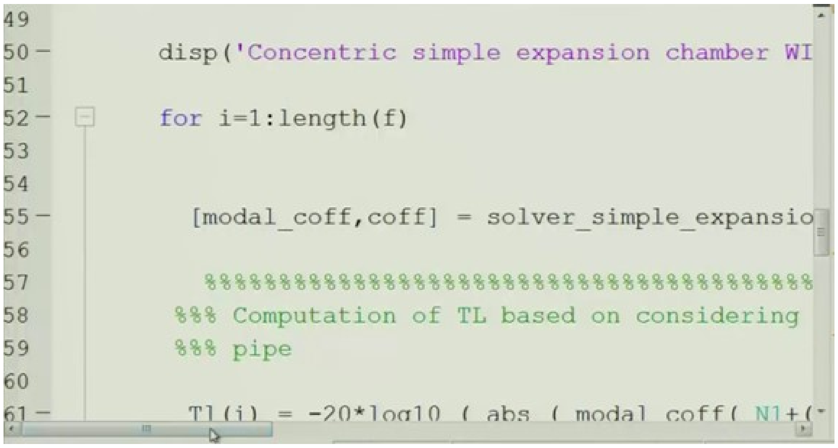
```
thout concentric duct...











ham_port_vel] = integrals(r_in,R_in,tw,N1,N2,N3)
```

So, once its evaluated its you know these are declared now and once they are calculated using the integrals file.



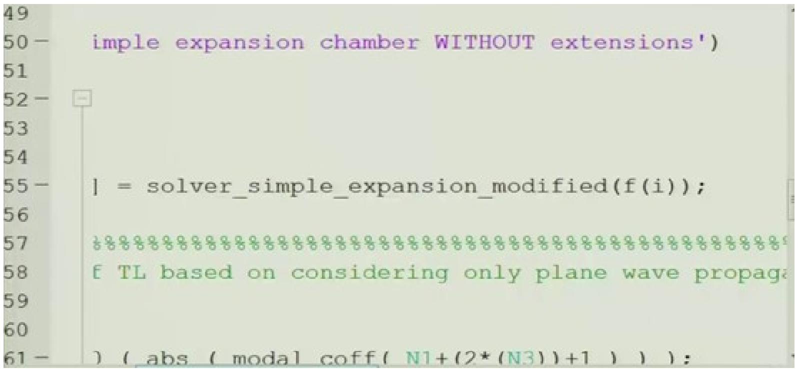
```
c0 = 346.1;  %%% sound speed at T = 20 degree ce

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f_lower=1;
fmax = 2500;
f_incr=1;

f=f_lower:f_incr:fmax;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (L_in==0) && (L_out==0)
```

And then sound speed I have taken 346. 1. The one that was taken in Selamets work. This is the frequency range as usual in steps of one frequency.



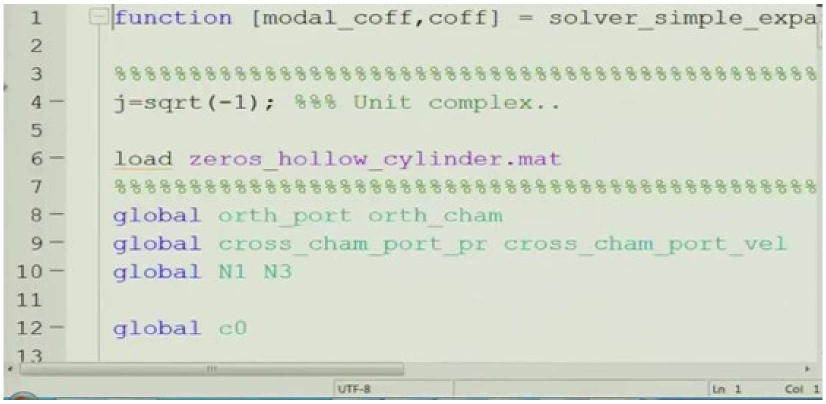
```
    disp('Concentric simple expansion chamber WI

    for i=1:length(f)


      [modal_coff,coff] = solver_simple_expansio

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
       %%% Computation of TL based on considering
       %%% pipe

      Tl(i) = -20*log10 ( abs ( modal_coff( N1+(
```

Now, we enter the loop. So, we just have to compute this value once and then that will be available in the solver file. So, this is very the actual inversion of the matrix the big matrix that I showed you in a while and that will happen. So, if there is no extensions then we get the simple Concentric expansion chamber case.



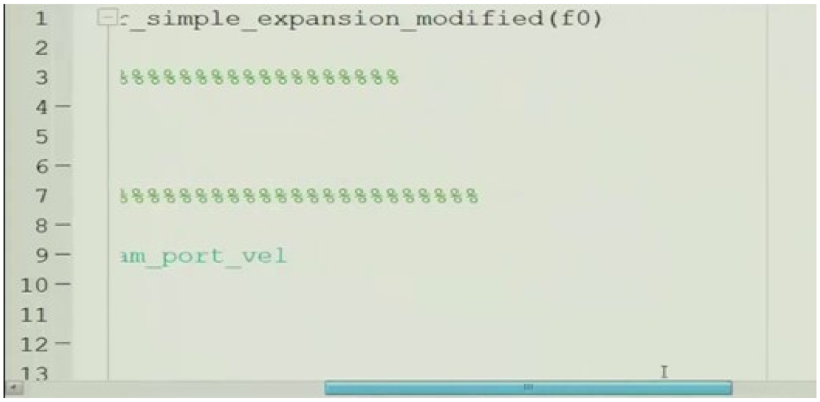
```
imple expansion chamber WITHOUT extensions')




] = solver_simple_expansion_modified(f(i));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f TL based on considering only plane wave propag


) ( abs ( modal_coff( N1+(2*(N3))+1 ) ) );
```

And it basically, so let us go to this file first solver underscore simple expansion modified.



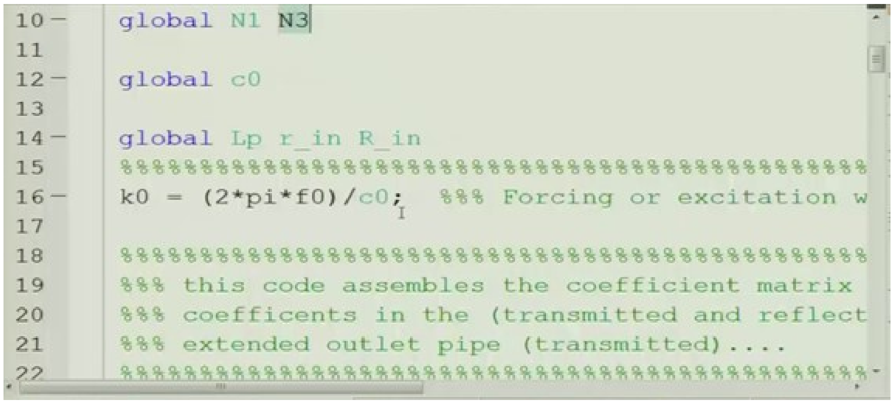
```
function [modal_coff,coff] = solver_simple_expa

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
j=sqrt(-1); %%% Unit complex..

load zeros_hollow_cylinder.mat
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
global orth_port orth_cham
global cross_cham_port_pr cross_cham_port_vel
global N1 N3

global c0
```

So, do not worry about the modified term just yet.



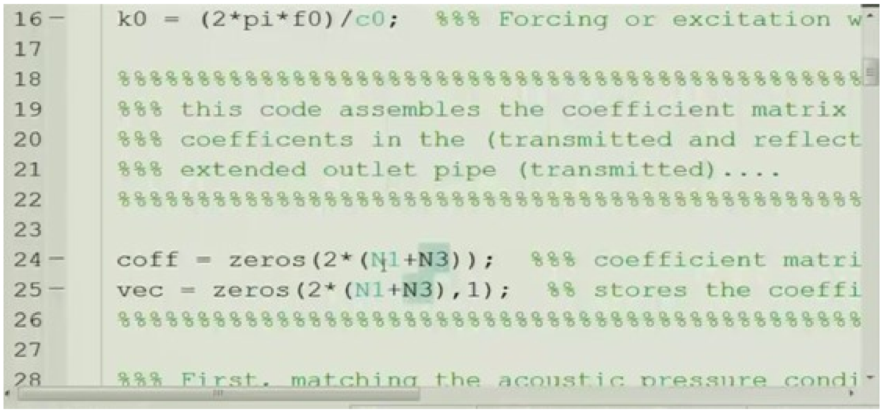
```
:_simple_expansion_modified(f0)

%%%%%%%%%%%%%%%%%%



%%%%%%%%%%%%%%%%%%%%%%%

am_port_vel
```

So, we are left with only the solver underscore simple expansion modified ok. It loads the zeros of this thing; obviously, here there is no annular cavity involved. Now, whatever functions we calculated in the run, run file run underscore code. The same function now are available just by we are not necessarily passing it, these functions are available just because these are global files.



```
global N1 N3

global c0

global Lp r_in R_in
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
k0 = (2*pi*f0)/c0;  %%% Forcing or excitation w

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% this code assembles the coefficient matrix
%%% coefficents in the (transmitted and reflect
%%% extended outlet pipe (transmitted)....
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

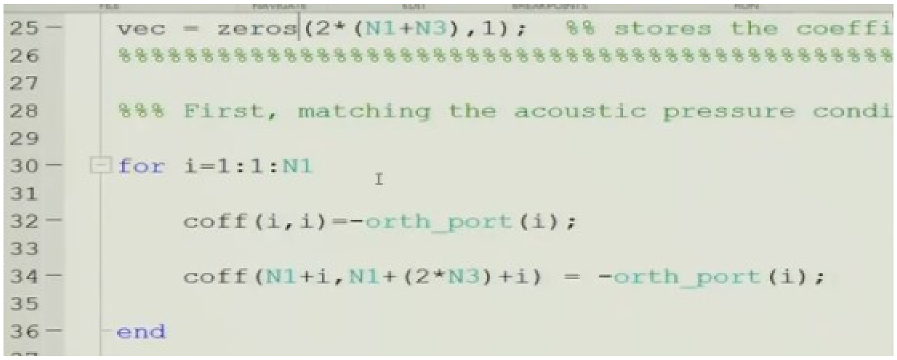
They defined here and the scope is also here.



```
k0 = (2*pi*f0)/c0;  %%% Forcing or excitation w

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% this code assembles the coefficient matrix
%%% coefficents in the (transmitted and reflect
%%% extended outlet pipe (transmitted)....
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

coff = zeros(2*(N1+N3));  %%% coefficient matri
vec = zeros(2*(N1+N3),1);  %% stores the coeffi
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%% First, matching the acoustic pressure condi
```

Now, frequency parameter frequency response and now this is the 2 instead of 2N + N3 +N2 we have 2N + N3 only, because of the N1 number of pressure matching conditions in the inlet port and 1 in the outlet port.

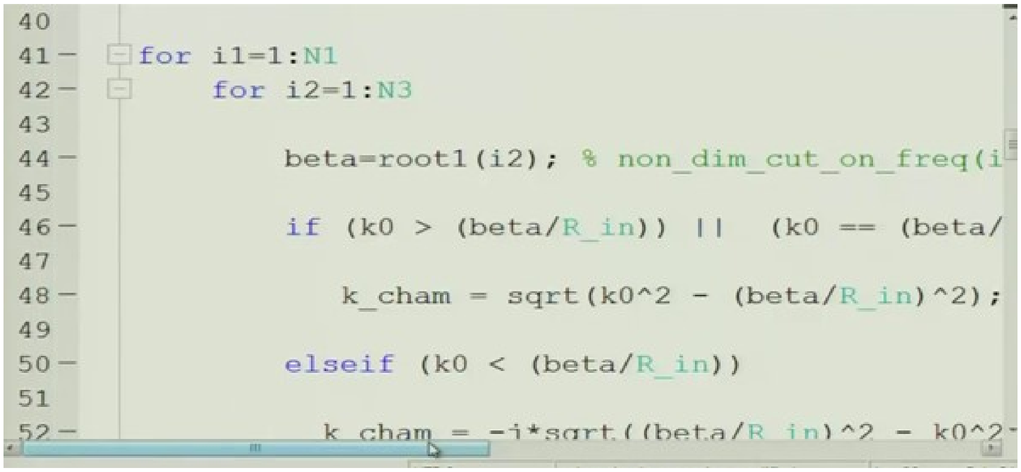
And N3 number of velocity matching conditions at the interface of the port and the chamber at the inlet and the same at the outlet. And vector this is defined at the load loading vector you know vector where and the size is only 2N + N1 +N3, 1 and the size of this is zeros 2N 1 + N3 this is a square matrix this is just a vector ok.



```
vec = zeros(2*(N1+N3),1);  %% stores the coeffi
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%% First, matching the acoustic pressure condi

for i=1:1:N1

    coff(i,i)=-orth_port(i);

    coff(N1+i,N1+(2*N3)+i) = -orth_port(i);

end
```

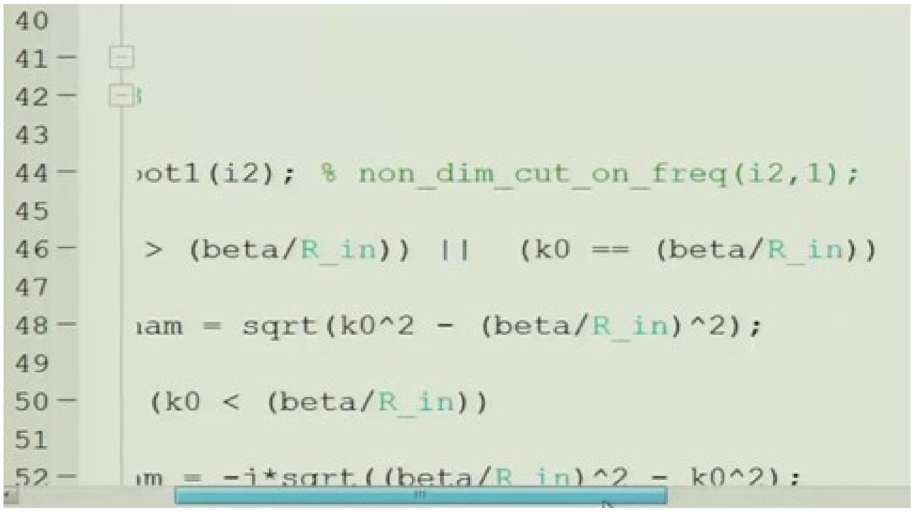
Now, based on all the equations that we discussed in the last; in the last class. So, what we do is basically we get a chunk we get a chunk of matrix. So, this is I mean this code this all this MATLAB coding involves a lot of book keeping ok. Eventually at the end of the day what we have to do is that we have to write down each of the matrix, write down each of the matching conditions and then put them in the particular go, go to individual rows and individual column to dump out the particular integral values.

So, these are the integral values I mean; so if you for a limiting case when I is equal to 1 here. So, this will become the plane wave mode and here only diagonal entries are evaluated. And similarly this is the outlet this these, these terms pertaining to the outlet conditions matching the pressure conditions at the outlet port.



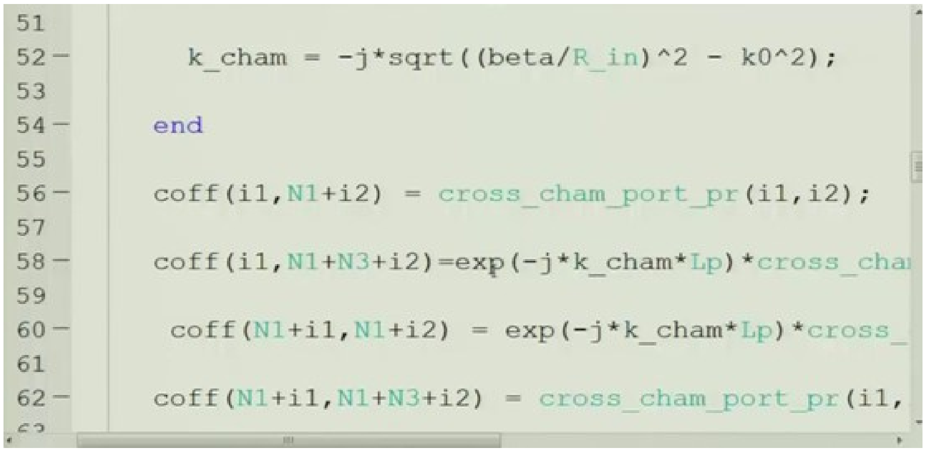
```

for i1=1:N1
    for i2=1:N3

        beta=root1(i2); % non_dim_cut_on_freq(i

        if (k0 > (beta/R_in)) ||   (k0 == (beta/

           k_cham = sqrt(k0^2 - (beta/R_in)^2);

        elseif (k0 < (beta/R_in))

           k_cham = -i*sqrt((beta/R_in)^2 - k0^2
```

Similarly, the large number of such terms and here this is the contribution in the a, a plus term that is the excitation plane wave excitation vector.



```



ot1(i2); % non_dim_cut_on_freq(i2,1);

 > (beta/R_in)) ||  (k0 == (beta/R_in))

am = sqrt(k0^2 - (beta/R_in)^2);

 (k0 < (beta/R_in))

m = -j*sqrt((beta/R_in)^2 - k0^2);
```

Now, here we employ a certain trick when k naught is less than beta by R underscore in.

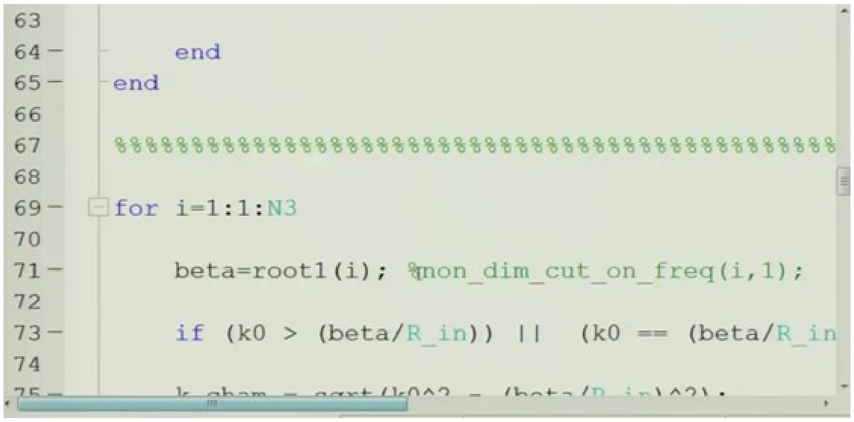


```

      k_cham = -j*sqrt((beta/R_in)^2 - k0^2);

    end

    coff(i1,N1+i2) = cross_cham_port_pr(i1,i2);

    coff(i1,N1+N3+i2)=exp(-j*k_cham*Lp)*cross_cha

     coff(N1+i1,N1+i2) = exp(-j*k_cham*Lp)*cross_

    coff(N1+i1,N1+N3+i2) = cross_cham_port_pr(i1,
```

We take a negative you know instead of taking it just j times this thing, we consider it minus j to because other if you do not consider this, there might be some numerical issues of inverting the matrix, especially even higher very large number of modes are considered. We might we might MATLAB may not be able to calculate that because of machine precision.

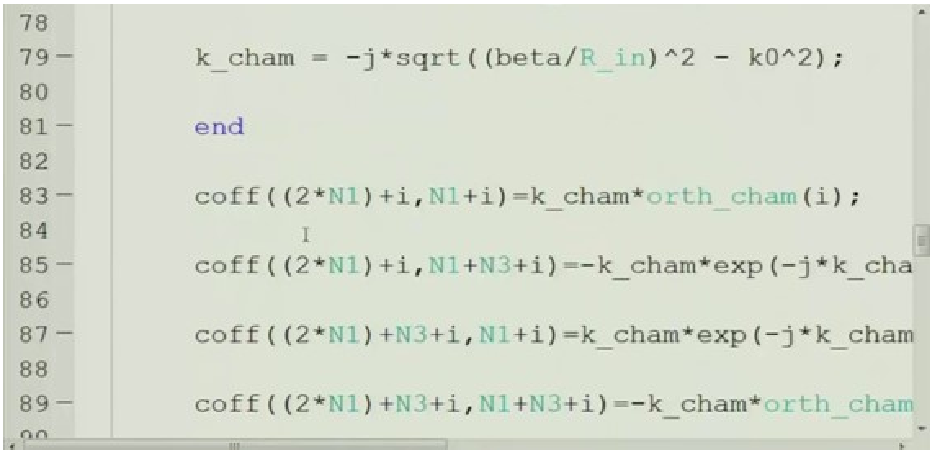
We employ this trick if this is less than if this is k_0 excitation frequency is less than the particular resonance frequency value, then this is negative. Then we instead of just positive imaginary we considered negative imaginary, because you have exponential term sitting in the modal term and then - j into - j that will become exponential negative.

So, there are certain tricks which you probably is best that you start coding and write your own codes you will get to know that. And then you again we dump out certain expressions in I mean these things goes in a particular row and column.



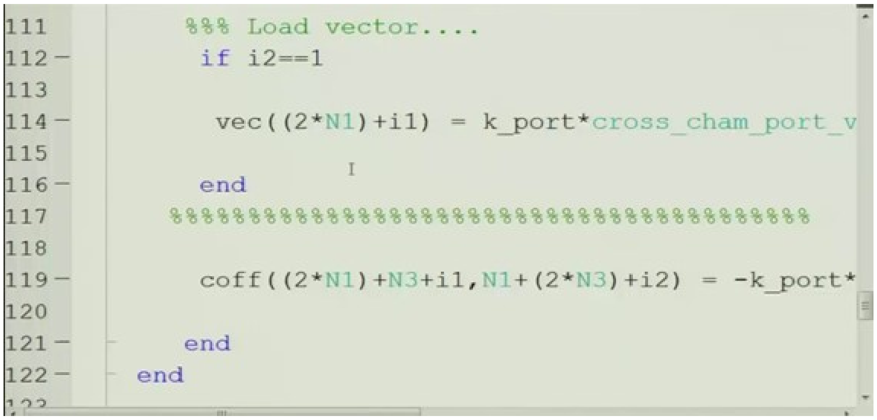
```
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:1:N3

    beta=root1(i); %non_dim_cut_on_freq(i,1);

    if (k0 > (beta/R_in)) ||  (k0 == (beta/R_in

    k_cham = sqrt(k0^2 - (beta/R_in)^2);
```

So, you know this is a solver file. Then similarly this is the velocity matching condition in the chamber.



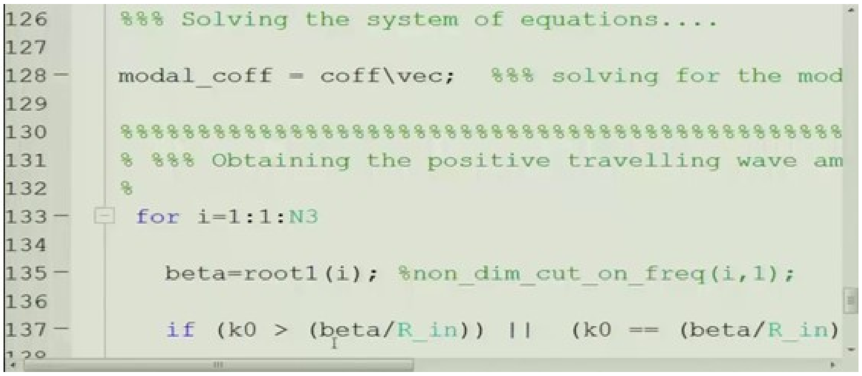
```
    k_cham = -j*sqrt((beta/R_in)^2 - k0^2);

    end

    coff((2*N1)+i,N1+i)=k_cham*orth_cham(i);

    coff((2*N1)+i,N1+N3+i)=-k_cham*exp(-j*k_cha

    coff((2*N1)+N3+i,N1+i)=k_cham*exp(-j*k_cham

    coff((2*N1)+N3+i,N1+N3+i)=-k_cham*orth_cham
```

And we access particular rows and column like this.



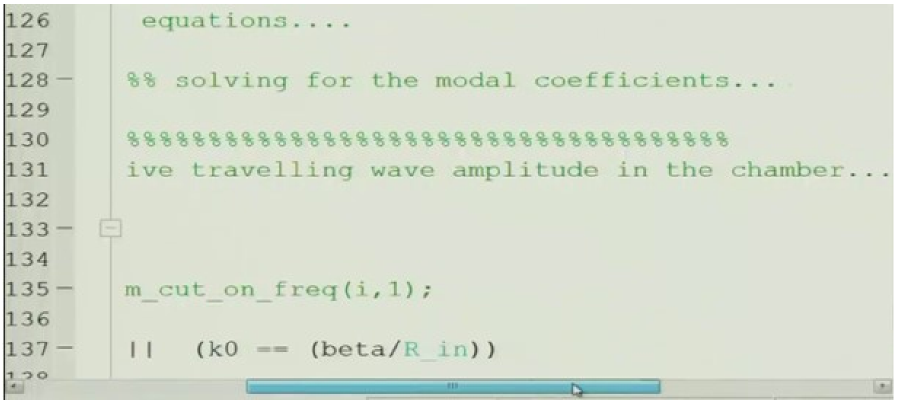
```
    %%% Load vector....
     if i2==1

      vec((2*N1)+i1) = k_port*cross_cham_port_v

     end
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

     coff((2*N1)+N3+i1,N1+(2*N3)+i2) = -k_port*

    end
 end
```

We follow this logic and finally, this is the vector where it is another contribution by the A plus term in the inlet pipe modal term and this is where we evaluate.



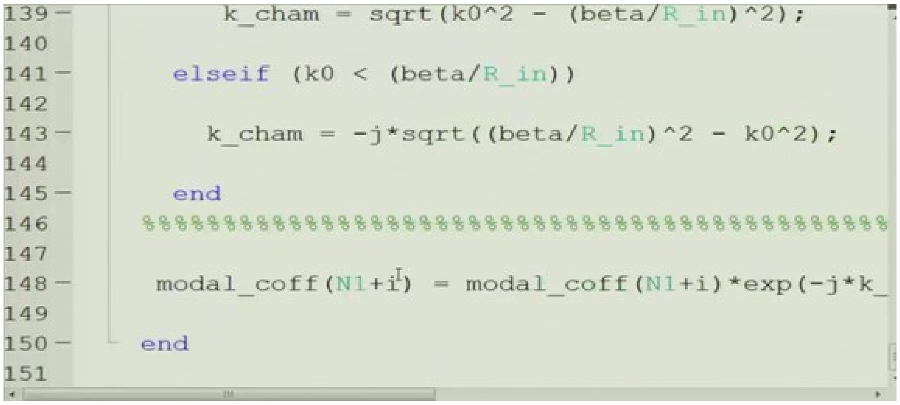
```
%%% Solving the system of equations....

modal_coff = coff\vec;  %%% solving for the mod

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %%% Obtaining the positive travelling wave am
%
 for i=1:1:N3

   beta=root1(i); %non_dim_cut_on_freq(i,1);

   if (k0 > (beta/R_in)) ||  (k0 == (beta/R_in)
```

And finally, what we do is that once we get the fully populated coefficient matrix, only some entries are non zero otherwise it is zero. Once we get coefficient matrix and vector we invert it you know coff. inverse, inverse of coff. times vector; so that big matrix inverse into the vector. So, that will give you the model coefficients.



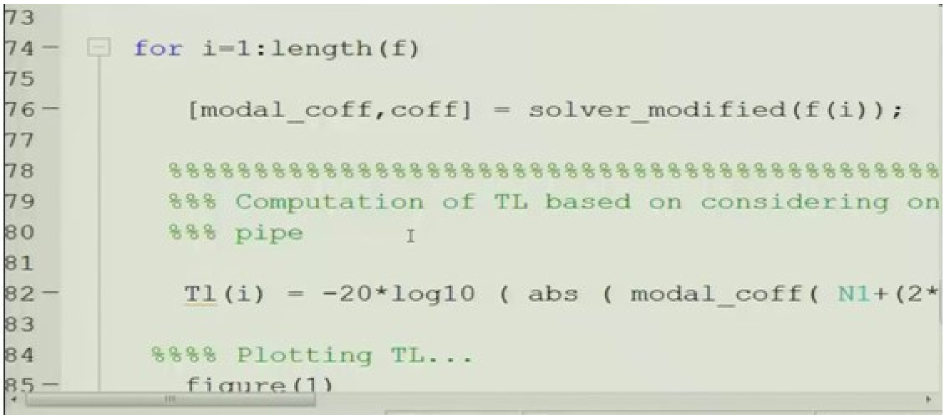
```
 equations....

%% solving for the modal coefficients...

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ive travelling wave amplitude in the chamber...



m_cut_on_freq(i,1);

||  (k0 == (beta/R_in))
```

Then we need to convert it back get the get back the positive travelling wave, only for a certain mode.



```
           k_cham = sqrt(k0^2 - (beta/R_in)^2);

        elseif (k0 < (beta/R_in))

          k_cham = -j*sqrt((beta/R_in)^2 - k0^2);

        end
      %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

       modal_coff(N1+i) = modal_coff(N1+i)*exp(-j*k_

   end

```

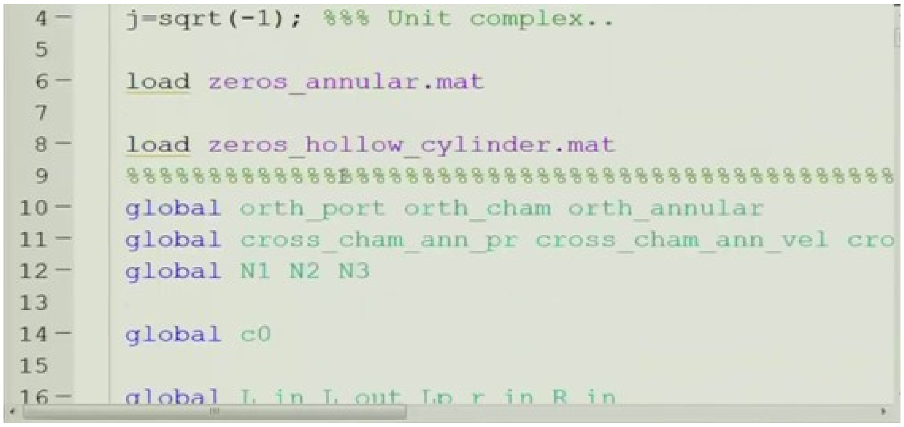
And so this modal c off is now passed on to the main calling function, that is your that is this function. So, we do not require necessarily require c offs just for checking if things are fine or not. And once we get the modal model coefficient what we do is basically, now evaluate the transmission loss; in the transmission loss by considering only planar wave propagation in the outlet pipe. So, this is what we get ok.



```

for i=1:length(f)

    [modal_coff,coff] = solver_modified(f(i));

   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   %%% Computation of TL based on considering on
   %%% pipe

   Tl(i) = -20*log10 ( abs ( modal_coff( N1+(2*

  %%%% Plotting TL...
   figure(1)
```

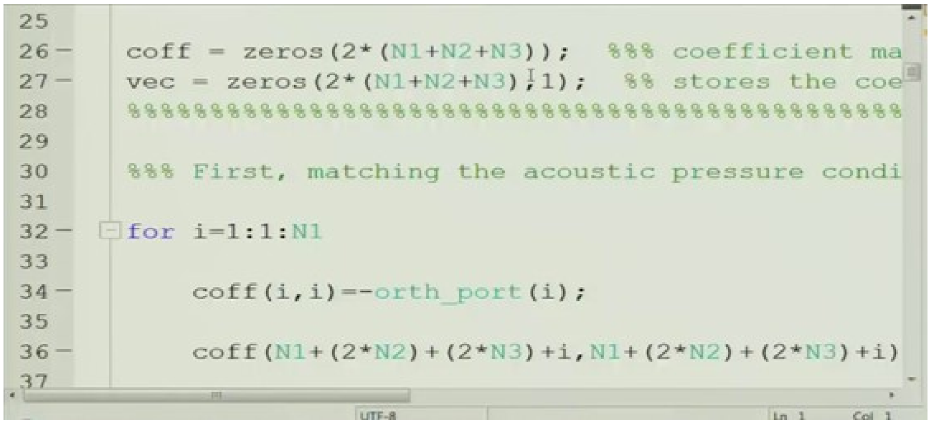
Now, we can plot it similarly when the solver modified that is for the extend inlet and when extensions are also involved.

Then what do we get? So, this is the file that we should basically, this is the file that we basically should open. So, just bear with me for a minute and.



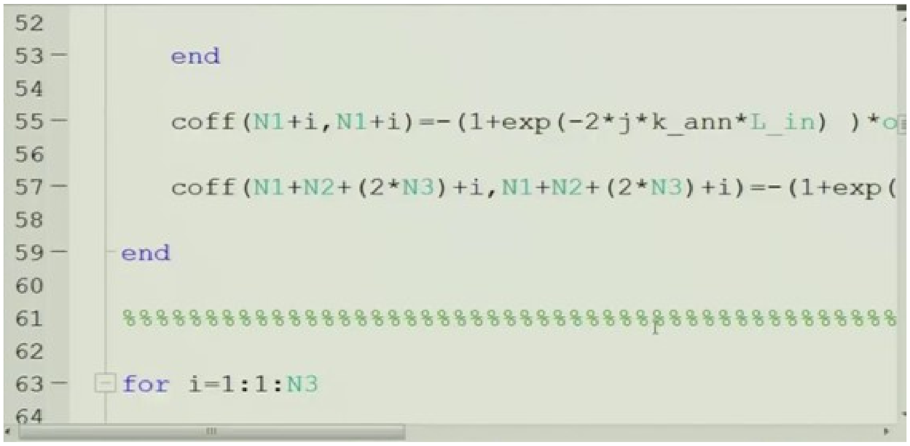
```
j=sqrt(-1); %%% Unit complex..

load zeros_annular.mat

load zeros_hollow_cylinder.mat
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
global orth_port orth_cham orth_annular
global cross_cham_ann_pr cross_cham_ann_vel cro
global N1  N2  N3

global c0

global L_in L_out Lp r_in R_in
```

So, here it is exactly the same other than the fact that we need to invoke extra a few more integers zero hollow underscore cylinder and those thing.



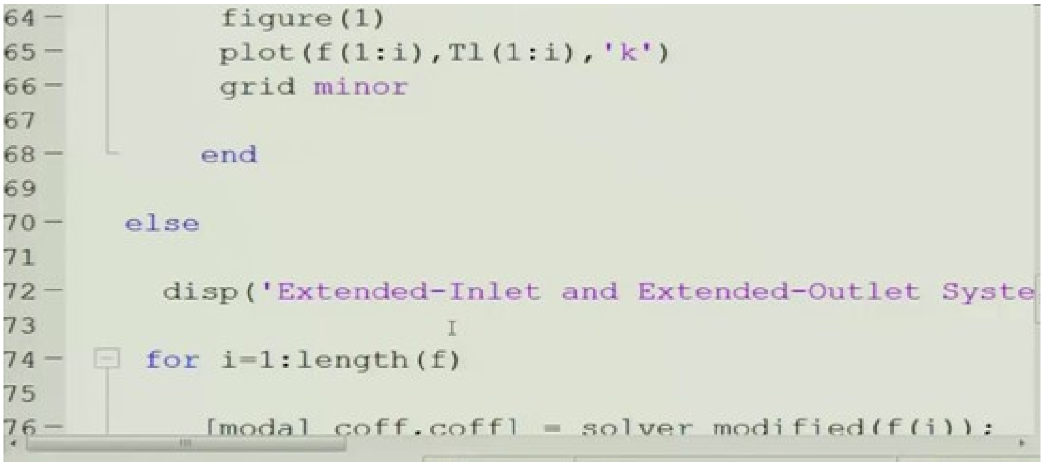
```

coff = zeros(2*(N1+N2+N3));  %%% coefficient ma
vec = zeros(2*(N1+N2+N3),1);  %% stores the coe
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%% First, matching the acoustic pressure condi

for i=1:1:N1

    coff(i,i)=-orth_port(i);

    coff(N1+(2*N2)+(2*N3)+i,N1+(2*N2)+(2*N3)+i)
```

Size has increased or the coefficient matrix in vector matrix everything else is sort of remains the same.



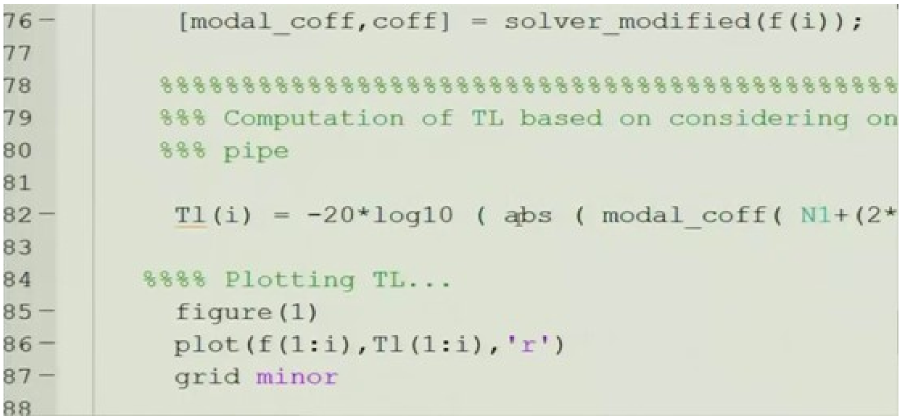
```

    end

    coff(N1+i,N1+i)=-(1+exp(-2*j*k_ann*L_in) )*o

    coff(N1+N2+(2*N3)+i,N1+N2+(2*N3)+i)=-(1+exp(

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:1:N3
```

And additional matching conditions are there at the inlet and outlet. So, these are something to do with the cavities ok.



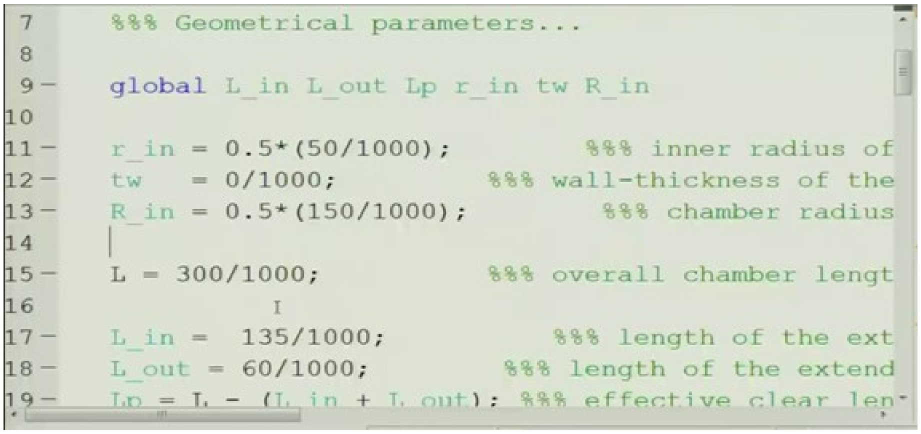
```
            figure(1)
            plot(f(1:i),Tl(1:i),'k')
            grid minor

          end

      else

        disp('Extended-Inlet and Extended-Outlet Syste

       for i=1:length(f)

          [modal_coff,coff] = solver_modified(f(i));
```

So, all these things are there eventually what; rather than explaining each and everything I think is best that you start coding and using the velocity matching conditions and following those papers, write your own codes.



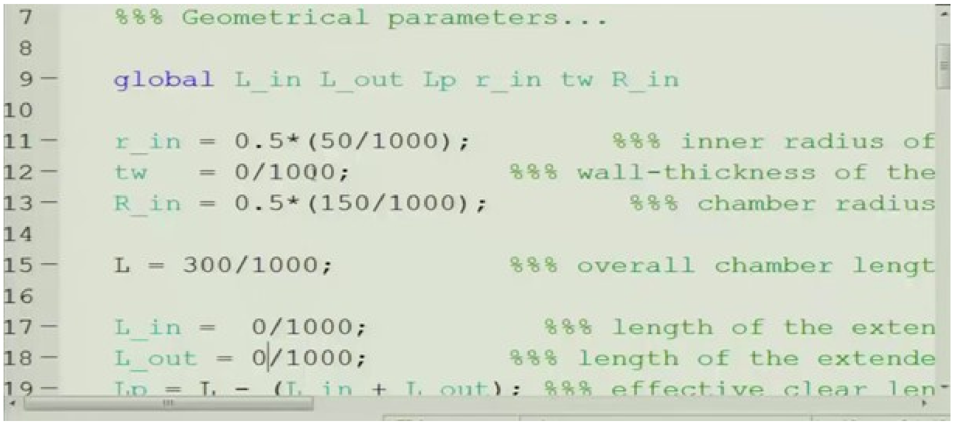
```
         [modal_coff,coff] = solver_modified(f(i));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%% Computation of TL based on considering on
        %%% pipe

         Tl(i) = -20*log10 ( abs ( modal_coff( N1+(2*

       %%%% Plotting TL...
         figure(1)
         plot(f(1:i),Tl(1:i),'r')
         grid minor

```

And whenever you are stuck up think about it or you can always Email me in the discussion forum and I am there to discuss.



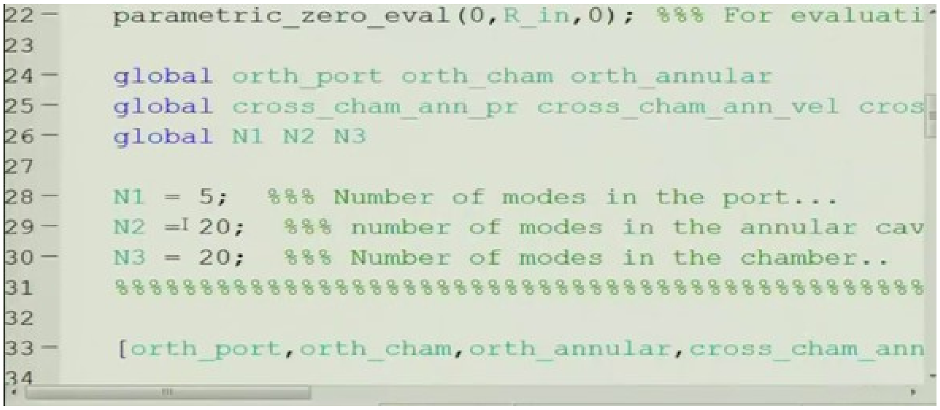
```
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);          %%% inner radius of
tw   = 0/1000;          %%% wall-thickness of the
R_in = 0.5*(150/1000);          %%% chamber radius

L = 300/1000;           %%% overall chamber lengt

L_in =  135/1000;           %%% length of the ext
L_out = 60/1000;          %%% length of the extend
Lp = L - (L_in + L_out); %%% effective clear len
```

So, what we can now do is basically you know run this file for particular parameters. So, let us begin our thing with 0 extensions. Let us see what we get.

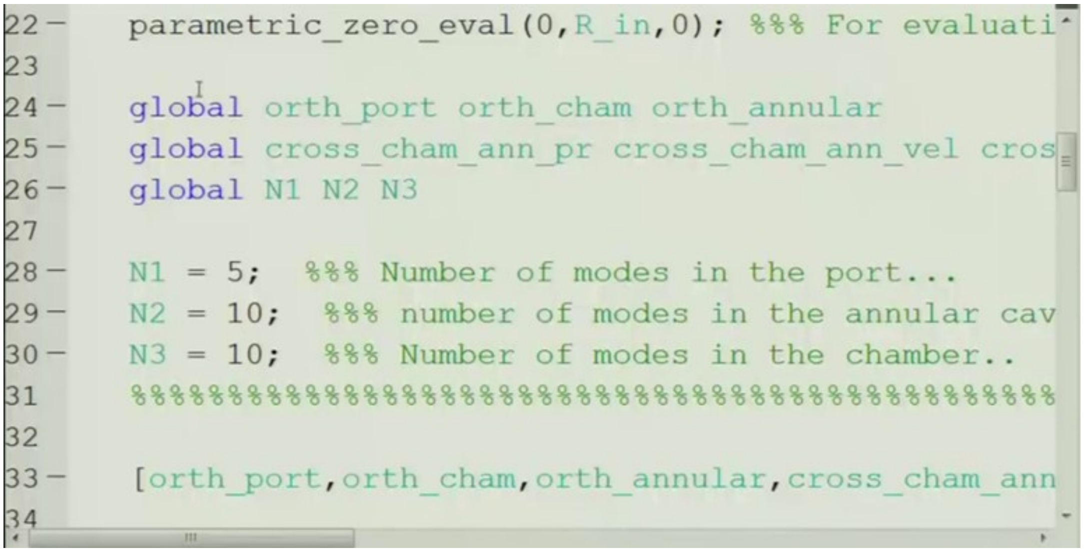


```
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);          %%% inner radius of
tw   = 0/1000;          %%% wall-thickness of the
R_in = 0.5*(150/1000);          %%% chamber radius

L = 300/1000;           %%% overall chamber lengt

L_in =  0/1000;           %%% length of the exten
L_out = 0/1000;         %%% length of the extende
Lp = L - (L_in + L_out); %%% effective clear len
```

We consider only; so this is the radius of the inlet pipe 25 mm and this is the radius of the chamber that is 75 mm. 150 mm is the diameter ok. And no extensions are considered.

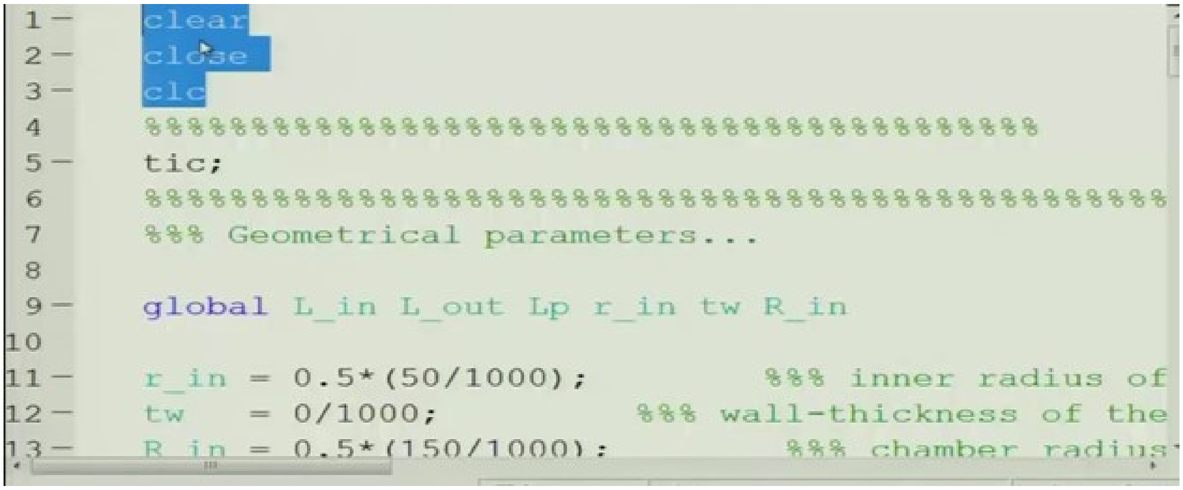


```
parametric_zero_eval(0,R_in,0); %%% For evaluati

global orth_port orth_cham orth_annular
global cross_cham_ann_pr cross_cham_ann_vel cros
global N1 N2 N3

N1 = 5;  %%% Number of modes in the port...
N2 = 20;  %%% number of modes in the annular cav
N3 = 20;  %%% Number of modes in the chamber..
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[orth_port,orth_cham,orth_annular,cross_cham_ann
```



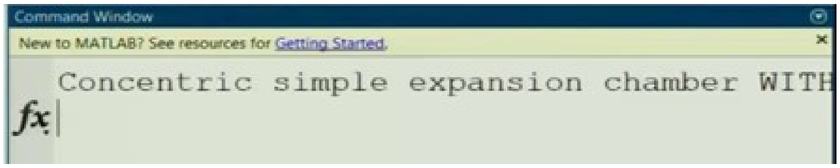
```
    parametric_zero_eval(0,R_in,0); %%% For evaluati

    global orth_port orth_cham orth_annular
    global cross_cham_ann_pr cross_cham_ann_vel cros
    global N1 N2 N3

    N1 = 5;  %%% Number of modes in the port...
    N2 = 10;  %%% number of modes in the annular cav
    N3 = 10;  %%% Number of modes in the chamber..
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    [orth_port,orth_cham,orth_annular,cross_cham_ann
```

Instead of considering 20 modes, let us consider only 10, 10 mode.

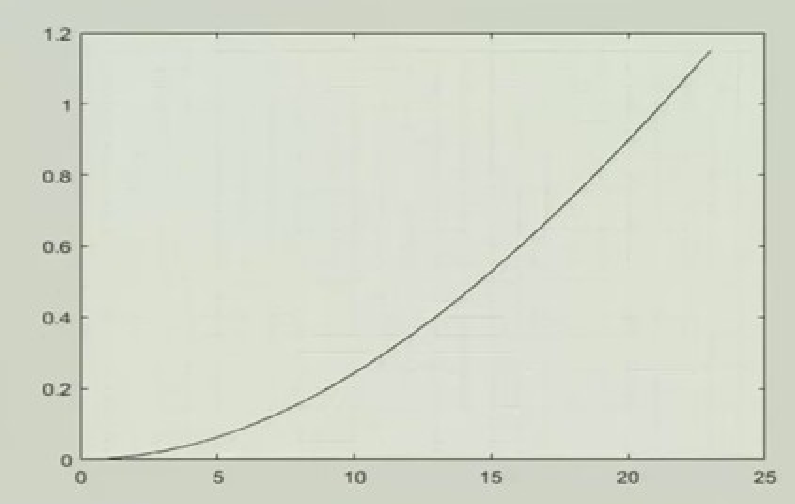


```
     clear
     close
     clc
     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
     tic;
     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
     %%% Geometrical parameters...

     global L_in L_out Lp r_in tw R_in

     r_in = 0.5*(50/1000);          %%% inner radius of
     tw   = 0/1000;             %%% wall-thickness of the
     R_in = 0.5*(150/1000);          %%% chamber radius
```

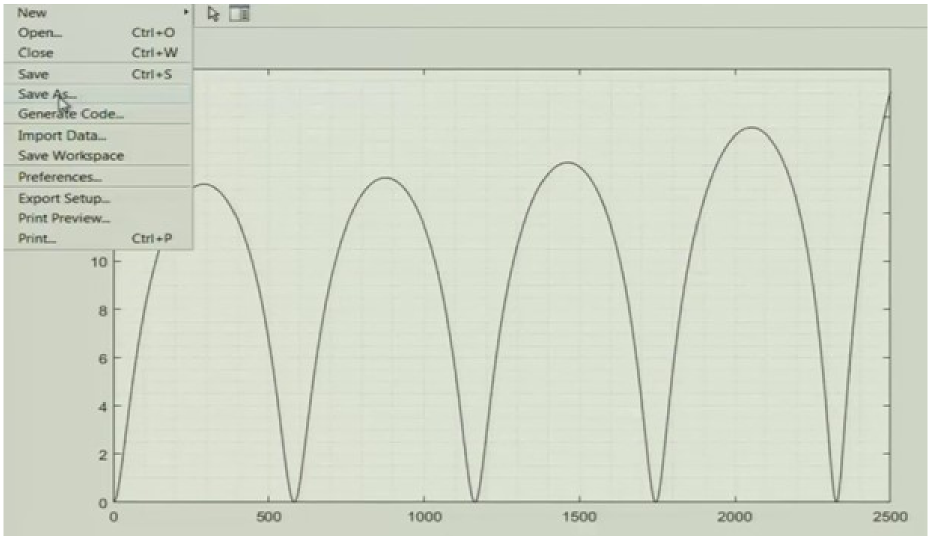
So, make it a bit quicker and run the code.



```
  Concentric simple expansion chamber WITH
fx
```



So, initially it will calculate, it is going to calculate the integral. So, now it is done. So, it flashes a message that we are doing concentric simple expansion chamber without extensions.

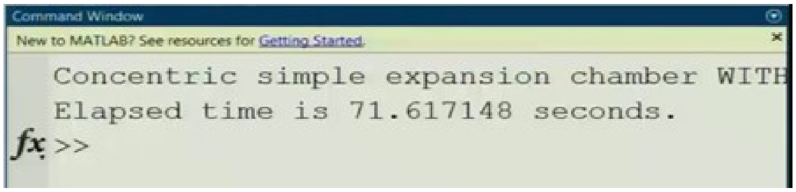


So, what we are seeing here is you know live plotting of transmission loss. So, you know you should be really getting for the 3100 mm length and 346 speed you should be getting the trough, trough here at 33 sorry 577 Hertz. And the peak is based on again the expansion ratio; we will work out all these things let it first compute.

And after a while now the 3D effects will start to kick in. Probably I should have considered a higher frequency range. You see these domes are unequal and suddenly a collapse will happen. So, I probably have to run it again for a higher frequency range.

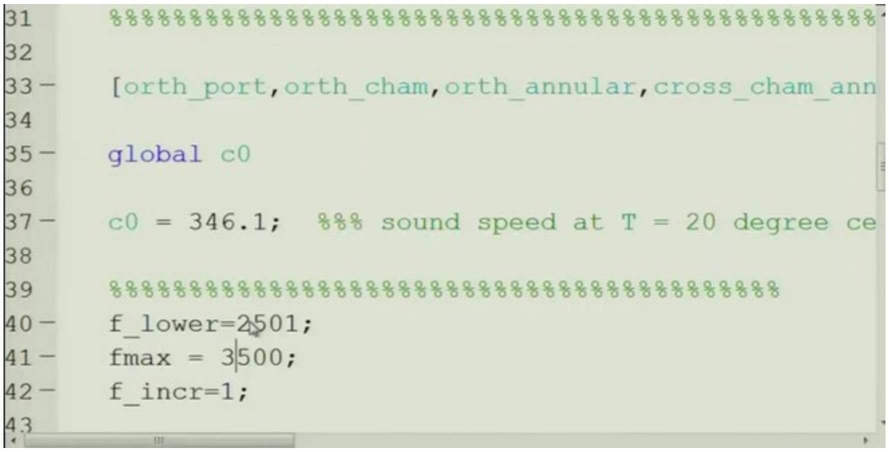
You see these are 3D effects coming in picture; the domes are you do not get some simple expansion chamber domes here, now the break breakdown will happen ok.

So, I have plotted only until this time. So, what I can do? I can actually save it, save the file going to take some time.



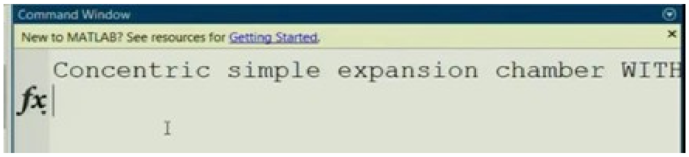
```
Concentric simple expansion chamber WITH
Elapsed time is 71.617148 seconds.
fx >>
```

And possibly run it. So, it takes about 70, 71 seconds only because you have pre-computed most of the integrals, employed this matching conditions; for a faster system it would have done a bit faster I mean if you have a faster system it would evaluate faster.



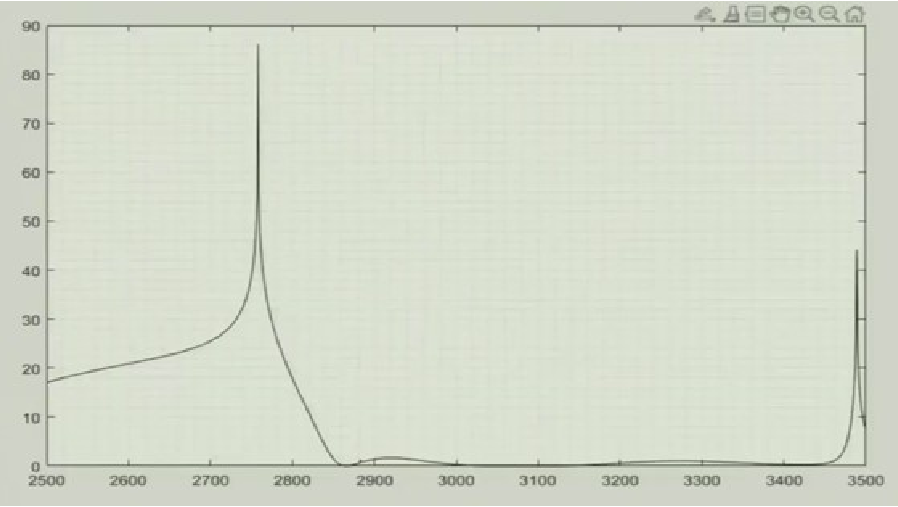
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[orth_port,orth_cham,orth_annular,cross_cham_ann

global c0

c0 = 346.1;  %%% sound speed at T = 20 degree ce

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f_lower=2501;
fmax = 3500;
f_incr=1;
```

So, now let us say we change the lower parameter and go up to 3500 Hertz.

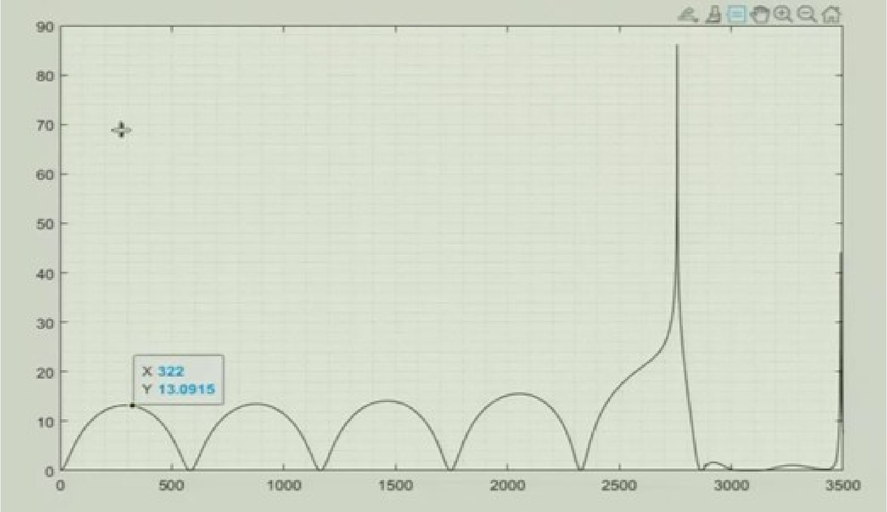


```
Concentric simple expansion chamber WITH
fx
```

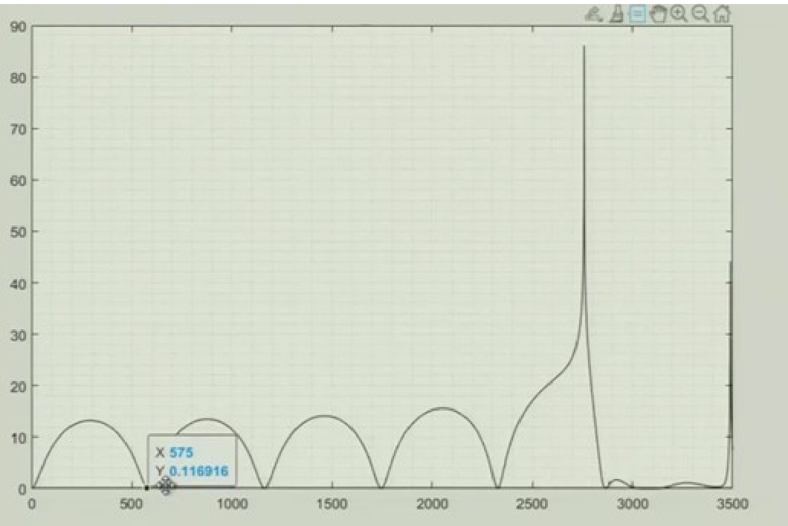
Note that the justification of planar wave propagation in the inlet pipe is still ok, because the first frequency is really at high things we will soon see.



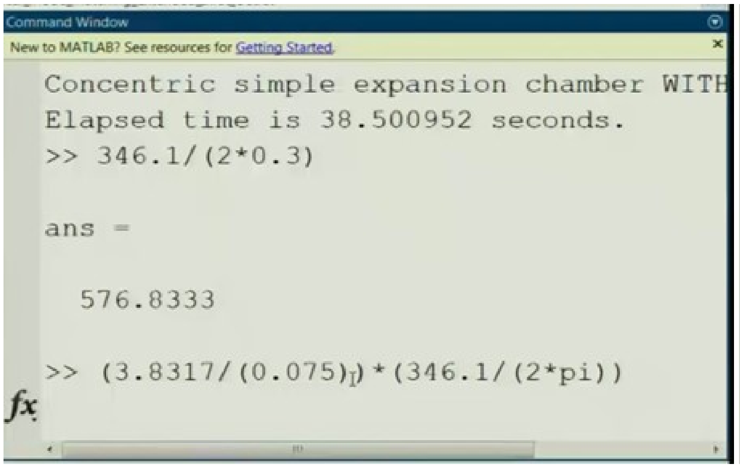
So, you see a complete collapse. We will put the both the pieces together and now we will run until 3500 Hertz. So, it gets a spike. So, what we are going to do? Let me close of this ok.



This is a complete picture, then let me save this guy please yes.

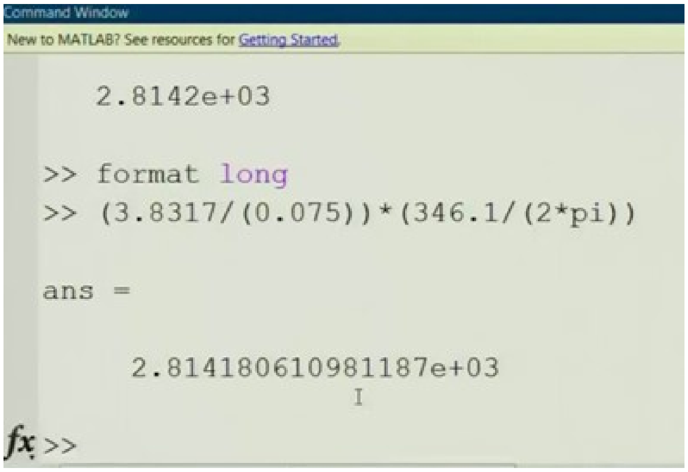


So, you see that this troughs always occur at a certain point at 577 roughly around, around about that point slightly more because of 3D effects.



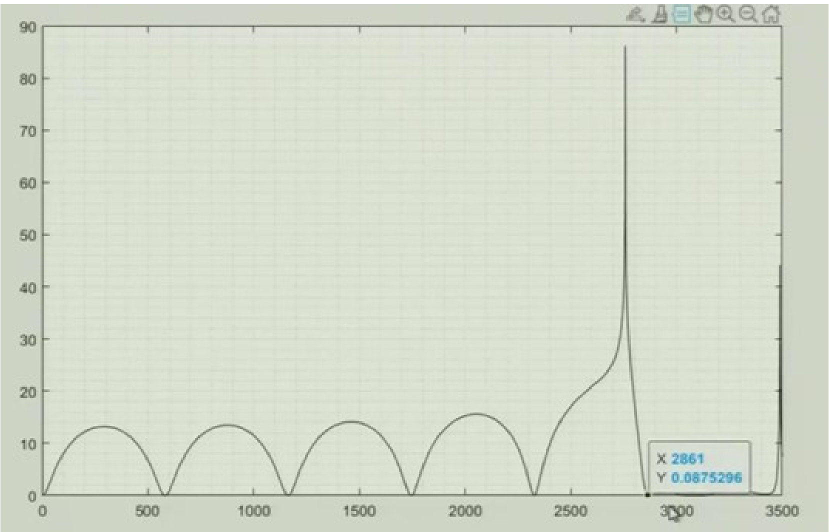
```
Concentric simple expansion chamber WITH
Elapsed time is 38.500952 seconds.
>> 346.1/(2*0.3)

ans =

  576.8333

>> (3.8317/(0.075))*(346.1/(2*pi))
```

And you can calculate because it is going to be something like 346.1 divided by 2 into 0.3, because c naught divide by 2L that is a formula right. So, its 577 roughly its because of 3D effects is showing at slightly higher things. And what is the cutoff frequency of the frequencies radial mode is 3.8317 and radius is your 0.075 for the chamber is not it and you need to multiply this thing by 346.1 and this is divided by 2 π.

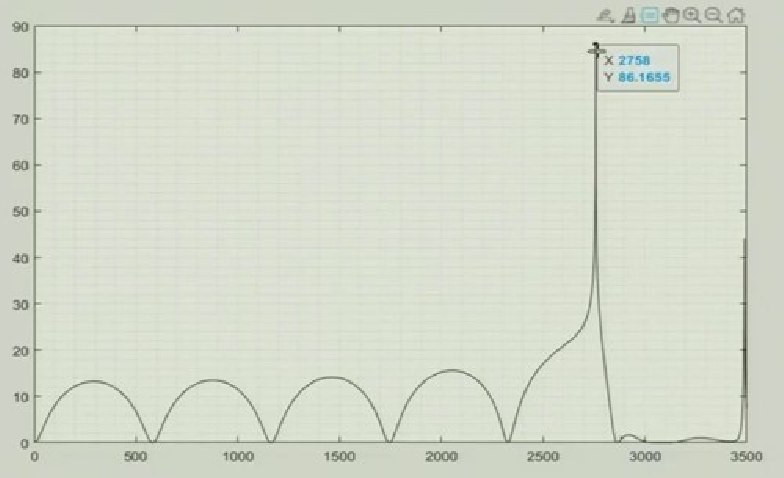


```
    2.8142e+03

>> format long
>> (3.8317/(0.075))*(346.1/(2*pi))

ans =

     2.814180610981187e+03

fx >>
```

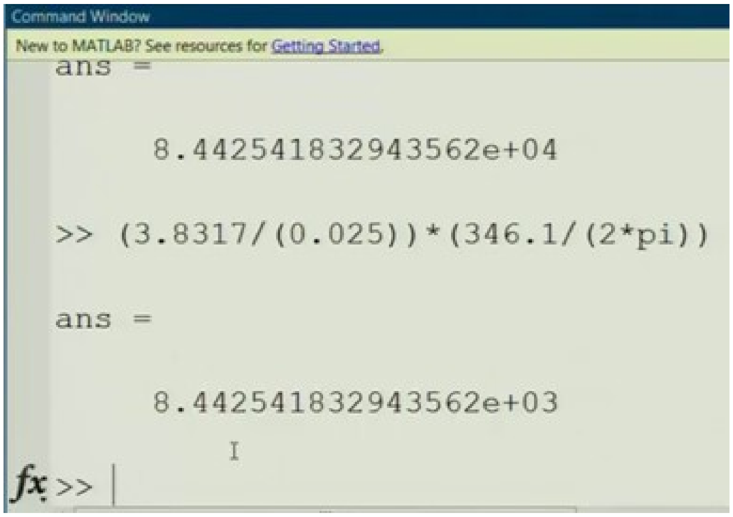
So, what we get is that 2814 Hertz is where the first radial mode happens in the chamber.



And so let us see what are we getting in, in regards to this? So, what it means that, before 2000 2814 Hertz this will definitely fail and that is what is happening. You know the chamber is just a little; just a little after that we are getting a complete collapse of the dome and trough feature. And before it collapses it gives you a massive peak.



You know just a bit before 2758 Hertz. And then after that there is a complete collapse and as you can see you know you getting number of kind of you know you will get a very spiky or a number of peaks and that kind of characteristics. So, you know we can do one more interesting thing actually we can actually increase the.



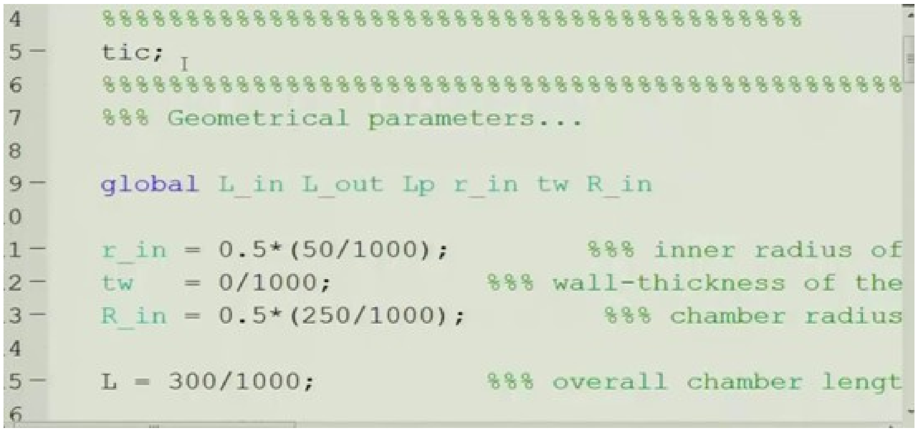
```
ans =

     8.442541832943562e+04

>> (3.8317/(0.025))*(346.1/(2*pi))

ans =

     8.442541832943562e+03

fx >>
```

And actually before we do that let us also evaluate whether we are justified in considering only plane waves in the pipe as far as transmission loss is concerned. Now, although all the model coefficients are computed using model coupling, but for calculating the transmission loss I think we are justified here in computing taking only the first term and that is because your here you have 0.0050 it was there now.

So, 025 first frequency we will start at sorry this is not 0025 just 0025, 8442 Hertz roughly 80443 Hertz. So, we are much below that. So, its perfectly fine if we consider only the first modal term for calculating the transmission loss alright.

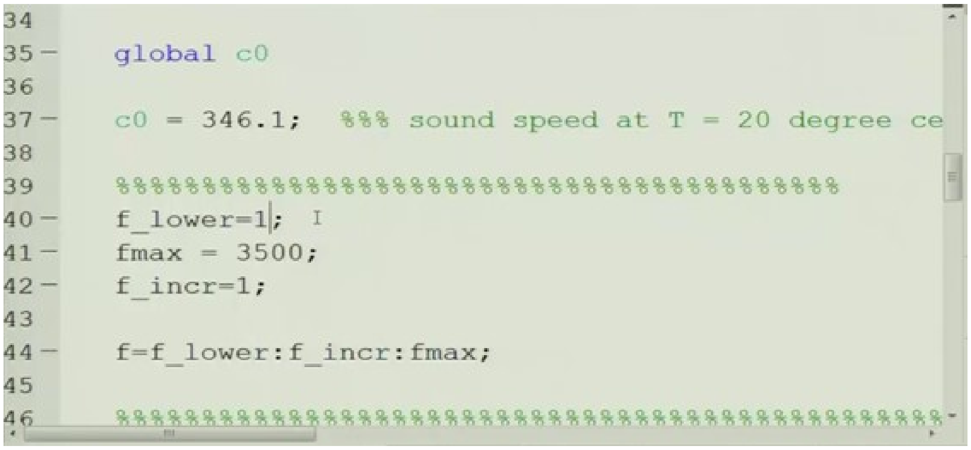
Although for you know this is a subtle, this is an important point you know in the pipe there will be higher order modes also, but then those will be evanescent. So, the coupling considers when you do the analytical mode matching the coupling between the modes in the port and the chamber are considered.

So, that is what that is how these well these model coefficients are evaluated, but now what we have really done is that we have just figured out the model coefficients in the pipe in the inlet and outlet pipe in the chamber as well, but for computing the transmission loss we justify to consider only the planar wave term that is the first model term in the pipe because we are well below the radial mode or the cut on frequency ok. So, that is; so that is what it is. So, this is one point I want to tell.



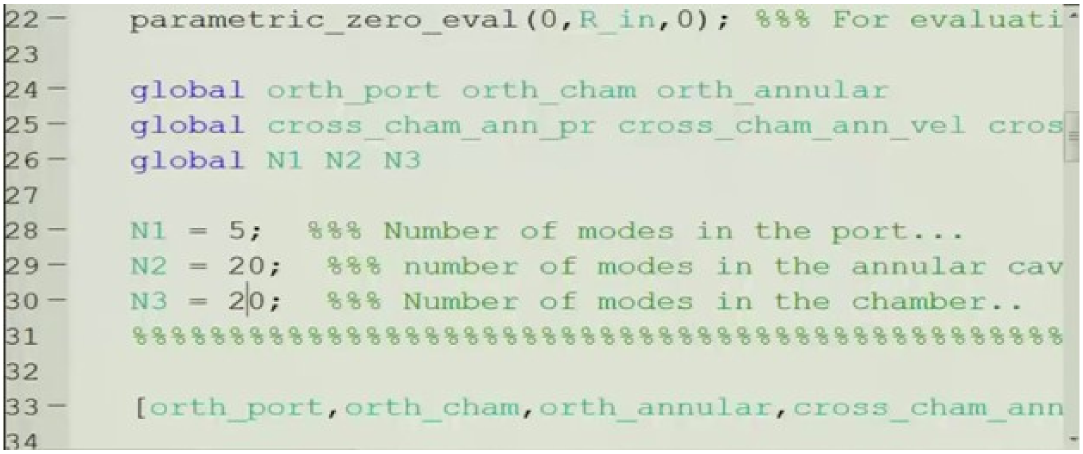
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tic;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);        %%% inner radius of
tw   = 0/1000;          %%% wall-thickness of the
R_in = 0.5*(250/1000);        %%% chamber radius

L = 300/1000;           %%% overall chamber lengt
```

Now, suppose let us increase this diameter ok. Let us make it 200 you know or maybe 250, 5 times the expansion ratio. Let us see and the length everything remains the same.



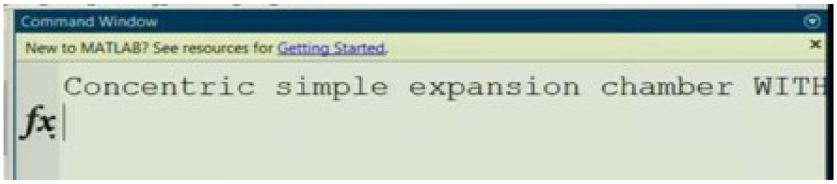
```

global c0

c0 = 346.1;  %%% sound speed at T = 20 degree ce

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f_lower=1;
fmax = 3500;
f_incr=1;

f=f_lower:f_incr:fmax;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

And now let us go from 1 to 3500 Hertz. Let us consider even more terms.

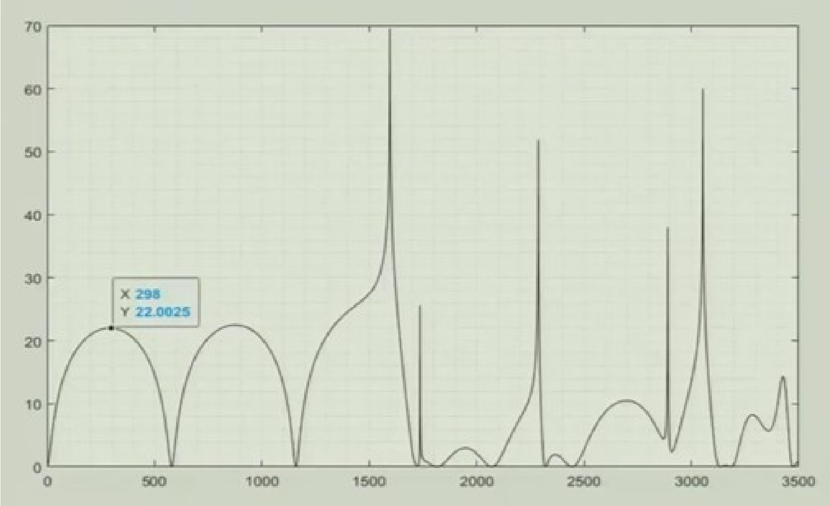


```
    parametric_zero_eval(0,R_in,0); %%% For evaluati

    global orth_port orth_cham orth_annular
    global cross_cham_ann_pr cross_cham_ann_vel cros
    global N1 N2 N3

    N1 = 5;  %%% Number of modes in the port...
    N2 = 20;  %%% number of modes in the annular cav
    N3 = 20;  %%% Number of modes in the chamber..
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    [orth_port,orth_cham,orth_annular,cross_cham_ann
```

Let us say 20 terms, because more number of modes will be excited. So, this comes from experience how do you select the modes, you should also establish convergence.



```
  Concentric simple expansion chamber WITH
fx
```

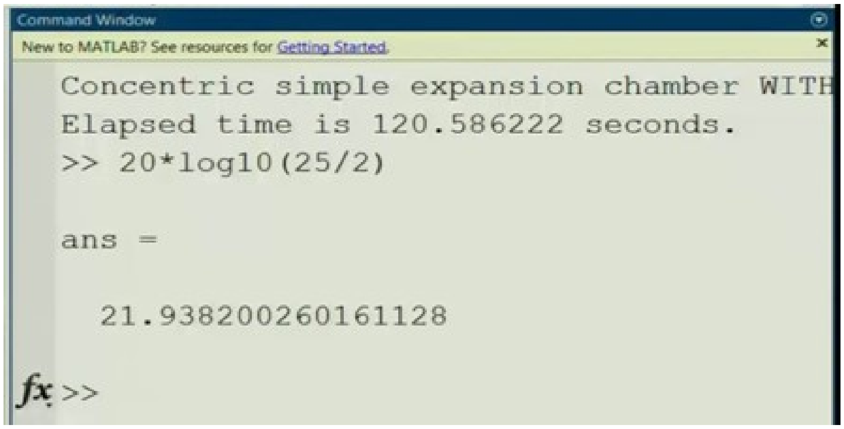


So, it is what it is doing now it is computing the pre computing the integrals so ok. So, we are on the track. So, now you see this 3D effects will come sooner into the picture and other thing that I want to tell is that now here the expansion ratio is 5. The expansion ratio is 5. So, the area ratio is 25. So, 25 divided by 2 is 12.5; log of 12.5 20 times log of 12.5, because 20 times log of m by 2 that is a maximum transmission loss.

So, wait for a while we will do all these hand calculations, but before that you see how much collapse in the transmission loss spectrum much before ok. You see a massive spike and after that the behavior will be completely peaky that will be large number of peaks and you know basically once the higher order modes once you allow the higher order modes to propagate you know it will be a bad thing, because then the transmission loss it will almost kill the transmission loss performance and that is what is happening here.

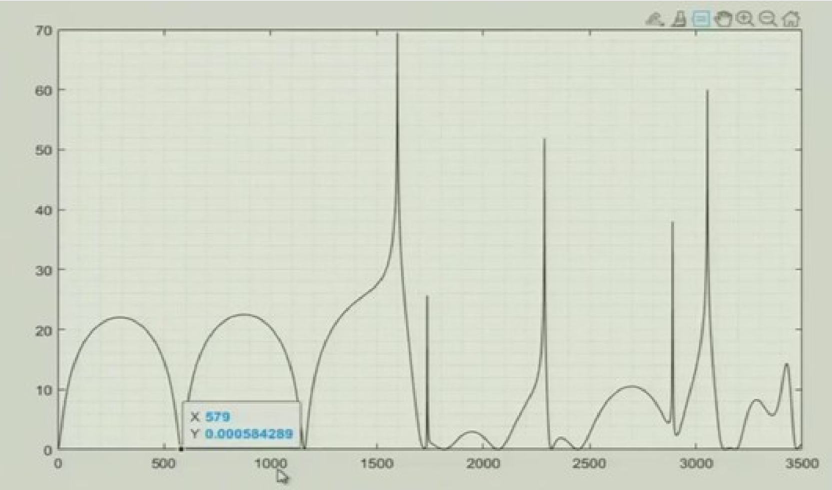
There will be number of dome straws peaks a mixture of that, but at least you know you know its not like this domain trough is very good you can do much better as we will soon see, but at least you know its still better this dome trough behavior is still better than this, this part because I think essentially the muffler is transparent at once you have the high order mode propagating for a simple expansion chamber muffler, the muffler is almost transparent.

Now, let us have a look at the maximum transmission loss that you are able to get ok. So, what is it that you are getting? 22.

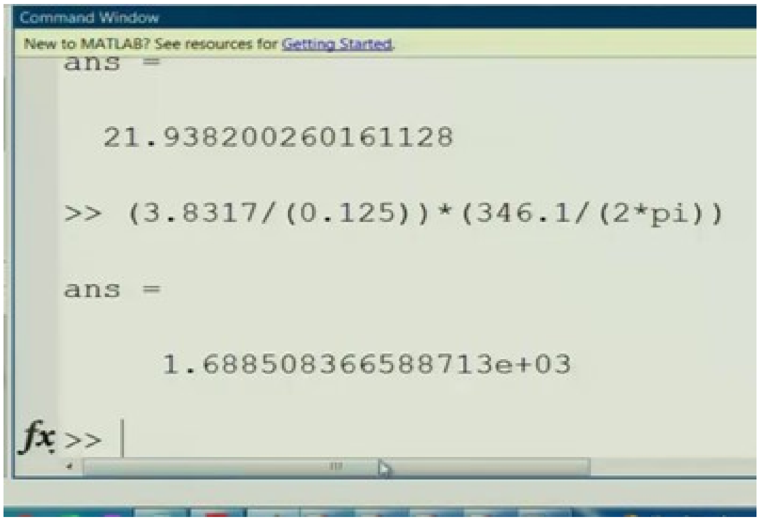


```
Concentric simple expansion chamber WITH
Elapsed time is 120.586222 seconds.
>> 20*log10(25/2)

ans =

  21.938200260161128

>>
```

And the formula says you remember the initial lectures m by 2. So, m is 25 because 250, 250 / 50 or 125 / 25. So, 5 you will get 5, 5 square is 25 and 25 / 2 you will get 21.93. And let us see what we what are we getting and 22.0025 is very close ok.



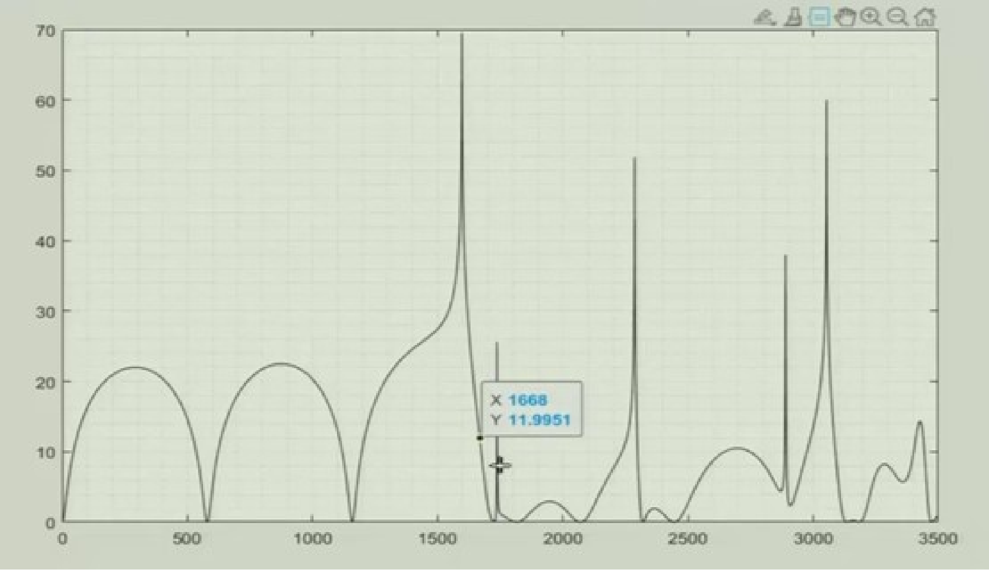
And you are getting a p trough exactly at 5 well 579 Hertz very close.



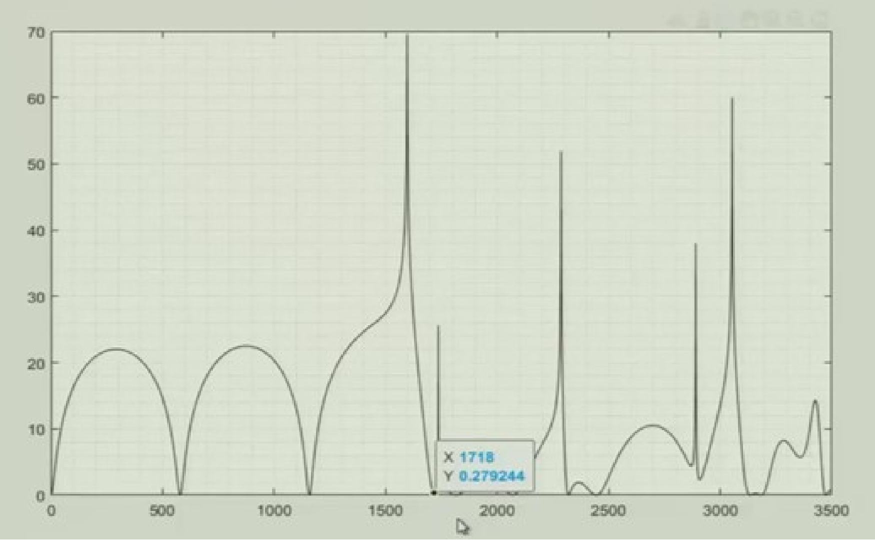
```
ans =

  21.938200260161128

>> (3.8317/(0.125))*(346.1/(2*pi))

ans =

     1.688508366588713e+03

fx >> 
```

And what about the; what about the radial frequency now. So, this becomes 125.

So, 1668 is where we get a the first at, at this frequency 1668 we definitely are we should definitely get a peak trough near about that 1668.

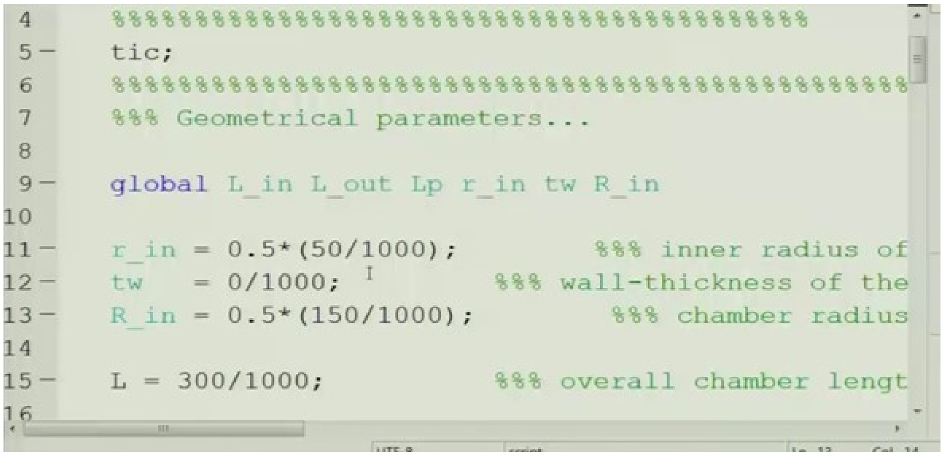


So, if we just go a bit back you know 1668. So, there is a modal coupling involved.



So, we are definitely going down and at trough occurs just immediately afterwards 1718 very close its characterized by a huge spike. Then after that beyond this first radial mode everything is pretty much useless. Although the values are accurate I mean to say the muffler performance goes down after the onset of the first radial mode for a concentric expansion chamber simple expansion chamber ok.

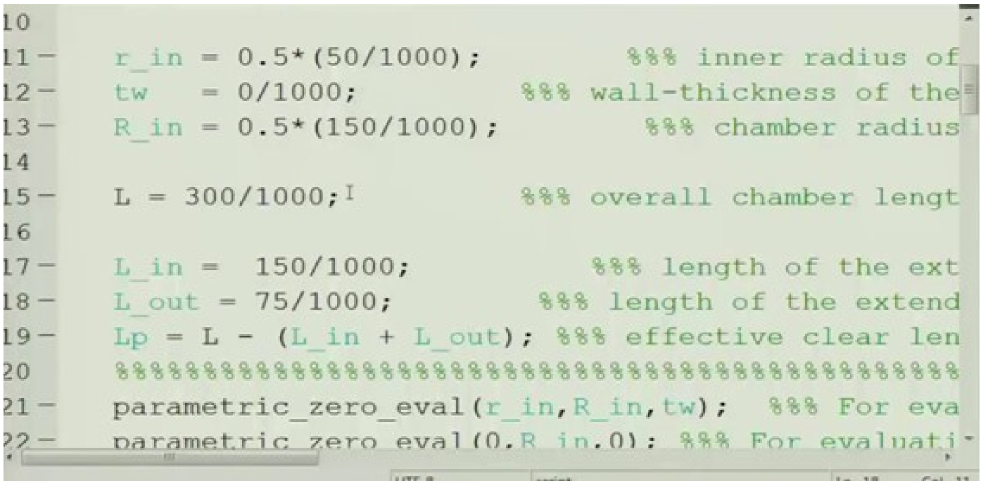
But for extended inlet and outlet we will see how these things can be further controlled. What is going to happen is that beyond the 01 radial mode you know you still get not a very good performance, but at least it will not be as useless as this thing. So, I am going to close of this file.



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tic;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);         %%% inner radius of
tw   = 0/1000;             %%% wall-thickness of the
R_in = 0.5*(150/1000);         %%% chamber radius

L = 300/1000;             %%% overall chamber lengt
```

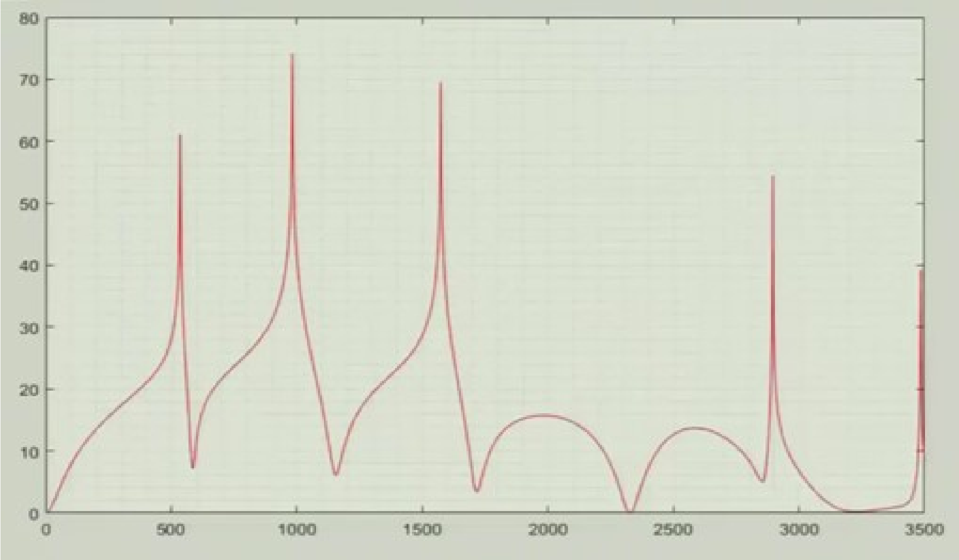
And now do a simple case let us get to this perhaps we can consider this 250 or probably 150 because we have saved this file the untitled.

So, we will compare it with this case and see a dramatic improvement that it gives ok. The ratio expansion ratio is only 3 for a year ratio is 9. The maximum transmission loss that we get is 20 log m by 2.



```
r_in = 0.5*(50/1000);         %%% inner radius of
tw   = 0/1000;            %%% wall-thickness of the
R_in = 0.5*(150/1000);         %%% chamber radius

L = 300/1000;             %%% overall chamber lengt

L_in =  150/1000;           %%% length of the ext
L_out = 75/1000;         %%% length of the extend
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);  %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati
```

But with extensions let us say half you know half the extension. So, what is half of one 300? This is 75. Now it is going to evaluate a different file ok. It is going to pre compute a few things.



And then so you will soon see at 5 instead of 577 instead of getting a trough you will get a non-zero value there; the trough is that is lifted, but then the tuning is not there. Now, we are entering into the principle of I am trying to do a MATLAB demonstration, how we how do we exactly double tune the chamber.

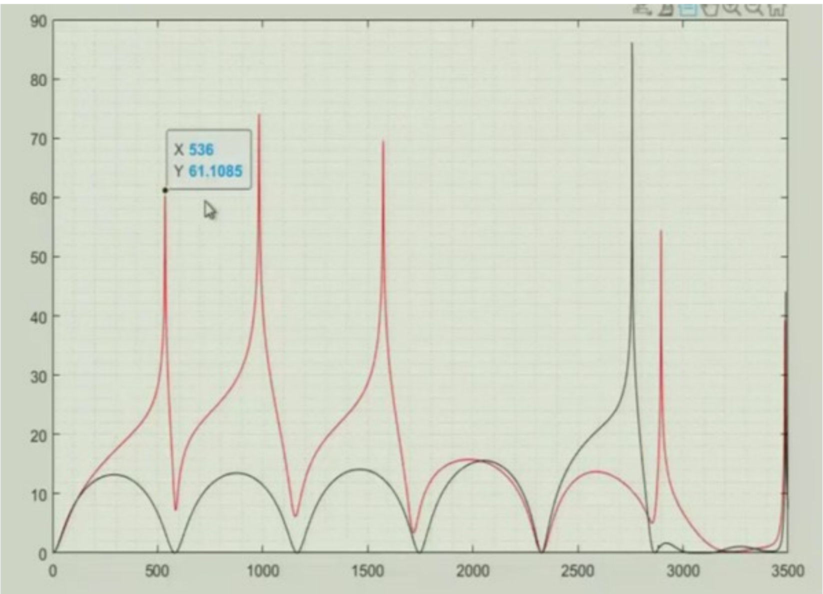
So, just by putting L/ 2; now let us go to the code briefly minimize this guy pops away anyways at L/ 2 and at L / 4 extension length is L /2 and the length of the extension at the outlet is L /4 it does not help, you know had it been plane wave it would have helped I can show you just by choosing only plane wave mode.

So, plane wave mode can be chosen just by putting you know N, N1 is equal to 1, N2 is equal to 1 and N3 is equal to 1, you just consider the plane wave mode. And so then this will work, but then that will not be accurate, because higher order modes are generated and you need to consider large number of modal terms until this until you get a convergent solution.

For our case I guess setting 20 number of mode terms in the annular cavity and 5 terms in the mode that that should suffice we can prove that. But what is important here to note that assuming this is all correct, which is the result have been compared with Selamets papers and also the ones by Chaitanya and Munjal and their number of other published work and using in which experiments were done or some finite elements full 3D finite element simulation was done.

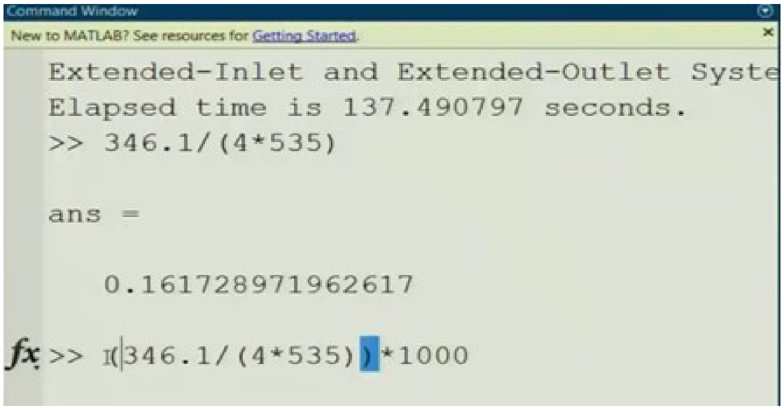
So, those good comparison was obtained using the analytical mode matching using this code, which is now pretty much standard. And we are able to get performance up to 3500 Hertz here, but here we get a missed tuned thing. So, let us do a good comparison I will stop this code stops here ok.

And I will open the untitled file here.



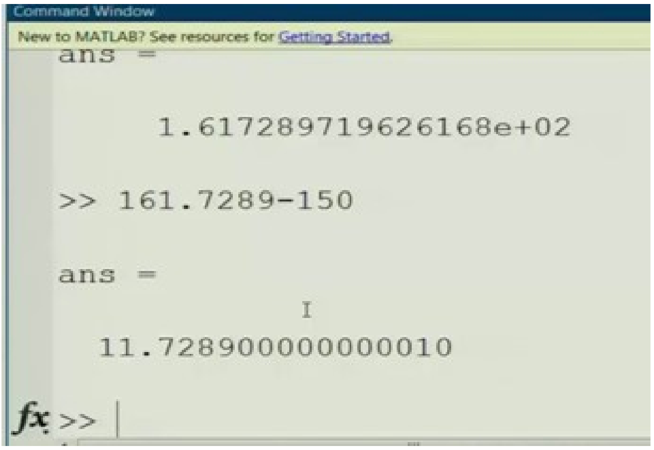
And well this was because ok. So, do not worry about the one that happens at the later part maybe you can kind of put it here ok. Now see a dramatic improvement and at least you are you are getting much better performance so, but these troughs are still not lifted. Because now let us do some fine tuning ok, some double tuning.

So, now effect here you know the reason we chose L/ 2 was because we wanted to ensure that the resonance frequency of the quarter wave resonator completely nullifies the first axial resonance peak. So, but then that is not happening it is not exactly cancelling this is the fact that you are getting a peak at 536 Hertz or 535 Hertz.



```
Extended-Inlet and Extended-Outlet Syste
Elapsed time is 137.490797 seconds.
>> 346.1/(4*535)

ans =

   0.161728971962617

>> (346.1/(4*535))*1000
```

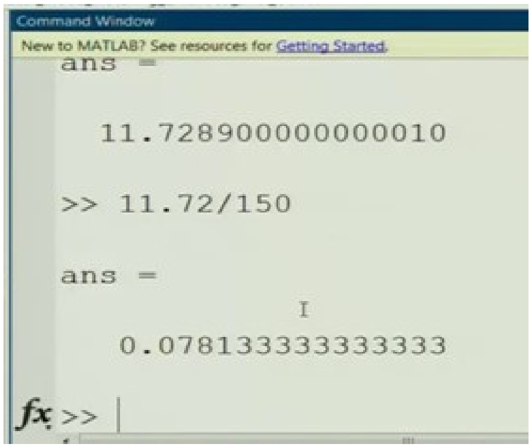
It shows that you should have really got a peak at how many Hertz? 577 or 578 Hertz. It is the fact that you are getting a peak at 535 Hertz mean that the effective length of the chamber is 346.1 divided by 4 into 535 is not it 535 yes, which effective length is let me sort of multiply this by 1000 to convert to mm.



```
ans =

     1.617289719626168e+02

>> 161.7289-150

ans =

  11.728900000000010

>>
```

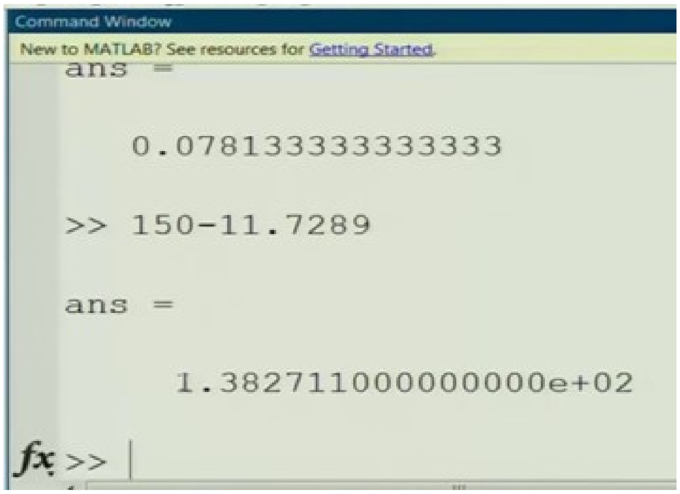
So, you get 161, 161 is effective length, but we chose only. So, the even though we chose the length to be 150 ok, but the extended inlet that the quarter wave resonator inlet is behaving of some as if its length is increased acoustic length is increased to 161 mm. So, you know what is the difference between 161.7289 minus 150? So, this is a; so that; so here now we have the 3D effects for the first time we have 11.72.

So, that is only for this case. So, roughly; so if you have a extension of the physical length to be 150 mm half of this thing or the chamber length, the acoustic length of the effective length is a bit more its about 11 mm more.



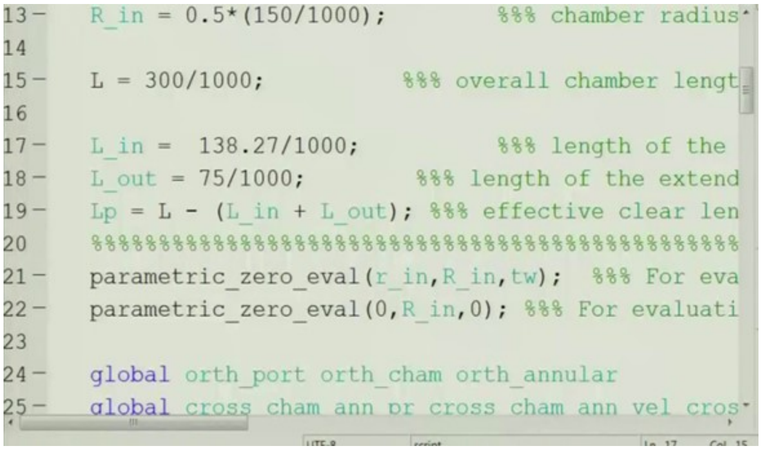
```
ans =

   11.728900000000010

>> 11.72/150

ans =

   0.078133333333333

fx >>
```

So, about 11.72 divide / 150 about, about 7 percent 7.8 percent more. So, basically what we do is that if we subtract this 11.7289 whatever the figure is.



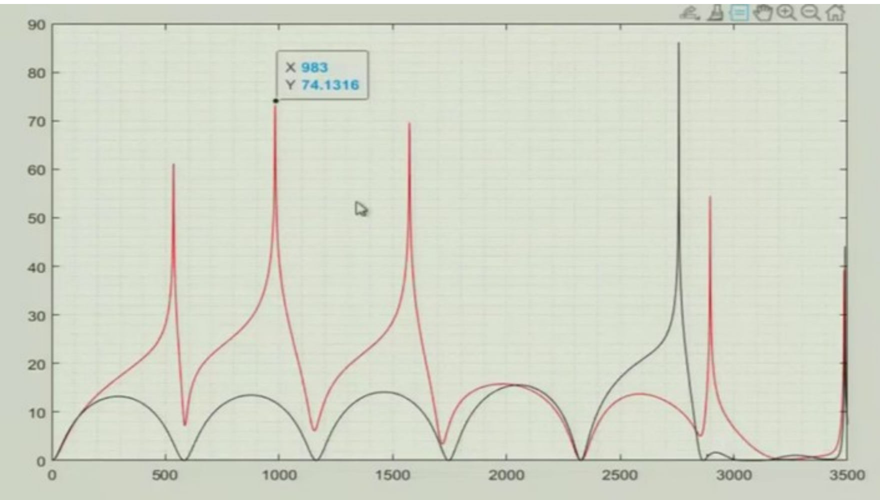
```
ans =

   0.078133333333333

>> 150-11.7289

ans =

     1.382711000000000e+02

fx >>
```

So, if we do 1 minus this guy or I would say 138.2711 ok.

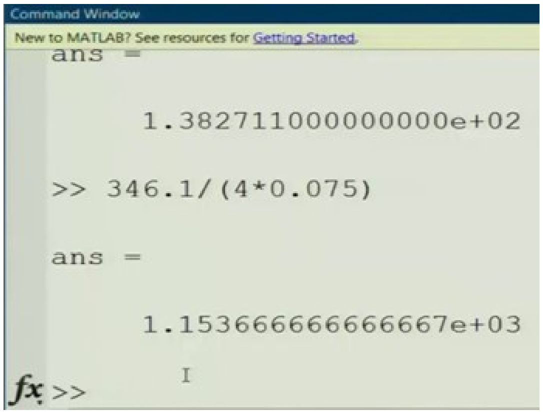


```
R_in = 0.5*(150/1000);         %%% chamber radius

L = 300/1000;           %%% overall chamber lengt

L_in =  138.27/1000;           %%% length of the
L_out = 75/1000;          %%% length of the extend
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);  %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati

global orth_port orth_cham orth_annular
global cross_cham_ann_pr cross_cham_ann_vel cros
```

We can put this value here and 138.271 you can just get rid of this guy.

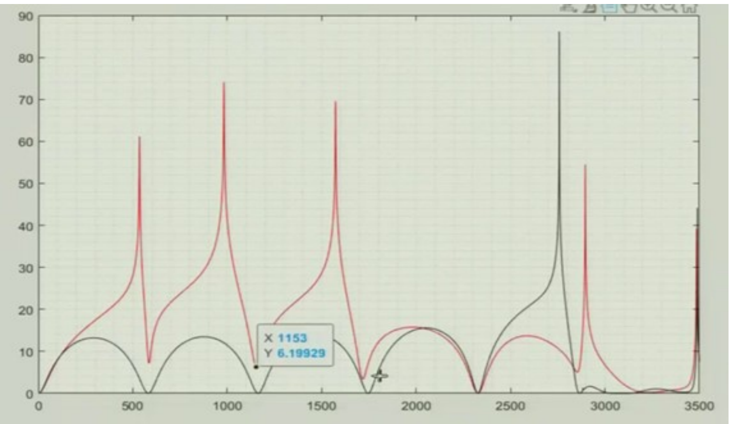


Now, what happens to the peak due to the extensions at the outlet? So, this is 983.

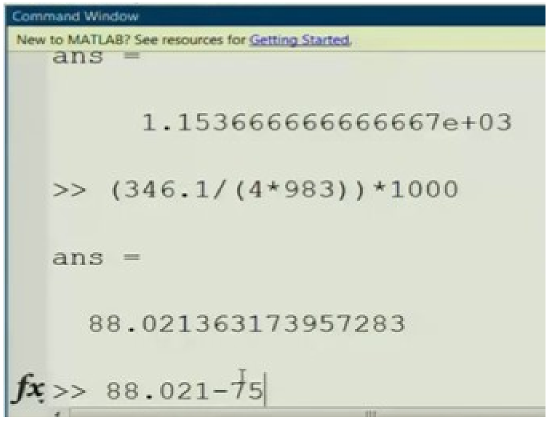


```
ans =

      1.382711000000000e+02

>> 346.1/(4*0.075)

ans =

      1.153666666666667e+03

fx >>
```

It should really have happened at 346.1 divided by 4 into 0.075 ok, 75 mm was the length right. It should have happened at 1000 153.6 Hertz.

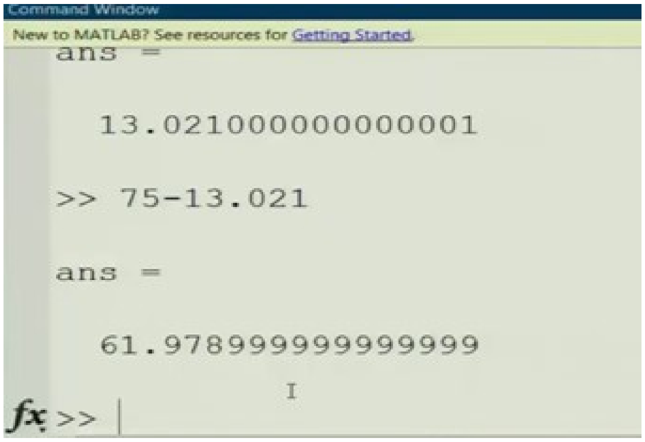


Which is basically your the trough frequency roughly ok. It did not happen it happened much before.



```
ans =

     1.153666666666667e+03

>> (346.1/(4*983))*1000

ans =

  88.021363173957283

fx >> 88.021-75
```

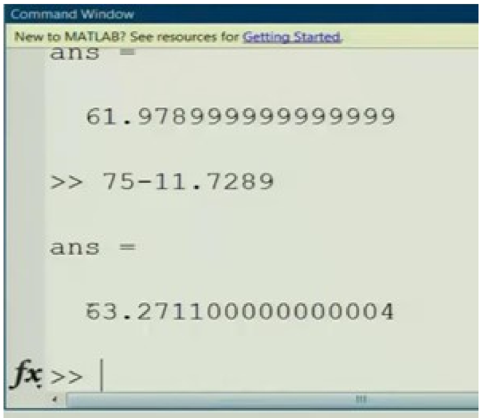
So, what is the effective length of this resonator then or the resonator at the outlet? It is what did we get? 983, 880.021- 75.



```
ans =

  13.021000000000001

>> 75-13.021

ans =

  61.978999999999999

fx >>
```

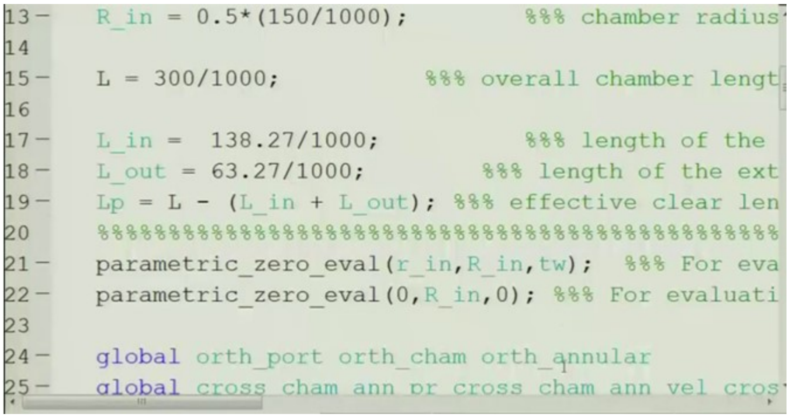
So, we get slightly more extension length 13. So, what we do is that you know we kind of 75 - 13.021 we can do that, but I would say that let us stick to 11 point, let us stick to whatever extension length you get at the inlet only and let us see if you want to avoid this thing.

Now, if you choose that you will also can you will also you can also tune this trough. This trough will now occur at the at this trough and the fourth thing it will it is going to fail here this you cannot prevent. So, what we are going to do is that we consider only 11.782 lengths.



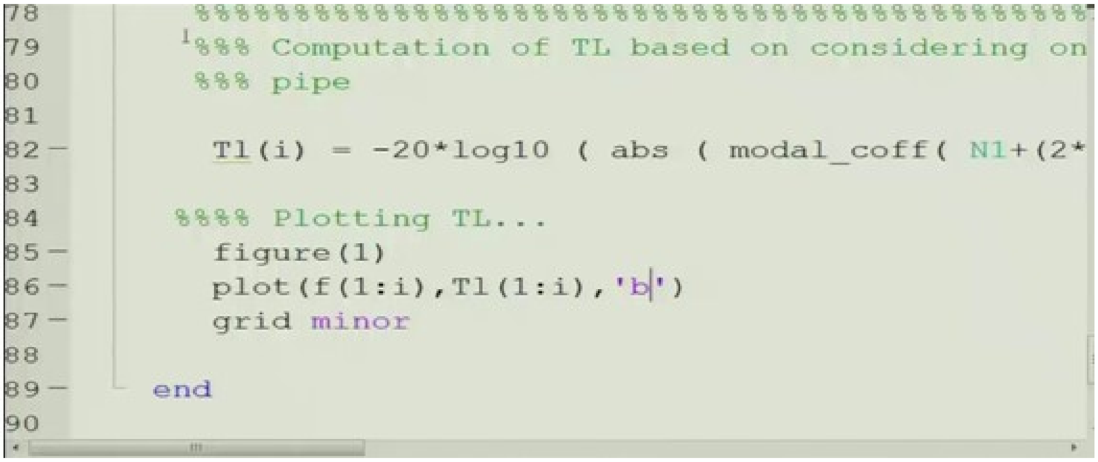
```
ans =

   61.978999999999999

>> 75-11.7289

ans =

   63.271100000000004

fx >> 
```

And so we will put 75 - 63.27.

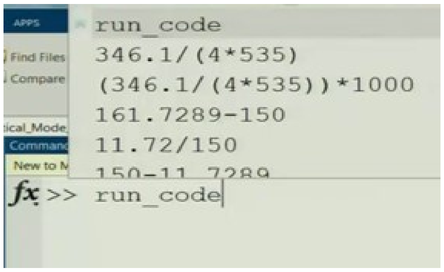


```
R_in = 0.5*(150/1000);          %%% chamber radius

L = 300/1000;            %%% overall chamber lengt

L_in =  138.27/1000;            %%% length of the
L_out = 63.27/1000;          %%% length of the ext
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);  %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati

global orth_port orth_cham orth_annular
global cross_cham_ann_pr cross_cham_ann_vel cros
```

So, 63.27 length if we choose here before running the code let me save this file. Save here as untitled and plot it in a different color, let us say plot in blue color. Let us see what happens.



```
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%% Computation of TL based on considering on
        %%% pipe

         Tl(i) = -20*log10 ( abs ( modal_coff( N1+(2*

       %%%% Plotting TL...
         figure(1)
         plot(f(1:i),Tl(1:i),'b')
         grid minor

     end

```



```
run_code
346.1/(4*535)
(346.1/(4*535))*1000
161.7289-150
11.72/150
150-11.7289
fx >> run_code
```

So, the good thing about analytical mode matching or these thing is that it makes parametric studies very easy ok. And otherwise you would have to mesh it again in finite elements or and do a lot of prose processing. For people who are you know very accustomed to the use of final element software for them it is very easy, but then this is like a proper kind of analytical work and which is kind of implemented in a computer software MATLAB.

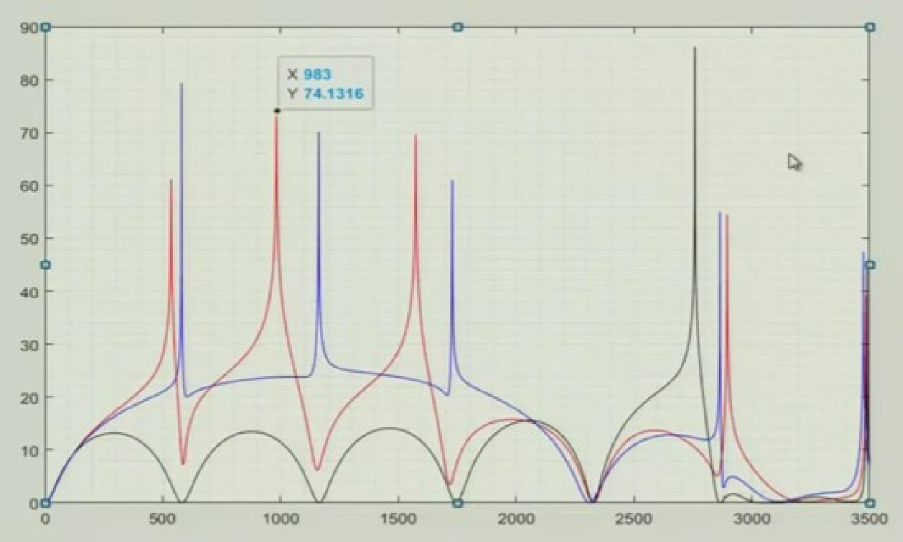
So, let us see what happens. Are we able to tune out the first actual resonance? The curve seems to be continuously increasing and I hope it does not kind of give a trough at 577 Hertz which is the first actual resonance, which probably going to completely annihilate this thing. So, its happening here, then again its going to come down.

So, it seems that we have cancelled the first axial resonance; we have nullified the first axial resonance by tuning the length of the chamber ok. So, and then what happens at 1153 Hertz that is the second axial resonance. So, the curve is behaving steady nearly giving over 20 Hertz attenuation value which is very good. And then we are exactly able to pick out and nullify that second trough ok, there is no peak there is no trough involved here.

And then the curve is going to come down a bit, but then again you see just by tuning providing a tuning at the inlet, this, this you know what I am trying to say is that this peak by tuning the axial by tuning the extension length we are not able to nullify the first axial trough, but also the third one by the virtue of tuning this thing.

By tuning this outlet extension length you know we are able to nullify only the second axial resonance trough, but the fourth one we are unable to do it we in that we have already discussed in the previous lecture we discussed the plane wave theory. So, this is the trough that you will get. So, the broadband transmission loss is limited until whatever frequency you get here. So, 577 times 4. So, that is roughly 2350 whatever Hertz.

And then after that you are going to get a number of spikes and curves. So, this is now where you will get your 2800 Hertz 600, 700, 800 and just 2000 I think it was 2830 or something Hertz, where the first radial mode starts propagating and just before that you get a massive spike and then you know things will happen.

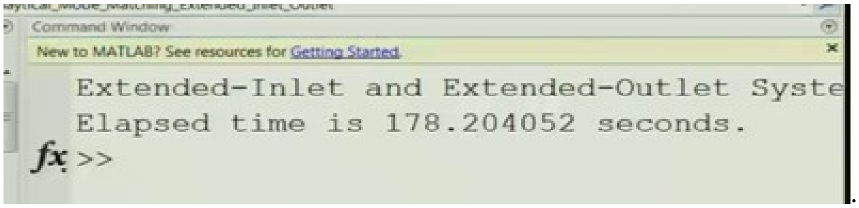


So, the basic idea is that now compare the muffler performance here with this one. Amazing we get up a compared compare the performance. So, the tuned double tuned extended inlet and outlet muffler with that of a simple concentric expansion chamber when there was no extensions. So, just by providing a tuned extension you can get a wonderful performance you know almost a broadband attenuation for significant frequencies of interest 2000 up to 2000 Hertz.

And most of the automotive noise is dominated for at low frequency 600 Hertz or 1200 up to 600 Hertz or 1200 Hertz. Basically your firing frequency and this first 3 or 4 multiples. So, what I am trying to say is that for by providing very good broadband attenuation up to 2000 Hertz and its going to give you almost 20 dB 20 dB of attenuation beyond roughly 400 Hertz or so until about 2000 Hertz.

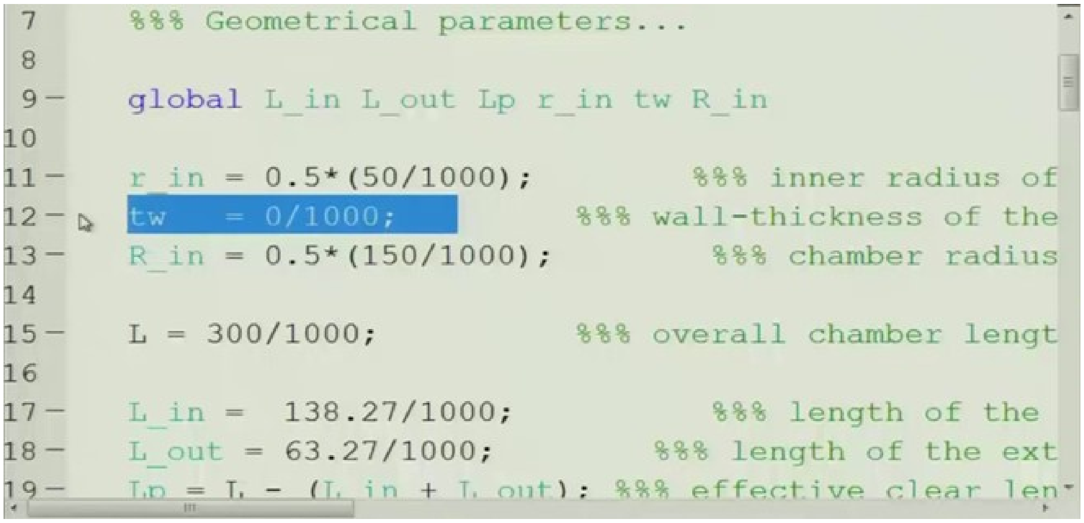
We are covering most of the firing frequencies, we can still do better here by getting some sort of a resonator attached to the Helmholtz resonator attached to the main to the as a side branch to the extended inlet and outlet muffler, but that is a separate discussion. Now, basically you know compare the performance with the simple expansion chamber and compare it also with the non tuned thing.

So, this peak is really cancelling out the trough. And so is the second peak cancelling out this trough. So, it is doing really good you know, but just by tuning we can get wonderful results. So, I will save this file also.



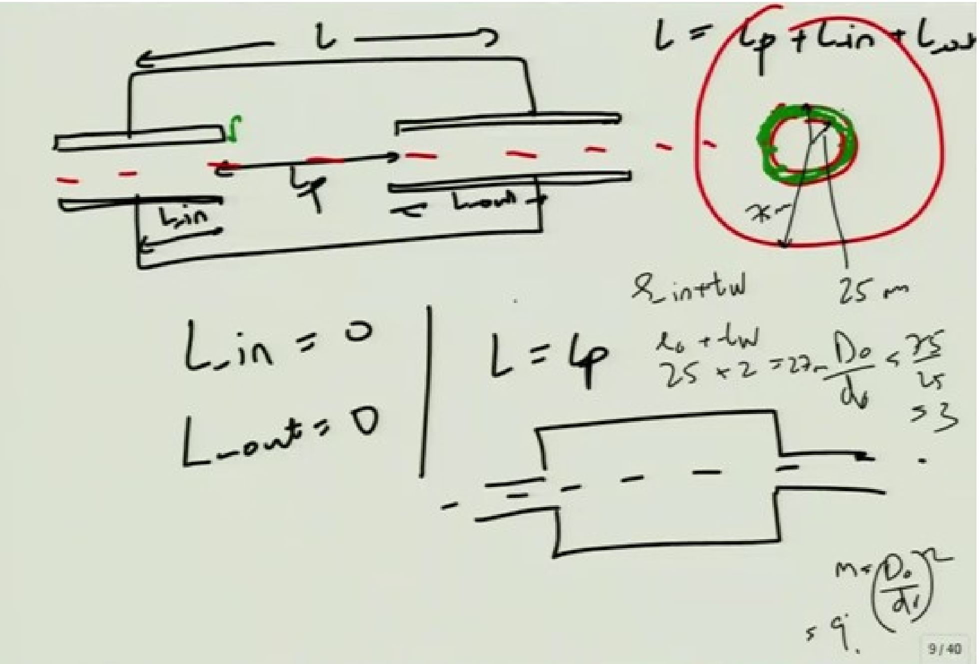
```
Extended-Inlet and Extended-Outlet Syste
Elapsed time is 178.204052 seconds.
fx >>
```

But now comes the critical question, what is that critical question?



```
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);           %%% inner radius of
tw    = 0/1000;           %%% wall-thickness of the
R_in = 0.5*(150/1000);           %%% chamber radius

L = 300/1000;           %%% overall chamber lengt

L_in =  138.27/1000;           %%% length of the
L_out = 63.27/1000;         %%% length of the ext
Lp = L - (L_in + L_out); %%% effective clear len
```

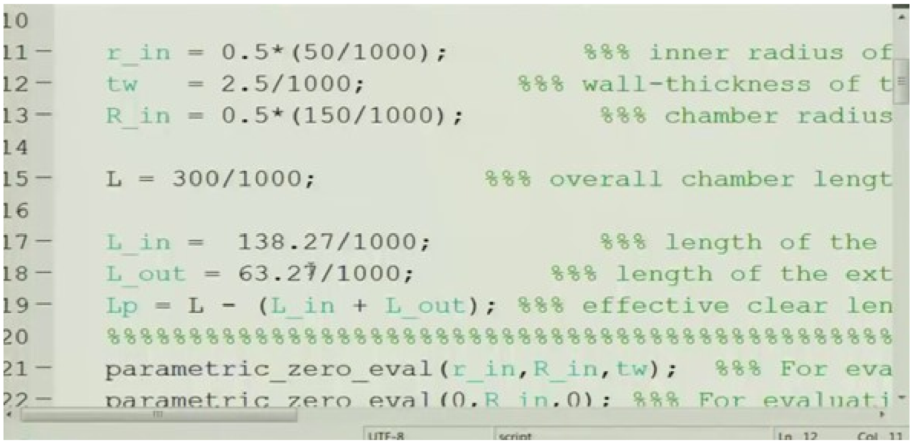
So, no surprises to tell you, you should you should guess I have set the thickness of the pipe to 0.



So, what it means let me go to the presentation. So, all this while we did not consider the thickness of this thing. Now, if we consider a finite thickness you know because if you draw the cross section view apologies for my bad drawing that is what it is. Now, if we draw a view of this. So, we get a thickness like this.

So, it has what I am going to says that the pipe must have certain thickness 1 mm, 2 mm, 3 mm or 5 mm in the extreme case, generally people do not use like 10 mm pipe or 15 mm pipe there will be too much. So, we will stick to about 2 mm or 3 mm pipe thickness. So, even a small pipe thickness let us say 2 mm pipe thickness if you do that in the if we make such changes in the code. Let us see what we are getting exactly.

So, if we these now these tuned values these values are tuned based on zero thickness. Mind you again I am repeating these tuned values L underscore in that is a in length of the extension the inlet. So, geometric length is 138.27 mm only, but because of acoustic, but because of higher order mode effects the acoustic length is as if it is exactly L/ 2 and that is why this trough is getting first and second axial and third axial resonance peaks are getting nullified.



```

r_in = 0.5*(50/1000);          %%% inner radius of
tw   = 2.5/1000;            %%% wall-thickness of t
R_in = 0.5*(150/1000);          %%% chamber radius

L = 300/1000;            %%% overall chamber lengt

L_in =  138.27/1000;           %%% length of the
L_out = 63.27/1000;          %%% length of the ext
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);  %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati
```

But, this is this tuning values are only for zero thickness I repeat. Let us consider a nominal thickness 2.5 mm, everything else remains same. So, what essentially does is that you know this pipe the inner diameter anyways I have considered inner diameter only. So, what I mean is that this length is about or this radius is about 25 mm and this guy the inner radius of the chamber outer radius can be thickness that is not relevant. So, this is 75 mm.

So, the expansion ratio is D_0 / d_0 that is 75 / 25 3 and area ratio is $(D_0 / d_0)^2$ that is 9. But now focusing our attention on this thing and what is the effective what is the effective inner radius of the annular cavity? So, that is r underscore in the code plus tw or algebraically if you want to say it is this. So, the effective radius is 25 +2 27 mm ok.

So, if we make this thing in the code, let us see what we are; what we are getting ok. So, let me get to the code. Now we are in a position to run this code. So, 2 and a half mm the thickness and I have already done this thing let me hit the button run.

So, let us see the kind of miss tuning that we get just by actually considering the finite thickness 2 and a half mm thickness which is not all that great I mean that is the nominal thickness you would usually encounter in all engineering applications.

So, after this pre computing the modal integrals and the resonance frequencies and all that we start with the life computation of transmission loss. We should be done as usual in the next couple of minutes or. So, you know let us focus our attention all these things are ok, you know we should really focus our attention on what happens right at the first axial resonance frequency you will be slightly off and the second axial resonance frequency there will be even more noticeable deviation.

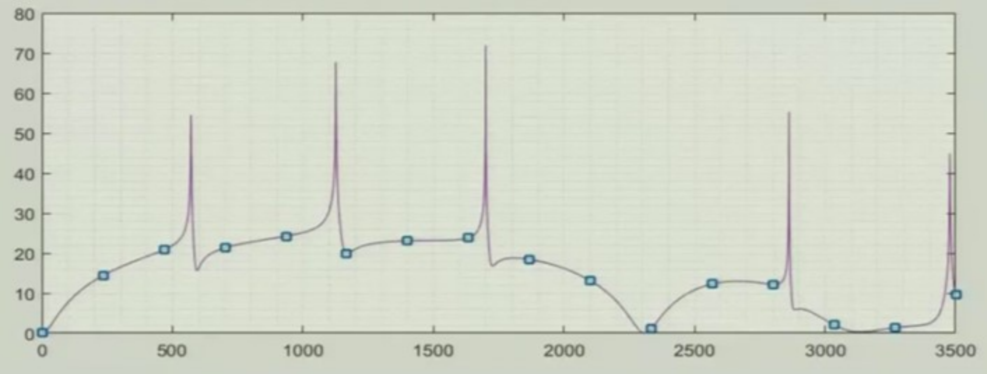
So, you do get a peak, but then you know we will compare the performance of this thing with the tuned double tuned expansion chamber that is the tuning at the extended inlet and outlet, but without considering the wall thickness the results that we saved just a while back. We will compare the performance of that configuration with the present curve in just a few minutes.

But we are going to discuss couple of things here, you know we are getting you see you know this was not the case here you were getting a sharp peak which was kind of very nicely annihilating the trough. Now it is also doing its job reasonably well, but looking at this trough and small looking at the speak I am sorry in the small trough you would kind of you are forced to think that this small amount of miss tuning alright.

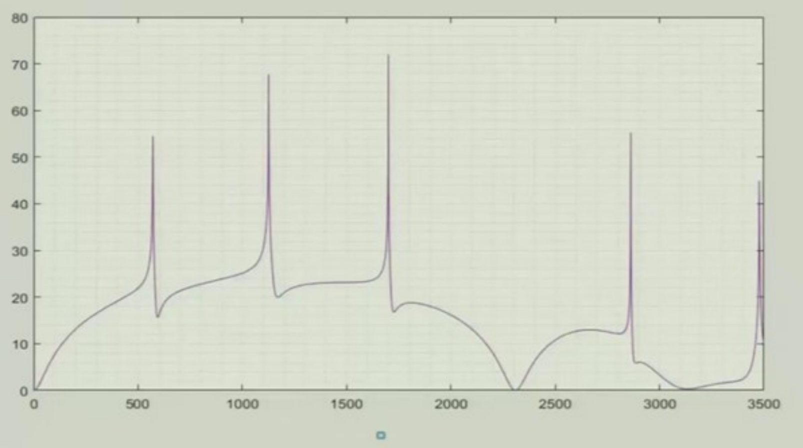
And so this trough should really been have been here. So, going to annihilate the trough here the peak should have been here and this is even more noticeable for the peak and the trough here, they should have been more towards the right hand side. You know and this is really because the thickness effect has not really been considered in the last case the tuning lens are obtained without due consideration of the finite thickness which is like I am saying which is very important in all practical applications.

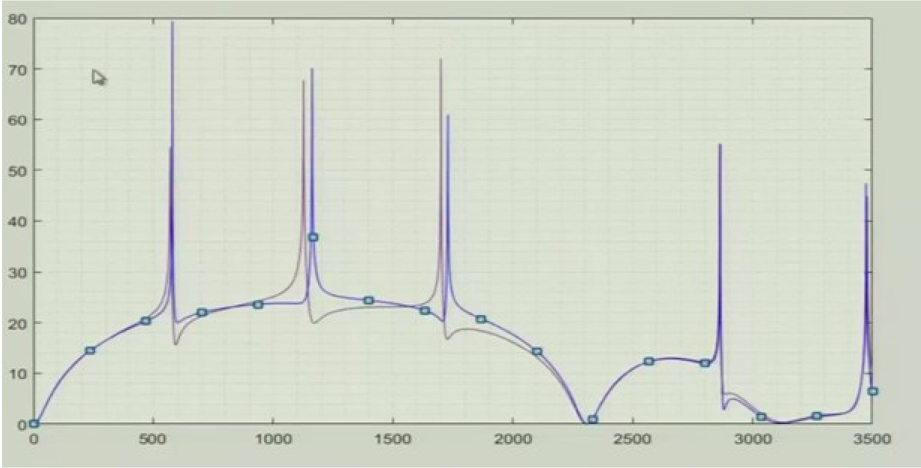
So, the end correction formula that was developed by Chaitanya and Munjal several years back they did a bunch of I mean a whole lot of parametric studies in commercial software and to evaluate the transmission loss performance of extended inlet and outlet muffler incorporating the wall thickness effect or the inlet and outlet pipe.

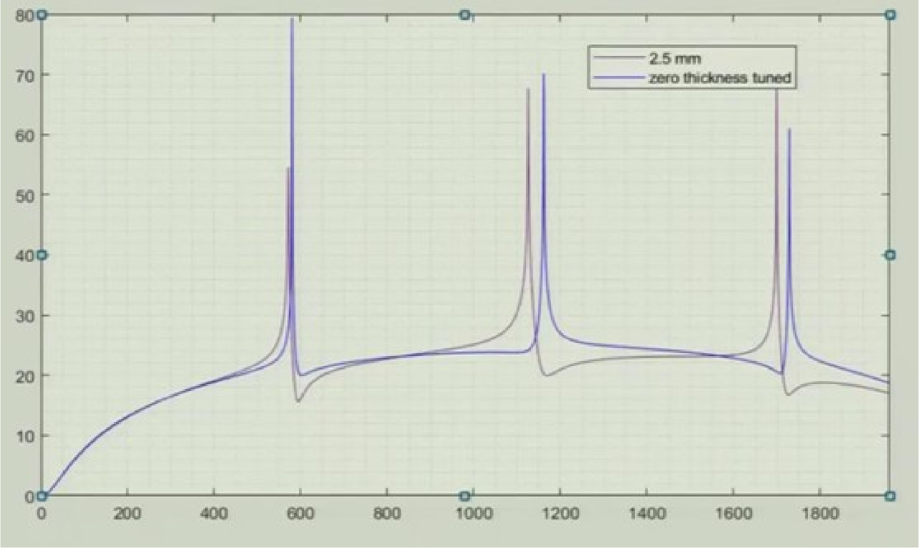
And they figured out that you know this is what is happens. And they developed an empirical relation which we can discuss just in a while we can I can just present to you that expression and now this curve is ready.



And I guess I should probably change the color because let me use another color perhaps this thing ok.



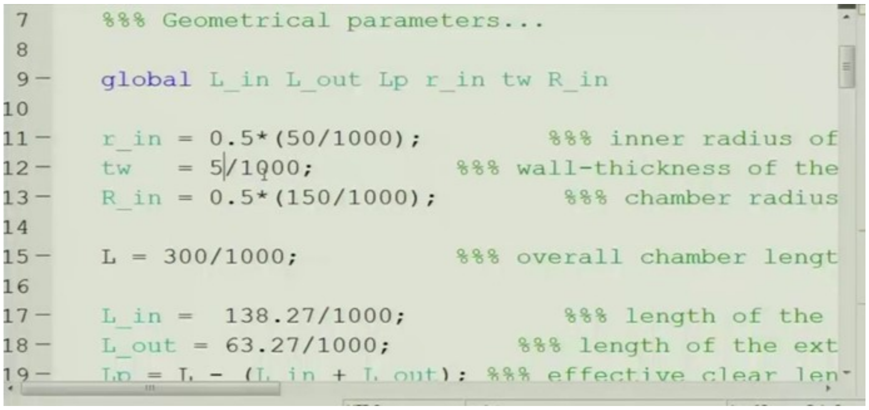




Well here it is. So, you see you know the blue colored curve really was much better in terms of tuning performance this trough was nearly cancelling that the peak was for nearly cancelling the trough, but this is more, but in the kind of a slightly miss tuned or the finite wall effect case this miss tuning is more kind of prominent and this is even more noticeable here. You know in the 0 wall thickness case this peak was kind of completely annihilating the trough, but now this trough is there.

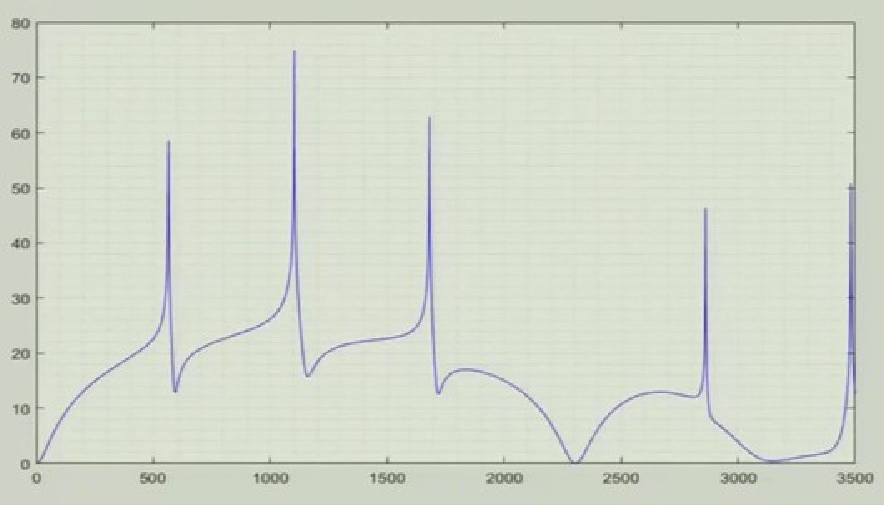
So, what I will do is that I will put a legend here I will put a legend and I will say this is also an opportunity for you guys to learn MATLAB 2.5 mm the wall thickness and the tuning was done for 0 wall thickness and let us see what effect are we getting. So, its probably saving the figure here you can zero thickness tuned ok and I will save this guy say untitled 2.

Now, let us get to even more extreme case, where we put this as 5 mm.



```
%%% Geometrical parameters...

global L_in L_out Lp r_in tw R_in

r_in = 0.5*(50/1000);            %%% inner radius of
tw   = 5/1000;             %%% wall-thickness of the
R_in = 0.5*(150/1000);            %%% chamber radius

L = 300/1000;              %%% overall chamber lengt

L_in =  138.27/1000;              %%% length of the
L_out = 63.27/1000;            %%% length of the ext
Lp = L - (L_in + L_out); %%% effective clear len
```

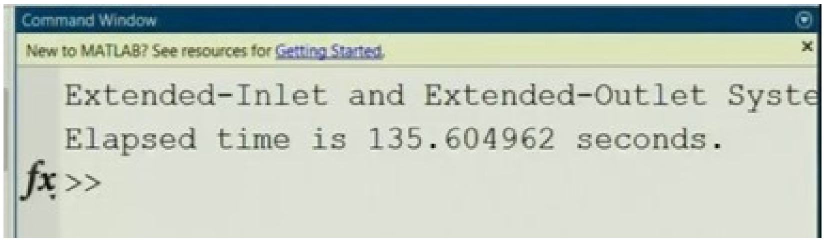
Which is really thick pipe such a pipe will not kind of vibrate such a big thickness pipe under flows.



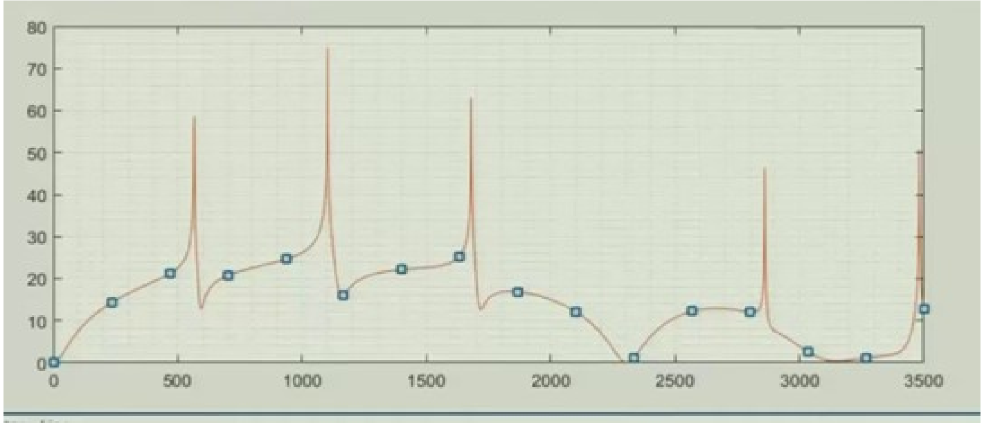
And let us see what we are getting. So, we should be soon be on with the computations. So, you see even more deviation that is pretty expected. So, all these things are ok, but what we really want to see is that, well you see the transmission loss has you know dropped down to about 12 12 and a half tb ok and you get a peak here.

The peak is not sort of able to annihilate the trough it seems there is a clear you can now you can clearly appreciate the missed tuning and its even more pronounced at the second axial resonance. So, this peak and the trough are sort of not quite tuned and they are not quite this thing although you know you would see just with a small tuning you could do better. So, what I will show you ways how to tune it in incorporating the wall thickness effect as well.

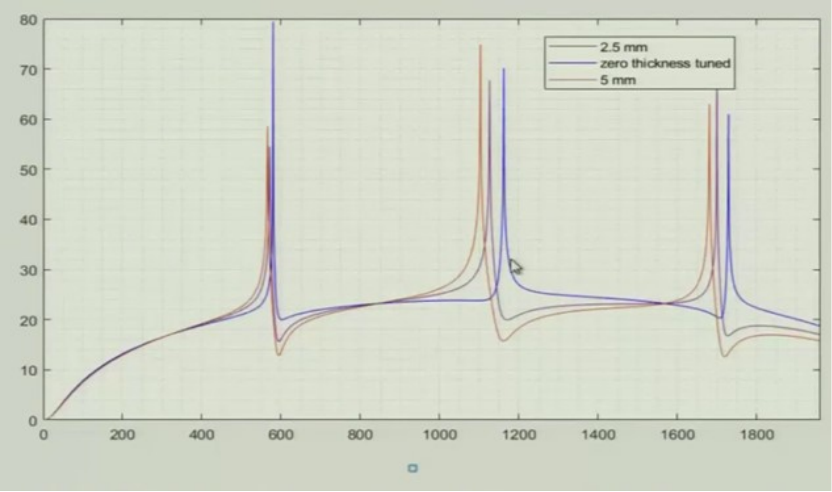
So, let the computations be completed and after that I am going to after that we really do not quite care. So, I am soon going to hit control c to stop the computation seems its not working its wants to do its full job. So, let it compute over the entire 3000 I guess 500 Hertz range or what was it? I guess it was 3500 yes alright does not matter ok.



```
Extended-Inlet and Extended-Outlet Syste
Elapsed time is 135.604962 seconds.
fx >>
```

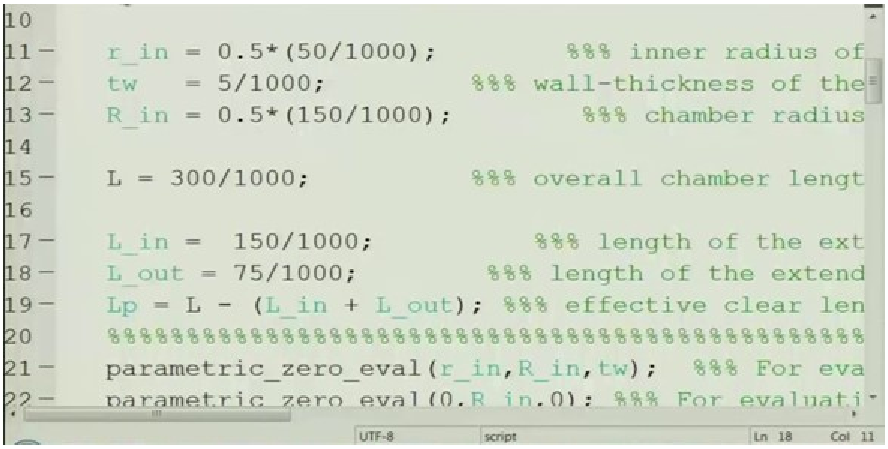


So, its done its job and what we are going to do, let me use some other nice color orange color ok.



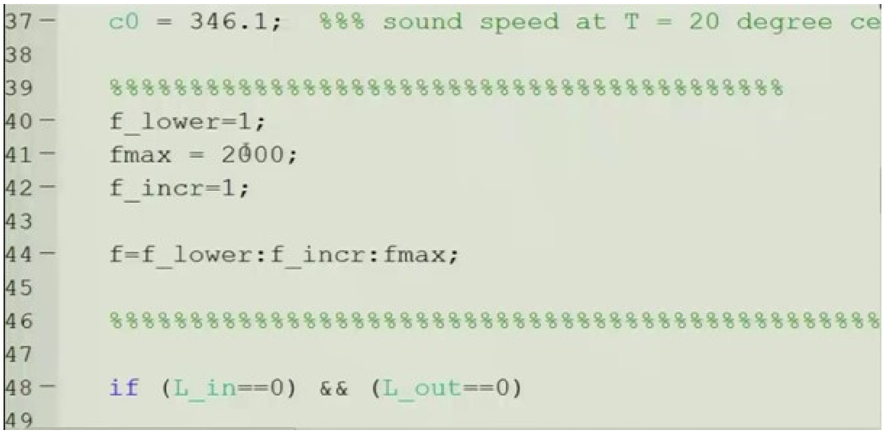
And 5 mm you see massive amount of let me save this figure first ok. You see clearly you know if you consider the tuning length based on 0 wall thickness and you know actually get the muffler manufactured or fabricated you will have problems. Because that probably not works specially if the wall thickness is as large as 5 mm which is still on the higher side I agree, but even something like 3 mm 4 mm you will have this small miss tuning problem.

So, what do we do now? So, when you have finite wall thickness. So, I will basically demonstrate a technique how to go about doing that; let us consider the finite wall thickness.



```
r_in = 0.5*(50/1000);         %%% inner radius of
tw   = 5/1000;           %%% wall-thickness of the
R_in = 0.5*(150/1000);         %%% chamber radius

L = 300/1000;            %%% overall chamber lengt

L_in =  150/1000;            %%% length of the ext
L_out = 75/1000;          %%% length of the extend
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);  %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati
```

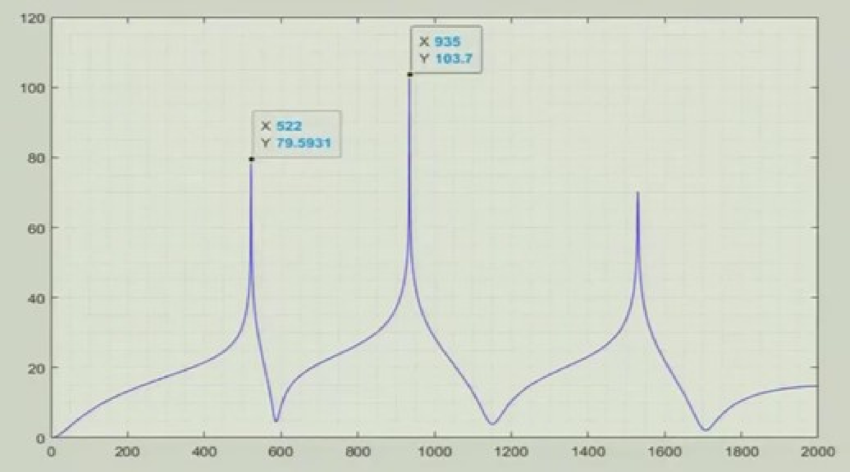
Let us say 5 mm only and L / 2 as usual we set it to L / 2. So, L / 2 is 150 and this is L / 4. So, this is about note that the correction here was 11.72 mm keep that in mind. But now in order to demonstrate the end correction we again get back this value 63.27. So, this will become 75 and instead of calculating over 3500 Hertz.



```
c0 = 346.1;  %%% sound speed at T = 20 degree ce

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f_lower=1;
fmax = 2000;
f_incr=1;

f=f_lower:f_incr:fmax;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (L_in==0) && (L_out==0)
```

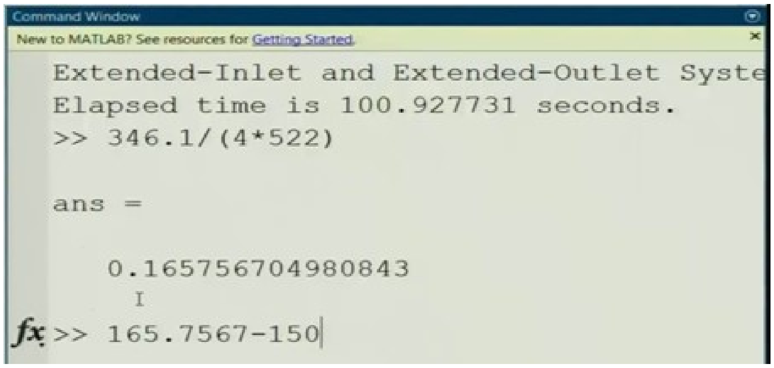
I would sort of stop at 2000 Hertz and ok rest all the things are same.

So, I will hit the button run and let it calculate and find out if the new tuned things let us see what happens.



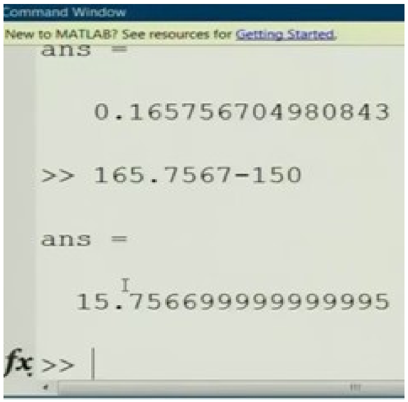
So, we are basically getting a peak at we getting a peak at something like 520 sort. So, we will soon figure out once this code is executed completely and let us wait for the computations to finish. So, you know we can as usual evaluate the effective length by noting by using the formula c_0 sound speed divided by 4 times the frequency at which the peak occurs.

So, that is the effective length and from that from the effective length we compute we subtract the axial geometric length to find out the net effect to find out the end correction length that is to be subtracted. So, that tuning can occur. So, it will be just be in a while you see lot of difference ok. So, this was the graph. So, as usual we will go to the frequency thing. So, this was our first 522 like I correctly kind of predicted down this 935 and well we do not worry about the other one.



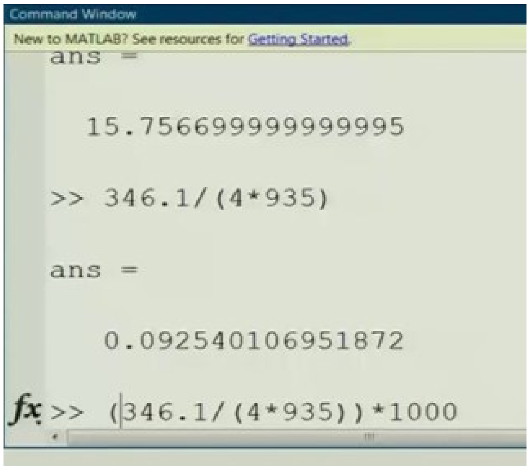
```
Extended-Inlet and Extended-Outlet Syste
Elapsed time is 100.927731 seconds.
>> 346.1/(4*522)

ans =

   0.165756704980843

>> 165.7567-150
```

So, 5 so 346.1 divided by 4 into 522; so that is what it was 165.75 is not it 165.7567 f minus 150 is the end correction length.



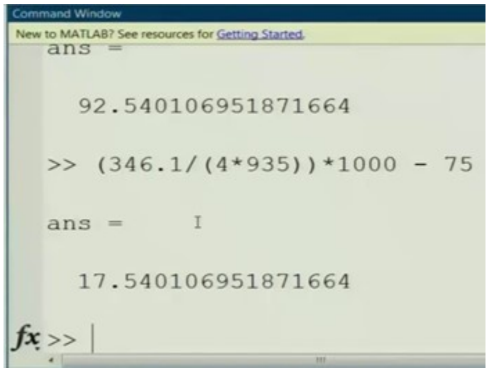
```
ans =

   0.165756704980843

>> 165.7567-150

ans =

  15.756699999999995

>> 
```

So, 15.75. So, there is definitely a difference between 15.75 and the other guy that was 11.72.



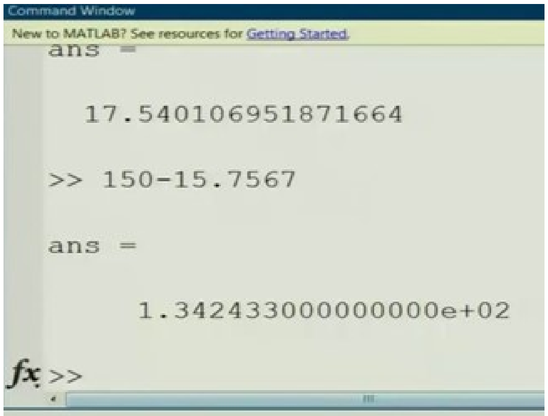
```
ans =

  15.756699999999995

>> 346.1/(4*935)

ans =

   0.092540106951872

>> (346.1/(4*935))*1000
```

What about the other one? Other one was 346.1 and 935 is not it. So, well.



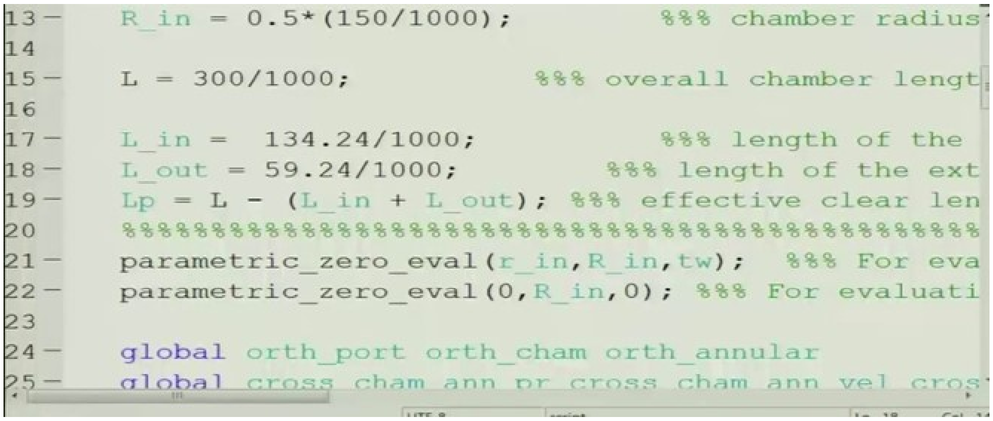
```
ans =

  92.540106951871664

>> (346.1/(4*935))*1000 - 75

ans =

  17.540106951871664

>> 
```

So, this one was 92.54 is not it, if you subtract 75 that 17.54 is the length. So, how about we just consider in the one at the extent inlet only you know 15.75.

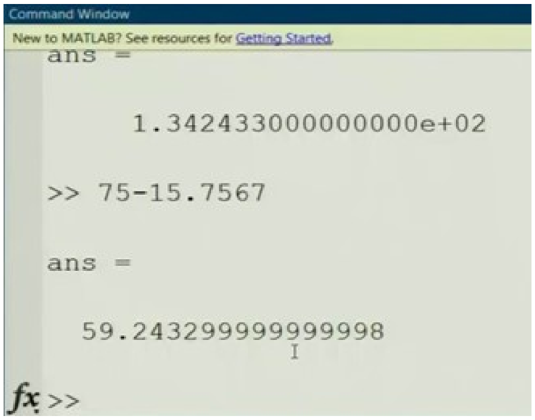


```
ans =

   17.540106951871664

>> 150-15.7567

ans =

     1.342433000000000e+02

fx >>
```

So, 150 -15.75; 150 - 15.7567 that is your 134.24.



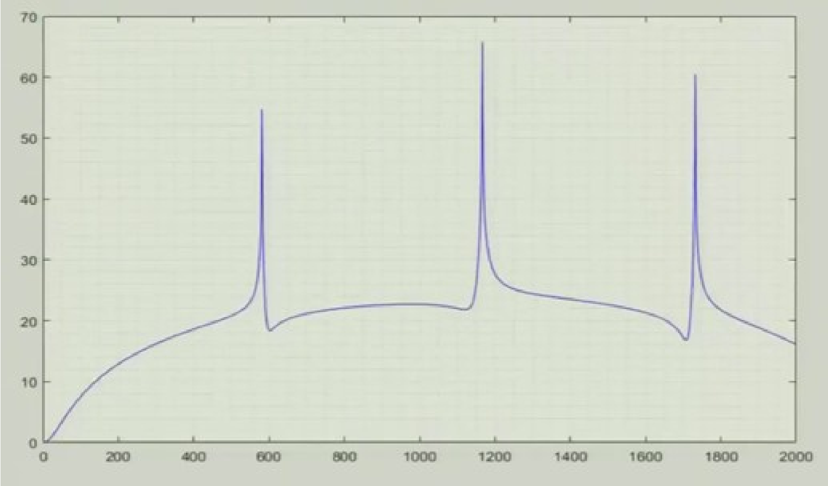
```
R_in = 0.5*(150/1000);                %%% chamber radius

L = 300/1000;                 %%% overall chamber lengt

L_in =  134.24/1000;                  %%% length of the
L_out = 59.24/1000;                %%% length of the ext
Lp = L - (L_in + L_out); %%% effective clear len
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
parametric_zero_eval(r_in,R_in,tw);   %%% For eva
parametric_zero_eval(0,R_in,0); %%% For evaluati

global orth_port orth_cham orth_annular
global cross_cham_ann_pr cross_cham_ann_vel cros
```



```
ans =

     1.342433000000000e+02

>> 75-15.7567

ans =

   59.243299999999998

fx >>
```

And the other thing was 75 -15.7567 that is 59.2432 59. So, let us see what are we able to get let us save this guy ok.

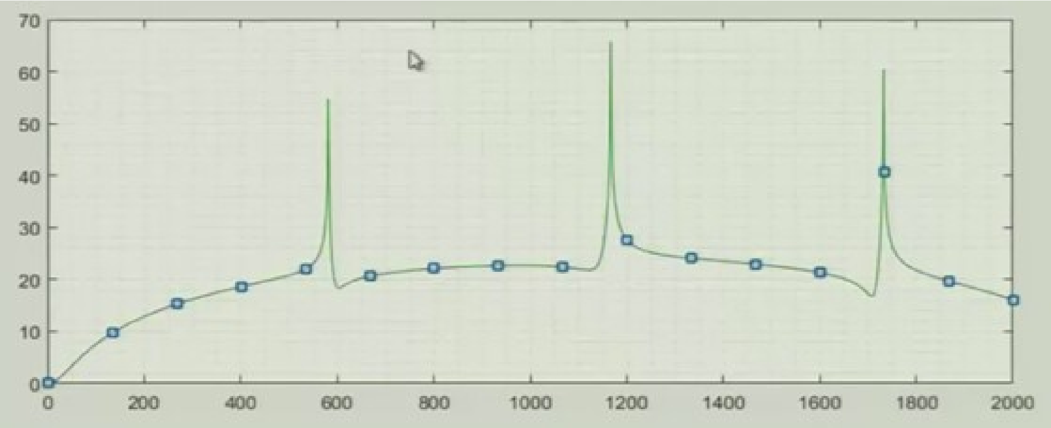
Let us see if we are able to tune it. So, clearly you know one thing that we immediately figured out that 15.75 mm and 11.72. So, roughly there is 4 mm difference you know in case of maximum or the extreme thickness that you usually encounter in engineering applications.



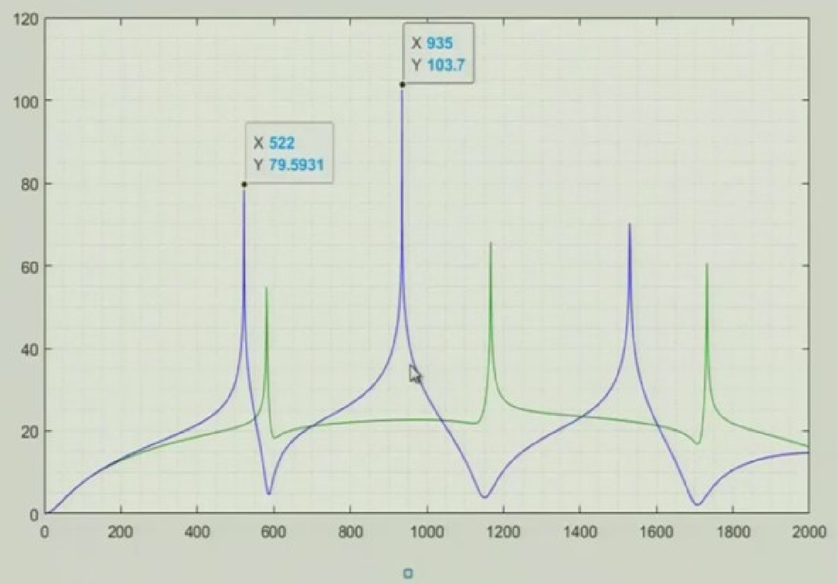
Especially here if you have more expansion ratio that is 5 or 6 you know what I mean is that the inlet diameter can be 50 mm. Outlet the chamber you know in the port diameter can be 50 mm, but the chamber diameter can be 20 250 mm or 220 mm or something like that.

So, the expansion ratio is 4, 5, 6 or something like that. So, then you will have even more effect of wall thickness although I am not going to show you those simulations because of lack of time. Let us just focus on this thing and a couple of things to discuss before we end today’s lecture. So, we get you know this is doing at least much better although you would sort of imagine you would think that this peak should have cancelled this guy. So, you are getting a peak here.

And now this trough is this peak is completely annihilating the trough here. And so and if you consider an average of 15.75 17 point whatever you are getting something like 16 is h you know you could do sort of even better. You could probably this can be you can actually take an average of this thing so not a problem. But this performance would be better than just L /2 and that sort of thing that much I can tell you. So, this is computation is done.

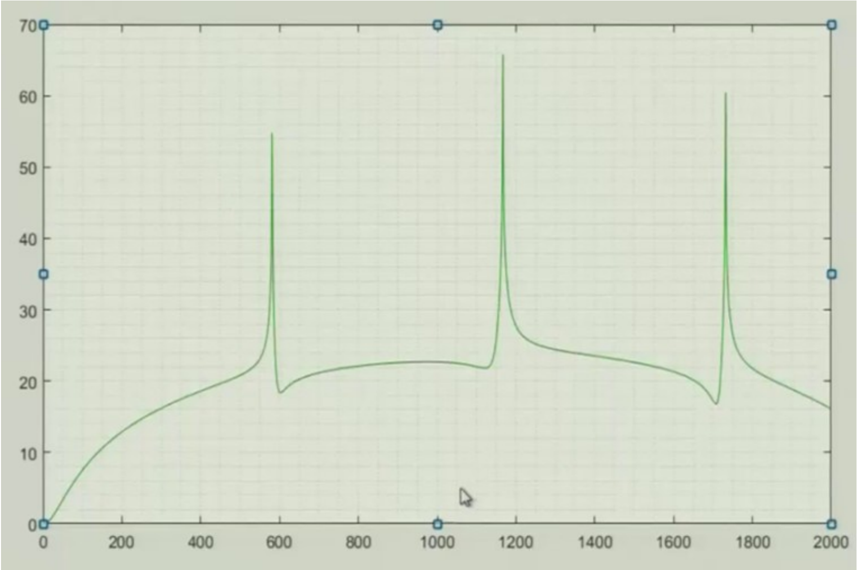


And this let me use another color green color perhaps and this is something like 5 mm. So, let me just superpose the curve here.

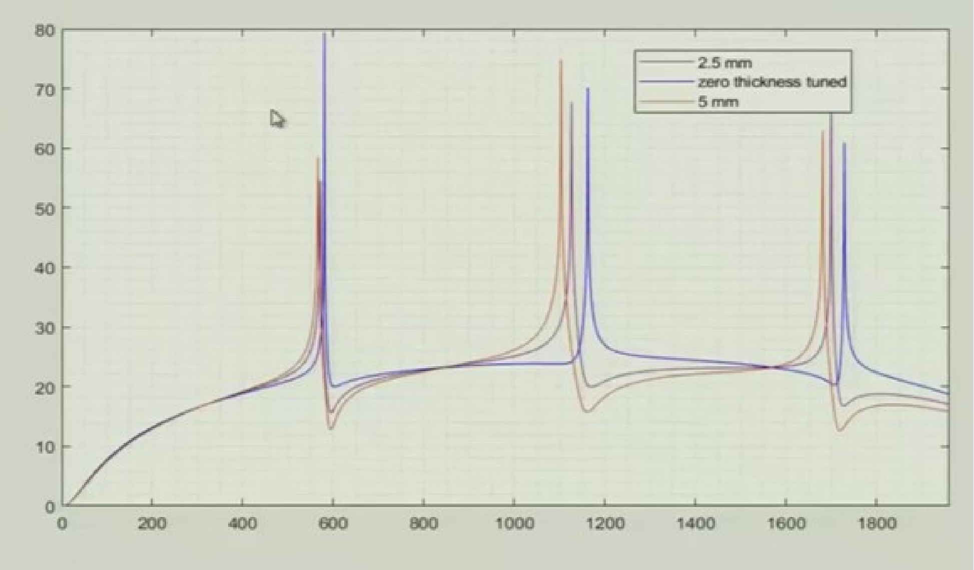


You see you know what a dramatic do not worry about these peaks, these peaks are not of much value. You know the amount of attenuation produced at the peak is not of significant importance.

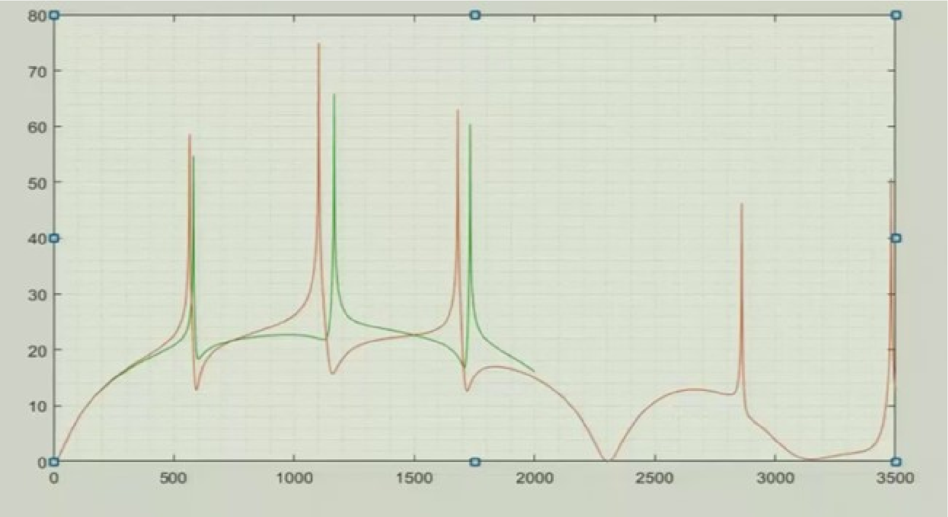
The main important thing is that it should have a broadband attenuation performance there you should minimize or avoid the troughs as much as possible. So, clearly the green colored cover the tune thing is much better than the blue colored one or the non tuned one.



Now, if we just on the curve you know.



Superpose the other curve you know 5 mm curve I guess, that was the curve with the tuning based on the zero thickness thing.

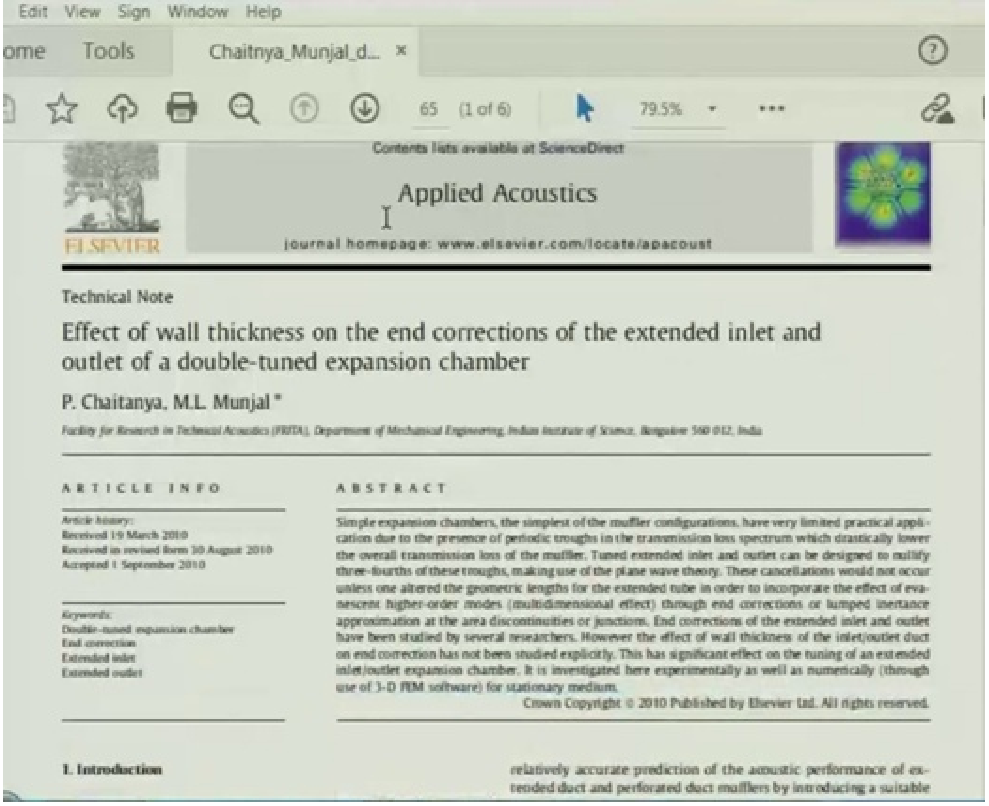


You see a difference you know let me just reduce the frequency range.

So, you see a massive difference. So, this is this green colored curve is actually the tuned one and this orangish curve is sort of missed tuned, because basically what this curve what this results demonstrates is that when you considering actual applications finite wall thickness or the extended inlet and outlet muffler should be considered to get you the actual tuned performance, when you actually fabricate the muffler and do the experimental testing whether it is able to get a broadband attenuation or not.

You know it may so happen that you know you are getting a very dominant tone of the engine noise somewhere here, but then because of the peak occurring right here and you are tuned the performance its able to cancel that and its very effective and this effectiveness will be reduced if you do not consider this. So, these are some of the important effects that I thought of highlighting here.

So, I will close off all these things. And what we could do is that you know actually Chaitanya and Munjal in their applied acoustics paper let me see if I have; if I have this paper readily available.



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Technical Note

Effect of wall thickness on the end corrections of the extended inlet and outlet of a double-tuned expansion chamber

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ABSTRACT

Simple expansion chambers, the simplest of the muffler configurations, have very limited practical application due to the presence of periodic troughs in the transmission loss spectrum which drastically lower the overall transmission loss of the muffler. Tuned extended inlet and outlet can be designed to nullify three-fourths of these troughs, making use of the plane wave theory. These cancellations would not occur unless one altered the geometric lengths for the extended tube in order to incorporate the effect of evanescent higher-order modes (multidimensional effect) through end corrections or lumped inertance approximation at the area discontinuities or junctions. End corrections of the extended inlet and outlet have been studied by several researchers. However the effect of wall thickness of the inlet/outlet duct on end correction has not been studied explicitly. This has significant effect on the tuning of an extended inlet/outlet expansion chamber. It is investigated here experimentally as well as numerically (through use of 3-D FEM software) for stationary medium.

1. Introduction

relatively accurate prediction of the acoustic performance of extended duct and perforated duct mufflers by introducing a suitable

So, this was the paper that I was sort of referring to the famous paper published in applied acoustics.

So, this is what they have done they found out the end correction length by finding out the effective you know effective length and subtracting the effective length from the geometric length to find out the end correction and subtracting that from the L / 2 length to get the fabricated length.

And so the effective length acoustic length will be L/ 2 in that case. So, this is the configuration that we are talking about finite wall thickness was considered and they eventually ended up with the empirical relation using which we can find out.

You know a relation between the small d is a diameter of the inlet and outlet pipe which are assumed to be equal diameters assumed to be equal. Capital D is a diameter of the chamber ok and t_w is the thickness of the pipe.

$$\frac{d_0}{d} = \alpha_0 + \alpha_1 \left(\frac{D}{d}\right) + \alpha_2 \left(\frac{t_w}{d}\right) + \alpha_3 \left(\frac{D}{d}\right)^2 + \alpha_4 \left(\frac{D}{d}\frac{t_w}{d}\right) + \alpha_5 \left(\frac{t_w}{d}\right)^2$$

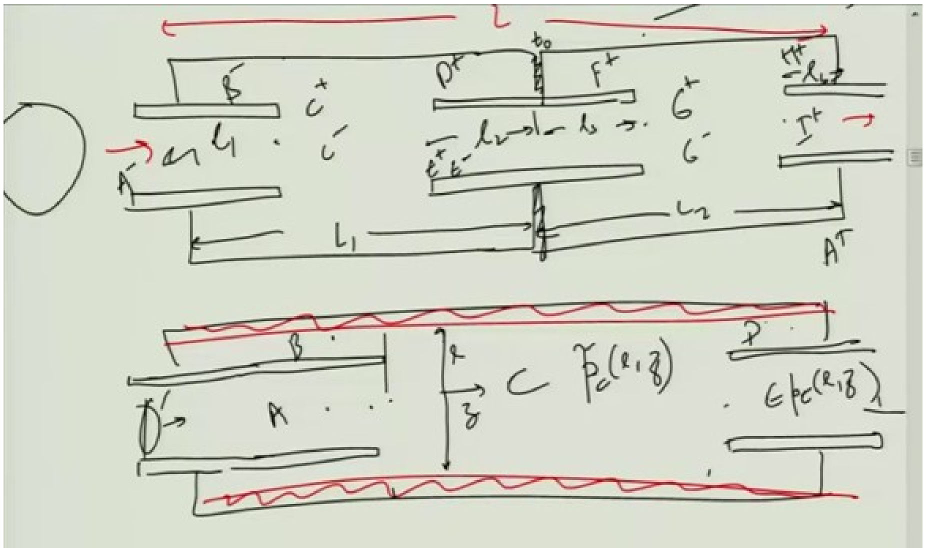
So, they developed a you know relation found using finite element simulations the relation between non dimensional and end connections that is delta a divided by d in terms of where the variables are the expansion ratio d capital D / d and the non dimensional thickness t_w/d diameter of the pipe as a quadratic by quadratic function or something like that.

And so all you need to do is that these constants are there. So, the you know this expression can also be evaluated using our analytical mode matching code or using finite element simulations or perhaps using experiments, when if you can sort of get those many muffler models fabricated.

So, people rely on simulations to find out these empiric relationships and within this within the you know within certain parameter range for example, expansion ratio varies from you know 3 to maybe 2, 2 to 6 or 7 perhaps between. And the diameter can for the pipe can typically vary from you know 30 mm to 60 mm range of this thing.

We can sort of get you know we can get a ray and wall thickness is from you know 1 mm to about 5 mm or whatever it is you can also set 0 mm. You know we can get a very good estimate of the end corrections that is to be applied at the inlet and outlet and using that we can tune it and get a muffler model fabricated or if you have certain commercial mufflers which are in which these ratios are within these limits you can very well corroborate the numerical finding or simulation findings with the experimental things ok.

And then we can easily get the end correction expression and we can I mean sort of we can easily figure out whether the end correction is working or not mostly it will the apart from a small deviations here and there. So, this is what the formula is and you know later on what we can also do not in this course maybe in advance courses.



Advanced course on muffler acoustics where you have this numerical and analytical techniques. We you know or may be this something like a self study, the analytical thing can also be you know used to analyze dual chamber muffler you know suppose if we have a limited length L and within that we need to attenuate maximum noise. So, by incorporating maximum reflection of sound back into the system.

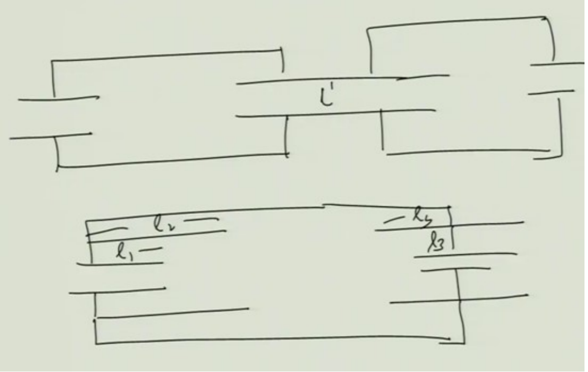
So, what we can do? Let me just sketch this guy for you know I am considering all finite wall thickness ok. So, you have this thing and all that. So, what we could do is you know the flow goes here it comes out and this is the length L original chamber was something like it was something like this, you know consider something like this.

So, you know the idea is that divide this length into L_1 part and L_2 part and again we consider you know just like we have done the mode matching between this section here, here and here and found out this big matrix.

Similarly such a thing can also be applied for this dual chamber muffler and you know by playing around with the length L_1 this will be you know small L_1, this will be small L_2 from here to here. Then it will let it have finite wall thickness you know t_0 and this is l_3 this is l_4 ok all this is there.

And so all these things are there. Now we when we have this thing we can do mode matching between this section and this section this and this and this and this and you know get the matrix to finally, find out all the coefficient like the reflected wave coefficients A - B - or something like that C^+ C^-.

Now, here you will have C^+ E^+ there will be reflections back. So, you cannot assume an anechoic termination a E^+ E^- F^+ G^+ G^- H and I. So, here you can assume an anechoic termination. And then finally, find out all these coefficient from $A^-{}_2$ I + in terms of A^+ and a matrix inversion of such a system usually you know one has to be careful because sometimes it is a problematic thing when you incorporate large number of modal terms.



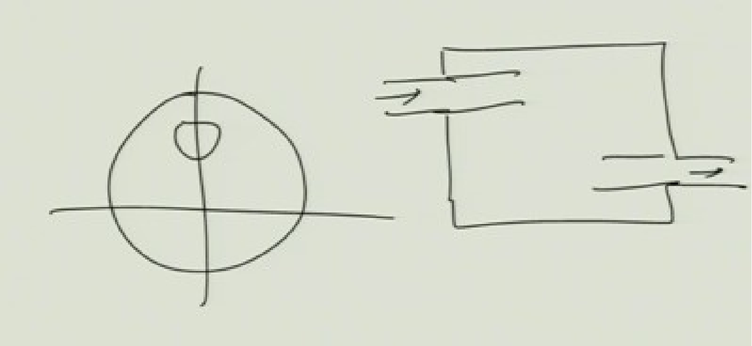
And you know just by this configuration can very well be considered you know equivalent to or you know just by once you have written the code for such a configuration using analytical mode matching it can very well be used to analyze such kind of a configuration ok and you know by.

So, this you know just you just have to instead of t_0 which is the wall thickness you need to consider which is about 2 mm, 3 mm wall thickness, you need to consider a much larger value whatever the length is replace t naught with say L' and you will the same code can be very well used to analyze this muffler configuration.

So, you know and so these analytical mode matching can do can help you to analyze a lot of things especially when you have you know one other thing that you can do is possibly have some sort of a lining on both side of the circular duct and this these are hybrid muffler configurations.

So, these linings can really kill the high frequency noise the moment from which the muffler starts behaving showing very exhibiting very peaky behavior number of troughs and domes beyond the cut on frequency of 0, 1 mode you can actually you know those beyond that the modes can really be controlled by you know incorporating this lining and that can really help you. So, the higher frequencies can be completely kind of killed off.

And other thing that you can also do using this mode matching is a folded resonator concept ok. So, that is something like this. So, there is a resonator *l*1 here *l*2 here the length is like this *l*3, *l*4. So, you know one can do a lots of things with this analytical mode matching thing provide that it is concentric circular.



If it is not concentric circular if you have an eccentric annular something like you know this kind of a thing then unfortunately this Bessel, then this can actually be used to analyze a simple expansion chamber that kind of a thing you know things like this it can be very well used to analyze such configurations, but the more as has been done in Selamets paper published several years back.

But then moment you have extensions here that probably would not be possible at least analytically numerically yes using numerical mode matching. So, all this will be a part of a higher I mean the next advanced level course on numerical and experimental techniques in muffler acoustics at a later stage and we will begin with this analytical advanced techniques of mode matching and number of advanced topics as well.

So, this is what it is and other thing finally, before we end this lecture we would have noticed one thing you know let me just get back to the; let me just get back to the MATLAB thing where we find out this modal coefficient is not it, we find out this modal coefficients when we get this.

So, use modal coefficients can be plotted you know this what I am trying to say is that if you consider in the cross section of the muffler what we can do really is that plot the pressure field in this region you know this is the this was the region A, B, C, D, E.

So, we can once we know the model coefficients we know the exact solution. So, over r and z plane theta is; obviously, theta there is no theta dependence is axis symmetric or radial problem.

So, using this 2D representation we can get the complete we can basically plot the p r p tilde c r z pressure field the you know we can get the rms field plotted over this distance and then rms field or this thing plotted here, similarly for this region, this region, this region for at a given frequency and then we can do a frequency swipe and to see basically how these pressures how this pressure contrast are behaving with time.

So, you will see whenever the peaks happen very less energy will be transmitted downstream and more energy will be transmitted and all the energy will be reflected back and at places where you will get troughs almost whatever power you are getting here acoustic you know the pressure contrast same contrast will be here; that means, the muffler is nearly transparent here at that frequency not much is happening.

So, all these things can give you more insight into muffler design just by just when you look at the pressure contrast ok. So, with this discussion what I will do is that I will end the today’s lecture and we will meet very briefly, we will meet very briefly for lecture 59, where I just discuss some of the dissipative muffler things although this will be covered in the later course like I said just you know once you how does the how do you evaluate the transmission loss of a muffler using an engineering approach you are given standard non dimensional curves in using that you find out certain values.

And also how does the transmission loss of fully lined muffler thing how does it look like and so on ok. So, we will do all those things very shortly.

Thanks.