Muffler Acoustics-Application to Automotive Exhaust Noise Control

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Lecture - 32
Aeroacoustic State Variables Transfer Matrix for a Tubular Element (Uniform Pipe)

Welcome to week 7 of our NPTEL course. We are in week 7 lecture 2. So, we are going to talk about the **Aeroacoustic State Variables** that I kind of just mentioned at the end of my last lecture. So, aeroacoustic state variables that are what we are going to do now. So, these obviously, come these basically the need for you know obtaining or defining new variables aeroacoustic state variables means that we want to add a few extra terms to the classical variables.

So, in order to better suit can cooperate the convective effects of mean flow Mach number m naught. So, the total acoustic energy flux or acoustic power that is carried by a mean flow is given by

$$W = M^{'}J^{'}$$

where the bar indicates really an average over the radius. So, M dash is your perturbation in the mass velocity.

$$M^{'} = (\rho_0 + \tilde{\rho})S \ \left(V_0 + \tilde{U}\right) - \rho_0 S\, U_0$$

So, now, when we multiply out the terms that

$$= \cancel{\rho_0 U_0 S} + \rho_0 S\tilde{U} + \tilde{\rho} S U_0 + \cancel{\tilde{\rho} S\tilde{U}} - \cancel{\rho_0 S U_0}$$

So, let us see what happens. This thing goes away and these terms again will drop out because and ρ_0 they are multiplied, they are second order quantities and they will be much sort of smaller than your other quantities that is your $\tilde{U}$ and this one and this one.

So, basically after a little bit of algebra what we get is the perturbation mass velocity is nothing,

$$M^{'} = \rho_0 S\tilde{U} + \tilde{\rho}\, S U_0$$

So, this is what we are going to get and this we can write as perturbation mass velocity plus your p tilde by

$$= \tilde{V} + \frac{\tilde{p}}{C_0^2}\, SU_0$$

So, we can further sort of simplify these guys and

$$= \tilde{V} + \tilde{p}\,\frac{S}{C_0}\,\frac{U_0}{C_0}$$

$$= \tilde{V} + \tilde{p}\,\frac{M_0}{Y_0}$$

So, we have an expression for the perturbation mass velocity when you have a nonzero flow which is this thing.

$$M' = \widetilde{V} + \tilde{p}\,\frac{M_0}{Y_0} \qquad (1)$$

Clearly, when M_0 you again get back m dash is V dash that is nothing, but acoustic mass velocity that is the perturbation velocity. The same thing we are getting here, but an extension of what is done in the classical variable. The classical variable is just V dash. Please understand, this point clearly the classical variable is just v dash, but when you have a mean flow of Mach number m naught, then you need to add an additional term which is of this thing.

Now, similarly we can also try to do in this short space what happens to the stagnation perturbation stagnation enthalpy that is head that iswe saw that only briefly in the last class

$$J' = \frac{p_0 + \tilde{p}}{\rho_0} + \frac{(V_0 + U)^2}{2} - \left(\frac{\rho_0}{\rho_0} + \frac{V_0^2}{2}\right) = \frac{\tilde{p}}{\rho_0} + U_0\widetilde{U}$$

.So, now we need to do some algebra, but I will sort of skip that because we have already donein the last class entire idea is about cancelling terms you know it is clear that these terms and this term will cancel away and this term will be retained and some second order terms will drop out. So, once we do basic things some simple algebra we are going to get exactly what we saw in the last class.

$$J' = \frac{\tilde{p}}{\rho_0} + \frac{U_0 C_0 \widetilde{U} S}{C_0 S}\,\frac{\rho_0}{\rho_0}$$

So, I need to go to the another page and kind of simplify what we just saw here.

$$J' = \frac{1}{\rho_0}\,\tilde{p} + M_0 Y_0 \tilde{V} \qquad (2)$$

So, that is what we are going to do and actually we are going to say something,

$$W_C = \frac{1}{\rho_0}\,V_C p_C$$

We will multiply and divide the numerator and the denominator and we are also going to do the same thing with the cross-section area S.

And, remember what we get here? There is no rho. So, if we multiply here by rho naught and there is U_0 times this thing. So, here we had also U_0 actually I am sorry and this was something like this. So, if we divide by density also and then happen to take the density common, ambient density mind you. So, this particular will term will be m naught is not it and C_0 by S will be 1 by Y. So, and this entire thing will be mass velocity.

So, we will get your $M_0 Y_0 \tilde{V}$. So, we have your (1) equation and (2) equation we have these kind of things. So, you know basically what we see again that when M_0 is 0 we again get back pretty much the classical variable, but it is just normalized by the ambient density for air it is 1.20 at standard temperature and pressure.

So, now we need to worry about what are we going to do with these variables. So, one thing that obviously, comes to our mind is used equations (1) and (2) basically back in the this particular main thing from where it all sort of started. So, clearly here as stagnation enthalpy is equal to the stagnation pressure divide by the ambient density, J dash is the perturbation stagnation pressure divide by the ambient density.

So, now what we are going to call then that this entire thing let me call this as p_c ok and similarly, this entire term is basically denoted as V_c. So, the reason that I sort of bought this out suddenly was that it will be very convenient to write out the power in terms of the convective state variables.

So, see here subscript c basically it means convective state variables. So, here we have this ok we have got this kind of a thing, alright. So, V_c is equal to 1 by rho naught into V_c p_c and we have a average over that ok. So, now, clearly we have defined these quantities and expressed the total power that is radiated in terms of the convective state variable.

So, how about we write you know in one dimensional muffler analysis as we have been noting from the beginning of this course the transfer matrices are probably one of the most easiest way to analyze a more complicated system. So, basically we must sort of develop a transfer matrix relation between the convective state variables,

$$\begin{Bmatrix} \tilde{p}_C \\ \tilde{V}_C \end{Bmatrix} = \begin{bmatrix} 1 & M_0 Y_0 \\ \dfrac{M_0}{Y_0} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p} \\ \tilde{V} \end{Bmatrix}$$

So, how do we do that? So, we are doing nothing, but we just calling this as V_c. So, V_c is this thing and your p c is this thing. So, we are just expressing p_c and V_c in terms of that is convective state variables in terms of your classical state variables. Now, clearly when M is 0, M_0 is 0 we get back what? The identity matrix and that basically your convective variable is equal to the classical state variable.

So, now, we need to do we need to probably go further. So, how do we go further? We will basically,

$$\tilde{p} = \vec{A} + \overleftarrow{B} \qquad \tilde{V} = \frac{A - B}{Y_0}$$

forward wave, backward wave and setting the coordinate system. So, when we do all those things and use these relations in the box here, so, we will get,

$$p_C = \tilde{p} + M_0 Y_0 \tilde{V} = A(1 + M_0) + B(1 - M_0)$$

So, now here is an opportunity to define yet another variable calling this as Ac and this guy as Bc ok. So, we have got this. Now,

$$\tilde{p}_C = A_C + B_C$$

$$A_C = A\,(1 + M_0)$$

In the similar manner where Ac is let mewrite it down again is nothing, but progressive wave variable, but it is now strengthened by the mean flow and this is nothing, but B, but this is strengthened by this thing.

$$= B(1 - M_0)$$

You know for a mean flow along the positive x direction significance of this is that the waves that travel in the forward direction they are strengthened by the mean flow M naught.

And, in the opposite direction it is natural that they will be weakened ok they will be weakened. Now, similarly what about v c?

$$\tilde{V}_C = \tilde{V} + \frac{M_0 \tilde{p}}{Y_0}$$

and this is after some algebra we will get this sort of thing.

$$= A\,(1 + M_0) + B(1 - M)$$

where we have pretty much the same relation between Ac, Bc with A and B. So, they are the same that is defined here ok.

So, basically the entire idea is that just like decompose this classical state variable pressure p into A plus B and classical mass velocity in terms of A - B the same thing we can directly decompose the convective state variables in terms of Ac and Ac Bc and similarly, vc volume velocity convective term can be decomposed as

$$\tilde{V}_C = \frac{A_C - B_C}{Y_0}$$

The idea is that to incorporate the convective effects of mean flow. Now, clearly we would let us sort of again go back to this particular expression and see and you know kind of relate or write down the acoustic total energy flux that is coming out of the duct in terms of this thing. So, we saw in the last class

$$W_C = \frac{1}{\rho_0} p_C V_C$$

$$= \frac{1}{2\rho_0 Y_0} \{|A|^2(1+M)^2 - |B|^2(1-M)^2\}$$

So, we have this kind of a thing ok what do we do for this thing? We have here W c given in terms of the convective state variables.

So, we as usual reflection coefficients were defined in the week 3 of 4 as B by A and now the convective reflected coefficients, it is natural to see that they are defined as Bc by Ac, is not it?.

$$W_C = \frac{1}{2\rho_0 Y_0} \left(|A|^2 - |B|^2\right)$$

$$R = \frac{B}{A}, \qquad R_C = \frac{B_C}{A_C}$$

So, what it would mean is that the total acoustic power that is carried by the waves in the duct with the mean flow, this guy can be written now, as

$$W_C = \frac{|A|^2}{2\rho_0 Y_0} \left(1 - |R_0|^2\right)$$

And, your Rc will become nothing, but

$$R_C = \frac{B_C}{A_C} = \frac{B(1 - M_0)}{A(1 + M_0)}$$

and this is what we are going to do Rc. We have this sort of a thing. So, reflective coefficient R c is R times 1 minus M naught divide by 1 plus M naught.

So, now, what it means is that in case of convective waves the reflection coefficient R c necessarily magnitude this is a complex quantity because remember R is also complex, butthe good thing is that the magnitude of

$$|R_C| \leq 1$$

Why? Because now when M is nonzero say let us say it is a you know we can ascribe any number we can sort of verify this.

We can put 1 M_0 typical values of 0.15 or 0.2 maximum value that sort of we can get. So, we get basically, $\frac{0.8}{1.2}$ or something like that. The point is that Rc will always be less than 1 because M_0 is positive, your denominator will be always be greater than 1 and numerator will be less than 1. So, this is just one sort of example since this guy has this relation Rc reflective coefficient is always magnitude of that is less than 1.

What we could further come to the conclusion is that

$$|R_C|\frac{1-M_0}{1+M_0} \leq 1$$

So, we have this sort of a thing. Reflection coefficient magnitude of

$$|R| \leq \frac{1-M_0}{1+M_0}$$

So, basically you know m like I said it is positive. So, in presence of your non-zero mach mean flow or non-zero Mach number all the small letters reflection coefficient can be greater than 1. It should be greater than 1 because your denominator is less than 1 and this is this thing.

So, that is one thing that obviously, comes out of all this calculations and clearly the total aeroacoustic power that is given through a tail pipe. So, if we have something like I mean by the virtue of all the expressions that we derived so far, the reflection coefficient for the classical wave variables can be greater than 1 when you have a non-zero mean flow in a duct, but in typical case it is less than 1 or equal to 1.

Now, going back to the previous slide we have R c which is always less than equal to 1. So, after doing all this thing what does it really mean you know it really means that for a moving medium when you have a reflection coefficient measured with the static probes that can exceed unity. You know that means, when you are measuring the reflection coefficient with the static probes the reflection coefficient in presence of a mean flow can exceed unity.

However, aero acoustic pressure can be picked up only through a total or stagnation probe facing the flow. So, another thing that I want to point out here is that the forward waves as we can see you know this is pretty obvious from the you know from the underlined terms the forward wave is really a amplitude of the forward progressive wave is A. So, it basically gets strengthened by 1 plus M. So, you have A into 1 plus M alright.

So, what it really means is that the forward wave gets strengthened with the mean flow by a factor 1 plus M, alright. So, and the power or the acoustic power associated with it also gets strengthened by $1 + M_0^2$. So, basically the incident power in the presence of flow basically gets it is increased by a factor of $1 + M_0^2$.

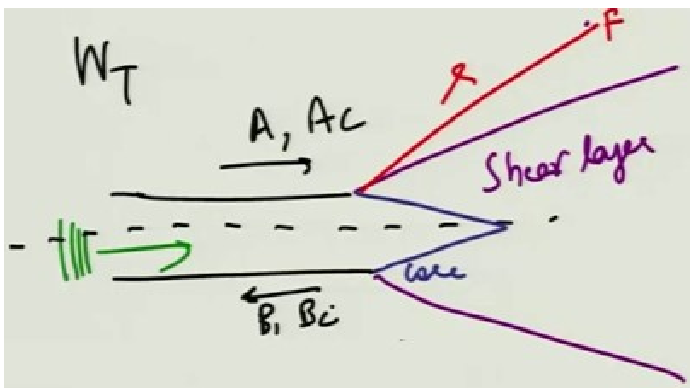
So, we can work out typically for a automotive muffler as we have discussed maybe 0.15 or 0.2 maximum can be the flow velocity Mach number. So, how much would be the power associated with it? We can do some calculation if

$$(1+0.15)^2 = 1.15^2$$

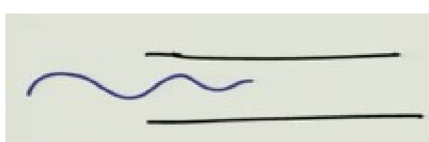
So, its 1.15 square and how much it is in let me figure out in a minute.

So, that is about $\simeq$ 1.32 basically a 32 % increase for a typical maximum flow velocity for automotive mufflers. So, that is quite a lot 32 percent increase. Another thing obviously, that is also obvious from these expression is that the wave that is going back is diminished by that much amount. So, we have B into 1 minus M.

So, the reflected wave is diminished by that much amount and the power is also diminished by a factor of 1 minus M square. So, that basically that would mean it is about $(1-0.15)^2 = 0.85^2$ which is = 0.73 you can say. So, it is about 27 percent reduction in the acoustic power that is carried by the backward propagating wave.



So, after having discussed all these things one thing that really comes to the to our mind look really we have a system something like this. You have a tail pipe and remember when we first started talking about acoustic real world application we had a situation like this in which you know waves are being incident and this is really an open end there is no flange and that is something what we did I guess in week 3 I believe.



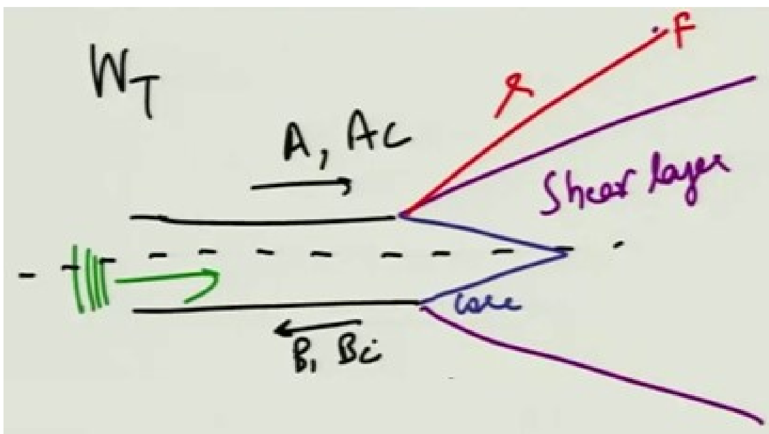
So, that was far more complicated. We have those expressions for the radiation impedance and that was basically given by,

$$Z_0 = R_0 + jX_0$$

are the reactants resistance and the reactants respectively and only for well low frequencies this is about

$$Y_0\,(0.5(k_0 r_0)^2 + j0.85 k_0 r_0)$$

So, this is what it was. So, this was the classical wave and you know due to the this is the radiation impedance imposed by the atmosphere. So, you have some waves that are basically reflected back from the atmosphere. So, now, a similar question that you know should come in our mind is that well, you know what happens when we have not just a stationary medium, but you have a flow mean flow along the direction along the axis of the duct. So, when we have such a thing, we would like to find out what happens to the total acoustic power radiated and what is the reflection coefficient in terms of this thing. So, these are sortof very much related to your resistance part of the atmospheric impedance. So, let us see howwe how we do that and before we before we begin discussing this let us first also understand that let us write down the expression for the total acoustic power that is radiated by the by theflow.



So, we have basically a potential core when the flow comes is it collapses and this is your potential core and then there is a shear layer there is a shear layer in which the rapid mixing of the jet happens. So, although it is like a you know by the time it is not a supersonic jet, it is pretty much a subsonic jet, but you still would have a potential core and you have a top hat profile and gradually becomes a Gaussian profile in the downstream.

So, you would have this is the mixing region really and or the shear layer as we say and shear layer and. So, if we want to measure at a certain point what happens to the at a certain radius or radiated we could do that and so, we begin again with a typical thing you have your A and Ac and B and B c your backward travelling reflected convective part. So, the total power that is radiated

$$W_T = \frac{1}{2\rho_0 Y_0}\{|A|^2(1+M)^2 - |B|^2(1-M)^2\}$$

So, we can actually write it a little more cleanly

$$= \frac{S}{2\rho_0 C_0}\left(\overrightarrow{|A_c|^2} - \overleftarrow{|B_c|^2}\right)$$

We have this sort of a thing.

And, let us see; let us see how we what do we how do we interpret all these things and why should it be useful. So, this W t is a total acoustic power that escapes from the pipe in the presence of flow and actually there is much more to it then what just meets our eye in this expression. This basically all this AB or A into 1 plus M, B into 1 minus M.

All these things basically tell you the sum total we are really interested in the net quantity and this is really the sum total of the total acoustic power contribution by monopole or dipole sources that they directly contribute to the far field and also because of the vortex shedding that happens in the wave. So, it is some total of all the direct field sources and the sources generated by vortex shedding in the wave that contributes to the far field noise that is observed at the point F.

And, another thing basically what people figured out from their analytical derivation is that W_F is which is basically the overall effect of the radiated power is a new expression.

So, I am so, it is basically if is the measure of the total power that is radiated from the jet. So, that is your spherical surface area into the intensity that is I_F. When you do that, this is approximately equal to W_T that is the theoretical power that is we obtained from this expression.

When you have an outgoing wave that is wave that goes in this direction the interaction of the outgoing waves with the shear layer or with the jet itself it produces some sort of a back reaction on the wave. So, as a result its not that the entire this power is radiated a fraction of that power is radiated and that is given by W_T empirically people have found out that this is given by

$$W_F = (4\pi r^2)IF \simeq \frac{W_T(kr_0)^2}{2M + (kr_0)^2}$$

The hypothetical spherewith the radius r and centered at say a point on the duct or probably it can be some point just at this point ok. So, we have this sort of a thing. Now, W_F is this. So, what it means is that the log of the difference if we basically were,

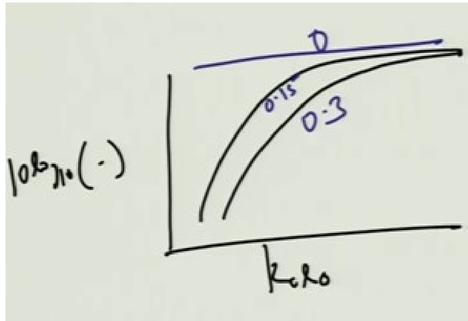
$$10\, log \frac{W_F}{W_T} = 10 \log \frac{(k_0 r_0)^2}{2M_0 + (k_0 r_0)^2}$$

This is the logarithmic difference of the total power theoretical or the actual power that is radiated by the jet to the theoretical possible power.

$$10\, log_{10} \frac{1}{2M_0}$$

So, you know people have plotted this thing and they what they observed is that you know basically for 0. Obviously, one thing that is clear when M_0 this quantity is this basically we have this quantity goes to 0 log of 1. So, basically then it is if there is no flow then W T and W F are the same, but for non-zero Mach number at very low $k_0 r$. Suppose let us put $k_0 r$ to be 0 tending to 0. So, we would get this as

$$10\, log_{10} \frac{1}{2M_0}$$



So, basically at very lowfrequencies let us say this is k naught r naught and this is your 10 times log of 10 entire quantity will go like this. So, the greater the Mach number the greater the difference will be inlow frequencies and then they will converge. So, this is the result for M_0 this let us say this 0.3, 0.15 and so on.

So, what basically it would mean is that the interaction of the sound wave and jet which attenuates the wave as it comes out amplifies the broadband noise. However, you know this is sort of inconsequential because even after amplification the jet noise remains insignificantly low compared to the sound wave. So, now, however, what really matters is the how the radiation real part of the radiation impedance in the presence of flow that is the resistive part. How does that vary with your flow and people have developed some expressions like Mechel.

Mechel has developed an expression in one of the references where he relates the radiation resistance in presence of flow which is a function of Mach number M_0 to the characteristic impedance of the duct as by given by the following relation.

$$\frac{R_0(M_0)}{Y_0} = \frac{R_0}{Y_0} - 1.1M$$
$$= \frac{(k_0 r_0)^2}{4} - 1.1M_0$$

Similarly, **Panicker and Munjal** a long time back they derived another expression another empirical expression which is given by the following relation you have this thing. So, what this would mean is that like this there you can find a number of such relation for the radiation resistance in terms of mean flow parameter. What it would really mean is that at sufficiently low frequencies this quantity or this quantity can be negative also.

$$\frac{R_0(M_0)}{Y_0} = \frac{R_0}{Y} - 2M_0^2$$

So, all these things will impact the radiated noise or the radiation or the atmospheric impedance in presence of mean flow, but I guess we probably have to move now more to the development of the transfer matrices in the presence of mean flow.

So, our next topic right in this lecture before we complete this lecture the couple of more things to be covered is basically development of at least the simple expression for the transfer matrix of a tube.

Transfer Matrix For a Uniform Area Pipe in Presence of Flow in The Presence of Flow. So, how would that happen? So, as we saw you know the pressure the $\tilde{p}(z)$ harmonicity is sort of is implied. So, what we would do is basically,

$$\tilde{p}(z) = Ae^{-jk_0^+ z} + Be^{jk_0^- z}$$

A from our previous theoretical developments that we did when probably in the 2nd weeks lecture the first couple of lectures the solution was something like this, is not it?

Assuming only planar waves no higher dimensional waves k naught z plus this. So, Iwould sort of put a thing here and

$$\tilde{V}(z) = \frac{1}{Y_0} Ae^{-jk_0^+ z} + Be^{jk_0^- z}$$

a mass velocity was given by 1by k_0 A ok. So, this was the case where what about k naught plus and k naught minus and here we are typically you could also consider more complicated or more sort of realistic situations such as damping or the absorption.

Basically you have in a viscous medium and a turbulent incompressible mean flow due to friction factors and due to absorption the wave attenuates also. So, if we were to discard all those things and consider a simple case of the non-zero absorption in the medium or basically, no dissipation in the medium lossless medium as we are always considering because a lossy medium a little more complicated to deal with will keep the matter simple and that is why above expressions will apply.

There is no changes in Y naught. Y naught is nothing, but you know sound speed divide by cross-section area of the duct c naught by

$$k_0^+ = \frac{k_0}{1+M_0}, \qquad k_0^- = \frac{k_0}{1-M_0}$$

So, basically what do we do with this expression? So, what we do let us consider

So, we can sort of write this as So, this I can introduce a new

$$k_0^+ = \frac{k_0}{1+M} = \frac{k_0(1-M_0)}{1-M_0} = k_c(1-M_0)$$

So, all these things will apply and then under such a situation these two equations that we have this one and this one can be written perhaps in a little bit more cleaner form

$$k_0^- = k_c \ (1+M_0)$$

$$\tilde{p}(z) = e^{jMk_c z}\left(Ae^{-jk_c z} + Be^{jk_c z}\right)$$

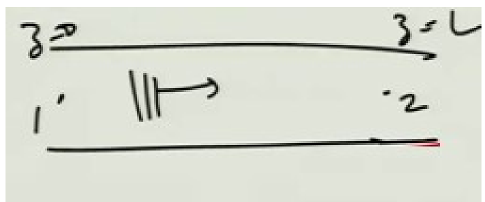
And, mass velocity will

$$\tilde{V}(z) = \frac{e^{jMk_c z}}{Y_0}\left(Ae^{(-)} - Be^{(-)}\right)$$

So, what happens is that now these expressions let us say this one and these two ones they are almost nearly identical with the case and there is no mean flow in a duct in a duct that is with a stationary medium because you see the forms A into e to the power minus jk_0z and B into e to the power j k_0z.

So, k_c is playing the role of k_0, where as we see k c is nothing, but this term that is encircled somewhere here is not it; k naught divided by $1 - M_0^2$. So, clearly when you have M_0 these expressions reduce to the old form. So, we need to recall our last well several lectures before what we did.

Using all this thing it still involves a little bit of maths, but there is a reason why we put these two expressions in the form that we are seeing that is basically to get transfer matrices derived. So, apart from this particular term apart from this term and this term everythingelse is pretty much the same.



So, one would naturally guess I would really encourage you guys to do the algebra. I am leaving the algebra you know for you guys to work out how will you basically eliminate A and B let us say you have a duct you know with a flow in this direction and one between 0.1 and 2 you would want to derive the transfer matrices is not it. So, how would you go about doing that?

So, what we probably could do is that apart from this term like I said we have pretty much the same thing. So, we could basically just write down between the points 1 and 2 you can say z is equal to 0, z is equal to L,

$$\begin{Bmatrix} \tilde{p} \\ \tilde{V} \end{Bmatrix}_{z=0} = e^{-jMk_c l} \begin{bmatrix} \cos k_c L & \mathrm{j}\mathrm{Y}_0 \sin k_c L \\ \dfrac{\mathrm{j}}{\mathrm{Y}_0} \sin k_c L & \cos k_c L \end{bmatrix} \begin{Bmatrix} \tilde{p} \\ \tilde{V} \end{Bmatrix}_{z=L}$$

So, the form this transfer matrix is the same k naught is replaced by k c and you have an extra term which is your exponential that you just saw somewhere here. So, again I am insisting that please work out the derivation I am just not showing with the steps involved. So, here you have L. So, let me sort of write it a bit more cleanly as. So, this is e to the power minus j M into kcl.

So, when M is 0 you get back your whole transfer matrix. So, this is the stuff that we are really aiming for to do in this class I mean one of the major objectives in this class. So, now, basically you know what we see is that we see a relation between the point 1 and 2 in terms of the mean flow.

So, the corresponding what we could also possibly do? You know if you recall the last lecture in this week you know we had this relation this particular guy. So, what does it do? It basically does it does relate the convective variables with the classical state variables. So, p and V are just the inverse of this into this, is it not?

So, what we could just probably do one thing? We could you know now that we have derived this guy what we could do that this is equal to the matrix that I had shown you that is the inverse of that and this matrix is we could basically relate it, is it not? So, I just write it down that matrix the matrix form sort of again what was given in slide 16.

So, the inverse of this matrix times this particular guy is this thing. So, basically what will happen is that I can instead of p and V at z is equal to 0, I can write this entire matrix inverse.

$$\begin{Bmatrix} \tilde{p}_c \\ \tilde{V}_c \end{Bmatrix}_{z=0}$$

This should be sort of quite clear to you guys that $\tilde{p}_c$ and $\tilde{V}_c$ of course, at z = 0.
So, we have this guy sitting here this thing. So, we have this, but actually before we do that we probably

$$\begin{bmatrix} 1 & M_0Y_0 \\ \dfrac{M_0}{Y_0} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_c \\ \tilde{V}_c \end{Bmatrix}_{z=0}$$

$$= e^{-j(-)} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} 1 & M_0Y_0 \\ \dfrac{M_0}{Y_0} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_c \\ \tilde{V}_c \end{Bmatrix}_{z=L}$$

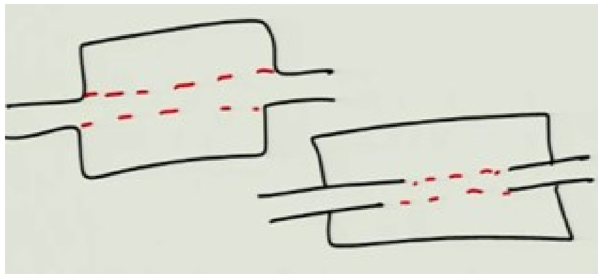
Now, we have so, this thing. So, we what do we do? Instead of this thing ok. So, let us go back to slide 16. So, this again we have to invert this. So, inverse of this times $\tilde{p}_c$ and $\tilde{V}_c$ at z = L is what is going to do the trick for us. So, here we have no choice, but to invert it and when we now we need to pre multiply this thing so, basically what it would mean is that. So, inverse of this, you need to multiply basically,

$$=> \quad \begin{Bmatrix} \tilde{p}_c \\ \tilde{V}_c \end{Bmatrix}_{z=0} = \frac{e^{-j(-)}}{1-M_0^2} \begin{bmatrix} 1 & M_0Y_0 \\ \dfrac{M_0}{Y_0} & 1 \end{bmatrix} \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} 1 & -M_0Y_0 \\ -\dfrac{M_0}{Y_0} & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_c \\ \tilde{V}_c \end{Bmatrix}_{z=L}$$

With the same matrix again. So, what we would basically end up is your e to the power this quantity this is there and here. Now, the inverse of this has to be written. So, when we do some linear algebra you could go to some symbolic package mathematica, maple or do some you know hand calculation and figure out for yourself that the inverse of this matrix.

And, of course, here it is $1 - M_0^2$, ok. So, after all this algebra we get this. After basically pre multiplying and all those sort of stuff the equation that I just sort of presented here, that will very cleanly reduce to the following form.

$$\begin{Bmatrix} \tilde{p}_C \\ \tilde{V}_C \end{Bmatrix}_{z=0} = e^{-jMk_cl} \begin{bmatrix} \cos k_cL & \mathrm{j}Y_0 \sin k_cL \\ \dfrac{\mathrm{j}}{Y_0} \sin k_cL & \cos k_cL \end{bmatrix} \begin{Bmatrix} \tilde{p}_C \\ \tilde{V}_C \end{Bmatrix}_{z=L}$$



So, we clearly see that even the convective state variables relation the transfer matrix between theconvective state variable that z is equal to 0 and z is equal to l, they remain intact.

So, it has pretty much the same form as this guy and then this basically can be used for a number of other things like you know connect variables between discontinuities and where we have to include loss factors where are which are basically found out through experiments in the literature.

So, we will discuss about couple of discontinuities in the next class like extend inlet and outlet discontinuity and how does that reduce to the sudden expansion type of discontinuity and probably you know get some simple expressions and perhaps why do we need to sort of why we do not really consider a discontinuity at as it is.

I mean to say you know in spite of all these things all the things that is taught in the last few weeks and this thing, we would probably never use the muffler you know just use something like this sort of a muffler or probably you would even never use an extended outlet and extended inlet and outlet just like this.

We do something here to connect this thing through perforated bridge otherwise flow creates a lot of problems; you would definitely need a perforated bridge. So, all these things are subject matter of the next lectures in this week. So, why do we need perforates and little bit background and all that.

So, till that time thank you, stay tuned.