Muffler Acoustics-Application to Automotive Exhaust Noise Control
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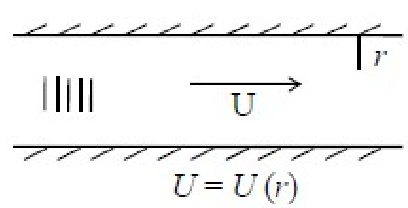
Lecture – 31
Acoustic Intensity (Energy Flux) in a Pipe with Mean Flow, and Transmission Loss Expression

Welcome to week 7 of our NPTEL course on Muffler Acoustics. Here, we have Application to Automotive Exhaust Systems. Now, in this week what we are going to do week 7, we are going to for the first time introduce the effect of mean flow on the acoustic performance of mufflers.

And before we do that, actually it is a little bit involved topic, because now, we are going to see the effect of mean flow on the perturbation quantity and how all these things are going to affect the muffler performance. So, what we need to do first is that we need to sort of develop new variables called aero acoustic state variables, and before even we do that what we need to do is basically consider some expressions for the acoustic intensity due to flow in a pipe, when we consider the mean flow effects.

So, let us see how we do it. What we will do is that the flow that is let us say M, being the rate at which the flow occurs, mass flow rate in a long pipe something like this. So, we have velocity u, u can actually vary, small u over the cross section.



$$M = \rho_0 \int_S U(r) 2\pi r dr$$

So, now, typically when you have flow over ducts, we have a profile you know let us denote the mean flow Mach number which is a function of radius as

$$S = \pi R^2$$

where u bar as a function of r is nothing but, your average velocity. And typically, this is usually taken as

$$M(r) = \frac{\overline{U}(r)}{C_0} = M_0 \left(\frac{r_0 - r}{r_0}\right)^{\frac{1}{7}}$$

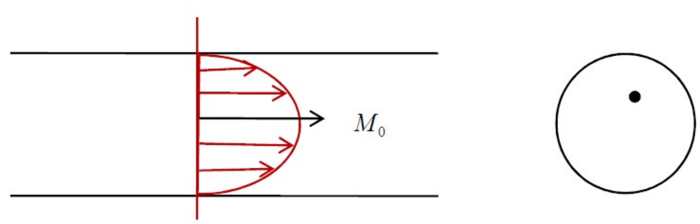
So, basically this expression, what it does? It gives us good idea about how the Mach number varies over the cross section radius r and taking into account the effect of boundary layers and all that. So, this is typically, valid for fully developed turbulent flows at the Reynolds number of the order typically,

$$Re \simeq 10^5$$

or one-tenth or you know something like that.

This is typical in internal combustion engine exhaust pipes of internal combustion engines. And what we also observe that the average value of the Mach number is typically about $0.85\ M_0$, Mach number observed at the right at the center.

So, the flow profile will be something like, if I draw allow me to draw an exaggerated view, so it will be something like this sort of a thing you know this. So, here we are talking about M_0 here.



So, here we have average Mach number. I am talking about is something.

$$M_{avg} = 0.85\ M_0$$

Now, what about expression for the average acoustic intensity for the waves that flow in a moving media? So, let us denote this by

$$E = \int N_i \, dS$$

the intensity over a small elemental area d S when you have this is the circular part, so something like this. So, N_i dS something like this. So, we have this kind of a thing, E is equal to N_i dS.

Now, basically, we have this. So, N_i is the **instantaneous energy flux that is normal to the surface element.**

We have that sort of a thing. We can write this N_i as enthalpy J M_i, ok, where M_i is the mass flow rate and J being the enthalpy.

So, what we do is basically, put energy as integral over the cross section area

$$E = \int_S Jm_i dS_i$$

where i really means some i-th element in a big complicated muffler. There is nothing very secret about the number i. And J is the stagnation enthalpy that is if the flow were to be very gradually bought down to rest.

$$J = h + \frac{V_1^2}{2} \; 1$$

$$M = \rho V_F$$

$$= \rho V S$$

we have this. And the mass flow rate is nothing, but density times volume velocity. So, this is nothing but rho u S. So, when we combine these two things we get this.

So, enthalpy stagnation enthalpy, I would, let me write down this thing more clearly. **Stagnation enthalpy** is the sum total of the enthalpy of the flowing fluid plus the velocity head term that is with the flow were to be brought down to rest and what about M i?

$$M_i = \rho V_i$$

So, we have really got two variables. Now, what we are interested in acoustics is really the perturbation quantities. So, how do we describe that or how do we? Let us first of all denote these perturbation quantities. So, we denote the perturbation quantities as

$$J', V_i = U_0 + \widetilde{U}$$

And let us first also look at the volume velocity term. We can write this as the mean flow velocity that is u naught that is that sort of an average velocity over the pipe plus your perturbation acoustic velocity.

So, if this situation is something similar to like a person, you are observing the person on the ground and the train is sort of moving and there is a there is a passenger in the compartment. So, it is doing oscillation from one seat to another seat. So, you would still see a net movement of the particle or of the person, but it is also oscillating. So, that is sort of an situation.

So, let us focus back on this equation. So, we have this sort of a situation then where V_i or the let me write it properly V_i is sum total of your u naught that is the mean flow mean flow velocity and your small perturbation velocity. We also used to call it u tilde, so, I will sort of retain the same convention. So, what happens then to the perturbation stagnation enthalpy?

So, you know when you have a small perturbation over the ambient thing then we can write this as,

$$J + J' = (h + h') + \frac{1}{2}\,(U_0 + \widetilde{U})^2$$

$$= h + h' + \frac{1}{2}\left(U_0^2 + \widetilde{U}^2 + \widetilde{U}\right)^2$$

$$=> \quad J + J' = h + h' + \frac{U_0^2}{2} + \cancel{\frac{\widetilde{U}^2}{2}} + U_0\widetilde{U}$$

So, then we can sort of regroup the terms and write these guys as. Another thing that I want to point out that $\widetilde{U}$ being small, $\widetilde{U}^2$ will also be very small, so we tend to sort of neglect that as we have done in the first week of the course when I introduced these perturbation quantities and talked about how small they can be. So, the second order terms will be you can neglect it with respect to the first order terms. And you can again sort of regroup this and

$$=> \quad J + J' = \left(h + \frac{U^2}{2}\right) + \left(h' + U_0\widetilde{U}\right)$$

And so, basically what we get after doing all this algebra is that we get

$$=> \quad J' = h' + U_0\widetilde{U}$$

So, we get this perturbation stagnation enthalpy. Similarly, we can also get the mass perturbation mass thing and that is M'.

$$M + M' = (\rho_0 + \tilde{\rho})S\left(U_0 + \widetilde{U}\right)$$

So, if we were to systematically multiply the terms, so this is your average thing and this is your perturbation thing.

$$M + M' = S\left\{\rho_0 U_0 + \rho_0\widetilde{U} + \tilde{\rho}U_0 + \tilde{\rho}\widetilde{U}\right\}$$

$$M + M' = S\rho_0 U_0 + \rho_0 S\widetilde{U} + \tilde{\rho}SU_0$$

$$=> \qquad M' = S\rho_0\widetilde{U} + \tilde{\rho}U_0 \qquad (1)$$

$$J' = h' + U_0\widetilde{U} \qquad (2)$$

So, we have another relation between the perturbation mass flux, is your M dash is equal to S times cross section times $\rho_0\widetilde{U}$ + $\tilde{\rho}U$ and this is these things. So, I will rewrite this stagnation thing again, perturbation stagnation enthalpy that is your h dash like this, ok. So, this is what we get.

Now, once we have these two quantities, so now, is a time when we go back to the expression for the or probably we write the equation for the energy total acoustic power coming out of the duct carrying a mean flow as something like this.

Average

$$< E > = \int_S (J' M') dS_i$$

So, intensity clearly means is

$$I = (J'M') \qquad (3)$$

So, we have this sort of a thing. So, what we need to do is basically, in order to develop an expression for the intensity that goes through the duct, you know acoustic power that goes through the duct, we will probably have to substitute equations (1) and (2) in equation (3). So, once we do that we will get your after some algebra I am going to sort of avoid doing that, we will do those things.

So, we will get

$$I = \tilde{\rho}\tilde{U} + \frac{U_0}{\rho_0}\tilde{p}\tilde{\rho} + U_0\,\rho_0(\tilde{U})^2 + U_0^2\left(\tilde{U}\tilde{\rho}\right)$$

Here p' and U' will be in general functions of the radius. It is quite natural to get that because; pressure and velocity will both gradually have a variation over the cross section area.

So, basically what we can do is that we can box this thing. So, this is the average time averaged acoustic intensity that goes out of the pipe, but then again i is a function of r, because these variables will depend on the radius. So, now, in order to find out the total acoustic power that is radiated, so, we just need to simply integrate the intensity over the cross section area. So, once we do that we will get the following expression.

$$W = \int_S IdS \;\; <> \rightarrow \;\; \text{time average or RMS}$$

So, we will get the expression which

$$W = \frac{1}{\rho_0}\left[< p\tilde{V} > + \frac{M}{Y_0} < \tilde{p}^2 > + MY_0 < V^2 > + M^2 < \tilde{p}V^2 >\right]$$

$$M = 0$$

$$M = \frac{U}{C_0}, Y_0 = \frac{C_0}{S} \qquad (4)$$

So, here v like I said is the acoustic mass velocity and M is the space averaged or the mean flow number that is averaged over the duct cross section area and y naught is your characteristic impedance.

This is the total acoustic power time averaged. Remember this is much more complicated or it is more tedious I would say as compared to the case where the flow was not there. So, now, as a check of self-consistency if we put M = 0 that is this term is not there, when you put M = 0 no flow and all these things will go away. And what will be left is just this term. And if you remember recall this is your acoustic power that is radiating out of the duct, when there is no flow involved.

$$\tilde{p} = Ae^{-jk_0x} + Be^{jk_0x}$$

So, now basically, what we can do? One would naturally like to use this expression that is developed for the case of nonzero mean flow. And then put suitable assumption like, let us say pressure field at a particular we will put you know typically, we

$$\tilde{p} = e^{-jk_0x} + Be^{jk_0x}$$

So, here, we will put x is equal to 0 or z is equal to 0 whatever suits us and we have this.

$$\left.\begin{aligned}\tilde{p} &= A + B\\ \tilde{V} &= \frac{A - B}{Y}\end{aligned}\right\}, \qquad \frac{B}{A} = R \Rightarrow B = RA$$

And your this velocity is nothing but, A minus B by y. And we will assume that B by A is equal to R, meaning that reflected wave or backward propagating wave is reflection coefficient times the incident amplitude wave. Now, R is the reflection coefficient which can be complex also. So, this can be complex quantity much like your the progressive wave variables p and A ok.

So, now, once we substitute these two guys in the big expression that we sort of developed here, and I am going to sort of avoid the algebra and just work on you know just present to you the final result of how it would look like. So, we will get the power, acoustic power that is radiated over duct as

$$W = W(M) = \frac{1}{2\rho_0}\frac{|A|^2}{Y_0}\{(1+M)^2\ \ |R|^2\,(1-M)^2\} \tag{5}$$

$$M = 0$$

So, we will get this sort of a thing. This is the expression for the total power radiated when you have nonzero mean flow. Clearly, when you have M = 0, so, this will and this should sort of reduce to the more familiar expression that

$$W = \frac{|A|^2}{2\rho_0 Y_0}\{1 - |R|^2\}$$

Now, if we just do some algebra take a inside this thing we will have mod A square minus mod of A into R whole square that is nothing your that is nothing but v.

So, this is something that we have seen in the very first few lectures of this course. So, let me box this expression. So, this is something that we kind of we will be using when you want to develop the expression for the transmission loss, in the presence of mean flow. So, let us do that now, before we move ahead to the case of air acoustic state variables. Before that we probably would like to make some comments on the nonzero Mach number.

So, as we see here from this particular expression let me call this equations (3), this is (4), this guy is your (5).

So, from 5 we get to see that W (M) or the acoustic power radiated out of a duct which is carrying a nonzero mean flow in general that is greater than the acoustic power radiated over the duct when you when you do not have any mean flow at all. So, that is for all M.

$$W(M) > W\ (0)\ for\ all\ M$$

$$R_{max}$$

And the error that would be caused by neglecting the mean flow for a given reflection coefficient would be positive and more significant around mod R which is approximately equal to 1 unity. Let us evaluate the expression for the maximum reflection coefficient R max. So, what we can do? So, R max then as we can see what is you know let us revisit equation (5) and figure out, what does this term mean.

So, as we discussed just a while back, this thing probably means that the acoustic power that is carried by the wave that is going in the positive z direction or along the duct as we can see and then so, this is what it is. And what about the wave that is travelling in the backward direction that is in this thing? So, that is basically or this part into this. That is your this guy is multiplied by this.

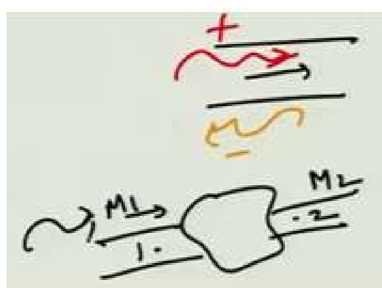
So, it has certain meaning. So, you know we will keep this thing aside for a minute, and worry about the acoustic power incident that is

$$W_{inc} = \frac{|A|^2}{2\rho_0 Y_0} (1 + M)^2$$

$$W_{refl} = \frac{|A|^2}{2\rho_0 Y_0} |R|^2 (1 - M)^2$$

So, we have this sort of a thing. Now, clearly reflection coefficient can be maximum only when the incident thing

$$W_{inc} = W_{refl}$$



That is whatever in a duct carrying a mean flow whatever acoustic power is being transported by a wave that is moving in the positive x direction, the same thing is being reflected back by the wave that goes along the negative x direction. So, when that will happen, so, we have to really equate these two terms.

So, when we do this obviously, your this will cancel and so, this will this, resulting.

$$(1 + M)^2 = |R|^2 (1 - M)^2$$

$$M = 0$$

So, this is your maximum reflection coefficient. Maximum because, I just explain the physical significance when this happens. Basically, you know your entire acoustic power that is being carried in the positive z direction is being reflected back. So, under such a situation

$$=> |R|^2 = \frac{(1+M)^2}{(1-M)^2}$$

So, basically, this would mean, so this would mean

$$|R|_{max}\ |R| = \frac{1+M}{1-M} = |R| = 1$$

So, this is your maximum reflection coefficient. Now, clearly there are few interesting things I would like to talk about here. When M = 0, so, R_{max} can just be equal to 1 when M = 0, is not it.

We have just said M is 0 and you will get reflection coefficient R, maximum R that you can get is 1 and that is in consistency with our physical explanation that whatever acoustic power is being carried by the forward propagating wave seen the entire wave is being reflected back. So, reflection coefficient is 1.

However, when you have a nonzero mean flow that is M, M is nonzero. It can be some number typically, you know in exhaust pipes it is about maximum is 0.3, 0.3 is really on the higher side, typically is about 0.15, 0.2. And once it goes in the chamber, because of expansion it is even smaller.

Although, we would not really allow it to expand due to a number of reasons which will discuss in this week's lecture, but all I am trying to say is that in the chamber it might be even lesser. So, under such a situation when you have a nonzero M, ok, so it can be even more, reflection coefficient can be more than 1.

Let us,

$$\frac{1.2}{0.8} = \frac{3}{2} = 1.5$$

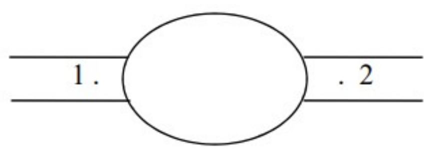
Reflection coefficient would be about 1.5. So, the point is here is that when you have a nonzero mean flow, it means that reflection coefficient in terms of the classical variables, classical progressive wave variables can be more than 1.

But that is probably the reason why due to number of all these things we it is a good idea to introduce some kind of a new variables, which are more suited for aero acoustic analysis. And we will refer to them as aero acoustic state variables, we will worry about all those things in the next set of lectures or next class probably. But what we can do now is just develop in the remaining time some expression for the transmission loss.

So, we saw from our I think from our week 4 or 5 I guess, probably week 4 I reckon. Week 4 we saw the transmission loss expression in terms of the 4 pole parameters that is you have your system.

$$\begin{Bmatrix} \tilde{p}_1 \\ \tilde{V}_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \tilde{p}_2 \\ \tilde{V}_2 \end{Bmatrix}$$

So, if we have a system which is being characterized like this, I would say 1, 2, ok, the transmission loss then is given by



$$TL = 20log_{10}\left[\sqrt{\frac{Y_2}{Y_1}}\ \frac{T_{11} + T_{12} + Y_2T_{21} + \frac{Y_2}{Y_1}T_{22}}{2}\right]$$

So, we have this. And here we have this sort of a thing is not it. This is for the case when you have no mean flow, that is 0 mean flow. But however, you like to extend all these arguments to the case when you have a nonzero mean flow, that is M is something like whatever we get typically in a automotive muffler. So, let us make use of our understanding for the using the expression that we just developed now, with the acoustic power carried by the wave when you have a nonzero flow.

So, let us say we when we have a system you know, it can be any complicated muffler system. So, we have inlet port here, outlet port here 2, and in the classical state variables that is p_1 V_2, p_2 V_2, the system is still characterized by this sort of representation ok. But here, you also have mean flow. In the inlet pipe let the mean flow be M_1 and here it will be M_2. So, it is pretty straight forward to see that the acoustic power that is carried by the incident wave variable that is the wave that goes in this direction is nothing but this term.

So, we will write this as

$$W_{inc} = \frac{|A|^2}{2\rho_0 Y_1} \quad (1 + M_1)^2 \quad \rightarrow$$

$$W_{trans} = \frac{|A|^2}{2\rho_0 Y_2} \quad (1 + M_2)^2$$

We will have this sort of a thing. And the one that is incident that is going in this direction is that is basically transmitted, is basically your 1 + M_2 whole square, is not it.

This is the only thing that you need to worry about A_1 and A_2 can be figured out from this thing when you substitute back the classical variables A_1 minus B1 by y1, and all these things here and same thing over here. So, basically, once you do that we will put these two expressions in the transmission laws and let us see how the expression for transmission loss here is modified.

This is given by,

$$TL = 20log_{10}\left\{\sqrt{\frac{Y_2}{Y_1}} \quad \frac{1+M_1}{1+M_2} |\underline{\hspace{3cm}}| \right\}$$

The big expression that we just mentioned here, this entire thing I am not going to write it here. So, you can also have a look at the book by Professor Manjal, in the third chapter where all these derivations are given in a greater detail. So, the point is that in the presence of mean flow expression for transmission laws will be pretty much the same, bearing the fact that there will be additional terms due to mean flow present in this thing.

So, naturally one corollary that happens that occurs is that when the Mach number is same in both the pipes, which can be the case when the pipe has the same diameters. So, then you have this term basically goes away, and you get back the classical expression of transmission loss. So, now I guess I will stop here in this lecture.

And in the next class, class 2 of week 7 what we will do is that we will introduce for the first time air acoustic state variables. So, M' and J' is what we talked about and obtained expression for acoustic intensity in presence of nonzero mean flow. So, what if we if we basically do some manipulation of the terms, we will see that we can probably relate $M'$and J' in terms of the classical variables, but then you have presence of mean flow, M terms, Mach number terms will be there.

And we will develop some you know again expression for the acoustic power radiated and carried by the forward moving wave and that carried by the backward moving wave and all that. And possibly, talk about transfer matrices for tubular elements or extended outlet elements and do all those sort of things. So, that will be the focus of the next lecture or something like that. Thanks. Stay tuned.