Muffler Acoustics - Application to Automotive Exhaust Noise Control
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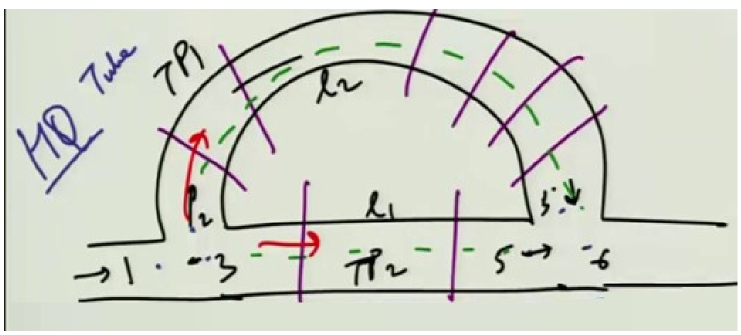
Lecture - 47
TL Analysis of HQ Tubes (MATLAB): Network Analysis and Analytical Formula

Welcome to week 10, lecture 2 of this NPTL course on Muffler Acoustics. The last lecture of this week. We just you know sort of set up the equations required to analyze a network connection, that is connection in which waves can travel through different paths that is the waves can travel through say for example, this path and this path. We set up the different equations and kind of figured out that simple cascading of transfer matrices cannot be really used.

One has to resort to write down the different equations that characterize each element and the junction laws itself.

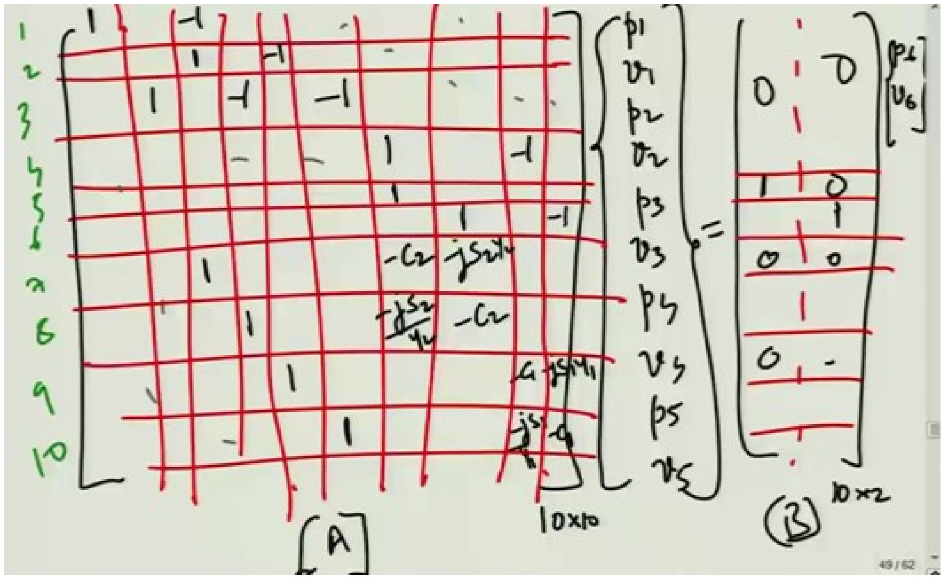


$$\tilde{p}_1 = \tilde{p}_2 = \tilde{p}_3 \qquad \tilde{V}_1 = \tilde{V}_2 = \tilde{V}_3 \tag{1}$$

$$\tilde{p}_4 = \tilde{p}_5 = \tilde{p}_6 \qquad \tilde{V}_4 = \tilde{V}_5 = \tilde{V}_6 \tag{2}$$

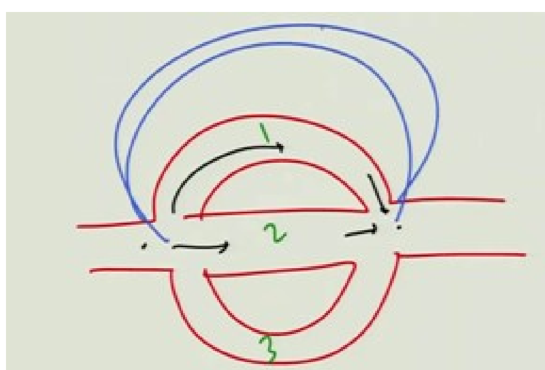
$$\begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix} = \begin{bmatrix} C_2 & jY_2S_2 \\ \dfrac{jS_2}{Y_2} & C_2 \end{bmatrix} \begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \tilde{p}_3 \\ \tilde{V}_3 \end{Bmatrix} = \begin{bmatrix} C_1 & jY_1S_1 \\ \dfrac{jS_1}{Y_1} & C_1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} \tag{4}$$



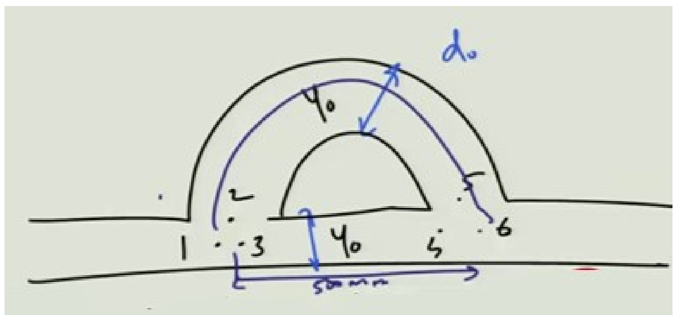
And, then assemble all the things into one big matrix like the one that we are seeing here and, this is represented in terms of the variable that is pertaining to the downstream ones, ok. So, this matrix needs to be inverted and then multiplied, alright.

So, what we are going to do in this lecture right now is basically to do to two things. First of all for such simple elements like the one that I have considered this one, you know we have this kind of a thing. So, between this point and this point what we figured out was that although it is complicated we can sort of develop an analytical formula at least for the two duct system.



Then there are bunch of papers, which talk about modified Herschel Quincke tube in which cases you have things like even more tubes attached. For example, things like this one; things like this one. So, there are basically like 1, 2 and 3 pipes or they could be more, they could be things like this one you know. So, they could be n number of such pipes each with different lengths and the constructive or destructive interference causes resonances.

So, let us actually get back to our simple configuration, that two duct configuration like the one that was presented here. So, what we can do is basically kind of develop an analytical formula for relating the upstream variable one with the ones that is presented a one that is mentioned here that is 6. So, let us see how we kind of do it. We again have the junction laws, but what we could possibly do is that start you know let me draw this guy again.



$$\tilde{p}_1 = \tilde{p}_2 = \tilde{p}_3 \qquad (1)$$

$$\tilde{V}_1 = \tilde{V}_2 + \tilde{V}_3 \qquad (2)$$

$$\tilde{p}_4 = \tilde{p}_5 = \tilde{p}_6 \qquad (3)$$

$$\tilde{V}_4 = \tilde{V}_5 + \tilde{V}_6 \qquad (4)$$

So, we have this kind of a thing and we have this kind of thing. So, of course, we assume that the pressure is uniform at this point. So, let us name this is 1, 2, 3, this is 5, 4, and 6 ok. So, we have this kind of a thing, that we already saw in the last class.

So, what we do basically is that we have also the other representation that is the transfer matrix representation, where

$$\begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix} = \begin{bmatrix} C_2 & jY_2S_2 \\ \dfrac{jS_2}{Y_0} & C_2 \end{bmatrix} \begin{Bmatrix} \tilde{p}_5 \\ \tilde{V}_5 \end{Bmatrix} \qquad (5)$$

So, this is I would just put it at C_2 that is basically, C I will tell you what it means. And another thing that I kind of forgot to mention. This is Y_0, this is also Y_0 meaning that cross-section area is the same for both the things. So, this is you know let us assume the diameter is d naught, ok.

$$\begin{Bmatrix} \tilde{p}_3 \\ \tilde{V}_3 \end{Bmatrix} = \begin{bmatrix} C_1 & jY_0S_1 \\ \frac{jS_1}{Y_0} & C_1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_4 \\ \tilde{V}_4 \end{Bmatrix} \qquad (6)$$

Now, one needs to combine all these things to ultimately get p_1 in terms of p_6 v_6 as we have already discussed. So, how do we go about it?

First thing with right now our focus to quickly develop analytical formula using some of these expressions and possibly going to Maple; Maplesoft and then try to write down a clean expression for the transfer matrix between the upstream and downstream variables. And, then get some you know basically figure out where the resonances or the troughs would occur based on the constructive or destructive interferences respectively and validate that using network analysis.

All this things we have done in the computational part will be done in MATLAB after we have developed the analytical formula by aid of Maple. So, now let us go to the let us see this is a bunch of equation (1, 1, 2, 3, 4, 5) and (6). Now, first let us see v1 is equal to $v_2 + v_3$. So, from this thing we can get v_2 in terms of p_5 v_5 and v_3 in terms of p_4 v_4. So, we can straightaway

$$V_1 = \frac{jS_2}{Y_0}\, p_5 + C_2V_5 + \frac{jS_1}{Y_0} p_5 + C_1V_4$$

So, we have this sort of thing. Now, we again invoke the relation here equation 3 where you know p_4, p_5 and p_6 they are all same. So, what we can do is that, we can just

$$V_1 = \frac{j}{Y_0}\,(S_2 + S_1)p_6 + C_2V_5 + C_1V_4$$

Where, C_1 C_2 again I am reminding you they are not constants they are your cos $k_0\, l_2$ and cos $k_0\, l_1$ respectively. Now, we have this kind of thing I am making it a box dashed box expression, because this is not the final one ok.

So, now another thing that we can also notice from this set of equation is that we can express this guy v_5 in terms of v_2 and p_5 or p_6 or rather yeah p_2, which is basically p_1 and p5 which is basically p_6. So, eventually v_5 that is the velocity mass velocity that comes in this pipe can be expressed in terms of the pressure here and pressure downstream. So, that is what we want. So, this can also be written rather cleanly.

$$\frac{p_1 - C_2 p_6}{jY_0 S_2} = V_5$$

$$\frac{p_1 - C_2 p_6}{jY_0 S_2} = V_4$$

These two guys can straightaway be substituted in this equation. So, it is going to be a little algebraically you know a bit messy, but that is the way the things are with your network a system.

$$V_1 = \frac{j}{Y_0}(S_1 + S_2)p_6$$

$$+ C_2\left(\frac{p_1 - C_2 p_6}{jY_0 S_2}\right)$$

$$+ C_1\left(\frac{p_1 - C_1 p_6}{jY_0 S_1}\right)$$

$$=> \quad V_1 = \frac{j}{Y_0}(S_1 + S_2) - \frac{C_2^2}{jY_0 S_2}$$

So, we get this. So, we can sort of again tend to simplify these guys. So, what will we get? We will get;

$$=> \ V_1 = \left\{\frac{j}{Y_0}(S_1 + S_2) + j\frac{C_2^2}{Y_0 S_2} + j\frac{C_2^2}{S_1 Y_0}\right\} p_6$$

$$+ \frac{j}{Y_0}\left(\frac{C_2}{S_2} + \frac{C_1}{S_1}\right) p_1$$

So, note that this is a minus. So, this will can be written as plus j square. So, this will be Now, plus you have. So, we are going to get these expressions these can be of course, we further kind of a simplified and what we can do possibly is that we can tend to write these guys as, and this is equal to the big expression here. I am not going to bother writing this thing again, but perhaps just mentioned this thing here that this is jY_0 into your jY_0 can be taken common.

$$V_1 - \frac{1}{jY_0}\left(\frac{C_2}{S_2} + \frac{C_1}{S_1}\right) p_1$$

$$= \frac{j}{Y_0}\ \{S_1 + S_2 +\ (-)\}\ p_6$$

And, $S_1 + S_2$ and like the other guys and this is equal this is multiplied by p_6. We have this kind of a thing expression and then you need to invoke another such identity for you know doing the analytical derivation. So, what do we do? What is the other identity that we have? The this is of course, one of it what is the other one?

$$V_6 = V_4 + V_5$$

$$= \frac{p_1 + C_1 p_6}{jY_0 S_1} + \frac{p_1 - C_2 p_6}{jY_0 S_2}$$

Now, v_4, v_5 we have already kind of figured out from the slides here. So, this is basically when we invoke such a thing we are using really your, the expressions here, right now we use the things here now we are using the things here to get another equation ok. Remember we in in the, this equation we have three variables v_1 p_1 and p_6. We need another equation so that we can have something like A matrix times p_1 v_1 is equal to B matrix times p_6 v_6 and then we can invert it. All these going to be quite messy we want to go to Maple and get our stuff done.

So, once we substitute these guys in here, this equation and this equation what are we going to get? We are going to get you know after a bit of a simplification well let me just write it down.

$$V_6 - \frac{1}{jY_0}\left(\frac{1}{S_1} + \frac{1}{S_2}\right) p_1$$

o, we have this. So, what we could do is basically you know get these guys in here

$$-\frac{p_6}{jY_0}\left\{\frac{C_1 S_2 + S_1 C_2}{S_1 S_2}\right\}$$

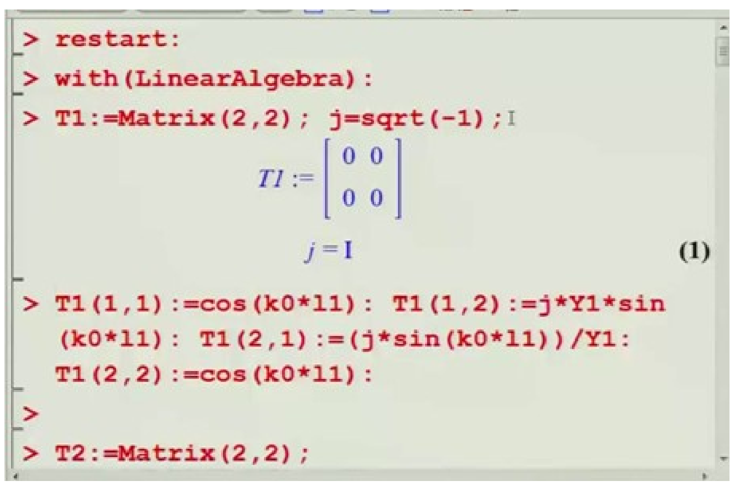
So, this will become your sin of

$$V_6 + \frac{p_6}{jY_0}\left\{\frac{sink_0\,(l_1 + l_2)}{S_1 S_2}\right\} = \frac{1}{jY_0}\ \left(\frac{1}{S_1} + \frac{1}{S_2}\right) p_1 \qquad **$$

So, you have another such relationship, ok. So, we have got this star you got this double star. So, we can obviously, write it in the form something like I was mentioning A matrix A sum matrix and

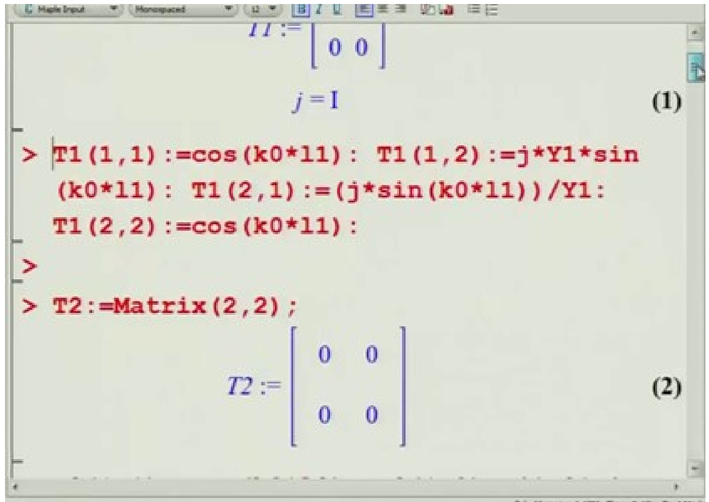
$$[A]\begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix} = [B]\begin{Bmatrix} p_6 \\ V_6 \end{Bmatrix}$$

So, you know based on our convenience we can do either do A inverse B, we can get the total or overall transfer matrix from upstream to downstream or we can if you want to express it relate downstream variables in terms of upstream variables. So, we can do B inverse A that is the inverse of the transfer matrix. So, what I suggest is that we can quickly go to Maple and because this is going to be algebraically a bit tedious, so, what we can do is that we can go to Maple and figure out things.



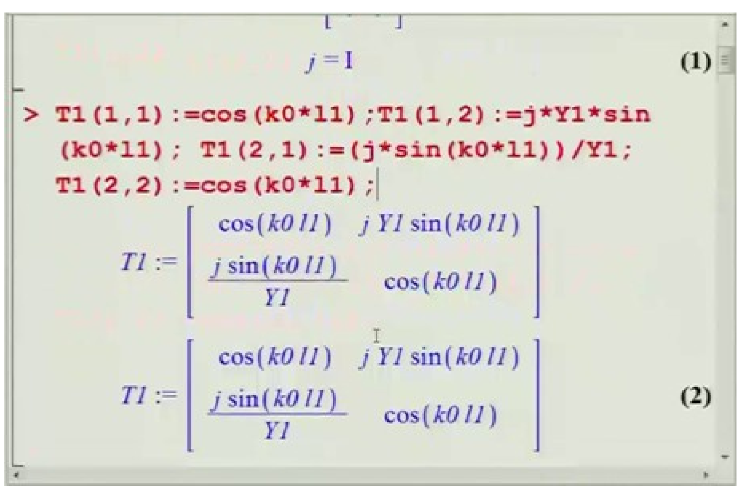
So, in we have opened Maple and in Maple there are some codes that have already typed. You know we did testing in some time in week 5 where we are doing some conical ducts and stuff like that and also in the weeks before week 5 likes probably week 4 or something like that when we did extended inlet and outlet mufflers for the first time.

So, we do restart as usual and we do linear with Linear Algebra and we kind of introduce these things. So, T1 is a matrix, we will say 2 cross 2 matrix and j is your root over minus 1.

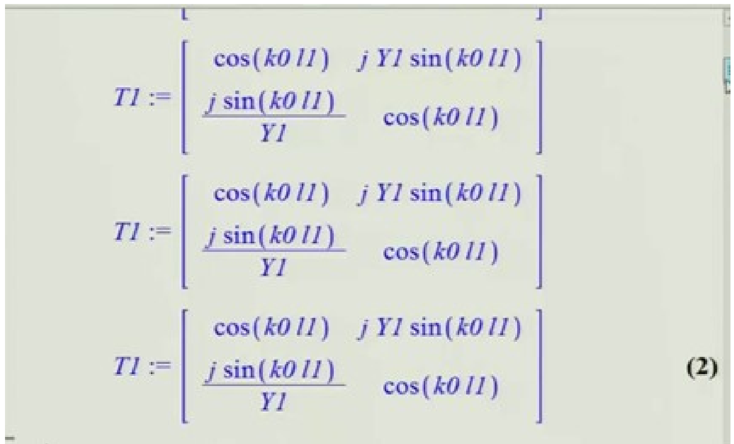


And, what we need to do is that if we go to our presentation, we will realize that this is the j Y naught and 1 by S 1 plus S 2. So, this can be written as S1 + S2 and another equation that we have is basically your this guy. So, here will be minus 1 by j this thing. So, in the numerator you will have your sin of k naught into l 1 plus l 2 and in the denominator you have S1 and S2.

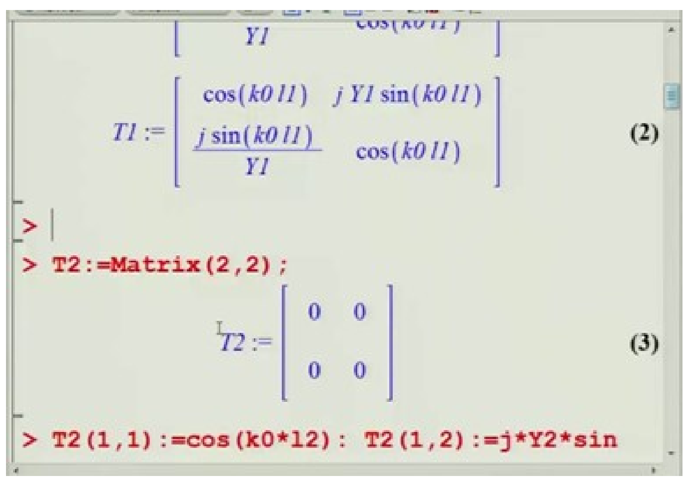
And, so, this will be, there will be a minus sin associated here and this will be 1 here and similar things exist here. So, basically after I am kind of avoiding the going through each and every step, but I am just trying to present to you the entire thing.



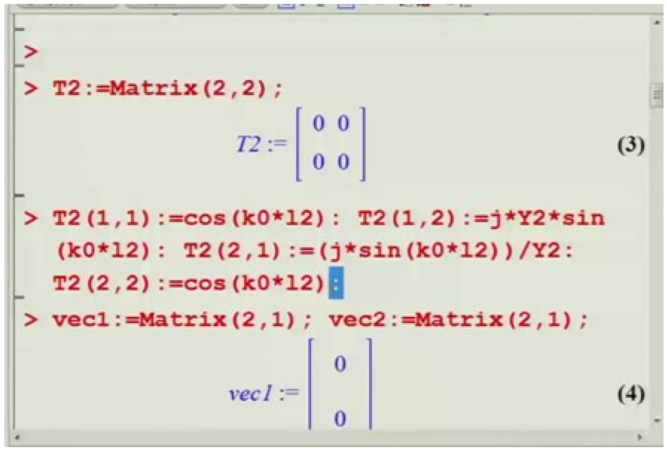
So, here we will get your T1 ah. So, let me just uncomment this.



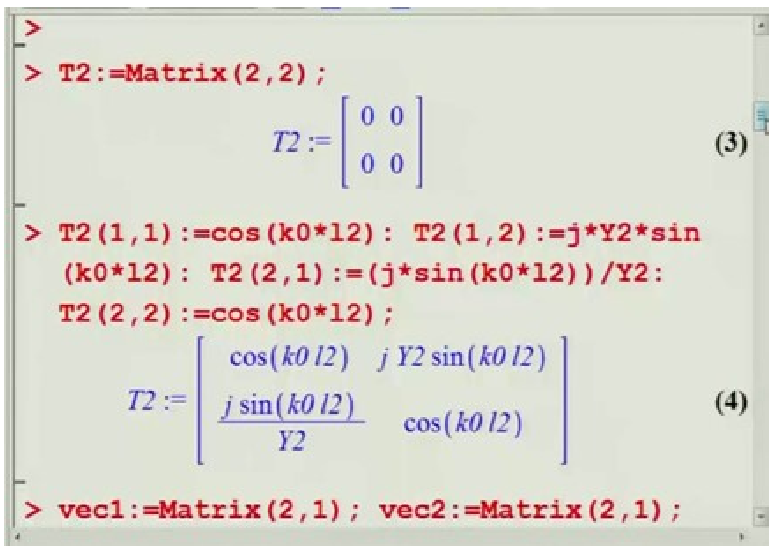
So, T1 is this this thing. So, T and this is like this, this guy will be, your this thing and this will be this thing. So, we get this T1 matrix something like this.



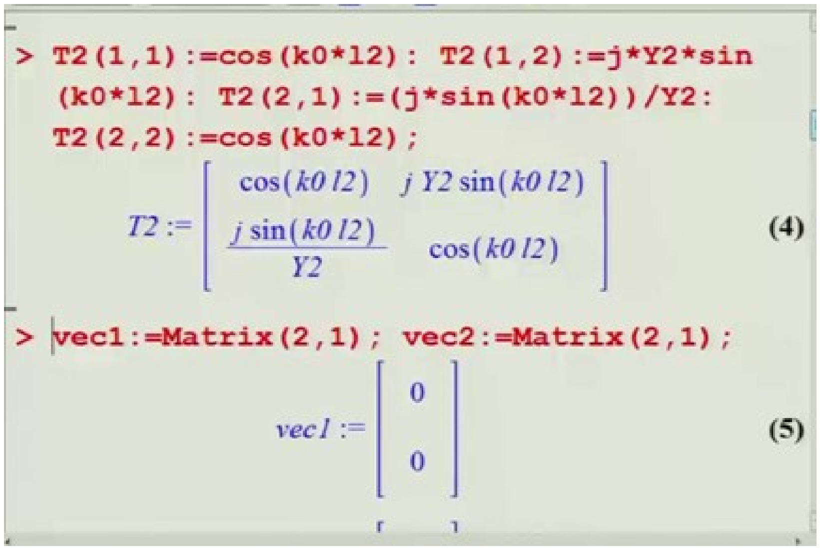
And, then your T2 matrix is also defined like this and you have basically this kind of a thing.



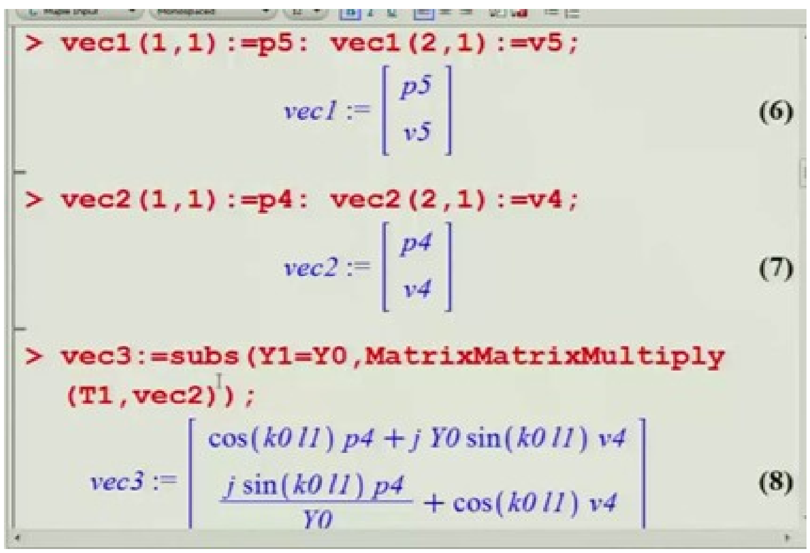
Now, I can just uncomment this guy.



And, just say cos of $k_0 l_1$ and all these terms. So, these are really your kind of transfer matrices. So, these are basically your transfer matrices for the tubular elements 1 and 2 not the A and B matrices.



So, and the vectors are your basically for your downstream variable that is your basically your I guess p_5 v_5 and p_4 and v_4. So, let us define it. So, this is p_5 v_5, we have initialized.



Now, what we do is basically multiply. So, vec3 is nothing, but your this will once you multiply the first transfer matrix with this one, you will get your the relation that is your I guess your expressing p_3 and v_3 in terms of p_4 v_4 and similarly, in this you are expressing p_2 v_2 in terms of p_5 and v_5.

$$vec3 := \left[\frac{j\sin(k0\, l1)\, p4}{Y0} + \cos(k0\, l1)\, v4 \right] \quad (8)$$

```
> vec4:=subs(Y2=Y0,MatrixMatrixMultiply
  (T2,vec1));
```

$$vec4 := \begin{bmatrix} \cos(k0\, l2)\, p5 + j\, Y0 \sin(k0\, l2)\, v5 \\ \frac{j\sin(k0\, l2)\, p5}{Y0} + \cos(k0\, l2)\, v5 \end{bmatrix} \quad (9)$$

```
> v1:=collect(subs(p4=p6,p5=p6,vec3(2,1)
  +vec4(2,1)),p6);
```

$$v1 := \left(\frac{j\sin(k0\, l1)}{Y0} + \frac{j\sin(k0\, l2)}{Y0} \right) p6 \quad (10)$$

So, then what we do is that we substitute in place of p_4 and p_5, p_6 that is what we have done here and vector 3. So, we have substitute this in vec3 2, 1; 2, 1 means the second row as for vec3 and second row of vec4. Why are we doing this? Because as we realize first it is v_1 is equal to v_2 plus v_3.

So, we are getting v_2 and v_3 in here and then we are we are you know doing all this algebraic manipulations. So, v_1 is we are defining v_1 like this here and substituting all these expression like the one that I talked about.

$$+ \cos(k0\, l1)\, v4 + \cos(k0\, l2)\, v5$$

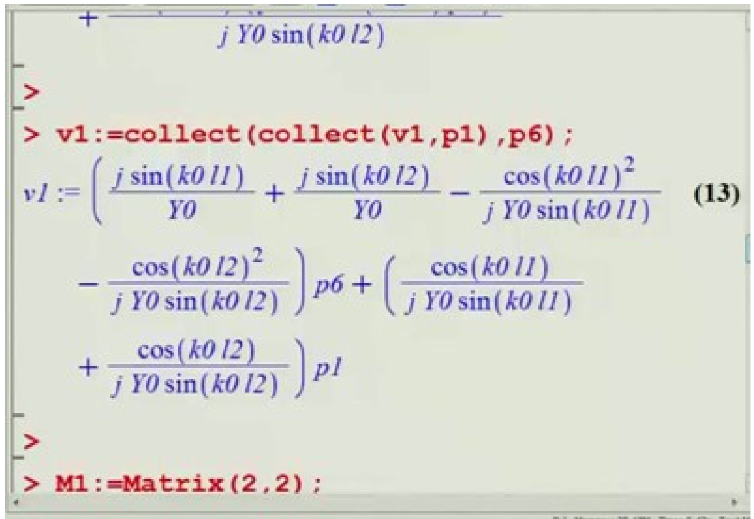
```
> v1:=subs(v5=(p1-cos(k0*l2)*p6)/(j*Y0*
  sin(k0*l2)),v1);
```

$$v1 := \left(\frac{j\sin(k0\, l1)}{Y0} + \frac{j\sin(k0\, l2)}{Y0} \right) p6 + \cos(k0\, l1)\, v4 + \frac{\cos(k0\, l2)\,(p1 - \cos(k0\, l2)\, p6)}{j\, Y0 \sin(k0\, l2)} \quad (11)$$

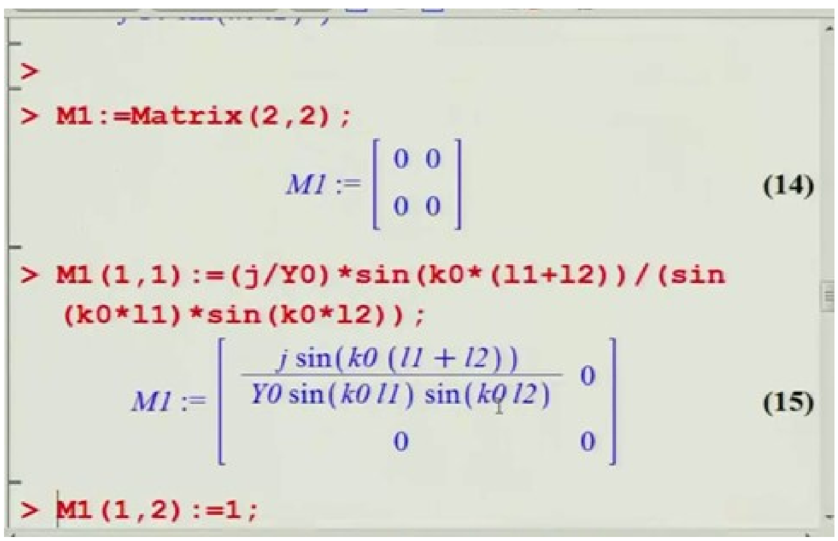
```
> v1:=subs(v4=(p1-cos(k0*l1)*p6)/(j*Y0*
  sin(k0*l1)),v1);
```

$$v1 := \left(\frac{j\sin(k0\, l1)}{\ldots} + \frac{j\sin(k0\, l2)}{\ldots} \right) p6 \quad (12)$$

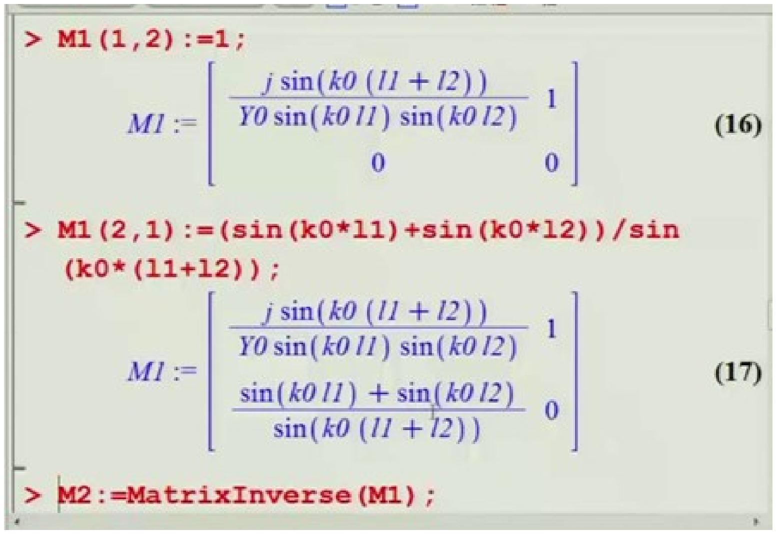
And, then we get this kind of a thing and then further this is further I guess simplification. So, we get v_1 here.



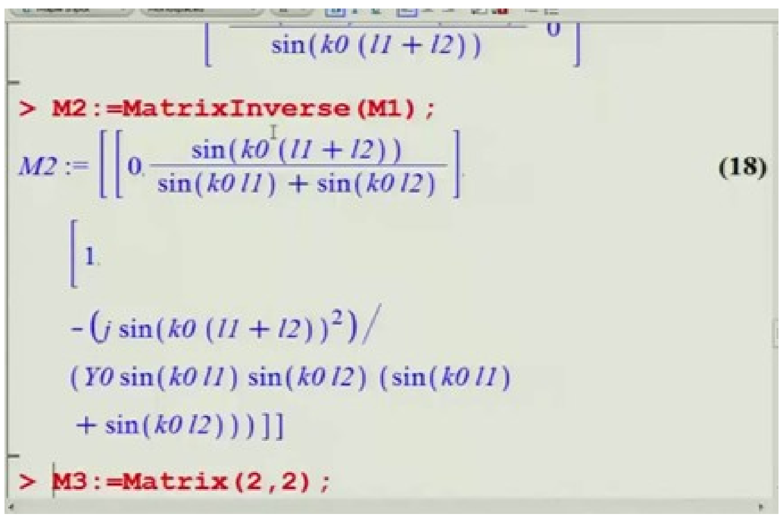
Now, we can sort of collect the term in p_1 and p_6 and then define a matrix. So, this is exactly of the form that you saw somewhere in here, is not it? Is the only difference is that well v_1 is equal to v_1 is equal to p_6 and p_1 you are seeing this. So, same thing we are getting here. So, we can avoid the hand calculation and go to Maple.



So, we defining M1 matrix and only putting certain entries like the one I have talked about.



M2 is 1 and M2 this is this guy I was I was talking about and this is 0 based on this thing, based on the variables if they are present or not.



So, M2 is again defined as M1 inverse because you need to read it upstream to downstream. So, we get this and M3 matrix is something that we get because of the things in here that is your, the first equation. So, this will be having coefficients all that is mentioned here and the coefficient of v_6 would be 1, ok.

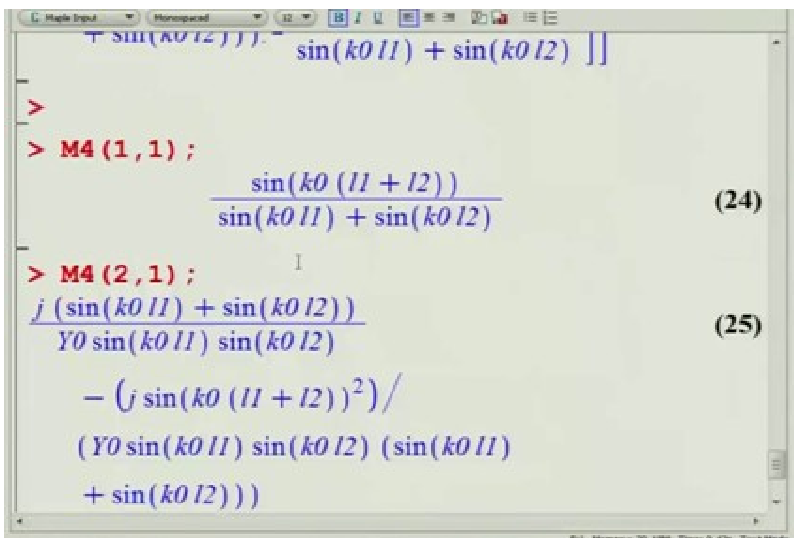
So, this is 2 cross 2 matrix, this is 1 again like this. So, we keep doing all this algebra and then there is the command Matrix Matrix Multiply in Maple. So, we get this to finally, get this tedious kind of looking expression.

So, what it means is that the first, so, this is basically nothing, but let me just go back to the to the presentation. So, eventually what we are successful in doing is

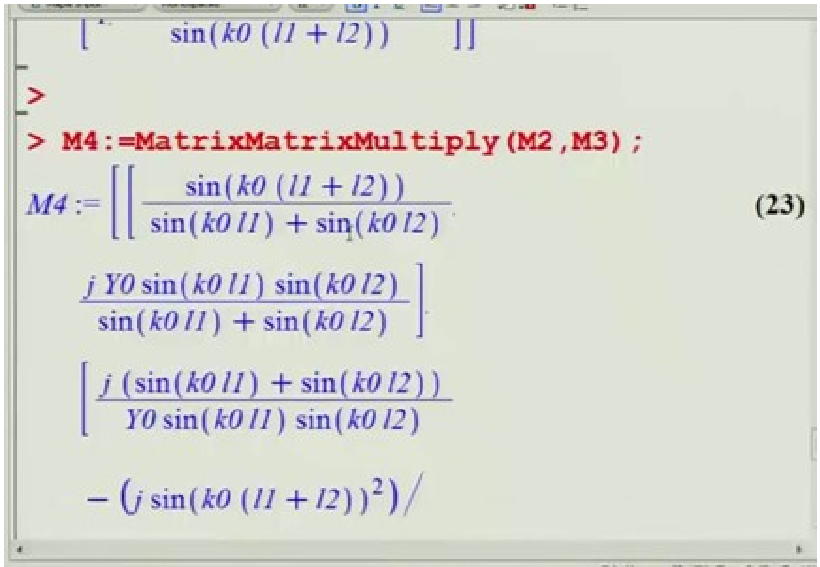
$$\begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix} = [A^{-1}][B]\begin{Bmatrix} p_6 \\ V_6 \end{Bmatrix}$$

$$= [T]\begin{Bmatrix} p_6 \\ V_6 \end{Bmatrix}$$

So, I am calling this guy as T matrix times this thing. So, this is the upstream relation between upstream and downstream variables after a lot of algebra. This can be verified from the book on ducts and mufflers by Munjal and also previous papers by and others where it was first discussed.



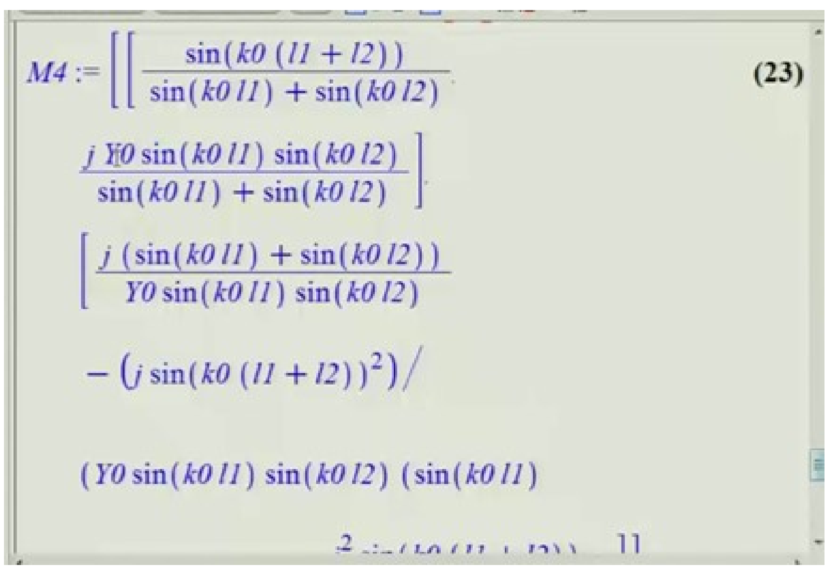
So, let us get back to your Maple and we figure out that M4 1, 1 is nothing, but sin of $k_0 l_1$ l_2 and this thing and this is nothing, but S_1 and S_2 and this is the most tedious looking expression we will worry about that in a bit. So, but other expressions are fairly simple in the sense that you have your M4 matrix here.



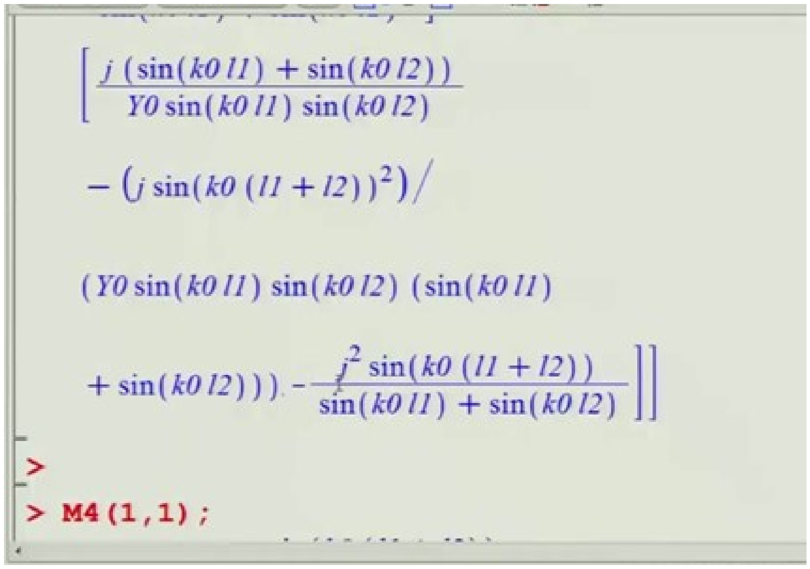
So, M M4 1, 2 that is basically

$$\begin{bmatrix} T(1,1) & T(1,2) \\ T(2,1) & T(2,2) \end{bmatrix}$$

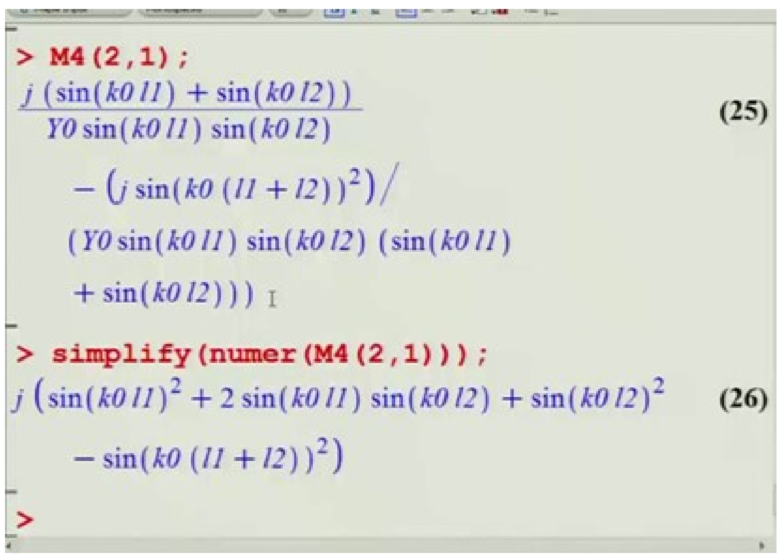
So, what is T1,1 and T2,2 ?



So, that is that is your basically jY_0 into S_1 S_2 divide by $S_1 + S_2$. So, $S_1 + S_2$ is always common and we can just take it out of the matrix.



So, and this is your minus j square. So, minus and do minus that will become plus sin of $k_0 l_1$ plus l_2 divided by $S_1 + S_2$ and this is the expression that that is present here is given by M4 2, 1.



Once you kind of simplify this expression, so, after even some standard algebra. So, what you will figure out is that this expression can be straightaway written in the following form and the, you know you have your S_1 and S_2 also in the denominator. I am sorry, it is basically here. So, and the numerator is this thing. So, basically what happens is that this expression when you simplify the entire thing.

This guy will become

$$T(2,1) = \frac{j}{Y_0}\frac{2(1 - cosk_0(l_1 + l_2))}{(S_1 + S_2)}$$

So, you will have this kind of a thing ok. So, what we can do is that quickly go to MATLAB and figure out things.

But, before that is important thing to mention here is that you have in the denominator S_1 and S_2 and so, what does it mean? It means that once you have related upstream variables to downstream variables, now if you were to do the opposite, that is relate downstream to upstream clearly you know S_1 and S_2 will occur in the numerator that is p_6 v_6 if you write in terms of T inverse you will get

$$\begin{Bmatrix} p_6 \\ V_6 \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix}$$

Now, this guy because this is S_1+ S_2 term is in the denominator for the T matrix in the numerator will be on the when you invert this T inverse it will be in the numerator. So, what does it mean? For transmission loss p6 and v6 both will become identically 0 when S_1 + S_2 is 0 for a particular frequency that is the if you apply your termination and figure out the waves that go in the propagate in the downstream or the termination. That will be identically sort of 0, provided that,

$$(S_1 + S_2)$$

$$sink_0l_1 + sink_0l_2 = 0$$

For a given frequency and such is the frequency where destructive interference happens, ok. So, let us go to that. So, when will that happen? When

$$sink_0l_1 = -sink_0l_2 \qquad\qquad M = 1,2,3, \dots \dots$$

$$sink_0l_1 = \cos((2M - 1)\pi)\, sink_0l_2$$

$$+ \sin((2M - 1)\pi)\, cosk_0l_2$$

This will happen when sin of $k_0 l_1$ there are two conditions of course. And here this will be 0 identically for all m being an integer. So, what the reason that I have written this is because this guy

$$sin k_0 l_1 = \sin\big((2M-1)\pi + k_0 l_2\big)$$

$$k_0 l_1 = (2M-1)\pi + k_0 l_2$$

$$k_0 l_1 - k_0 l_2 = (2M-1)\pi$$

So, what it means is that when you know k naught and $k_0 l_1 1$ is the, and $k_0 l_2$ the difference between them is the difference is really the path difference between the waves that come you know path difference between the waves that arrived from here.

And, the ways that propagate here when this path difference is equal to 2 m minus 1 times pi where m ranges from 1, 2, 3, 4 whatever it is some integers, then you have a destructive interference and then your, S1 and S2 is cancelled. So, this is one relation that we have.

Now, another way in which the destructive interference can happen is when

$$k_0 l_2 = 2n\pi - k_0 l_1$$

$$=> \quad k_0 l_2 + k_0 l_1 = 2n\pi, \qquad n = 1,2,3$$

You know this is what you get. How do we get this guy? Because here you just like instead of minus as put cos here you can actually also do it the other way around.

So, this will happen when you take minus inside. So, you what you can get is basically,

$$sin k_0 l_1 = 2n\,(-k_0 l_2 + 2n\pi)$$

$$k_0 l_1 = 2\pi n - k_0 l_2 \qquad n = 1,2,3$$

Where n is equal to your same thing here ok. So, this basically this relation will fetch you this one.

And, so, basically what it means that when the sum of the phases or the waves that propagate in both the ducts is equal to 2 n π, where n has to be 1 at least to make it nontrivial then also destructive interference will happen and then we will get almost 0 power transfer downstream resulting in sharp attenuation peaks. So, you know the frequency corresponding to this is easy to work out. So, this is

$$\frac{2\pi fp}{C_0} = \frac{2n\pi}{l_1 + l_2}$$

So, 2, 2 is cancelled and so, is π. So, giving

$$=> \quad fp = \frac{C_0 n}{l_1 + l_2}$$

You are getting this relation for the corresponding to this condition. What about the other condition? So, other condition will fetch you the following relation. So, this is

$$fp = \frac{2n\pi - 1}{2} - \frac{C_0}{l_1 + l_2}$$

Depending on whichever is greater; m is 1, 2, 3, 4.

So, we can work out couple of interesting cases. So, what we can do is that you know for the configuration, that I have shown here. So, what we get really is that suppose we take this length to be 0.5 meters that is about 500 mm from here to here, and this guy is about just half the this thing. So, it becomes π 2 pi r. So, it just pi r ok. So, π r means π d by 2 and this is your 500.

$$\pi \frac{0.5}{2}$$

$$\pi \times 0.25m$$

$$= 0.785.4m$$

So, we get this kind of thing. So, we can substitute these l_1 l_2 lengths this is l_2 this is l_1 and the diameters you know you can consider this something like a nominal diameter 40 mm or so. And, now we can quickly go to MATLAB and figure out what is going to happen at the transmission loss.

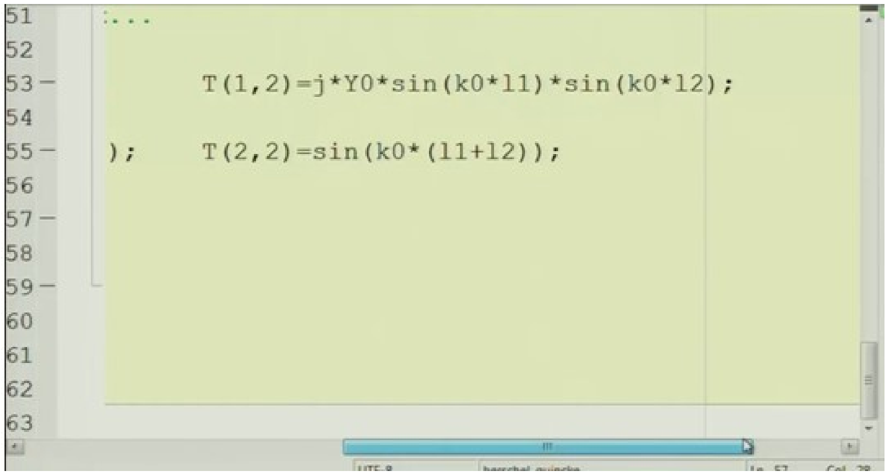
```
function [T]=herschel_quincke(d0,l1,l2,k0)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% this code finds the transfer matrix for a He
%%% both pipes have the same diameter...
%%% l1,l2 are the lenghts of the respective duct
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
j=sqrt(-1);
c0=346;  %%% sound speed...
S0=(pi/4)*(d0^2);
Y0=c0/S0;  %%% characteristic impedance of the p
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% now, the connectivity matrix is to be define
```

So, here we have your MATLAB script that I have written in advance. So, I am not going to do it live here these records were written some time back. So, I am assuming the sound speed of 346 here and diameter is something that is given by the user l_1 and l_2 are the lengths as I have mentioned, this is a characteristic impedance. So, this is the connectivity matrix.

What I am going to do is that I am just going to comment out the entire thing here that that much I can do. And, just use the analytical expressions the 1 that we derived in Maple using the help of Maple and some simplification.

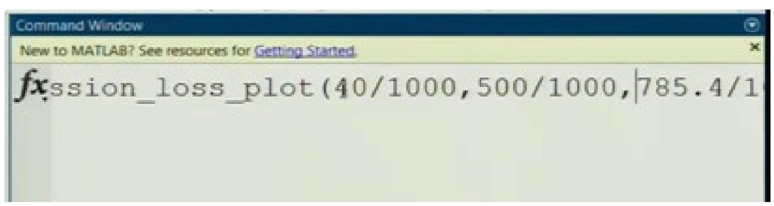
```
%% T is the required impedance matrix...

T(1,1)=sin(k0*(l1+l2));                    T(1,2

T(2,1)=(j/Y0)*2*(1 - cos(k0*(l1+l2)) );    T(2,2

den=sin(k0*l1) + sin(k0*l2);

T=T/den;
```

So, you know these are nothing, but sin of k_0 $l1$ plus $l2$.

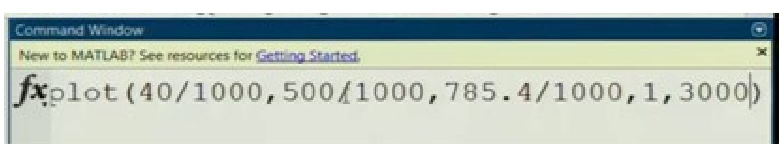


Your T_1, 1 parameter this is jY_0 into sin of S1, S2; this is your again S1 sin of k_0 l_1 plus l_2 this is nothing, but the simplified expression and denominator is just 1. So, we divide the entire matrix by this.

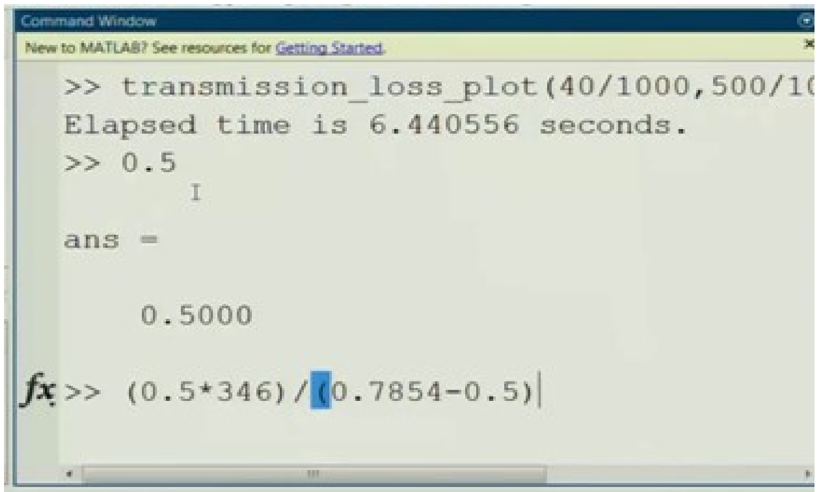
So, once we get this then these things this particular function is invoked by the transmission loss function which is invoked by the transmission loss plot.



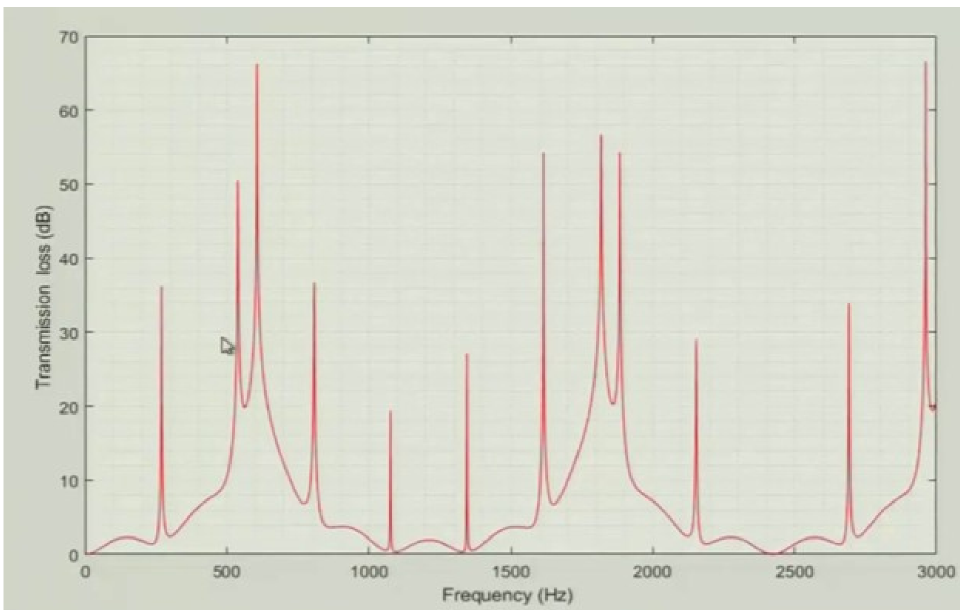
So, what we are going to do for the parameter range that I talked about I can just set this to be 40 40 mm and length can be 500 mm. The other guy like I was mentioned like I mentioned was 785.4 mm everything is in meter.



So, I can go from you know 1 to well 3000 hertz because really for such things only very low frequencies will propagate.

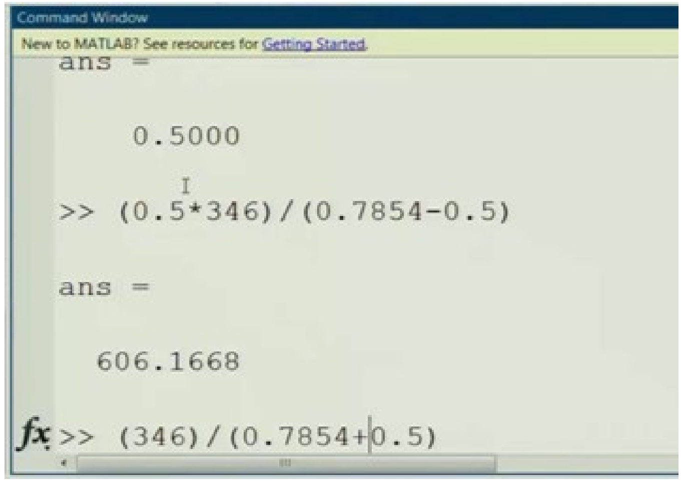


So, we have this kind of a thing. So, it is going to take little bit time.

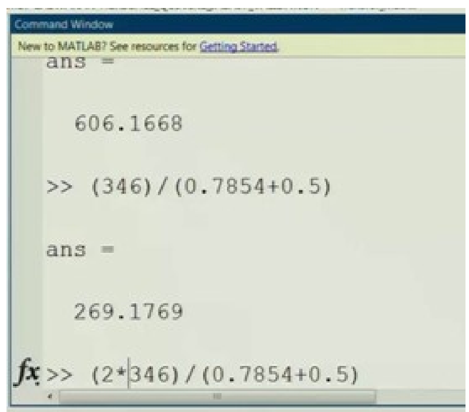


And, we get this kind of a transmission loss. So, what is important here to note is that let us first analyze two or three important peaks that would kind of occur we will go to the basic data tip. So, we get your 269 at frequency 269 we are going to get one peak, another peak that occurs is at 538, next one that occurs is at 606 and so on.

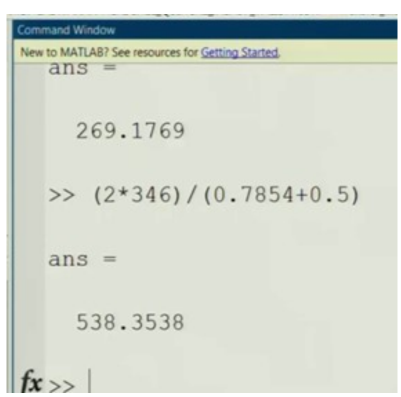
We can easily do the maths I mean in the sense that if we figure out like m is equal to 1, so, 0.5 times 0.5 times 346 and the denominator will be 0.7854 minus 0.5.



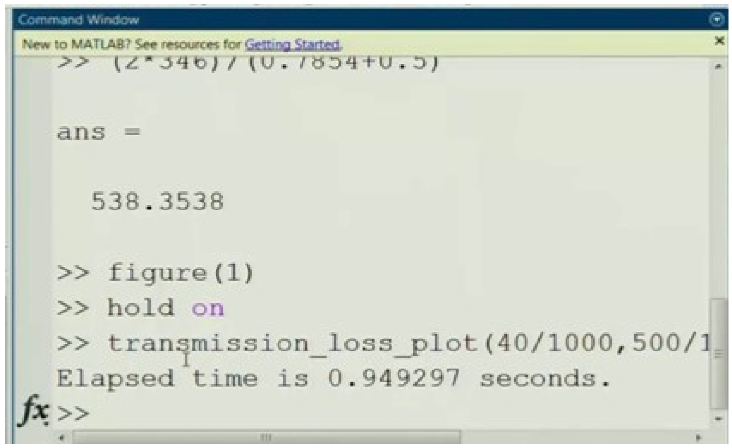
So, let us analyze where do we get 606, ok. So, this is a first peak that we are getting to do destructive interference and what about the other one peak due to the other term?



So, that is basically your C naught that is your 346 divide by l 1 plus l 2 and n is 1 in the other formula that is 269, ok. So, you are getting a peak at 269, and the other one is probably the multiple of that which is nothing, but if you just multiply the numerator by 2, you should be fine.



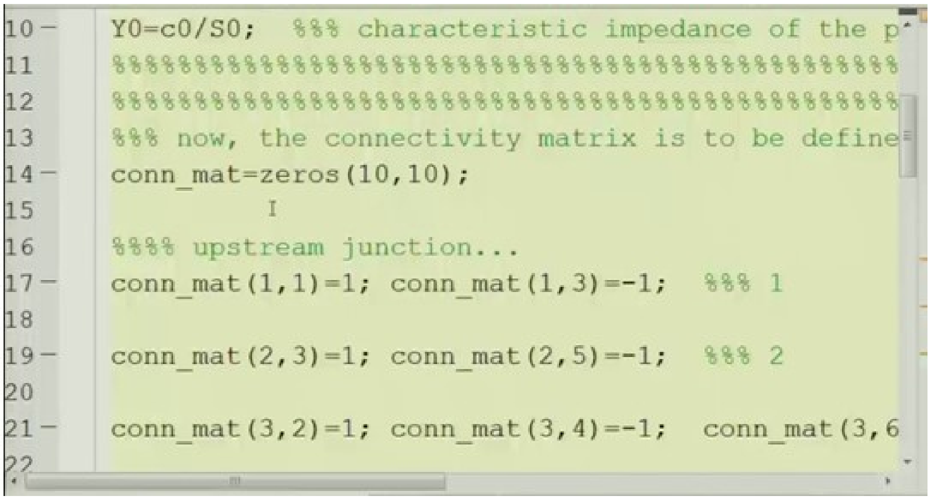
So, this is 538 what we are going to be what we are getting. So, you know like this we can completely cover the entire thing. Now, we can basically figure out why these peaks are occurring wherever they occurring.



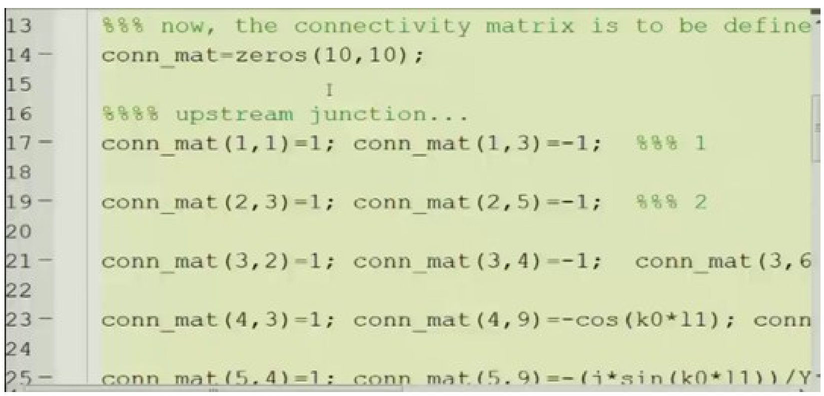
Now, another way to basically compute this is using a connectivity matrix approach. The analytical one I talked about because I want to demonstrate as is usually the case with all courses in science engineering, that you consider whenever you develop a new kind of an approach we tend to develop we consider we tend to consider a system which is only slightly more complicated than the usual one so that at least somehow you can get a feel of the analytical formula. So, that is what we have done here.

But, for you know much more complicated systems like I will talk about that I will draw that and probably show you some figures some muffler configurations published in previous papers. So, in such configurations almost impossible to figure out the transfer matrix that is over all transfer matrix between upstream and downstream variables, analytically like we have derived here. So, we have to go to the connectivity matrix approach.

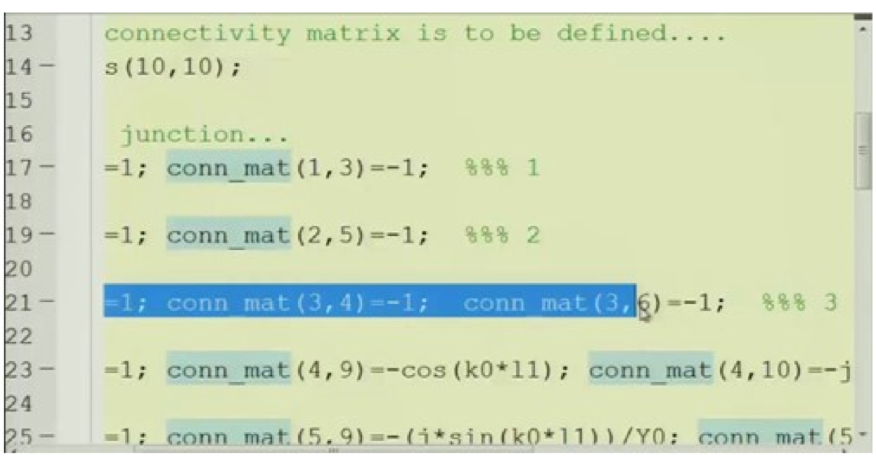
So, it becomes quite important that we kind of go to the connect use the connectivity matrix approach and demonstrate that it works very well even for I mean just to begin with for such a system. So, we are going to comment out the entire thing and I am just going to walk you through the code. You can I encourage you also to start coding. It can be any language, not necessarily MATLAB.



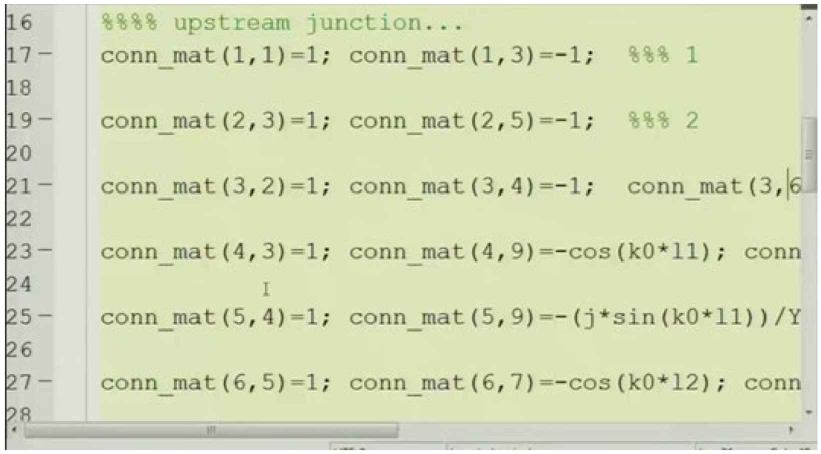
So, we define a connectivity matrix like 10 cross 10 where you know remember we have we want to invert you know A inverse B. So, we are getting the C matrix and we are taking only a chunk of that, that is, only first few rows to get stuff.



So, this is your 10 cross 10 matrix and you know here this is your first row is your first p_1 is equal to p_2, p_2 is equal to p_3 that kind of relation.

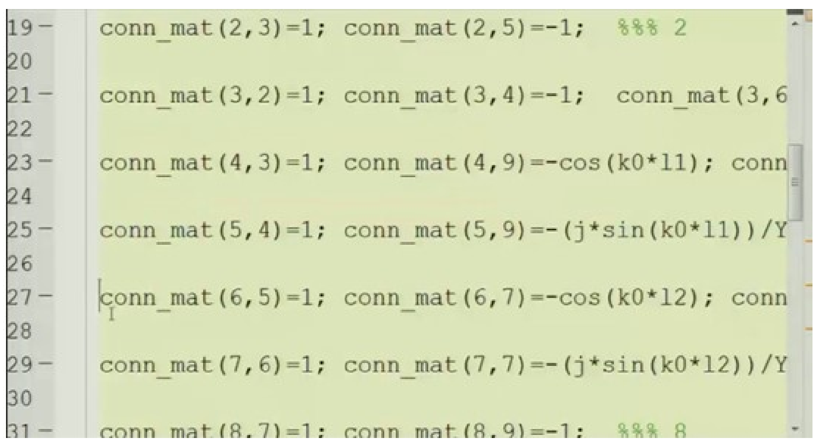


Then is a mass velocity conservation equation.



```
%%%% upstream junction...
conn_mat(1,1)=1; conn_mat(1,3)=-1;   %%% 1

conn_mat(2,3)=1; conn_mat(2,5)=-1;   %%% 2

conn_mat(3,2)=1; conn_mat(3,4)=-1;   conn_mat(3,6

conn_mat(4,3)=1; conn_mat(4,9)=-cos(k0*l1); conn

conn_mat(5,4)=1; conn_mat(5,9)=-(j*sin(k0*l1))/Y

conn_mat(6,5)=1; conn_mat(6,7)=-cos(k0*l2); conn
```

And, then this is a relation between your, you know variable from 2 to 5, the tubular element. So, let us go back to the code. So, that is what we have done in rows 4 and 5.



```
conn_mat(2,3)=1; conn_mat(2,5)=-1;   %%% 2

conn_mat(3,2)=1; conn_mat(3,4)=-1;   conn_mat(3,6

conn_mat(4,3)=1; conn_mat(4,9)=-cos(k0*l1); conn

conn_mat(5,4)=1; conn_mat(5,9)=-(j*sin(k0*l1))/Y

conn_mat(6,5)=1; conn_mat(6,7)=-cos(k0*l2); conn

conn_mat(7,6)=1; conn_mat(7,7)=-(j*sin(k0*l2))/Y

conn_mat(8,7)=1; conn_mat(8,9)=-1;   %%% 8
```

And, then similarly row 6 and 7 we have related things for the straight thing ok.

```
conn_mat(5,4)=1; conn_mat(5,9)=-(j*sin(k0*l1))/Y

conn_mat(6,5)=1; conn_mat(6,7)=-cos(k0*l2); conn

conn_mat(7,6)=1; conn_mat(7,7)=-(j*sin(k0*l2))/Y

conn_mat(8,7)=1; conn_mat(8,9)=-1;   %%% 8

                  conn_mat(9,9)=1;   %%% 9

conn_mat(10,8)=1; conn_mat(10,10)=1;   %%% 8

%%%% downstream junction...
```

And, then your junction laws p_4 is equal to p_5 p. Now, p_5 is equal to p_6 we can easily get that and this is your mass velocity v_4 plus v_5 is equal to v_6.

```

conn_mat(10,8)=1; conn_mat(10,10)=1;   %%% 8

%%%% downstream junction...
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
vec=zeros(10,2);

vec(9,1)=1;
vec(10,2)=1;
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% sol_mat=inv(conn_mat)*vec;
% %%% we want the acoustic pressure and velocity
% %    |p1|   = |T11  T12||p6|
```

And, similarly vector this thing will only be initialized with 10 cross 2 system, but with zeros. But, only few entries couple of entries in this case will be nonzero in that will be your p_4 is equal to p_6 and v4 plus v_5 is equal to v_6.

```

vec(9,1)=1;
vec(10,2)=1;
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sol_mat=inv(conn_mat)*vec;
% %%% we want the acoustic pressure and velocity
% %    |p1|   = |T11  T12||p6|
% %    |v1|     |T21  T22||v6|
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T(1,:)=sol_mat(1,:);        %%% p1....
T(2,:)=sol_mat(2,:);        %%% p5....
%% T is the required impedance matrix...
```

Then it is as simple as sort of inverting writing a command sol mat that is basically your C matrix is, a inverse b kind of a thing. So, we get this. So, in practical system this is 10 cross 10. It can be 10000 cross 10000 also. This can be pretty challenging.

```
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
sol_mat=inv(conn_mat)*vec;
% %%% we want the acoustic pressure and velocity
% %   |p1|  = |T11 T12||p6|
% %   |v1|    |T21 T22||v6|
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T(1,:)=sol_mat(1,:);          %%% p1....
T(2,:)=sol_mat(2,:);         %%% p5....
%% T is the required impedance matrix...

% T(1,1)=sin(k0*(l1+l2));                    T(1
%
% T(2,1)=(i/Y0)*2*(1 - cos(k0*(l1+l2)) );    T(2
```

And, then we take only the first few rows first two rows because remember it is about also about proper bookkeeping. So, we might. So, one possible idea of one possible area of research is to develop an algorithm which will just see the stuff and you know it will put you know it will define the connectivity matrix in such a manner it start dumping the elements at appropriate places using proper bookkeeping.

So, these are all part of numerical techniques or training that one must go through. So, T1 and T2 we are getting like this sol mat and then the idea is just to run the code again.

```
omega=2*pi*f;
k0=omega/c0;

n1=size(f); n=n1(1,2);
for i=1:n
    Tl(i)=transmission_loss(d0,l1,l2,k0(i));
end
plot(f,Tl,'k');
grid minor
xlabel('Frequency (Hz) ');
ylabel('Transmission  loss (dB) ');
toc;
```

So, I have already put hold on; hold on mean is save the previous result and plot it using a different colour. So, right now, mind you have commented out this part and I am going to just use another colour say black k and plot it is going to take some time 0.94 seconds. Where is it gone? Well, it is almost identical. It is identical. So, this is a numerical way of doing stuff, it matches very well with the analytical formula. Same thing I will not repeat again. So, this is I have taken out and pasted it again.

So, we are getting multiple resonances due to destructive interferences and there are some truss also occurring because of constructive interference, there are frequencies at which the entire acoustic power is transmitted as it is.

So, these are all part of multiple connected mufflers and then there are large number of such connections. For example, what I am going to probably talk about in the next class perhaps is the some few connections which are much more intricate much more complicated. So, till that time goodbye. I will see you in the next class.

Thanks.