Muffler Acoustics-Application to Automotive Exhaust Noise Control
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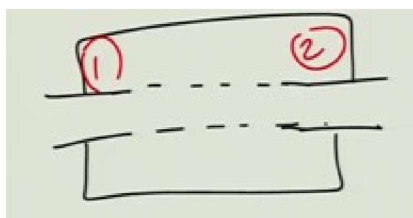
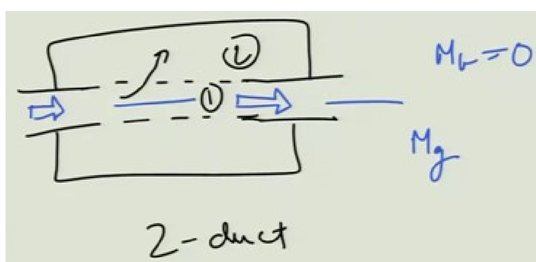
Lecture - 41
Cross-Flow Elements: Setting-up the Equations

Welcome to week 9 of the NPTEL course on Muffler Acoustics. So, as promised at end of the last lecture that is lecture 5 of week 8. In this week we will be primarily focusing on cross flow elements. So, let us let us try to understand what the Cross-Flow element is, and before moving on to more complicated things like multiply connected mufflers.

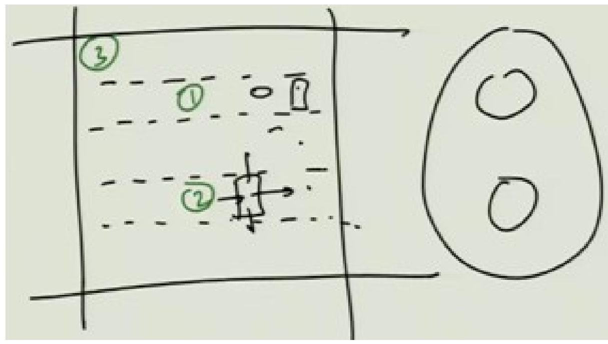
So, you know we in the last class we saw, before we actually talk about cross flow things let us you know go in a very systematic manner and try to draw this thing ok. So, this was ECTR. So, extended concentrative resonator with extension that you know 1 and 2 ok, something like this.





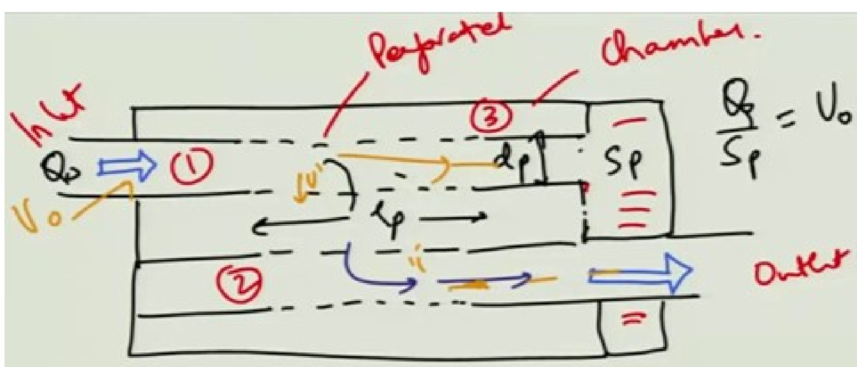
Now, suppose we have this element and then you know the flow really just sort of grazes the boundaries ok, the flow just grazes the boundary. The flow is not allowed to go through the element. Basically the only way the flow can go through the pipe. So, the here the only thing that is important is the grazing flow impedance. So, here only the Mach number Mg the grazing Mach number important Mb or the bias flow Mach number is 0.

So, what is bias flow? So, bias flow is when the flow has to necessarily go out of this and there is another duct and the flow again enters the other duct which I will draw soon and then it goes through this thing. So, basically the flow the velocity that goes through the ducts if that is nonzero then you then you have a nonzero bias flow or you have a cross flow elements.



Let us then get to the cross flow elements, let us see how they look like. First of all they must have at least a three duck interacting system. All this while like in this figure we have a two duct system ok, because this is the first one, this is the second duct ok. Now if we have a three duct system something like this ok something like this.

So, let us see how we go. So, this is your first duct, this is the second duct and this is your third duct; that is the duct or the main chamber that houses both the pipes ok. So, this is a common section that you are seeing from here and from here ok something like this. So, basically how does the flow interact and what sort of element can be form?



So, we can sort of draw the configurations. Suppose we have this kind of a thing, if the flow comes in here and it can be a circular section and you have a perforated section. So, you have this. So, you have this kind of a thing. So, and it goes like this. The flow has to necessarily come out here and again grace and become like a grazing flow in the duct 2.

So, we can name this duct, duct 1 inlet ah, you can name this as duct 2 outlet and duct 3 which is the chamber itself ok. So, these are all perforated sections perforated section and notice there is a there is an overlapping perforated section it need not be always, but you know for simplicity we are assuming and there is one wrong thing that I did here, it should be a cavity like this

So, this is called a across flow expansion chamber, because the flow really has to expand and its like a straight through kind of a configuration. Not exactly a straight through, because you know the flow comes like this and it kind of it has to you know, we have to understand the difference between this configuration and this one. So, the flow need not go out or the mean flow itself, need not go out of the perforates in the E C T R case.

But in this there is no there is no other way that the mean flow can leave the chamber cleanly ok. The flow has to whatever if Q_0 is the flow velocity here and you know if S_p is the cross section area. So, clearly Q / S_p is the velocity U_0 in the pipe.

Now a simplified hypothesis; obviously, this needs to be a proper way of dealing with evaluating the mean flow distribution would be either through lumped resistance analysis which is given in some recent papers or maybe if one wants to you know do it very properly then C F D analysis would be even better.

Because these flow distributions would be rather non uniform, given the fact that the flow has to come out of the perforates, again there will be some non zero velocity radial velocity like in this direction and ok and then it has to again enter and go.

So, basically the idea is, that you know this Q_0 is split into two parts; one is going through the grazing or the you know the actual direction and it gradually reduces. For simplicity you can assume a uniform a grazing flow in the pipe 1 and then it comes out and sort of reattaches again.

So, basically your velocity Q_0 here, it is you can equate this to the area of the perforate. So, what is the area of the perforate?

$$Q_0 = \left(\sigma.\pi d_p\ lp\right)U' = U_0 Sp$$

So, this is your area curved area into the porosity that will give you the effective area of the or the net area of the total number of holes.

So, σ into this into if you multiply this entire thing by U'. So, this if we equate to say we know U_0 we know Sp diameter. So, one can figure out easily the U' or the velocity U' is really in this direction that is the velocity coming out to each other holes again this U' will tend to be more to as the flow you know goes along this direction for this configuration and you know will tend as we approach this hole will tend to be 0.

And finally, you have a grazing flow tends to really be you know significantly different from nonzero that is this value, approaches value more towards this direction, again there will be some losses. So, whatever U_0 we are seen you know here that will not really be the case here.

There will be you will get a slight lesser velocity here, because of losses, that is what happened in a real world. But nevertheless is a very gross or a first approximation I guess it will be a decent approximation if we were to you know do something like this.

$$U' = \frac{U_0 Sp}{(\pi d_p\ lp)\sigma}$$

So, we can, this is just an approximate value of the grazing thing.

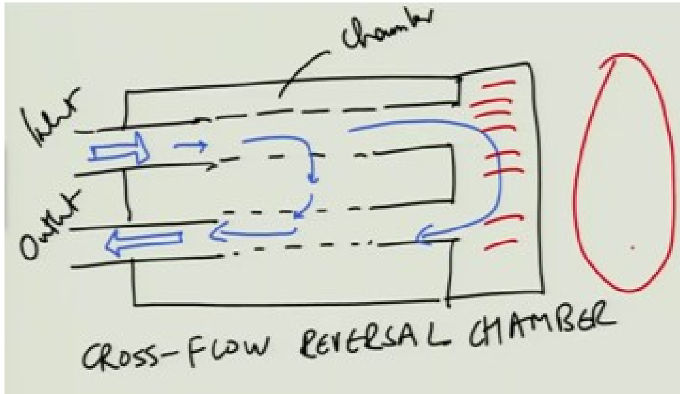
$$U' = \frac{U_0 \pi d_p^C}{4.\pi d_p\ lp} = \frac{U_0 d_p}{4 l_p}$$

$$U' \simeq U_0 \frac{d_p}{4 l_p}$$

$$M' = M_0 \left(\frac{d_p}{4 l_p}\right) \qquad 0.15$$

So, generally your diameter of the pipe is pretty small about 40 mm 50 mm for automotive mufflers and *l*p is your length of the perforated pipe which is usually more than the diameter.

So, again this quantity and plus it is divided by 4. So, this and Mach number again it is about this maximum value is about 0.15 or something like that. The point is that the bias Mach number will be even smaller. So, and mind you these kinds of configurations that, this called a cross flow expansion chamber ok cross low expansion chamber.



So, and you yet you have another such simple configuration is called a reversal flow, something like this you know something like this. So, the flow again has to enter here, go like this, come out and re attach itself. So, this is called a cross flow reversal chamber. So, here also the flow has no other option, but to you know navigate through these perforated holes and you know come out of this.

So, this can be your inlet, this can be your outlet and this is the chamber itself you know. So, there is only one way the flow can go. Now I know this is the topic for a little later, but typically what one can also do, is that one can suppose if we were to have such a kind of a thing, you know this kind of a thing.

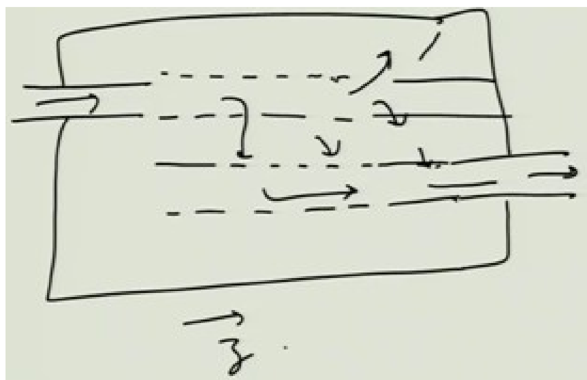
So, then the flow has multiple paths really. So, this can be your you know you can have a flow like this and then you can also, the flow can actually take a complete U turn smoothly and then it can go through like this. So, you know these configurations and these are called your short end chamber. Remember we are discussing about wave propagation along the transfer direction.

So, this can be a elliptical shape or circular shape. I am not drawing here because of lack of space. Similarly, you can also have something like resonator here and things like this, waves can also go like this, but again there will be unique path or you can also sort of get things like this thing here or maybe just a port. I am sorry you can just have a port like this. So, we can have multiple such things.

So, then you will have really multiple waves in which the waves can in which the waves can go. I am sorry for that. So, be like this. So, you can have a port here, port here and you can have multiple ways in which the waves can go, the waves can go like this and it come out like this or maybe we can directly go through this. So, there will be, the flow might just have one path, but the waves can definitely take no unique part in this case the waves can actually can go like this or it can go like this and number of things it can do.

So, such elements are called multiple connected elements. This is just one a you know sort of very simple sort of example, they are much more complicated examples of multiple multiply connected mufflers.

So, then of course, you know talking about such configuration there are many more such configurations what one can probably deal with. Another configuration that we sort of have, is your extended non overlapping perforated thing. So, this can be a something like this.



You know you have a pipe like this and they are non uniform thing like this, like these and you have perforates all across like this and you have things like this kind of a thing and this. So, flow comes here, it has to go to this, it has to go through this and finally, leaves ok.

So, here the flow can of course, expand and again do like this kind of a thing. And you can also have things with an overlapping section. So, in such a case you can have this kind of a thing. So, these are called these are your open end open ended perforated elements, because flow is just allowed to expand. So, here there is no guiding thing, such things are not there ok.

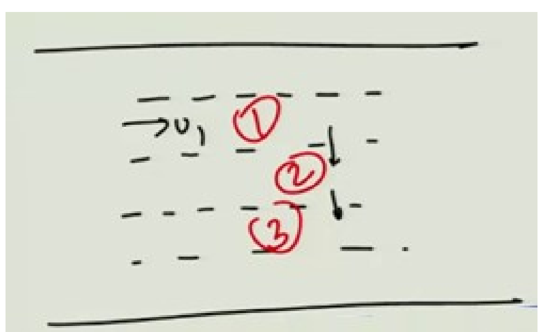
The little bit drawback about this configuration is that it has a back pressure or head loss, because the flow is just allowed to expand. Who knows it might be more much more counter productive, compared to the case where you have this thing were the flow is easily guided at least the flow cannot go through then there will be there has to be resonances that are developed in this part.

So, their there are pros and cons for each of the muffler configuration depending upon the your flow velocity and your thing that you want to analyze you can have different sort of configurations. So, now, I guess with this bit of understanding what one can do, and mind you these are all of uniform cross section area. So, the area of the circle or the ellipse you know does not really sort of change.

Now, what one can really do, is basically develop or set up the system of equations that are really necessary to go ahead with our analysis. So, what we will do? You know we will go back revered back to our you know configuration we all sort of yes rub this or probably I will draw this again. Anyways I have to talk about this configuration.

So, basically you know let me just write down the equations for this duct with the understanding that we still use a control volume approach. You know we have to take a control volume here, you know find out the fluxes that enter inside, go outside and then the ones that go here and equate them to the temporary or temporal rate of change.

See then you have momentum equation here, you have your momentum equation here, continuity here, momentum here, and continuity here by using a control volume approach. So, this is going to be a little, but algebraically tedious, but hopefully when we do a systematic job we can overcome all the challenges associated with it. So, let me just write down that was slide number 17.



So, I will go to slide number 23 and start writing down the equations for different ducts So, you have this and. So, and then we have this kind of a thing.

So, I will number the duct 1 section 3, this duct as 2 ok. So, the continuity equation for duct 1. So, we

$$C_1 \ \rho_0 \frac{\partial \widetilde{U}_1}{\partial z} + U_1 \frac{\partial \tilde{p}_1}{\partial z} + \frac{4}{d_1} \rho_0 \widetilde{U}_{1,2} = -\frac{\partial \tilde{p}_1}{\partial t} \qquad (1)$$

the velocity let us say that goes here and.

So, I will tell you what do I mean by this guy, $-\frac{\partial \tilde{p}_1}{\partial t}$. So, $U_{1,2}$ U_1 you know this is nothing, but the difference in the pressure

$$U_{1,2} = \frac{p_1 - p_2}{\rho_0 \ C_0 \zeta} \qquad (2)$$

I would say I would just assume equal perforate impedance that is what the case is in most of the commercial mufflers. So, its fairly general to keep it xi and not xi 1 it will not make any not make much of a difference. So, and then you have your momentum equation for the duct 1.

$$\rho_0 \left\{ \frac{\partial \widetilde{U}_1}{\partial t} + U_1 \frac{\partial \widetilde{U}_1}{\partial x} \right\} = -\frac{\partial \tilde{p}_1}{\partial z} \qquad (3)$$

So, I am dropping off on the tilde sign. So, apologies for the any confusion. So, this is your let us say this is capital U_1, So, this is your momentum equation ok. This is a momentum equation for the inner duct 1.

And for the duct 2 that is your. I mean the main duct of the chamber that houses both the things. So, for such a thing is

$$\rho_0 \frac{\partial \widetilde{U}_2}{\partial z} + U_2 \frac{\partial \widetilde{U}_2}{\partial x} - \frac{4 d_1}{d_2^2 - d_1^2 - d_3^2} \ \rho_0 \ \widetilde{U}_{1,2}$$

$$+ \frac{4 d_3}{d_2^2 - d_1^2 - d_3^2} \rho_0 \ \widetilde{U}_{1,3} = -\frac{\partial \tilde{p}_1}{\partial t} \qquad (4)$$

$$\widetilde{U}_{2,3} = \frac{\tilde{p}_2 - \tilde{p}_3}{\rho_0 C_0 \zeta} \qquad (5)$$

Because this is really your annular area into row U_0 , where $U_{1,2}$ is something that we have just seen a plus 4d3, because you know see in this configuration whatever comes out it also you know sort of enters here.

So, you know when we do the, I am just leaving out the details because we have a fair bit of ground to cover. So, I am just simply writing out drown down these equations. So, $U_{1,3}$. So, I will tell you what this thing means. So, this is something like So, this is what it is ok. And this we have already seen. So, we have this thing. And then you have your momentum equation in this duct which is of course,

$$\rho_0 \left\{ \frac{\partial \widetilde{U}_2}{\partial t} + U_2 \frac{\partial \widetilde{U}_2}{\partial z} \right\} = -\frac{\partial \tilde{p}_2}{\partial z} \qquad (6)$$

So, maybe its good idea to introduce those tildes just to sort of prevent any confusions. So, I just have to go back and keep doing this tilde business ok, this is like this you. So, now, things should be more clear this is like this and ok.

And

$$\rho_0 \frac{\partial \widetilde{U}_3}{\partial z} + U_3 \frac{\partial \tilde{p}_3}{\partial z} - \frac{4}{d_3} \rho_0 \widetilde{U}_{1,3} = -\frac{\partial \tilde{p}_3}{\partial t} \qquad (7)$$

so we get this sort of a thing ok. And here you have your

$$\rho_0 \left\{ \frac{\partial \widetilde{U}_3}{\partial t} + U_3 \frac{\partial \widetilde{U}_3}{\partial z} \right\} = -\frac{\partial \tilde{p}_3}{\partial z} \qquad (8)$$

So, this is your momentum in the second duct and this is your continuity equation in the third duct. And this is your $\widetilde{U}_3$ and this is your z ok in the third duct. So, we have this sort of thing.

Now we know really we have how many 6 equations actually more, because let us start numbering them 1. I would say you know 2 because we need that many relation. So, 2, 3, 4, 5, 6, 7, 8. So, you know $U_{2,3}$ and $U_{1,\,3}$ that can be simply be replaced by putting an appropriate expressions like you know 5 and your 2 ok.

So, your equations that we are numbered like until I guess 8 that gets reduced to 6 ok. So, when we have such a thing we have you know we get 6 equations and the number of variables that we have. How many variables do we have? We have $\tilde{p}_1 \rho\ \widetilde{U}_{1.}$ I am sorry $\tilde{p}_1 \rho_0\ \widetilde{U}_{1.}$ here, and you have your such thing for the second duct and your third duct. So, basically 9 variables. Now, we have only equation (6). So, then what we do is that we use isontropicity relation

So, that is you know now it is should be fairly simple or straightforward for you guys to notice to make use of such expressions and have this. So, you have additional three equations; (9) variables. I would say you know substitute all these guys into the respective places wherever they occur.

So, as a result you have your variables down to 6 and 6 equation is namely,

$$\tilde{\rho}_1 = \frac{\tilde{p}_1}{C_0^2}, \qquad \tilde{\rho}_2 = \frac{\tilde{p}_2}{C_0^2}, \quad \tilde{\rho}_3 = \frac{\tilde{p}_3}{C_0^2} \qquad (9)$$

So, we have 6 equation and becomes algebraically very messy. So, I hope to share with you the only the you know the way it is finally, written in a [Laughter] maybe in a new slide. But this time around in this particular fashion or this kind of a thing. So, now, notice,

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{p}_3 \\ \rho_0 C_0 \widetilde{U}_1 \\ \rho_0 C_0 \widetilde{U}_2 \\ \rho_0 C_0 \widetilde{U}_3 \end{Bmatrix} = [\,]' \frac{d}{dz}$$

the same dimensions as $\tilde{p}_1$ and that is why you know choosing this will eliminate any ill conditioning problems in MATLAB, because anywhere you are going to use the exponential function to sort of integrate. So, you know you have your dash. What does dash mean? Dash means d d z spatial derivative with respect to the axis.

So, this was your mind you this was your axis z is not it. So, you have your derivative and this. So, I will sort of just rub this guy and with understanding that you have a derivative sitting here, and this is your gigantic [Laughter] equation.

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{p}_3 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \\ \rho_0 C_0 \tilde{U}_3 \end{Bmatrix}' = \begin{bmatrix} A_{11} & A_{12} & \cdots & \cdots & \cdots & A_{16} \\ A_{21} & \cdots & \cdots & \cdots & \cdots & A_{26} \\ A_{31} & \cdots & \cdots & \cdots & \cdots & A_{36} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A_{61} & \cdots & \cdots & \cdots & \cdots & A_{66} \end{bmatrix}_{6\times 6} \begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{p}_3 \\ \rho_0 C_0 \tilde{U}_1 \\ \rho_0 C_0 \tilde{U}_2 \\ \rho_0 C_0 \tilde{U}_3 \end{Bmatrix}$$

So, you have, one can sort of populate the entire thing not a issue. So, we can do that.

$$\begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}_{z=0} = expm\ (-[A]L) \begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}_{z=1}$$

And again the procedure then is rather straightforward you know you basically, so let us say we have this big variable. So, at sections z is equal to 0 we equate by exponential e x p m and I would call this is A matrix over the length L times this entire thing. So, this is a road map at z is equal to l that is what we sort of get. So, its a big matrix you know namely 6, 6 cross 6 matrix as compared to 4 cross 4 matrix, because it is a fully couple thing ok.

And now you know what happens is that couple of things that I want to mention here. First of all if you have you know these equations are already looking very nasty. These equations [Laughter] are looking really sort of a big a bit difficult to handle in the sense of you know doing this hand calculation.

So, the hand calculations are anyways ruled out, you have to use a program some MATLAB, Fortran and all the things. In MATLAB it life is a bit easier like I said. Now what we have to do here is that a further simplification can possibly be done by understanding that you know going back to these equations, possibly noticing these things you know I was talking about Mach number being very small if it is 0.15 in the main duct where the flow enters.

So, its going to be even smaller in the annular cavity and the bias its going to be even smaller. The point I am trying to make is that the convective effect of mean flow in the duct as well as in the chamber that is duct 1 duct 3 and the overall chamber duct 2 can be simply be ignored while retaining the dissipative effect of the perforates in the perforate impedance expressions.

So, because that is what matters the most. Even if you were to ignore the convective effects and just fit in the dissipative effects it would be ok to get at least some kind of decent approximation, because remember perforate impedance tends to have a non zero t l part of resisted parts when the flow is there, you we saw from the expression given an Elnadis paper that we have been using in MATLAB codes.

So, we have pretty involve expression and you have to take the real part and imaginary part and all that. So, the point is that the mean flow definitely has adds to the real part, as we can you know sort of I can pull out the that equation this was the one ok

```
% It evaluates and returns the perforate impedanc
% pipe by using Elnady et al. expressions. (see E

% poro = Porosity of the inner pipe in fraction
% tw = Perforated pipe thickness in m
% dh = Diameter of perforated hole in m
% k0 = Wave number in 1/m
% c = Speed of sound in m/s
% mg = Grazing flow Mach no.
% mb = Bias flow Mach no.
% mu = Dynamic viscosity of the exhaust gases in
% rho = Density of the exhaust gases kg/m^3
% Cd = Coefficient of discharge
```

```

mu1 = 2.179*mu;
nu = mu/rho;
nu1 = mu1/rho;

K = sqrt(-1j*(k0*c)/nu);
K1 = sqrt(-1j*(k0*c)/nu1);

B = besselj(1,(0.5*K*dh));
J1_kd = besselj(1,(k0*dh));
B1 = besselj(0,(0.5*K*dh));
F_mu = 1-(4*B/(K*dh*B1));

B2 = besselj(1,(0.5*K1*dh));
```

```
B2 = besselj(1,(0.5*K1*dh));
B3 = besselj(0,(0.5*K1*dh));
F_mu1 = 1-(4*B2/(K1*dh*B3));

% -----------------------------------------------
% REAL PART:
% -----------------------------------------------
re1 = (1j*k0/(poro*Cd))*((tw/F_mu1)+(del_re*f_int
re1 = real(re1);
re2 = (1/poro)*(1-(2*J1_kd/(k0*dh)));
re3 = 0.3*mg/poro;
re4 = 1.15*mb/(poro*Cd);

theta = real(re1+re2+re3+re4);
```

So, so this was the MATLAB script, MATLAB port. So, this is the real part and you see means flow is there.

```

theta = real(re1+re2+re3+re4);

% -----------------------------------------------
% IMAGINARY PART:
% -----------------------------------------------
im = (1j*k0/(poro*Cd))*((tw/F_mu1)+(0.5*dh*f_int/
khi = imag(im);

xeta = theta+1j*khi;

end

```

So, it definitely does contribute to the resistive losses or dissipative losses in the mean flow thing. So, especially when the if you. So, be the point is that if you include these nonzero mean flow in the expression, and sort of ignore or purposely do not consider this U_3 big U_3 or U_2 for that matter, U_3 or you know U_1 you will be still better off. You can simplify things and yet retain the most important effects.

So, what we do in the next class that is tomorrows class, lecture 2, I mean, is that we will set up these equation with a little bit of simplifications and you know do some MATLAB coding and you know do some MATLAB demonstration on for simple configurations and try to evaluate the transmission loss, at least for the case of non zero a for the case of 0 mean flow by assuming suitable expressions, perforate and impedance expression.

And possibly you know h doing some calculations for non zero mean flow a non zero grazing and bias flow in the pipes. So, we will do all those things and hopefully that should set us up for you know advanced configurations in this week and possibly in the next one. So, still that time.

Thanks lot and I will see you tomorrow.