**Muffler Acoustics - Application to Automotive Exhaust Noise Control**

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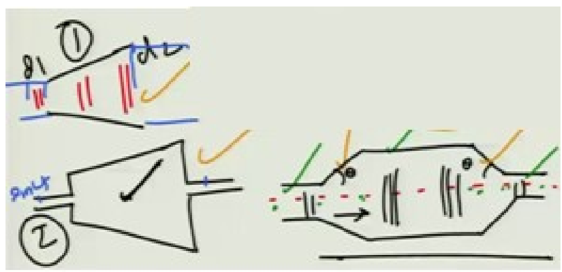
**Lecture - 30**
**Segmentation Approach for Analysing Gradually Varying area Ducts (MATLAB)**

Welcome to lecture 5 of week 6 on the NPTEL course on Muffler Acoustics. So, in the last lecture, we stopped at the analytical solution of a conical duct and we did some live examples on the transmission loss of different conical mufflers.



Just to remind ourselves what we did in the last class was basically you know analyze configurations like these ones where the ones that are noted here. So, this was a conical chamber is more like a gradually varying area flare with a constant slope or a constant flare angle.

And it could be used as a simple expansion chamber muffler like the one that is mentioned here, configuration 2 where I am pointing or it could be use something like a diffuser kind of a thing and obviously, the this particular thing has a much better transmission loss performance, but then, their configurations also which where conical flares are used the ones where I am highlighting here.

So, now, the thing is we also talked about since this course is specifically focused on only mufflers and not on horns in general or not on you know physical acoustics, we would probably like to stick to only those gradually varying area configurations, which are sort of commercially more used.

Now, having said that conical mufflers are used, but then such mufflers probably can also be used in a number of interesting ways for example, let us talk about a configuration something like this you have your extended inlet, but instead of a constant cross section area, you have a gradually varying flare and you have things like this. So, what I would do is basically draw something like this, ok.

So, now, we have already seen a very similar configuration in our last few lectures, but what is so special about this one? We will get to know that in just a minute. So, as we can clearly see that, you know the radius is changing.

**Numerical Method for Varying Area Ducts**

So, if it is $R_1$ at the inlet and $R_2$ at the outlet or the other side a vice versa. That would mean that the area changes finite or it is varying gradually, it is varying according linearly because it is a cone.

And so, what we intend to do here is to basically analyze a transmission loss performance of such configurations. Often such configurations are used in your two-wheelers or basically other places where only limited area is available in one, at one place and, but gradually more space is available towards the end. Two-wheelers are this at the beginning of this week and they use popularly such mufflers there.

The problem really here is that to analyze the wave propagation using Webster's horn equation, when you try to solve this for sections like this, unfortunately there is no analytical solution for such a system, analytical solution does not exist, basically for areas in which wave propagation is in the annular region where the annular region is comprising of a space that is bounded by a uniform duct and gradually varying conical flare.

If you fix your x coordinate somewhere like here x so, what will be your area?

$$where, \qquad S(x) = \frac{\pi}{4} D(x)^2 - \frac{\pi}{4} d_0^2$$

So, the flow comes here, goes like this.

So, again your basically the idea is to use $Q_1$ and $Q_2$, where Q or $Q_{W1}$ $Q_{W2}$, where $Q_{W1}$ and $Q_{W2}$ are like the gradually varying area quarter wave resonators formed at the inlet and outlet. Now, it is difficult to find out the analytical solution for this guy you know for this equation, where to gradually varying area flare that is a conical area minus the cross sectional area of the inlet pipe.

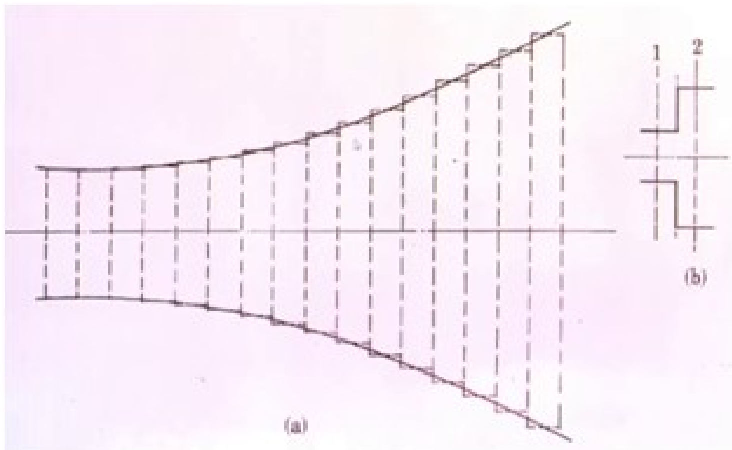
$$\frac{d^2\tilde{p}}{dx^2} + \frac{1}{S}\frac{dS}{dx}\frac{d\tilde{p}}{dx} + k_0^2\,\tilde{p} = 0$$

Because once you put this guy here differentiate so, analytical solutions probably would not exist and you might consider expanding this as a power series expansion and you know truncating it to a finite number of terms based on the frequency and all that sort of a thing.

So, you can define certain series and you can get some you know an approximate analytical solution for such things, but for a grad for a general case, what we have to do is basically look for more numeric, more general techniques, which can be applicable to any sort of a problem. Power series definitely will help, but then again, the one needs to look for the convergence and you know expressions becomes little tedious as we go up for high order terms and so on.

So, basically, we need to solve this equation of course, there is no mean flow involved here so, and for the general case that is your duct is like this, it need not be conical, it can be general and then, we apply it to us you know popular configuration like this one and later on something like this configuration, where this is bounded by a perforated bridge to guide the flow and yet and make it sort of acoustically transparent the perforates and yet your resonators will do the job.

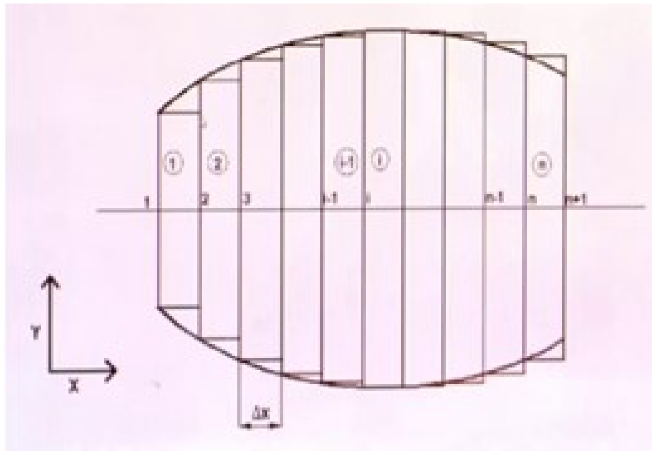
So, these, this is the perforated section and your flow is there. So, but we will probably worry about this later when we talk about perforates in the next week perhaps, but for now, we are not probably interested in doing this. So, I rather rub it off and focus only on this problem.




So, how do we go about it? How do we exactly solve the problem? So, to do such a thing, let us consider the figure shown here is a schematic, where you know we see a gradually varying area duct, this discontinuities are really notional, they do not quite exist, but what we have done is that you know if I just zoom this figure, we see that you know this is a gradually varying area duct and its divided just for our simplicity into a large number of small parts; such that each part is a uniform duct.

So, what is happening at the interface between say this duct and this duct, what is happening? So, basically what is happening is there is a discontinuity; there is a sudden area discontinuity.

So, otherwise how would we model of course, there couple of more techniques to model such gradually varying area plays of a general nature numerically, but right now, this method is based on dividing the entire length into small-small segments; such that this length is very small.



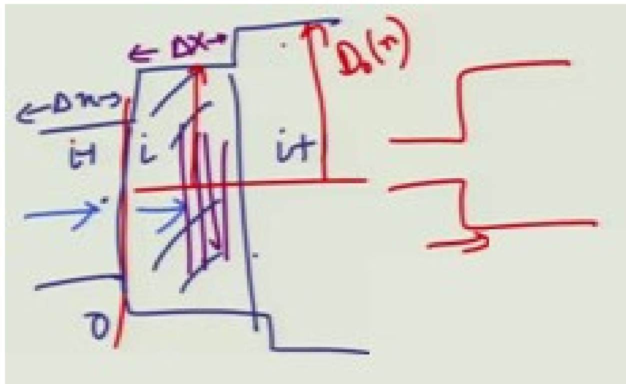

So, let us look at this figure. So, this is pretty much you know sort of equivalent to the other figure that I had taken from some reference from some article. So, here we have a general gradually varying area duct and divided into small parts each of length say $\Delta x$ and 1, 2, 3, 4 like this n, n are the notional segments each of uniform cross section. And 1, 2 the one, the numbers that are written not in circles, they are just the nodes at or the discontinuity number.

So, what happens really at the discontinuity or a sudden area jump that we have to see. Basically, you know all these things are kind of a system of small concentric circular ducts or any shape duct, but with area discontinuity at the interface that is the key. So, this such a method in which model a gradually varying area flare with short concentric uniform pipes with jump this area or jump discontinuities at the area interface is called the **SEGMENTATION APPROACH.**

Let me reiterate here that there is no mean flow here. So, mean flow would obviously, complicate things in terms of how do you model things, basically variation of state variables at the interface, but right now, we will probably just think along the lines of a stationary medium. So, what happens at the discontinuity? Let us look at this figure again.

So, let us call maybe probably we can have a look at this as well. Let us call the point that is just here, somewhere here or maybe just have a look at this one. So, let us focus on the node 2. So, the point or the node at or the basically the plane at which we are looking at the state variable the acoustic pressure and velocity that is located in the duct 1, but just at the interface.

Similarly, there is another node that is located in the duct 2, but it is pretty much at the interface. So, basically, we are worrying about things that are happening at the interface named number 2 here where I am pointing, and we like to relate the state variables within the duct 1 and 2 how does it vary? So, basically it is the same thing.



Let us introduce a convention that pressure in the ith so, you have a system like this, things like this. So, let us introduce this is say $i$ - 1, and $i$ +1. So, at this point, what happens is that $\tilde{p}_{i-1} = \tilde{p}_1$ at i or acoustic pressure tilde and the mass velocity is the same $\tilde{V}_{i-1} = \tilde{V}_1$that is whatever mass that goes here is going here.

So, this is basically, we are kind of considering the discontinuities here, the notional discontinuity is just as if it were a regular discontinuity. So, the regular relations that would be valid for a sudden discontinuity like this what we were analyzing so far that also holds for this one.

So, pressure across this interface between duct $i$ - 1 and $i$ that is the steam planar wave front propagate and your mass velocity or the fundamental mode that is also the same. Mass velocity or volume velocity you know we can easily get it, when we integrate the acoustic particle velocity and then in either of the chambers and integrate also over the annular region, we can clearly show that this relation also holds. So, what it would mean in terms of if we write let us

$$\begin{Bmatrix} \tilde{p}_{i-1} \\ \tilde{V}_{i-1} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \tilde{p}_i \\ \tilde{V}_i \end{Bmatrix}_{|x=0}$$

$$[I]$$

So, this is essentially an identity matrix that is the acoustic state variables, acoustic pressure and acoustic mass velocity that is the same so, they across the discontinuity, across the jump, they are related by a identity matrix, there is no difference, this is a continuity. Let us say the length is $\Delta x$ ok.

So, what happens across or over the wave propagation that occurs here? How is the thing related? So, as we know let me go to another slide perhaps.

$$\begin{Bmatrix} \tilde{p}_i \\ \tilde{V}_i \end{Bmatrix}_{|x=0} = \begin{bmatrix} cosk_0\Delta x & jY_i sink_0\Delta x \\ \frac{jsink_0\Delta x}{Y_i} & cosk_0\Delta x \end{bmatrix} \begin{Bmatrix} \tilde{p}_i \\ \tilde{V}_i \end{Bmatrix}_{|x=\Delta x}$$

Now, we get this, after this what? We know what happens to pi and $V_{i-1}$ say let us say at x is equal to delta x here will be some $\Delta x$ for this duct and this is x is equal to 0 and that is for this duct and here, we have this thing. So, basically what happens is that if you were to relate the acoustic pressure and mass velocity, oneth segment with the things at the other discontinuity.

So, what will we get? We will get

$$\begin{Bmatrix} \tilde{p}_{i-1} \\ \tilde{V}_{i-1} \end{Bmatrix}_{|x=\Delta x} = \underset{2\times 2}{[I]} \begin{bmatrix} C & jY_i S \\ \frac{j}{Y_i} S & C \end{bmatrix} \begin{Bmatrix} \tilde{p}_i \\ \tilde{V}_i \end{Bmatrix}_{|x=\Delta x}$$

So, this I would be multiplied by, pre-multiplied with this thing. So, I would just rather get rid of i and so, the idea is that p i and v i at this discontinuity is related to the one here by using a simple transfer matrix, which is valid for a uniform duct; you know which is valid for a uniform duct using this thing and where Yi most important is Yi where Yi is the characteristic impedance for the i-th duct that is sound speed divide by the cross sectional area of the i-th duct that is for this guy, this guy here .

$$\begin{Bmatrix} \tilde{p}_{i-1} \\ \tilde{V}_{i-1} \end{Bmatrix}_{|x=\Delta x} = \begin{bmatrix} C & jY_i S \\ \frac{j}{Y_i} S & C \end{bmatrix} \begin{Bmatrix} \tilde{p}_i \\ \tilde{V}_i \end{Bmatrix}_{|x=\Delta x} \qquad (1)$$

$$Y_i = \frac{C_0}{S_i}$$

So, what happens for the other thing? So, $Y_i$ is note that you know the diameter is increasing, diameter is a function of the distance D = $D_0$ ($x$) that is your; that is your radius or the, of the axial distance some $x$, ok. Now, the thing is that this has to be updated. So, basically what we get,

$$\frac{C_0}{\pi D_0(x)^2}$$

so, 4 goes in the numerator and $D_0(x)$ for we can evaluate it right in the middle based on some functional rule that its satisfies. We can illustrate this example for a conical duct or for the annular cavity.

Now, when we go for the this thing, this guy, it is $D_0(x)$ also, but you know the diameter has sort of increased. So, in such a case, what will happen is that we can easily.

$$\begin{Bmatrix} \tilde{p}_i \\ \tilde{V}_i \end{Bmatrix}_{|x=\Delta x} = \begin{bmatrix} C & jY_{i+1}S \\ \dfrac{j}{Y_{i+1}}S & C \end{bmatrix} \begin{Bmatrix} \tilde{p}_{i+1} \\ \tilde{V}_{i+1} \end{Bmatrix}_{|x=\Delta x}$$

Basically, what we get is that here,

$$Y_{i+1} = \frac{C_0}{S_{i+1}} \qquad (2)$$

So, basically how do we go ahead with this particular thing? Now, we need to probably multiply this thing so, let me write down the cascaded sequentially multiplied form.

So, we have relation in $p_{i+1}$ and $V_{i+1}$ at $x$ is equal to $\Delta x$ here and now, we also have the relation, now we get this particular thing and now, we have this particular relation . This is actually i +1 so, I will just get rid of this guy and $x$ is equal to $\Delta x$.

So, now, it is just a job of multiplying. So, you see let us say let me call this as equation 2, let me call this as equation 1. So, i -1 that the things that happen at $x$ is equal to $\Delta x$ for i-1 that is i minus oneth duct, the notional duct that is related to the i + oneth duct by this thing.

So, the idea is here we saw the things that happen here, now we are trying to relate this, this with this is known and this with this is known. So, we can just simply multiply this matrix pre-multiply this matrix with the other one or post multiply this matrix with the other one because this particular thing we know that pi, Vi, this particular thing we know for the ith duct at *x* is equal to delta x, we know in terms of things that happen at i+1 at *x* is equal to $\Delta x$. So, in such a case, we will get let

$$\begin{Bmatrix} \tilde{p}_{i-1} \\ \tilde{V}_{i-1} \end{Bmatrix}_{|x=\Delta x} = \begin{bmatrix} C & jY_i \\ \frac{jS}{Y_i} & C \end{bmatrix} \begin{bmatrix} C & jY_{i+1} \\ \frac{jS}{Y_{i+1}} & C \end{bmatrix} \begin{Bmatrix} \tilde{p}_{i+1} \\ \tilde{V}_{i+1} \end{Bmatrix}_{|x=\Delta x}$$

$$C \equiv cosk_0 \Delta x$$

$$S \equiv cosk_0 \Delta x$$

So, what happens now? We have to you know probably stop here and introspect what is happening here? Why does this method work?

You know this is like relating things from well so sorry this is plus 1, this is plus 1 and plus 1. So, when we divide this duct into this part, and this is your, this thing so, we are relating things just here with the things at this point.

So, we are able to multiply these two things and like this, we can continue to cover the entire duct and you know account for whatever variation we are able to see.

So, now, let us get back to this figure where you have this kind of a discontinuity. If you were to write the pressure here, pressure and acoustic velocity here with one that is here so, it will be simply multiplying matrices with the characteristic impedance being constantly updated at each cross section.

$$\begin{Bmatrix} \tilde{p} \\ \tilde{V} \end{Bmatrix}_{x=0} = \begin{bmatrix} C & jSY_1 \\ \frac{j}{Y_1}S & C \end{bmatrix} \begin{bmatrix} C & jSY_2 \\ \frac{j}{Y_2}S & C \end{bmatrix} \cdots\cdots \begin{bmatrix} C & jSY_2 \\ \frac{j}{Y_1}S & C \end{bmatrix} \begin{Bmatrix} \tilde{p} \\ \tilde{V} \end{Bmatrix}_{x=l}$$

and it is divided into n number of segments. So, n what are those n segments? Those n segments are here. So, it is as simple as.

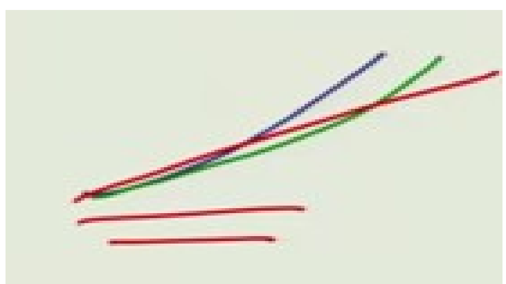
$$Y_1 = \frac{C_0}{S_1}, \qquad Y_2 = \frac{C_0}{S_2}, \qquad Y_3 = \frac{C_0}{S_3}, \cdots\cdots$$

$$Y_n = \frac{C_0}{S_n}$$

$$S = S(n)$$

Where now, you know $S_1$, $S_2$, $S_n$ these are all the cross-sectional areas of the uniform duct. So, when we have; when we have such a system in here so, you know let me zoom it so, $S_1$ is the cross-section area, let me call it this area where I am pointing of the duct 1, $S_2$ is the cross section area I measured from here till here, like this we traverse the entire duct.

And you know in principle, S can vary arbitrarily, it can be a exponential horn, it can be a parabolic horn, it can be a conical duct, it can or it can have some annular cavity x with external at an outlet in, large number of you know such things can be done.



So, it is about gradually writing a simple for loop code I can sort of write a pseudo code or you know I can probably demonstrate that in the MATLAB just in a while, some simple area discontinuity I mean basically cascading of area discontinuity matrices.

And then, multiplying it with the duct transfer matrix for uniform duct and generating the entire system of equations, deriving the overall transfer matrix or the overall four pole parameters for a gradually varying area duct of any profile that can be easily done in a simple MATLAB code and then, we will do that just in a while.

And then, there are other techniques also which is known as a matricent approach in which basically you know although we will not be covering that just now, maybe not in today's lecture, but just to get you a feel of that, I will show some nice photo here where you get this kind of a thing.

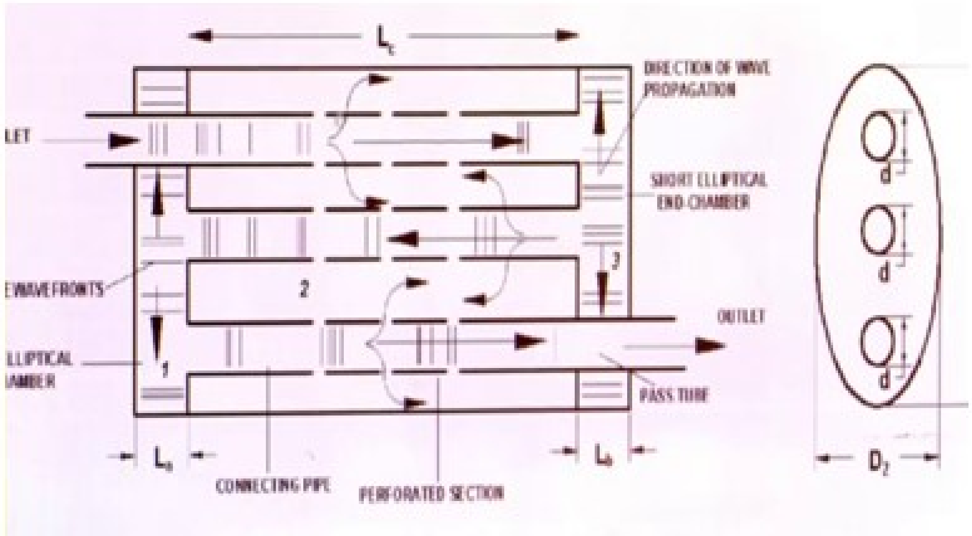
So, here, what happens essentially here this is yet the same duct but divided in again notional segments; such that there is no jump discontinuity. So, what happens? Essentially, you know we will probably worry about the maths later on in the probably in the week 8 or probably towards the end of week 7 when we have just introduced a CCTR, a Conical Concentric Tube Resonator in which you know such techniques often are handy.

So, as a steppingstone to the main CCTR problem, we will be considering again the gradually varying area duct problem without any perforates using the matricent approach where the ducts are modelled as notional segments, but with small segments of exponential profile so, this is approximately next short exponential profile, such that the discontinuity is not there.

And for an exponential profile, we saw we know the transfer matrix. So, then it is just about matter of you know gradually you know multiplying the transfer matrices and getting the solution so, we could do that and analyze a CCTR using a matricent approach also.

What we will do now? We will have a look at a simple conical duct example exactly what we saw in the last lecture and possibly for the annular cavity as well and there is yet another system that is used that is a short end chamber cavity in which the wave propagation is along the transverse direction. So, I will probably talk about that after finishing that case.

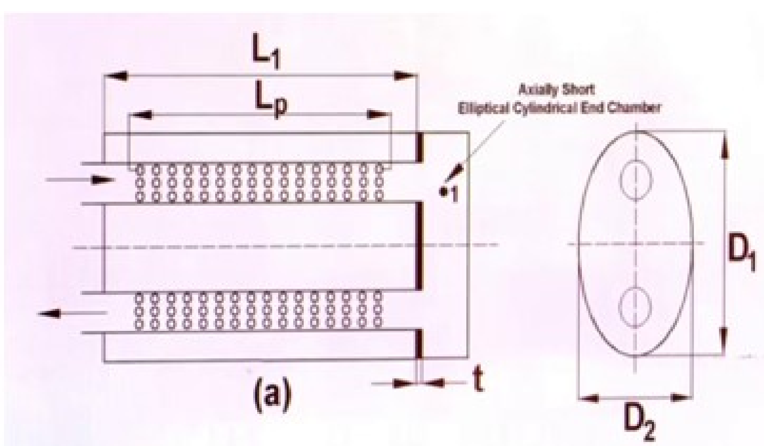
Look at then this figure. Well, you know this is something like a, also known as a multi-pass perforated duct silence, basically you have a outer shell, you can also have a look at perhaps this figure.



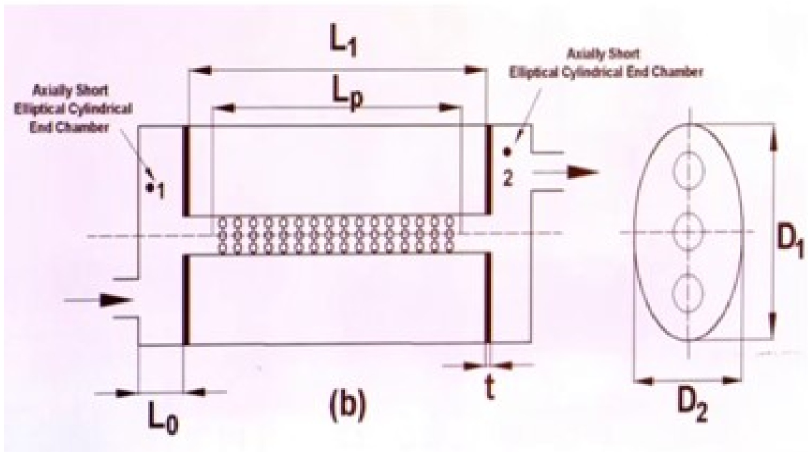

So, you know what it does is basically, it shows you there are two configurations. The overall shell is elliptical in shape and you have two perforated duct also in general known as multi pass perforated silencers.

So, such silencers are used in invariably in a lot of modern day commercial vehicles and you know the import the good thing about the silencer is that it is able to pack in a lot of transmission loss, lot of attenuation, good attenuation performance even when you have limited space because you know the waves have to go through multiple perforated pipes and there are number of end cavities got away resonators, but then, it allows the flow it the way it does is basically, it forces the flow to basically undergo a number of turns.

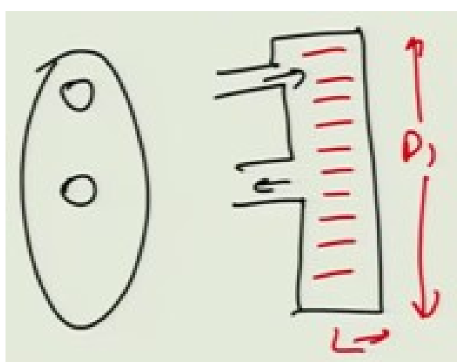
So, if you have a look at this figure, this configuration, it basically takes at least one reversal.



So, it is called the flow reversal configuration also. Here, it is able to is basically forced to take three reversals, 1 and then 2 and actually 2 reversal, but then there are three perforated pipes because most of the thing is used in the middle chamber at the end generally less space is available although, there is no hard and fast rule that the end chambers have to be small.



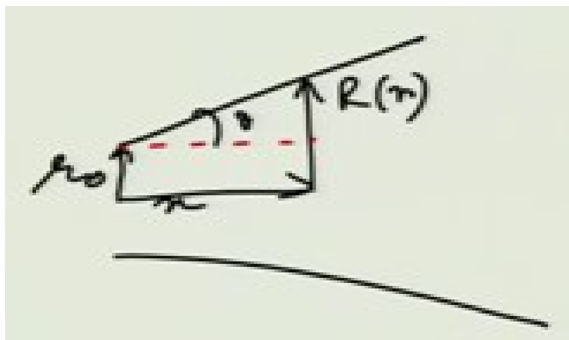

But generally they are the end chambers where I am pointing here this one and this guy here is you know generally short in length. So, such cavities are called end chamber short end chamber cavities, they are of elliptical shape because of the overall shape, because of space constraints or they could also be circular in shape.



So, in such a configuration specially, when such a thing is an elliptical in shape highly eccentric ellipse so, for such a thing if you have one port here and one port here and you have a configuration you know like this, something like this, flow comes in here and it leave here.

So, the entire idea is that the wave propagation is along the transverse dimension, but again it is not a uniform section if you do it like this, it is uniform only along the axis L, but along this direction along the major axis, you can consider this duct as a duct of gradually varying area duct. So, for such a thing, we again have to resort to numerical approaches like segmentation approach or a matricent approach.

So, based on the time permitting, we will probably take up this case. Now, probably in the later parts of the course, when I talk about some design criteria, we can do that as well. Now, before we end this lecture, let us do a quick a demonstration of a gradually varying area duct a conical duct, analyzed using the segmentation approach.



So, let us analyze this a conical duct. Before we go to MATLAB, let us also look at the general relation, let us say we measure the distance x here and at this point, we are you know having a radius *r*. So, what happens? What is the radius at a distance say *x* here? So, let us say this is R of *x*.

$$\frac{R(x) - r_0}{x} = tan\theta$$

$$=> R(x) = r_0 + x \ \ tan\theta$$

So, we get this kind of a relation. Now, we will keep this handy and let us now, it is time now to finally, enter into MATLAB thing.

```
function [] =transmissionlossplot(fr1,fr2,r,l,r_inlet,r_out,theta,m,choice,ch,chl)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% this function plots the transmission loss(in dB) versus frequency for
%%% ANY muffler configuration as it requires only TL value(in dB) at a particular
%%% frequency................
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tic
f=fr1:10:fr2;
n1=size(f);
n=n1(1,2);
for i=1:n
    Tl(i)=transmissionloss(f(i),r,l,r_inlet,r_out,theta,m,choice,chl);
%     i
end
figure(1)
plot(f,Tl,ch)
grid minor
xlabel('FREQUENCY in HZ)');
ylabel('TRANSMISSION LOSS(in dB)');
%%title('TRANSMISSION LOSS FOR A CYLINDRICAL DUCT OF RADIUS AND LENGHT  AND MEAN FLOW ');
```

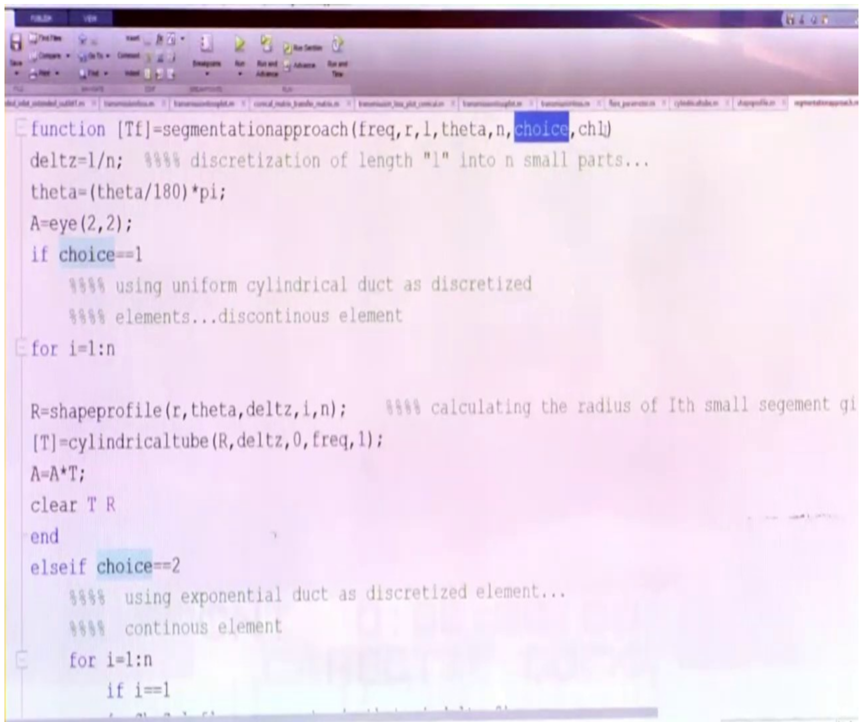
This is the transmission loss. So, the so, like I have been mentioning, MATLAB scripts are usually best written as function files. Also known as subroutines in Fortran, but we will work with MATLAB and this is a transmission loss plot more like an overarching routine which calls yet another routine transmission loss where the transmission loss for a given frequency is computed and which is your this part.

```
function [Tl,Tf]=transmissionloss(freq,r,l,r_inlet,r_out,theta,n,choice,chl)
% global google Tfout
P0=1.013*10^5; %%% ambient pressure
rho0=1.20545011; %%% density at 20degree centigrade... so that the speed of sound is exact
c0=sqrt((1.4*P0)/rho0);
Si=pi*(r_inlet^2);
Yi=c0/Si;
Sf=pi*(r_out^2);
Yf=c0/Sf;
[Tf]=segmentationapproach(freq,r,l,theta,n,choice,chl);

% if google
%     Tf=Tfout;
% end

v1=Tf(1,1)+(Tf(1,2)/Yf)+Yi*(Tf(2,1)+  (Tf(2,2)/Yf) );
v2=sqrt(Yf/Yi)*v1*(0.5);
Tl=20*log10(abs(v2));
```

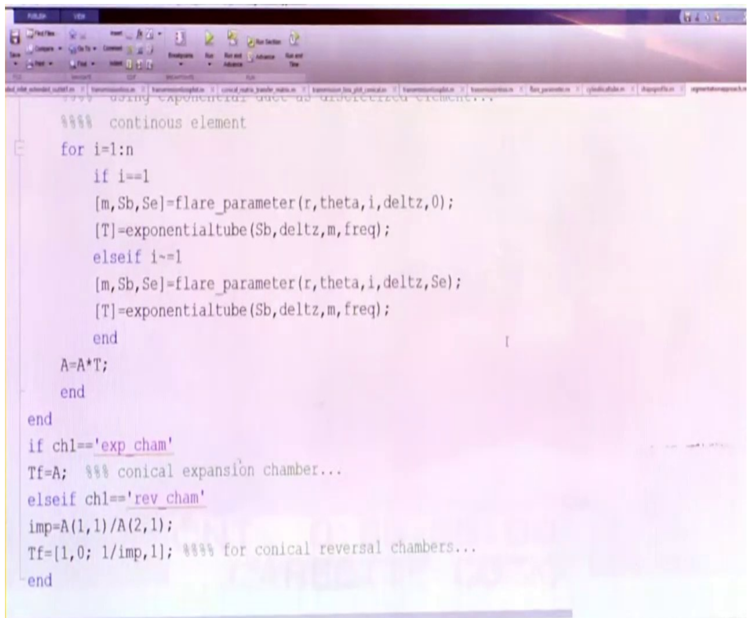
And this calls the most important routine, the heart of the segmentation approach, the file is named as segmentation approach so, this is what it is.



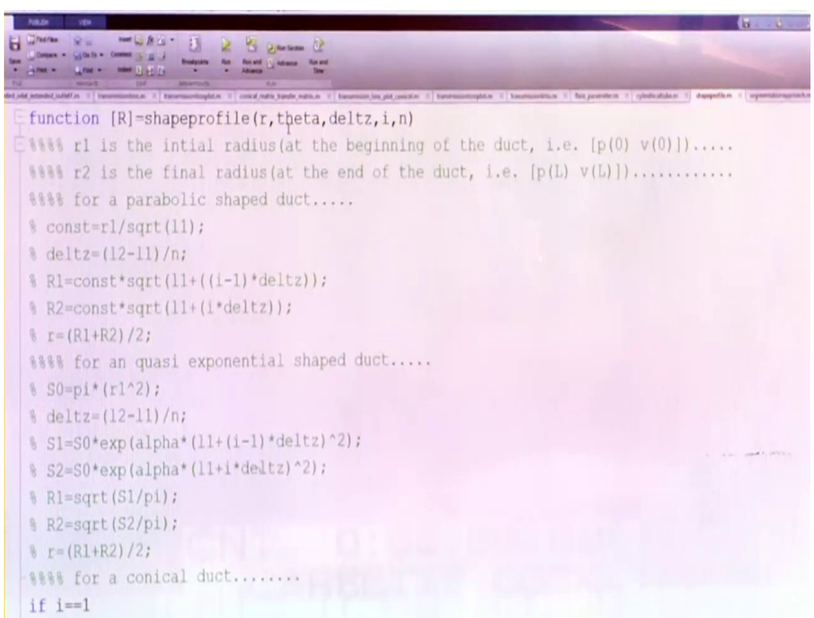

And segmentation approach is basically the this is the file, now it takes in few parameters namely the frequency $f_0$ or freq at which the transmission loss is to be computed, radius $r$ which is the radius at the right at the inlet $r_0$ and $l$ that is at the section 1 where the flare just begins and $l$ is the overall length over or the conical section, theta is the flare angle.

You know usually if it is more than 4 degree or 5 degrees, flow separation would happen. So, it is generally not more than 4 or 5 degrees, it will take probably the case of 4 degrees for numerical purpose. N is the number of parts so, number of small segments into which the conical duct is notionally divided so, it can be 100, 200, we can do some convergence analysis on the transmission loss.

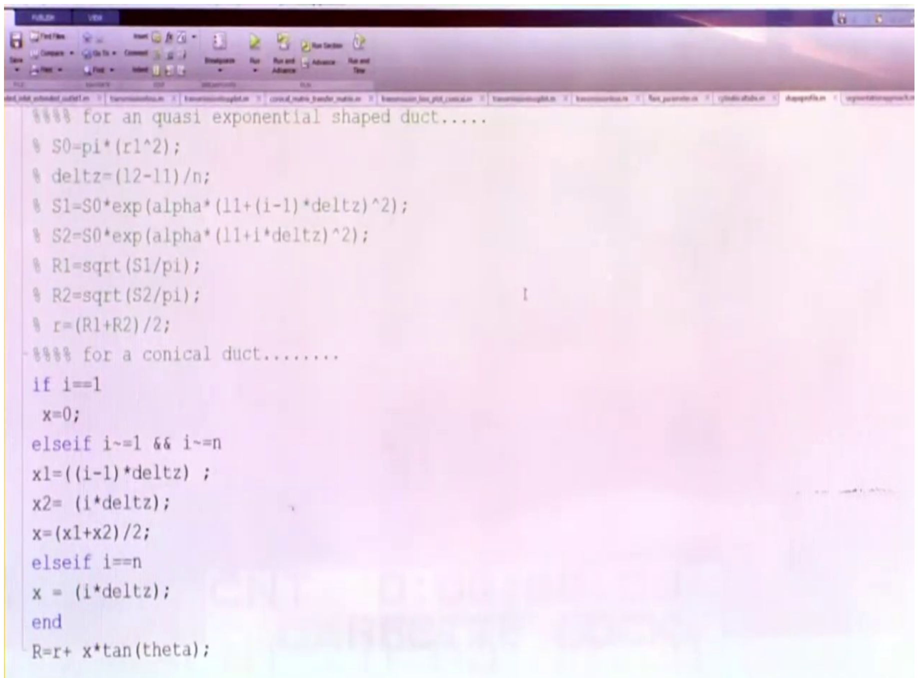
Choice is nothing but you know basically whether you want to do it using a segmentation approach or a matricent approach, we will not be worrying about matricent approach right now, so, we will just use the choice 1 which is for the segmentation approach.




Ch 1 is nothing but you know is more like which kind of a configuration we would like to analyze. We would probably like to analyze a conical expansion chamber of configuration where ports are on the opposite end faces, reversal type of configuration is something where the ports are located on the same end face so, we will not discuss that in this lecture, we will worry about only the expansion chamber in ports are located in the opposite faces. So, this routine in turn call the shape profile.

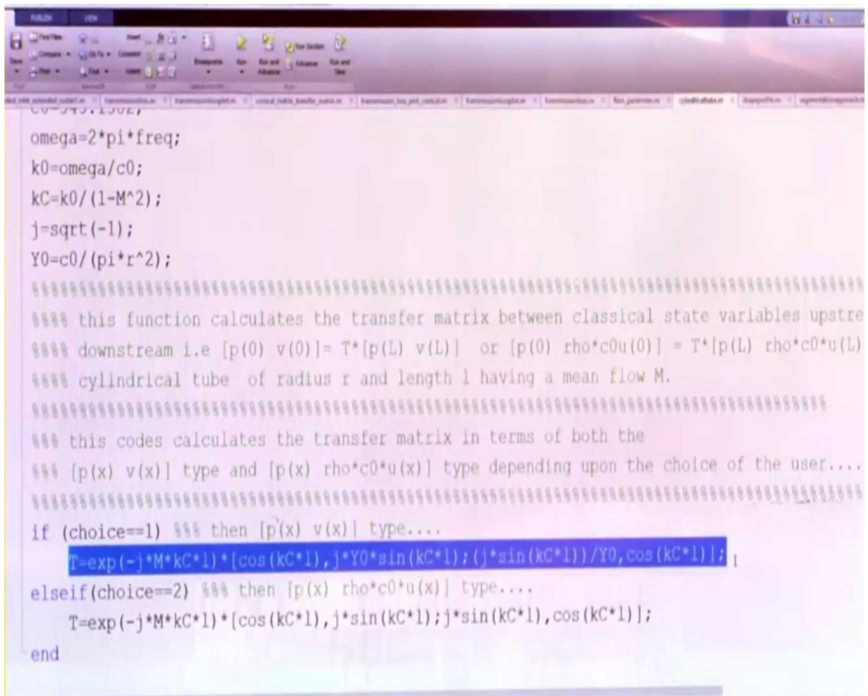


So, what does the shape profile do? Shape profile basically, these are all commented parts on the top, it again takes in the few parameters the r which is the radius at the inlet, theta is the flare angle, delta z is a small part over which the duct is assumed to be uniform. So, a delta z is already defined here I guess, delta z is l by n divided by n parts as I noted before and i is the section for which we need to compute the radius that is the ith part and n is the number of your such parts.



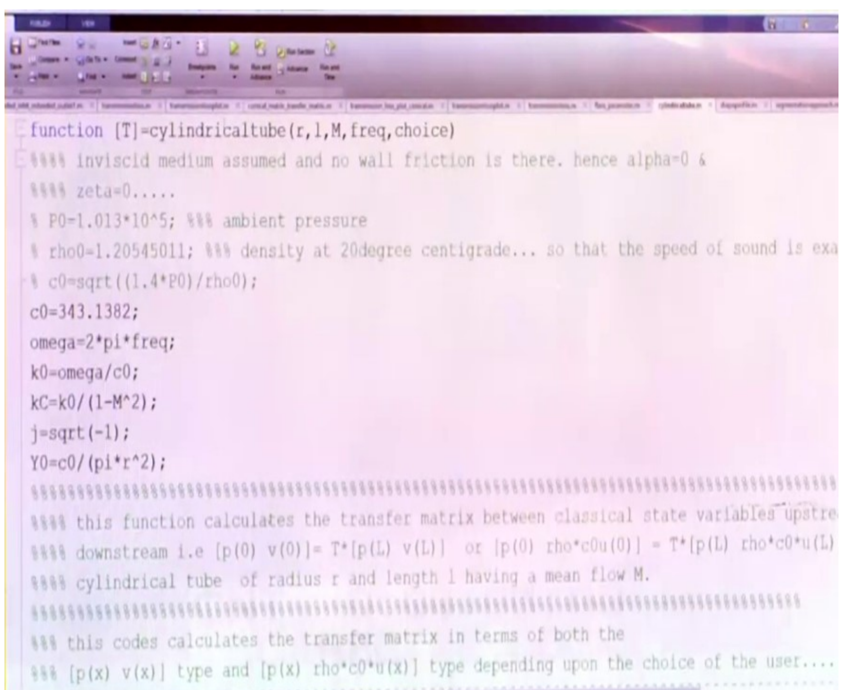

So, what happens now? When at the first section $x$ is equal to 0 so, you know the radius becomes at equal to $R_0$ or R itself, but if i is between if it is not equal to 1 and not equal to the last part, then we take the middle part of that as the approximate thing that is $x1$ is equal to i -1 and $\Delta$z and i into $\Delta$z, the average of that for safety we are taking that as the approximate x value over which the increment in the radius has to be computed.

And when x is i is equal to $n$, $x$ is equal to into $\Delta$z. So, the radius at any section R, capital R as a function of $x$ is r, small $r + x$ into tan into $\theta$. So, theta you have to be careful, it is given in terms of the degrees, but we have to convert into radians before we use this. So, that is what we have done here.



```
omega=2*pi*freq;
k0=omega/c0;
kC=k0/(1-M^2);
j=sqrt(-1);
Y0=c0/(pi*r^2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% this function calculates the transfer matrix between classical state variables upstre
%%%% downstream i.e [p(0) v(0)]= T*[p(L) v(L)]  or [p(0) rho*c0u(0)] = T*[p(L) rho*c0*u(L)
%%%% cylindrical tube  of radius r and length l having a mean flow M.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% this codes calculates the transfer matrix in terms of both the
%%% [p(x) v(x)] type and [p(x) rho*c0*u(x)] type depending upon the choice of the user....
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if (choice==1) %%% then [p(x) v(x)] type....
    T=exp(-j*M*kC*l)*[cos(kC*l),j*Y0*sin(kC*l);(j*sin(kC*l))/Y0,cos(kC*l)];
elseif(choice==2) %%% then [p(x) rho*c0*u(x)] type....
    T=exp(-j*M*kC*l)*[cos(kC*l),j*sin(kC*l);j*sin(kC*l),cos(kC*l)];
end
```

And this gets you the shape profile radius and then, it gives you, it basically computes a transfer matrix for a cylindrical tube here that is basically of this part cos j sin Y, j sin / Y, cos $k_0$l all these things its computes in the form of p pressure and the mass velocity. If it is a matricent approach, it follows a different sort of a slightly different form of a transfer matrix. So, anyways we will worry about work only with this part.



```
function [T]=cylindricaltube(r,l,M,freq,choice)
%%%% inviscid medium assumed and no wall friction is there. hence alpha=0 &
%%%% zeta=0.....
% P0=1.013*10^5; %%% ambient pressure
% rho0=1.20545011; %%% density at 20degree centigrade... so that the speed of sound is exa
% c0=sqrt((1.4*P0)/rho0);
c0=343.1382;
omega=2*pi*freq;
k0=omega/c0;
kC=k0/(1-M^2);
j=sqrt(-1);
Y0=c0/(pi*r^2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% this function calculates the transfer matrix between classical state variables upstre
%%%% downstream i.e [p(0) v(0)]= T*[p(L) v(L)]  or [p(0) rho*c0u(0)] = T*[p(L) rho*c0*u(L)
%%%% cylindrical tube  of radius r and length l having a mean flow M.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% this codes calculates the transfer matrix in terms of both the
%%% [p(x) v(x)] type and [p(x) rho*c0*u(x)] type depending upon the choice of the user....
```

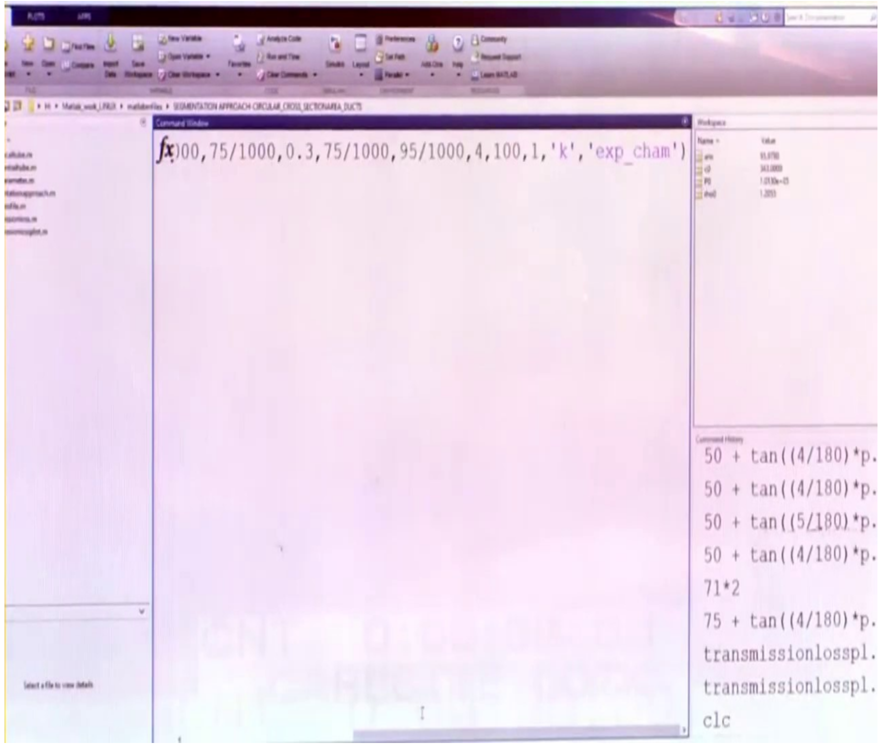
So, this function gets cylindrical function gets invoked it finally, throws up the T parameter, transfer matrix parameter for the small notional segment. Now, here is the heart or the holy grail of segmentation approach. So, once we get T, now A is initialized with i, matrix is nothing but a unity matrix 1, 0, 0,1.

So, initially, let us say when i is equal to 1 that is we are traversing just the first segment so, identity matrix times the T matrix for the first segment so, it be A becomes T after the end of this operation and these things are cleared. And the loop goes back here, the radius is updated and the transfer matrix, which with appropriate characteristic impedance is obtained and that is multiplied by A, A was Ti where i is 1 so, T1 into T2.

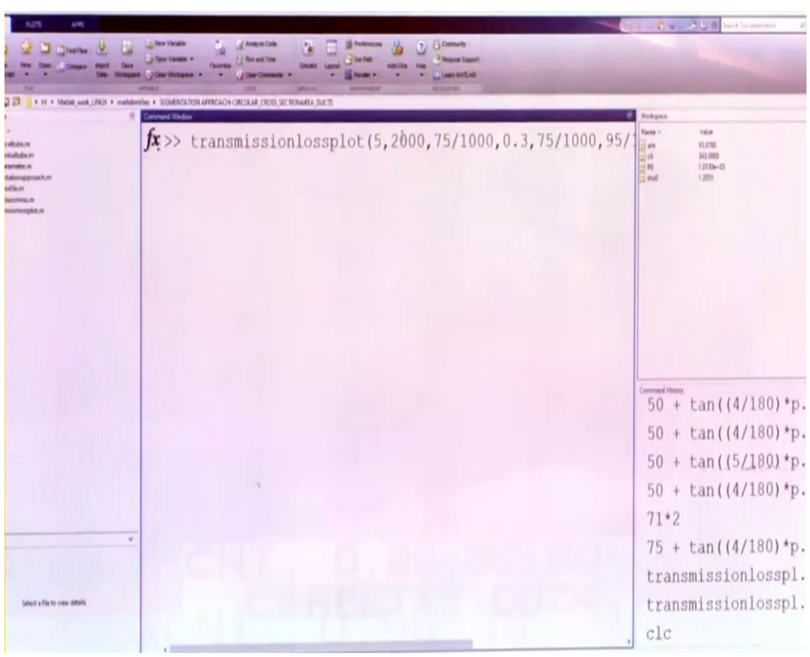
So, A becomes T1 into T2 and then again, the loop enters here R is computed for the third section, then transfer matrix is obtained and A is equal to A1 into A2 into A3 and so on. So, like this once we do that, we get the transfer matrix, or we get the transfer matrix of the overall duct. So, then once the program exits the loop for the expansion chamber overall transfer matrix is nothing, but the cascaded matrix A, A itself.

Now, once we do that, we can pass this to the function the calling function which calls segmentation approach named as transmission loss, it basically computes your transmission loss as usual what we have been doing in the last classes and some summation of the transfer matrix parameters appropriately scaled by the characteristic impedance.

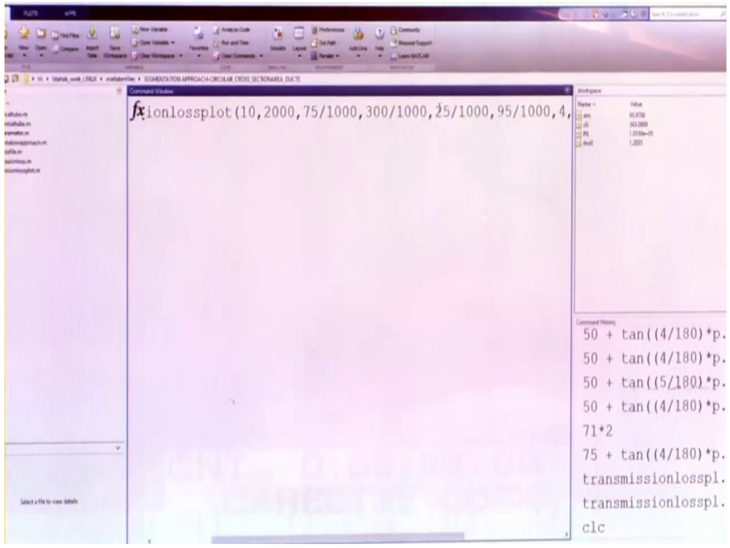
Now, here r inlet and r outlet are the radius of the duct and the outlet duct inlet and outlet duct which can be same or different from the radius of the inlet and outlet section of the conical duct. So, this is the plotting thing which we have been discussing. So, r inlet and r out I what I mentioned, r we have you know basically these two and the theta parameter in a hand, r inlet and r outlet we can also fix.



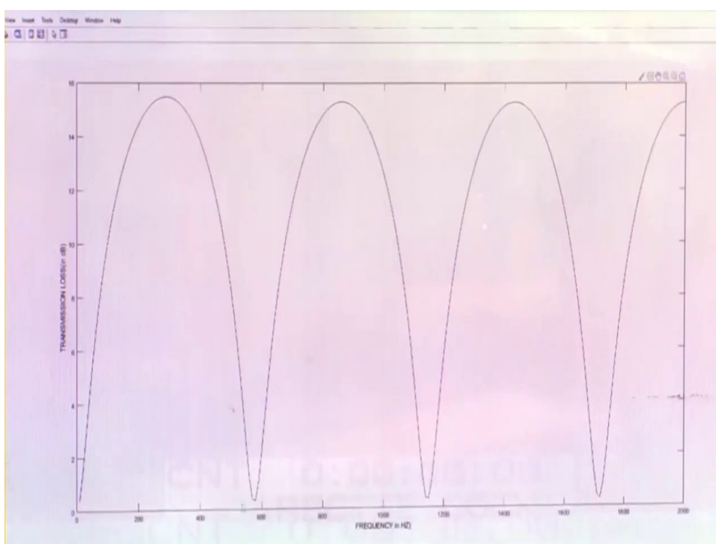

So, let us do some parametric studies we will there something, which I have already done. Expansion chamber I mentioned, k is the colour of the graph and 100 parts I have mentioned that and this is the segmentation approach what I have asked the function to do.



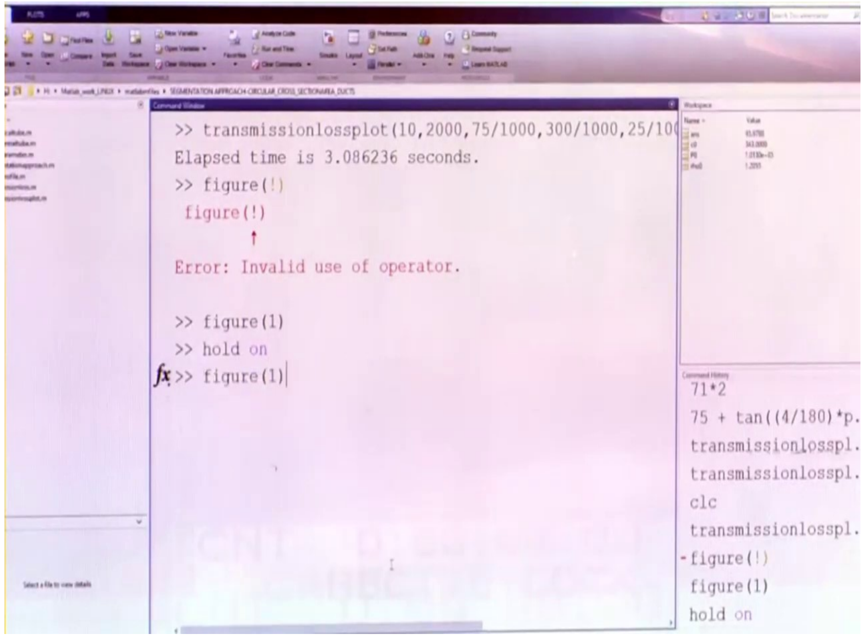
Frequency is from 5 hertz, 2000 hertz in step of or maybe 10 hertz we can do.



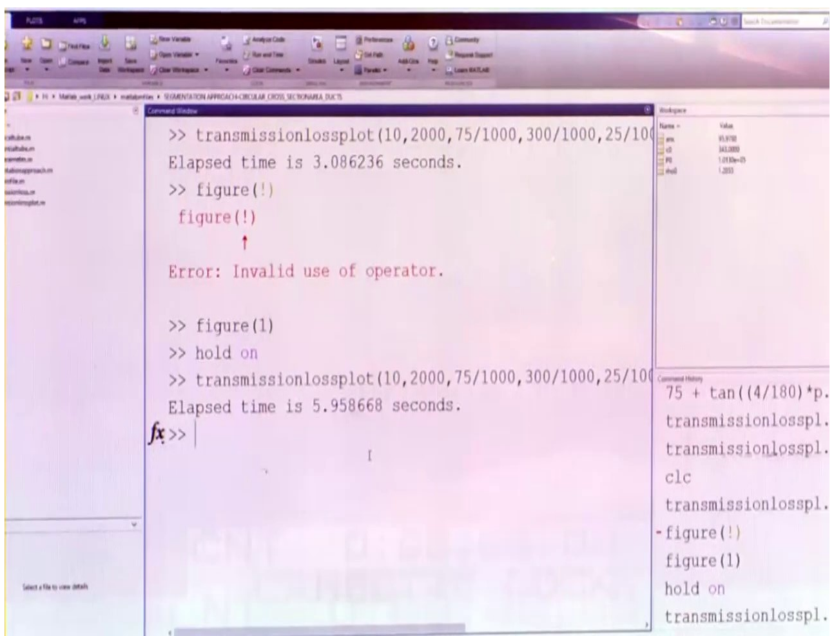

In steps of 10 hertz just for demonstration purpose and 75 is let us say the inlet R naught value basically 75 mm. So, 75 by 1000, 300 mm we could have written it like this and same and let us make it more practical 25 mm is the radius of the uniform pipe at the inlet and 25 mm is also the radius of the outlet pipe at the larger section and 4 is the flare angle which is of course, is supplying that in degrees.



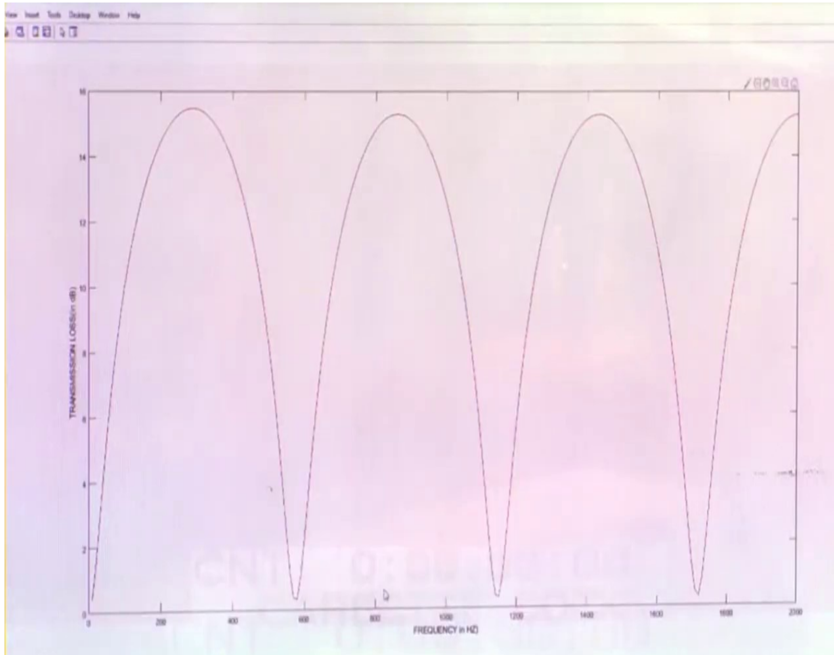
So, let us evaluate the transmission loss for this thing, it will take a few minutes based on the processor speed. So, we get you know again the same dome and troughs of configuration, but the troughs are you know lifted up, they are not the same.



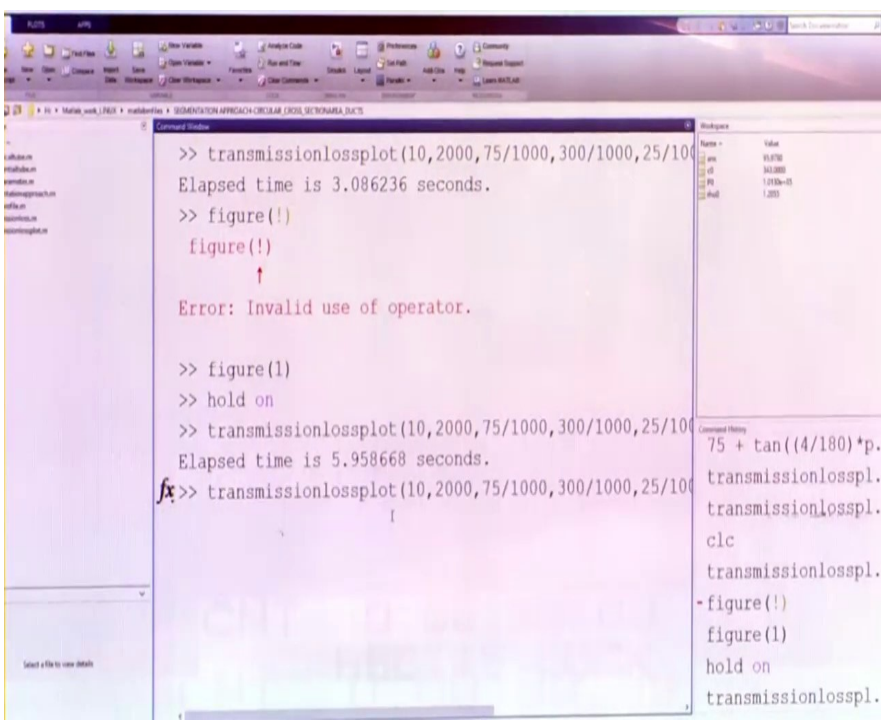

And as the flare value increases the angle or the radius at the outlet section, these values would actually go up.

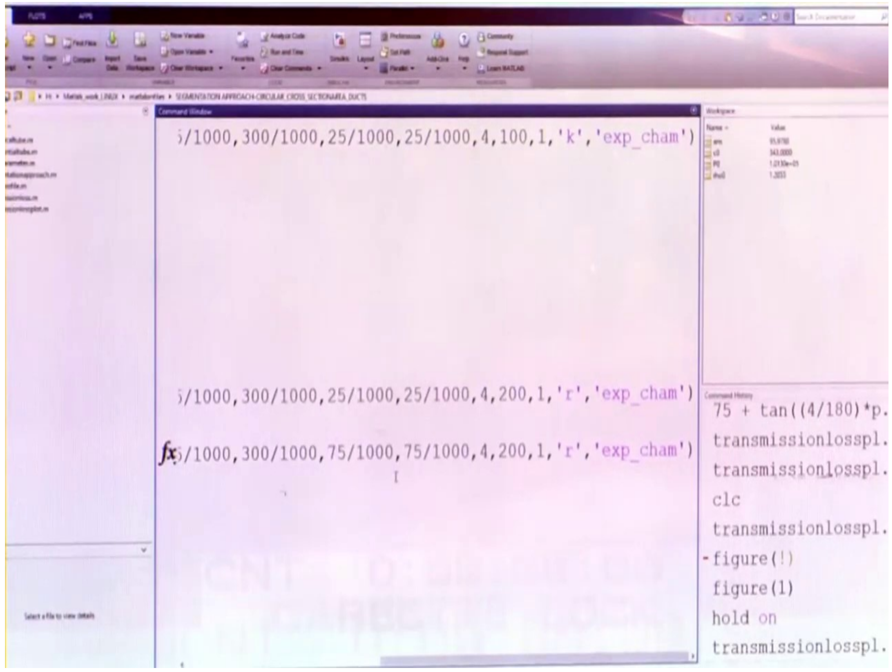



So, we can demonstrate that, but what we will probably do is that we will hold on to this figure sorry, we will hold on to this figure by command hold on and you know plot it with a different colour, but larger number of segments 200, double of the last one. So, the conical duct is divided into a exactly the double number of segments.

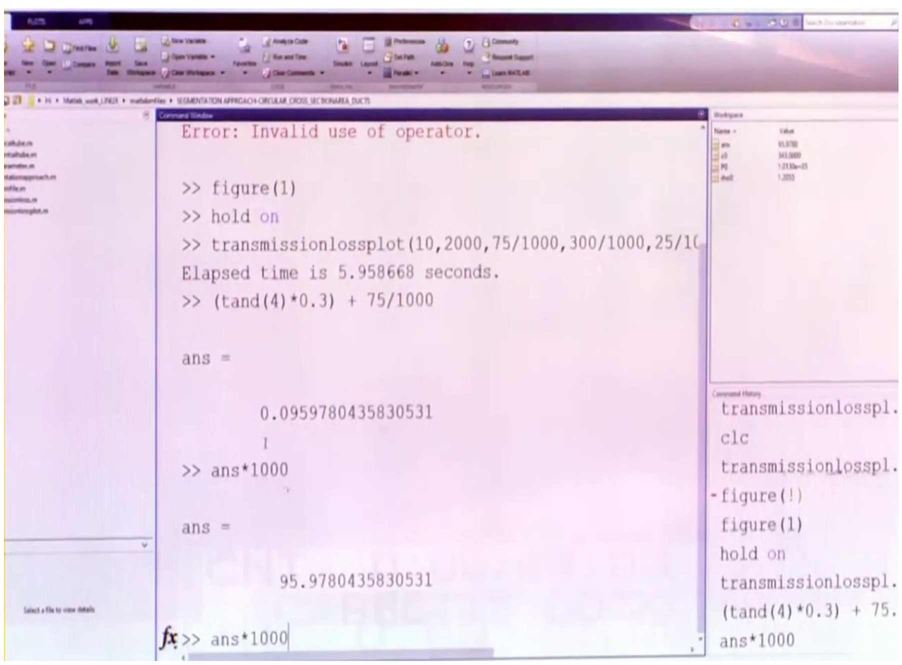



So, the graphs are nearly identical, we I can just sort of remove or get rid of this part and you know they are identical. I mean like this if we do for 400, 800, we will see the results have converged and it will match exactly the analytical thing although, we are not sort of comparing it right now. So, this is one part.

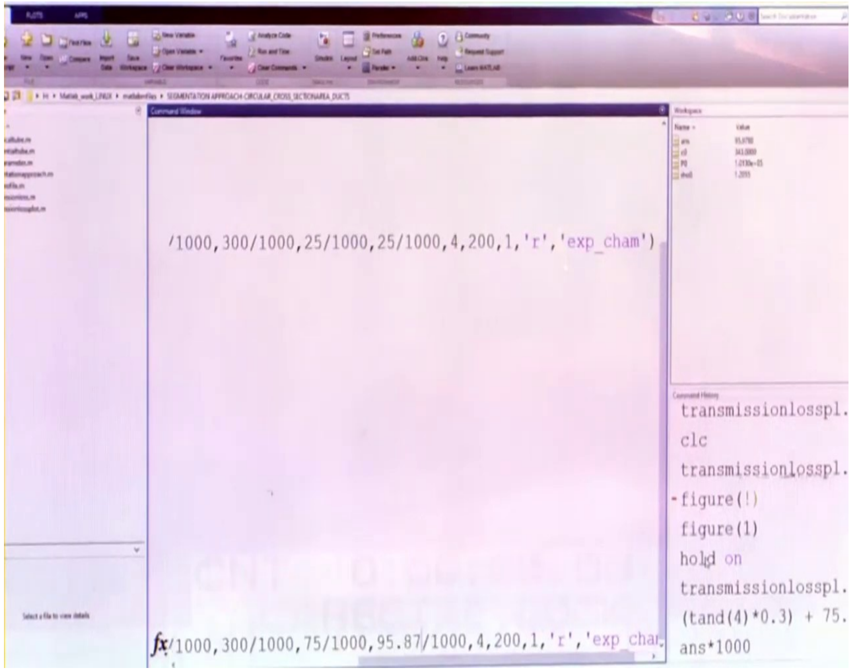




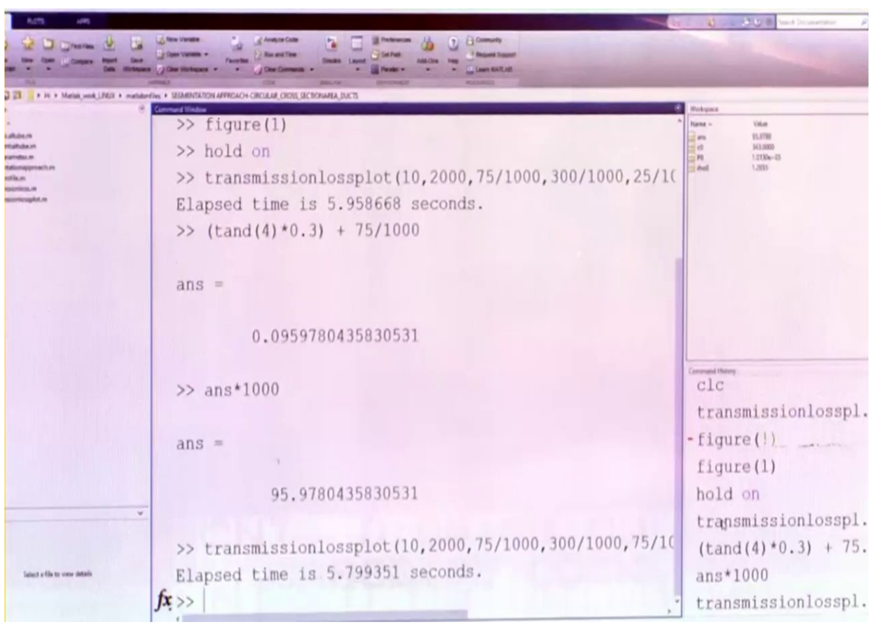

And another thing what I wanted to quickly mention here is that if we go with 75 mm here and 75 mm 75.



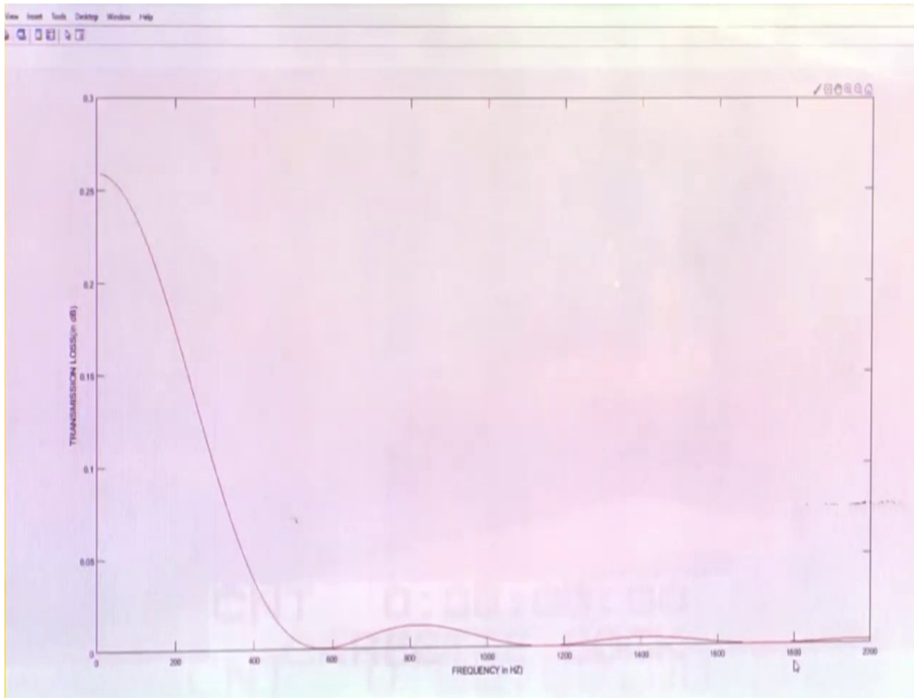

Look, the thing is that let me just compute it for you. The angle is tand 4 degrees into the length is 0.3, and this is the inlet radius is 75 so, what will be the radius of the outlet? So, it is basically your answer into a 1000 that is your 95 mm, 95.7.




So, why am I telling you this? Now, let us say we look at the other configuration what we discussed, inlet section, the radius of the inlet pipe is the same as the inlet section so, we have the radius 75 and this radius is 90 point what it was probably 95.87, we will plug in this value and have a look at the things here.




You will get you know decaying curve not your dome and troughs, you will be evident in a minute.




So, we see you know the same decaying thing. So, this approach is actually indeed working fine and like this, we can in principle evaluate the complicated sections like I mentioned at the beginning of this lecture, which is your this section, annular duct we just have to get take care of this expression and update the area change accordingly and get the overall transfer matrix for from this part to this part compute the impedance here.

Similarly, for this part compute the impedance here and then, regular cascading of transfer matrices, we will multiply this. You know in principle, it is possible to analyze any duct using the segmentation approach and it is a very good numerical technique robust one.

We will stop the lecture here. And what we will do in the next week, next week is going to be very important week 7 where we will introduce the effects of mean flow more formally. Although, we have done it to a certain extent in the lecture week 2 you know the last few lectures of week 2 where we talked about the convective Helmholtz equation involving the convective effects of mean flow.

But, basically what we are going to do is that we are going to talk about air acoustic state variables which basically involves the effect of mean flow where Mach number M naught is much less than 1 typically, 0.1, 0.15 that is a max that happens in automobile or automobile exhaust systems.

We will talk about the air acoustic state variables, we introduce that and then, we will kind of have a look at different perforated duct configurations and different expressions for perforate impedance. Basically, across the perforate, there is a jump, there is a pressure jump. So, we are going to model that using the appropriate functions.

And then, once we do that using a planar wave analysis, we will start analyzing simple CTR's Concentric Tube Resonators and you know duct with a concentrated perforated tube and so on and then, we will keep analyzing different cross flow configurations, straight through configuration and all those kind of things some of that would probably involve MATLAB coding and then, gradually build up or build upon the stuff that we have learned in week 6 on gradually varying area duck.

So, week 7 and I guess to a certain extent of week 8 is going to be very interesting in terms of because we will talk about directly about the perforates a very very practically important a huge a very important element so, till that time stay tuned and keep looking at the lectures and noting down your doubts and all that sort of a thing. We will probably meet in the next week, week 7, we will introduce perforates.

Thanks.