Computer Integrated Manufacturing
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Lecture 09
Computer Graphics (Part 4 of 4)

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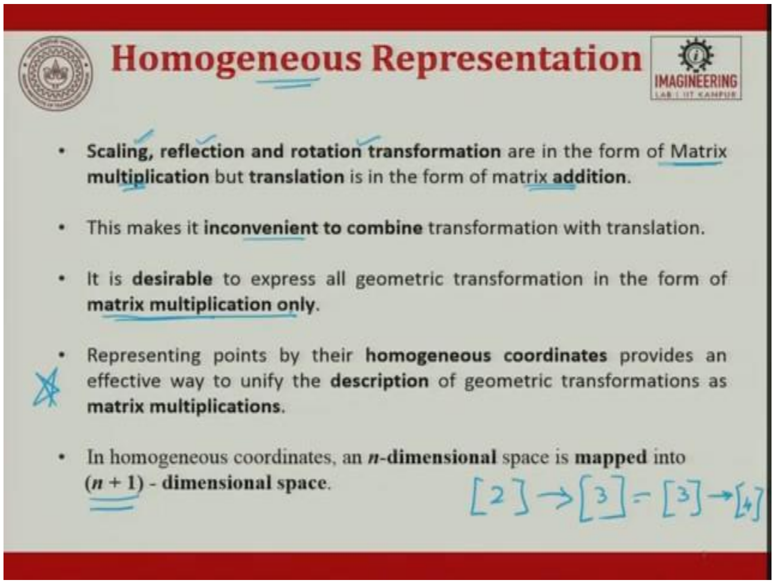

So in CAD, the most important thing what we have to understand is homogeneous representation. Till now, what all transformation did we saw, we saw translation, scaling, mirror or reflection, rotation. So, now what we will see is, we have to come up with a homogeneous representation why is it required? Scaling, reflection and rotation transformation are in the form of matrix multiplication, translation is in the form of matrix addition. So, these 3 are in terms of matrix multiplication and translation in terms of addition.

It makes it inconvenient, because many a times you will be doing step by, step by, step by step operation. When you do that, in the next topic of discussion on concatenation, then it becomes extremely difficult for solving it mathematically. So, this makes, so multiplication and addition, this makes it inconvenient to combine transformation with translation. So, it is desirable to express all geometric transformation in the form of matrix multiplication only. Matrices is the most easiest form of doing translation or transformation, whatever it is.

So, if we could convert everything into a standard matrix multiplication form, so that gives us a benefit. So, homogeneous representation is only moving towards this. Representing points by their homogeneous coordinates provides an effective way to unify the description of geometric transformation as matrix multiplication, very, very important point what we have discussed. In homogeneous coordinates, an n-dimensional space is mapped into n plus 1 dimensional space.

This is what is homogeneous representation. If you have 2 X 2, you will try to write it as 3 X 3, you will try to represent it as 4 X 4. 2 X 2, 2 dimensional you write it in 3, 3 X 3 matrix, when you have a 3 dimensional object you write it in 4 X 4. That is what is. So here, we are converting into matrix multiplication form.

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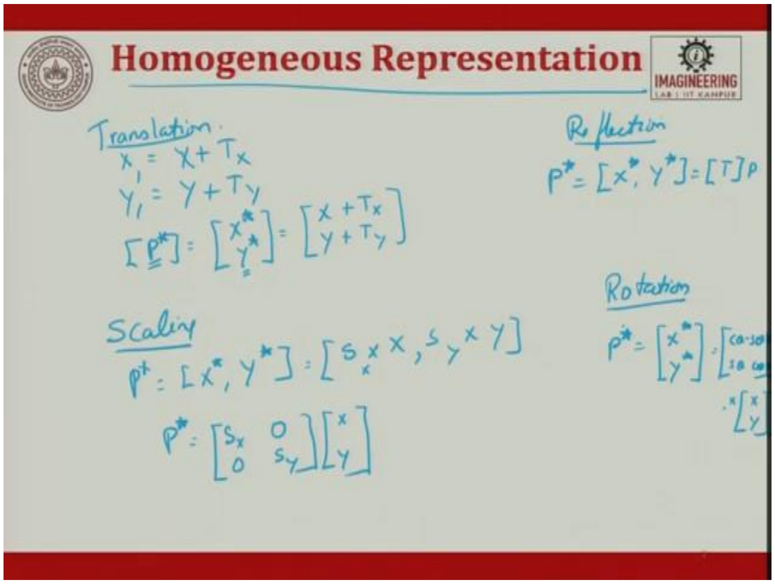

So, let us look into the homogeneous matrix representation. Whatever we have studied till now on various operations like, translation is nothing but X1 is X plus Tx, Y1 is nothing but Y plus Ty. So now, P^* can be written as,

$$P^* = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

the new matrix point, P star can be represented as X star, Y star, which is nothing but X plus Tx, Y plus Ty. So, these are the points and these are the converted points. These are X and Y which will be there, which gets converted.

So, when we try to do reflection, it will be P^*, will be X star comma Y star which is, can be written as transformation matrix into P. When we talk about scaling, it is P star equal to X star comma Y star, which is nothing but S of X into X comma S of Y into Y, so which is represented as P, Sx, 0, 0, Sy, we try to get it into X and Y.

When we talk about rotation, it is P star, these are the transformation which we saw, cos theta, minus sin theta, sin theta, cos theta which will be multiplied by X and Y.

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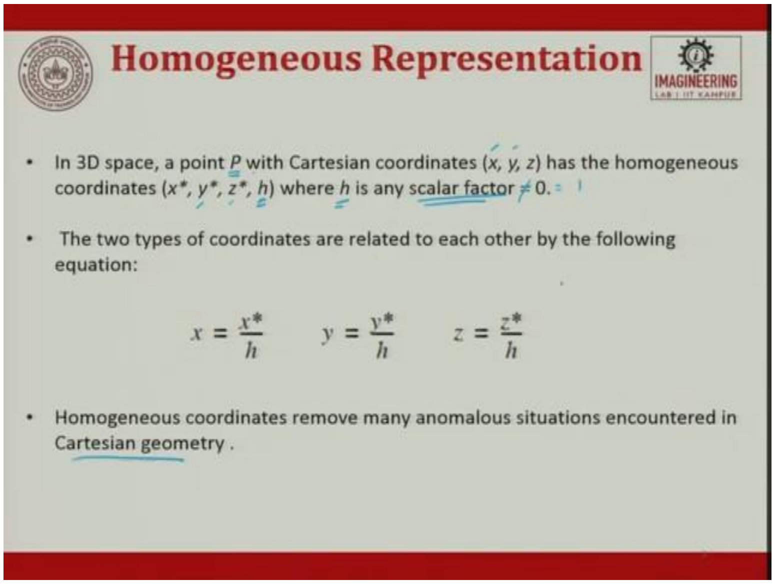
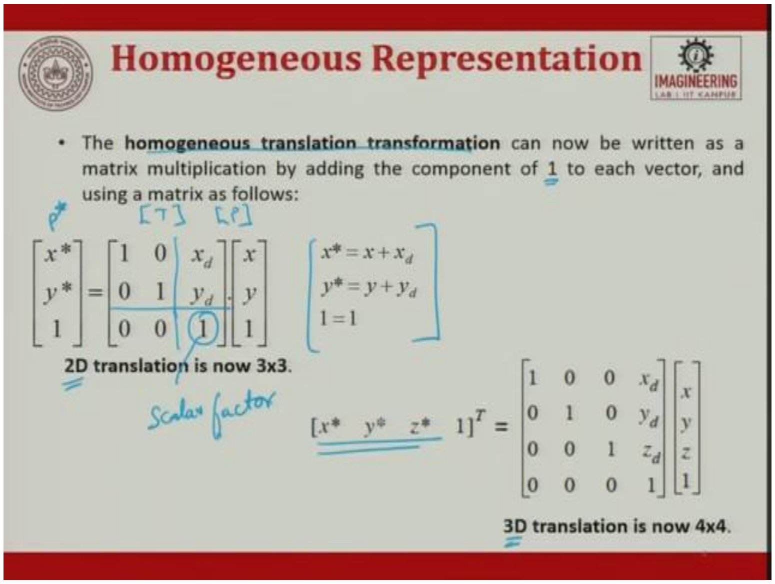
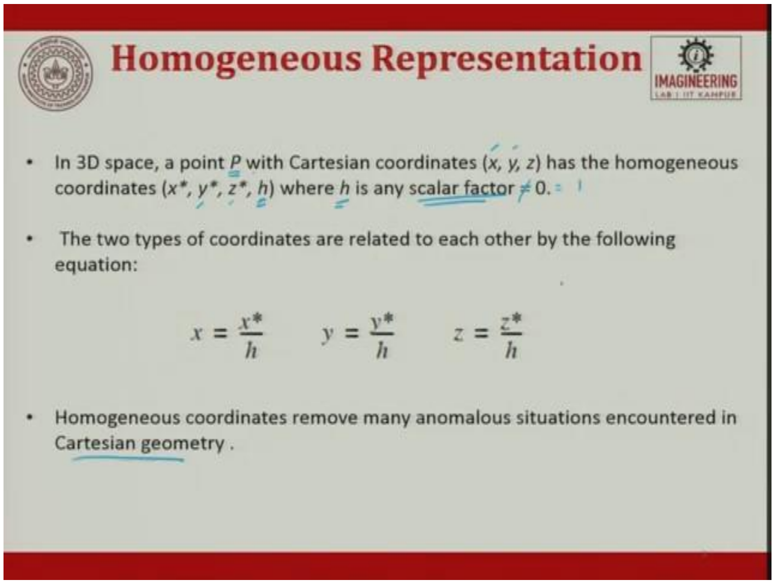


So, in 3D space, a point P with a Cartesian coordinate x, y, z has the homogeneous coordinates of x^*, y*, and z* with a term called as h, where h is the scaling factor or a scalar factor, it should not scaling, it is scalar factor which should never be equal to 0. Normally, we take the value of 1, the 2 types of coordinates are related to each other by,

$$x = \frac{x^*}{h} \quad , \quad y = \frac{y^*}{h} \quad , \quad z = \frac{z^*}{h}$$

Homogeneous coordinates remove many anomalous situations encountered in the Cartesian geometry. So, that is why we to write it in the homogeneous representation. (Refer Slide Time: 06:19)





So, the homogeneous translation transformation can now be written as a matrix multiplication by adding the component of 1 to each vector, and using a matrix as follows: P star y star 1, which is final destination points is equal to transformation matrix into the point P. So, which is nothing but this is the transformation homogeneous matrix. This 1 is a scalar, which we studied here, scalar factor.

So, the translation homogeneous translation matrix is now x which was 2 cross 2, is now expressed as 3 cross 3 and we try to get the homogeneous translation matrix like this. x^* and y^* is given by,

$$x^* = x + x_d$$

$$y^* = y + y_d$$

When you use a 3 X 3, this is 2 X 2 that is 2D, when you write it in 3D it is going to be, you will have a 4 cross 4 matrix multiplied with the points x, y and z and then you try to get the output.

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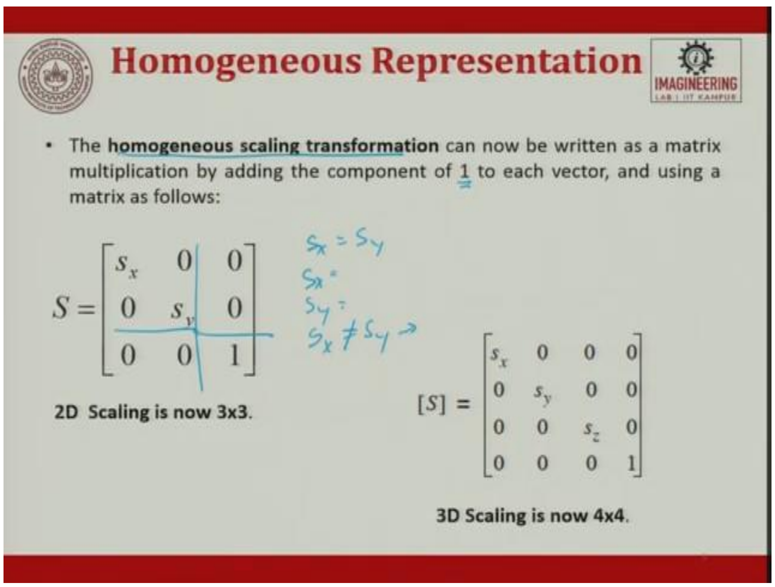

When we try to write the homogeneous scaling transformation can now be written as a matrix multiplication by adding the component of 1 to each vector, using a matrix as given below,

$$S = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Same for 4 X 4, as I earlier said Sx can be equal to Sy, Sx can only be taken, Sy can only be taken, Sx need not be equal to Sy. The matrix is given by,

$$S = \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, these are the possible cases which you can have in scaling. So here, this is something like sharing matrix, sharing operation you want to do, you can try to do.

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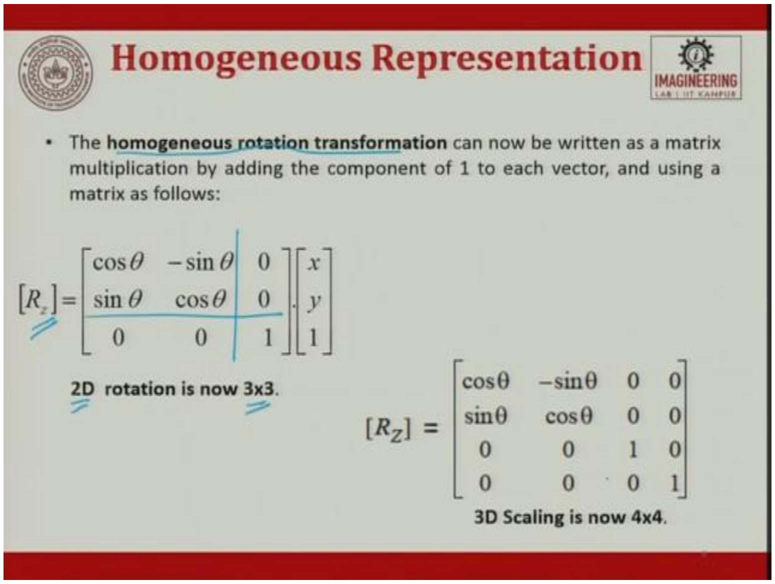

When we have to talk about the homogeneous rotation transformation can now be written in multiplication by adding the component 1. This was 2 X 2 which you studied, now that 2D homogeneous representation is called 3D, so you try to express it in this form. This is for rotation about z, for the same rotation about the z in 3 X 3 form will be something like matrix equation R_z

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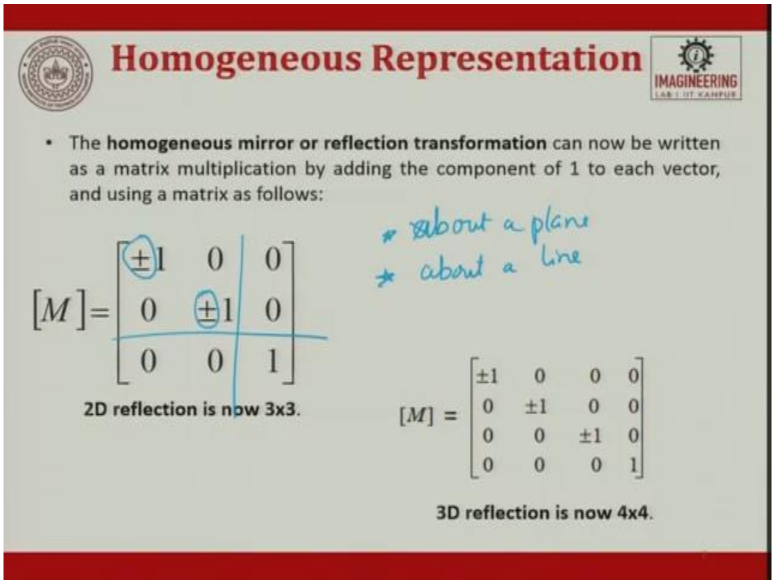

So now, it is very clear in the examination, you can expect problems in this transformation to be solved. You will have some assignment problems to be solved and the examination you will have small problems to be solved. So, remember those matrix, transformation matrix.

When we talk about the mirroring, so again the same way here, we can have plus or minus depending upon the value. So, as I told you about a plane, you can have a mirroring, you can have about a line, you can have. So then, you will have this plus minus values, so depending on that you choose it and then you write it. So, this is the homogeneous mirror or reflection transformation in the homogeneous representation form.

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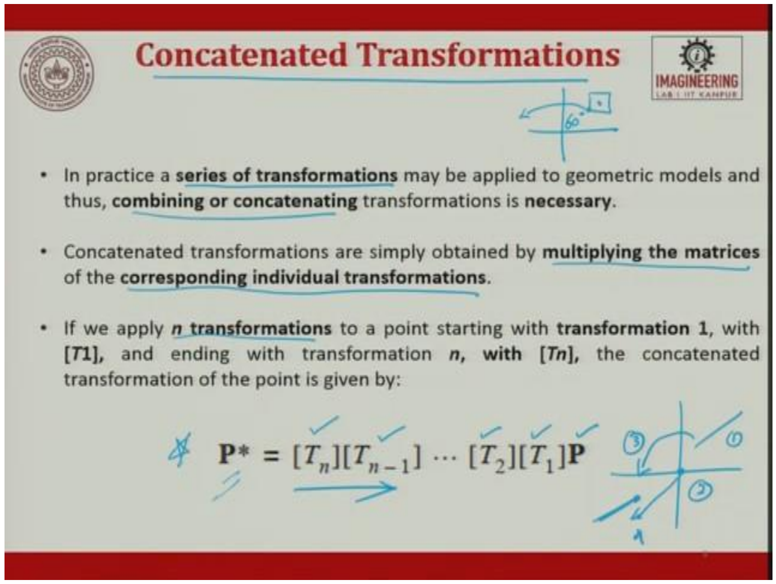

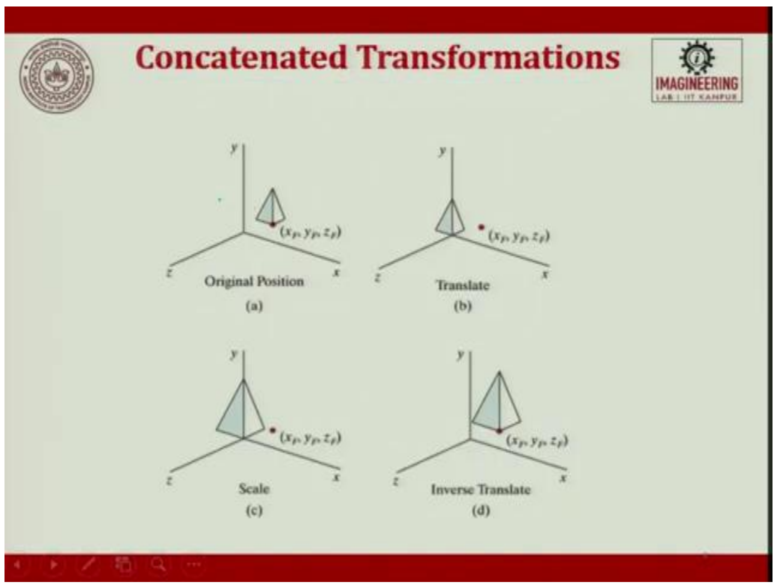

Now, we have seen only 1 step transformation, but in reality, what happens is there are several steps an object will undergo to go towards its final destination. For example, let us take here is a square, this square suppose I wanted to move this square about its axis, the center about its axis and I wanted to rotate the corner by another 30 degrees or 60 degrees in the clockwise direction. So now, I have to rotate about this axis, and then do the second.

So, there are 2 steps or let us try to take, I have a line, this line I am trying to move it to this point like this. So, step 1 is I try from here I will move towards the origin step 2, then I will try to rotate this to this point this will be step 3, and then I will try to translate it to here that will be step 4. So, these are 4 steps now involved. So, if there are several steps involved, now you have to follow a sequence of operation. So, that is called as concatenate transformation or concatenation.

This is followed in robotics, this is followed in CAD also. This is also followed when you start doing with respect to drones. Any moving object, when there are several links where the transformation has to happen to get the final coordinates, we use this concatenation. In practice, a series of transformation is done, is applied to a geometric model and thus combining or concatenating transformation is necessary. That is why it is called as concatenation or concatenated.

Concatenated transformation are simply obtained by multiplying the matrices to the corresponding individual transformation. So, you will do individual multiplication and then you will try to, arrange them in sequence such that you try to get the final spot whatever you want. If we apply n transformations to a point starting with transformation 1, with T1 and end the transformation n, with Tn, the concatenated transformation of the point is given by

$$P^* = [T_n][T_{n-1}] \dots [T_2][T_1]P$$

Last one becomes the first step, then the previous, then the previous, then the previous, then previous, so that is what is told here. If we can apply n transformation to a point starting with a transformation 1, with T1 and end with n, end with n with Tn, the concatenated transformation of the point is given by P star is nothing but Tn, Tn minus 1 going up to P. So, this is very, very important. Please keep this in mind.

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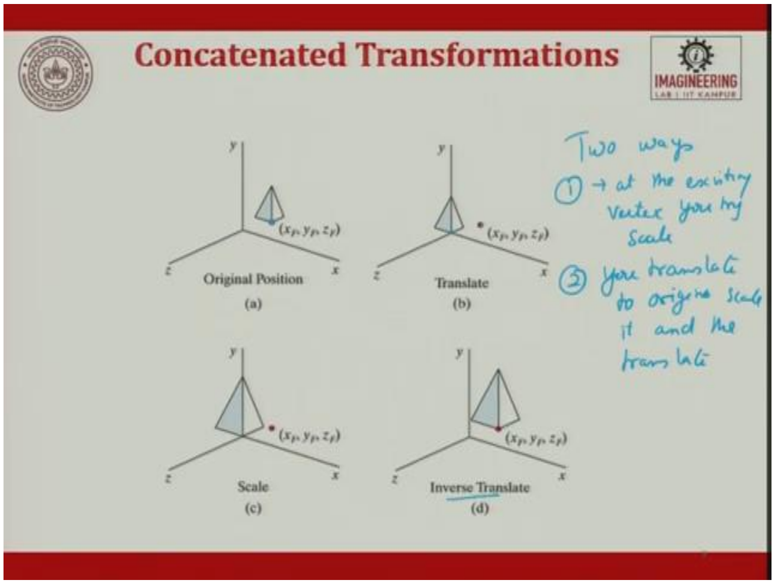

We will see an example next. Suppose, let us assume, there is a prism and a corner of a prism, you have located this corner, it has to be translated. So, and then it has to be scaled, and then you have to come back to the original position. One way of doing it is, at this point itself you multiply it with a scaling factor and then you try to put that in the same point, that is one way of doing it.

The other way of doing it is, I try to take the point, try to move this point or the vertex to the origin, and try to multiply and do scaling at the origin keeping origin as a base and then now I transform it to this point. That is also possible. I have told you 2 ways, first way is at the existing point itself, existing vertex you try to scale. Second, you translate to origin, scale it and then translate to that point.

When we do this there are several steps which are followed. So, first is this point, then from this point you are moving to here, then you are trying to do scaling and then you are trying to bring back, inverse translation.

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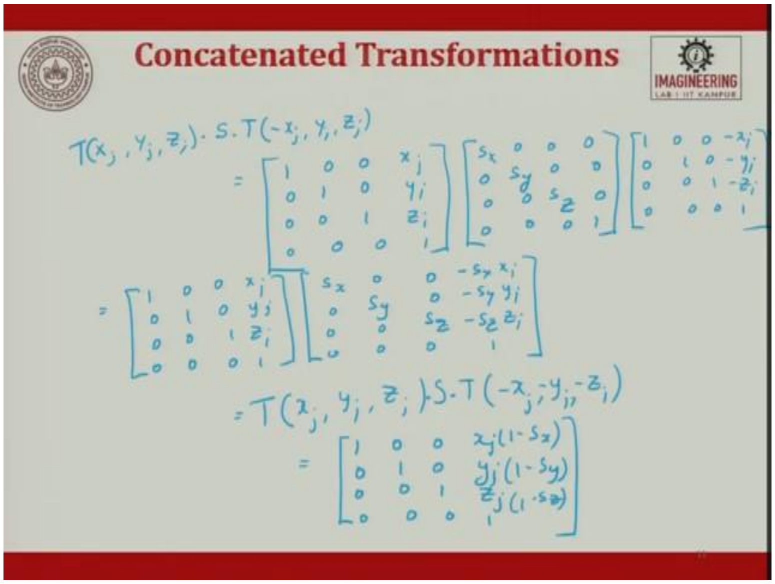

So, if we wanted to represent this in the matrices form, we can try to write it as $T(x_{j,}\ y_{j,}\ z_j).S.T(-x_{j,}\ y_{j,}\ z_j)$

$$\mathrm{T(xj,\ yj,\ zj).\ S.T(\text{-}xj,\ yj,\ zj)} = \begin{bmatrix} 1 & 0 & 0 & \mathrm{xj} \\ 0 & 1 & 0 & yj \\ 0 & 0 & 1 & zj \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \mathrm{\text{-}\ xj} \\ 0 & 1 & 0 & -yj \\ 0 & 0 & 1 & -zj \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

On solving, we get,

$$\mathrm{T(xj,\ yj,\ zj).\ S.T(\text{-}xj,\ yj,\ zj)} = \begin{bmatrix} 1 & 0 & 0 & \mathrm{xj(1 - Sx)} \\ 0 & 1 & 0 & \mathrm{yj(1 - Sy)} \\ 0 & 0 & 1 & \mathrm{zj(1 - Sz)} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This will be represented this, where we have started, we have done the concatenated transformation for the given object.

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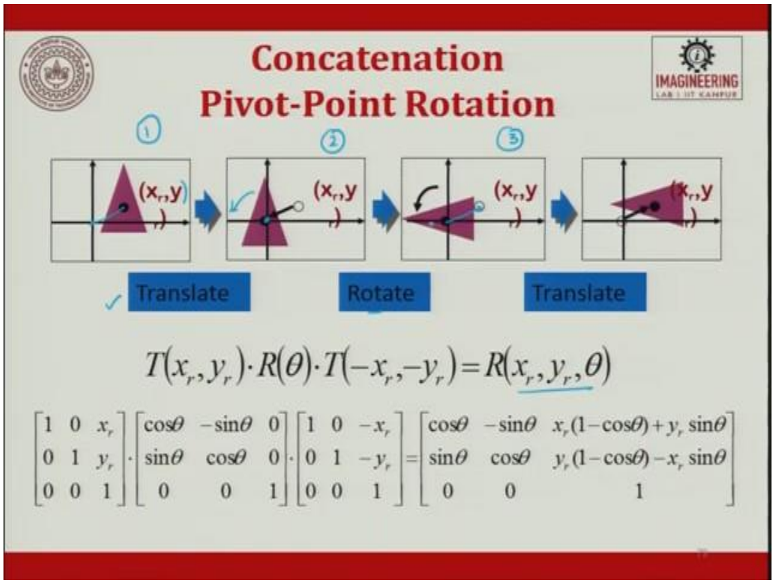

Since we are already discussing about concatenation, here we will try to do it systematically, such that you will try to see how does it happen mathematically in a CAD software. So here, this example is taken to discuss about a pivot point rotation, so here is a triangle. We have asked you to first find out the center of the triangle, and now that we consider is as a pivot point. So, now what we do is we try to first move the pivot point to the origin, so that is translation.

After it is moved to the origin, now we are trying it to rotate, so that is the next step, we rotate it. And after it is rotated, we would like to translate it back to the point. So, we will have x, this is first step, this is the second step, and this is the third step, we will try to do. So, in concatenation you already know we will start solving it from last to first. So here, the T, translation matrix is x, y translation matrix into rotation whatever it is and then you will have a translation back to point, so you will have this.

So, this is the final matrix whatever is there. So, we will have this translation, then rotation, then translation, when we multiply all the 3, we get into this form.

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r \sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r \sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

So here, is a simple example where multiple operations are done to get to the final answers. So here, there are several events which has to happen to do one operation. So, when we do an example, you will have a more clarity on it.

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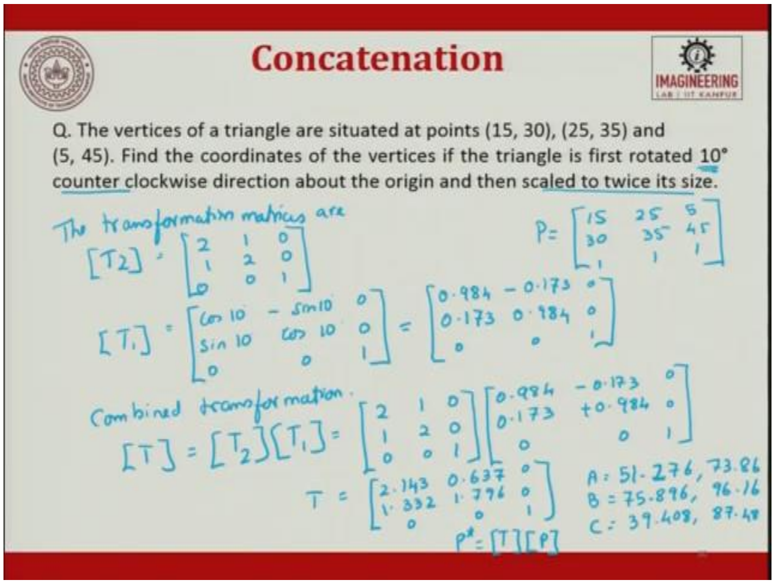

So here, let us take an example, the vertices of a triangle are situated at points (15, 30), (25, 35), and (5,4). Find the coordinates of the vertices if the triangle is first rotated 10 degrees in counter clockwise direction, you should see this, counter clockwise direction. If I have not given in the examination, you have to assume it as counter clockwise only, about the origin and then scale it twice.

You rotate it and then you scale it with respect to what? With respect to the coordinates or the vertices if the triangle is rotated 10 degrees about the origin and then scale. So now, let us start doing, solving the problem. So,

$$P = \begin{bmatrix} 15 & 25 & 5 \\ 30 & 35 & 45 \\ 1 & 1 & 1 \end{bmatrix}$$

1^{st} transformation matrix,

$$[T_1] = \begin{bmatrix} \cos 10 & -\sin 10 & 0 \\ \sin 10 & \cos 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.984 & -0.173 & 0 \\ 0.173 & 0.984 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Second transformation matrix,

$$[T_2] = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$[P^*] = [T_1][T_2][P]$$

On solving, we get, the 3 vertices are going to be, A is going to be (51.276, 73.86), then B is going to be (75.896, 96.16), C is going to be (34.408, 87.48). So, this will be the answer.

So here, we have done only 2 steps, you can also have multiple steps. In the examination, you can expect 2 step transformation or 3 step transformation. So basically, you have to bring your calculator and you have to be quick in doing this matrix multiplication. Please do practice and then come for the examination. Please try to do matrix multiplication, have lot of practice.

In the examination, online you will have a calculator, you will have to use that and you have to solve the problem. The steps are very easy, but if I give you a 3 translation form, then you have to be quick in solving that problem and here you will, it will be binary. So, do yes or no. So,2619 if you get the right answer, you get, and the choices we will try to give as close as possible. So, until and unless you solve, you cannot take the correct answer.

And we can also have link questions, so we can have a link saying that first step you solve it we will display the answer and then we will do the third step, you will multiply and then get the answer. So, if the first question is right, then the second question also will be right. So, you have to solve all the steps and use the online calculator for solving it. Thank you.

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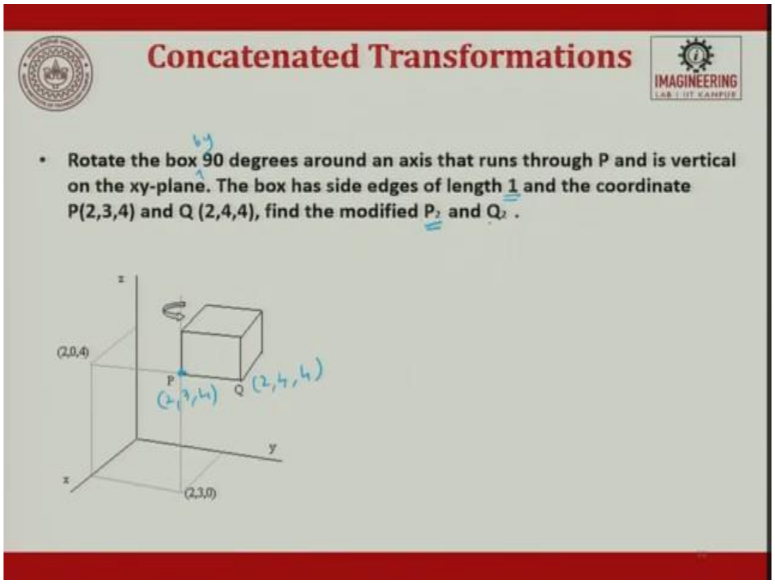

So now, let us take an example and try to solve the example. Rotate the box by 90 degrees above, around an axis that runs through P, point P and is vertical on the xy-plane. The box has side edges of length 1 and the coordinates P is nothing but (2, 3, 4) and Q is nothing but (2, 4, 4), find the modified P2 and Q2.

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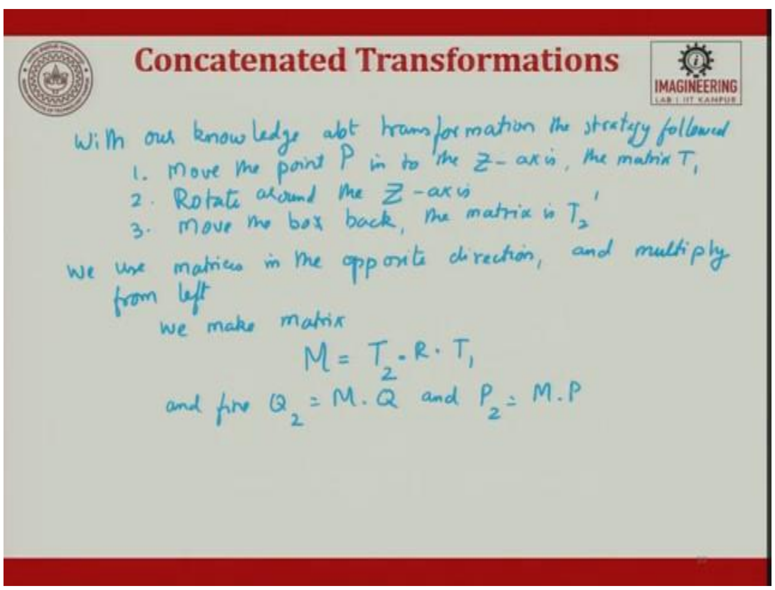

With our knowledge about transformation, the strategy followed, first, move the point P into the Z-axis, the matrix is T1. Two, rotate it, rotate around the Z-axis. Move the box back, the matrix is T2 dash transformation. So, we use matrix, matrices in the opposite direction and multiply from left. So,

M = T2. R. T1, and

Q2 = M. Q, and

P2 = M. P

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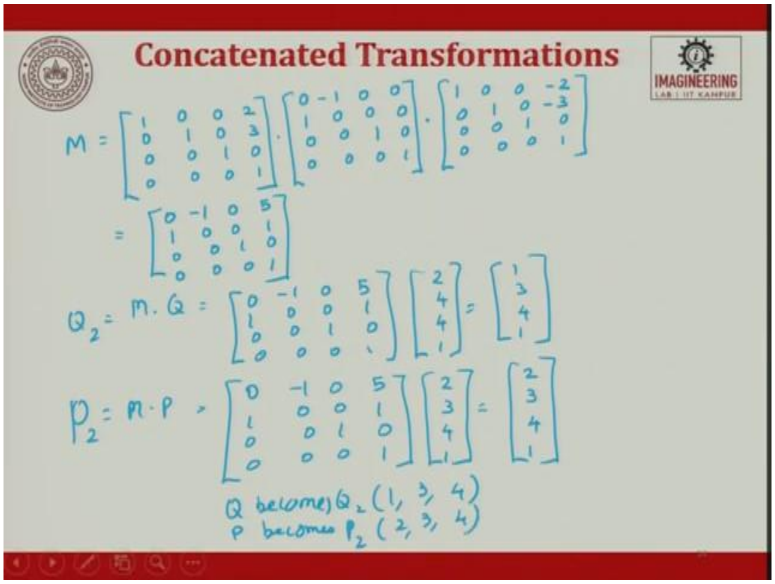

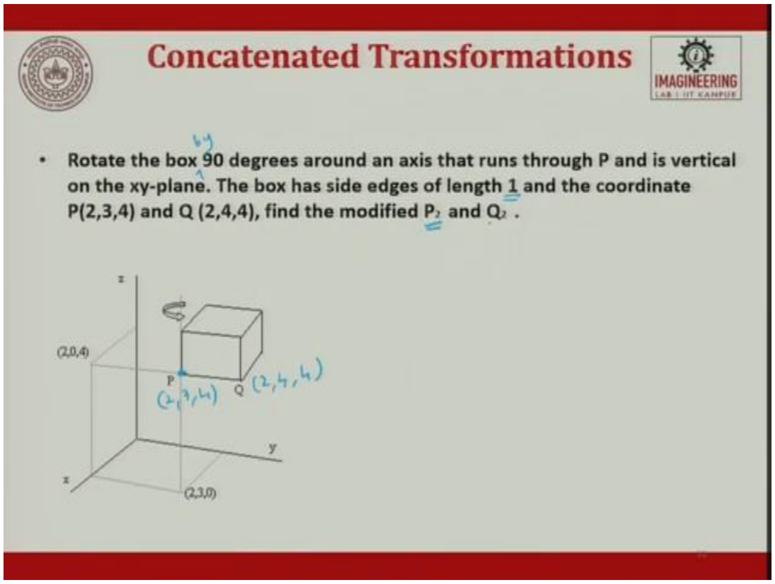

So,

$$[M] = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So now, on solving for Q2,

$$[Q2] = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

Similarly, on solving for P2,

$$[P2] = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

So, this is how you will try to solve the problem if you have a simple problem like this, a 3 dimensional object rotated. You can also have in your examination 2 dimensional rotation also.

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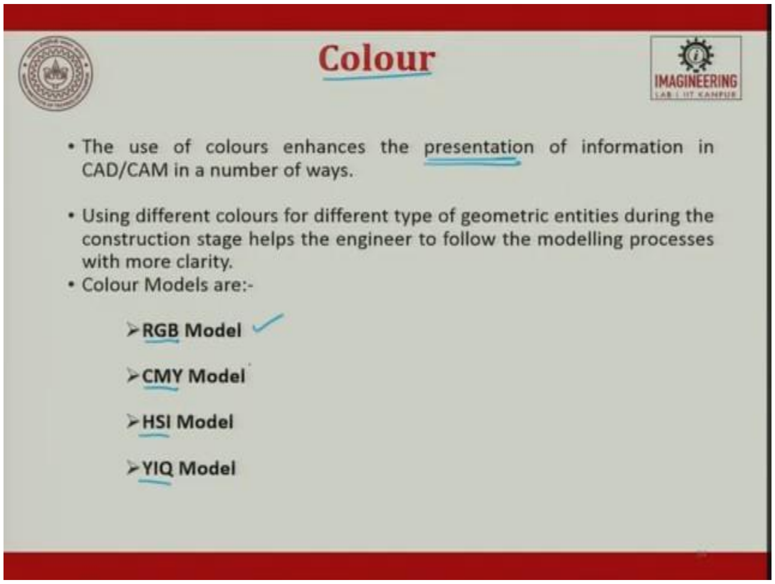

The last topic to be discussed is, colors. The use of colors enhances the representation of information in the CAD, CAM. Even in a cartoon figure, when a cartoon is drawn and if you give a red color to it, it looks as thought the cartoon character is angry. So, just by giving the color and some features, you can try to give the representation information. So, color plays a very important role. When you do finite element analysis, we always look at the points where there are more number of stress it will be red in color.

So, use different colors for different type of geometric entities during the construction stage, helps the engineer to follow modeling process with more clarity. So, the color models are, you have RGB, you have CMY, HSI, and YIQ. These are the 4 models. This is very common, red, blue, green model which we do.

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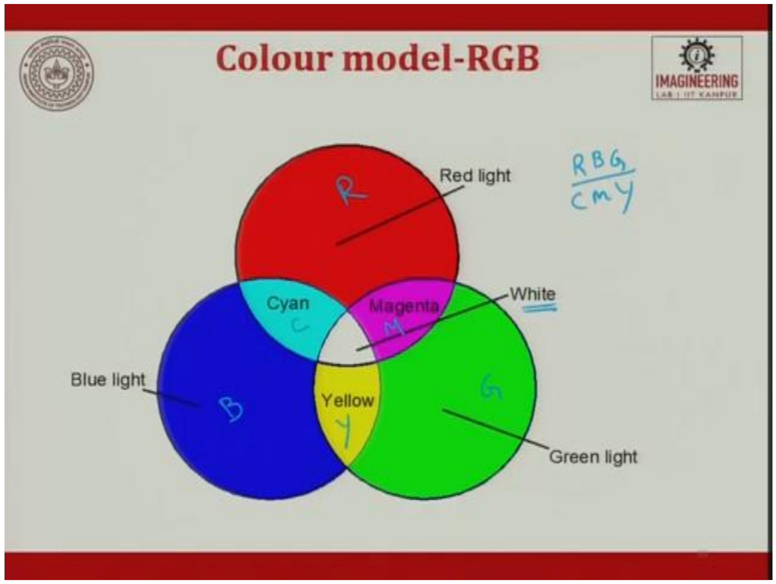

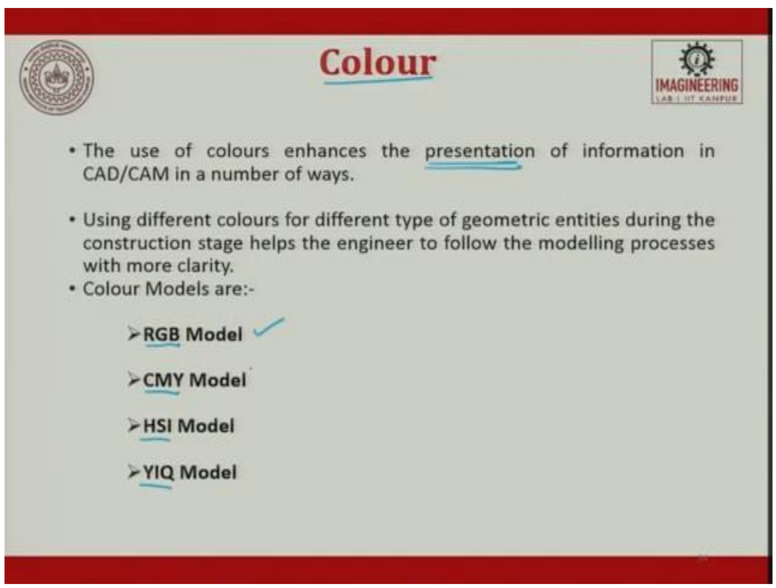

And this model, when we try to do this R, B and G, so this is R, B and G. When we try to have merging of A union B, you get C, Y and M model. Look at it, CYM model.

When a combination of all these things happen, you get a white light. So, you can follow either this model for coloring, or CMY model for coloring. So, each has its own advantage and disadvantages.

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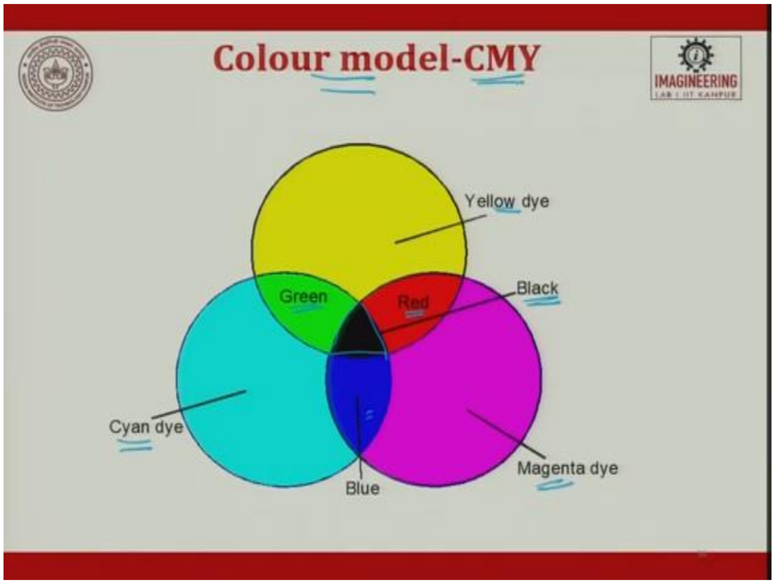

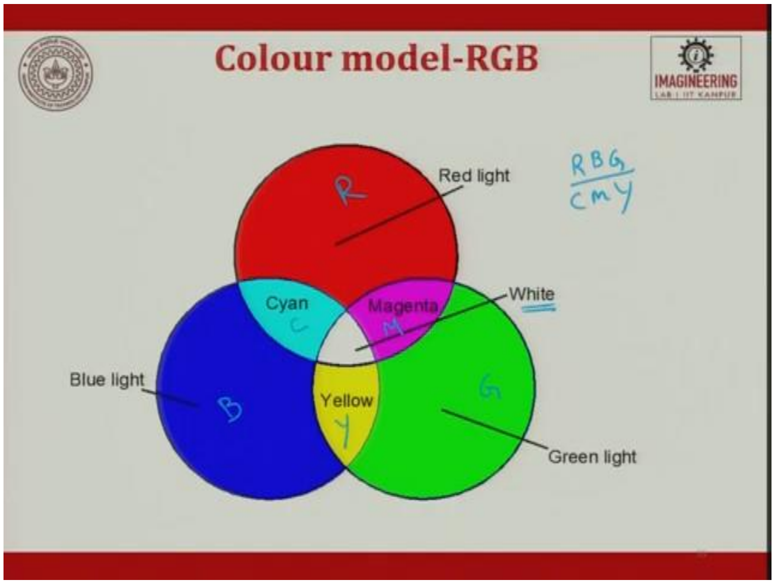

So, this is CMY, when we try to do. CMY is, C becomes Cyan, then Yellow, then Magenta. So, whatever we had, green, red, blue, when we merge these 2, you get these green, red and blue. Now, what you get in the center portion is black. When you look into the previous one, the center portion was white.

And, now what happens this CMY dye you apply, the center portion is black. So, this is the second coloring model, CMY model.

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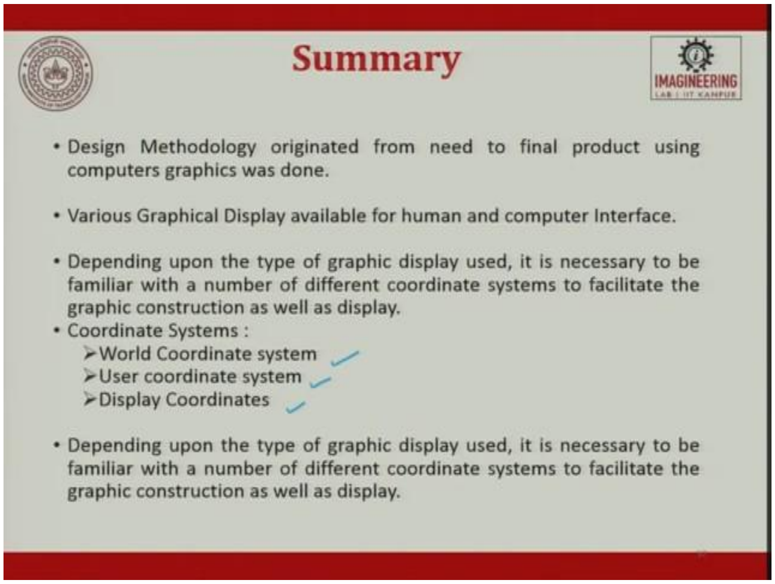

To summarize, what we saw in this particular chapter, we saw design methodology originates from need to final product using the using computer graphics. Various graphic displays available for human and computer interface we saw. Depending on the type of graphic display used, it is necessary to familiarize the number of different coordinate systems to facilitate the graphic construction.

So, we had world coordinate system, user coordinate system and display coordinate system, these are the different types of coordinate systems we saw. Depending upon the type of graphic display used, it is necessary to familiarize with a number of different coordinate systems to facilitate the graphic construction as well as display.

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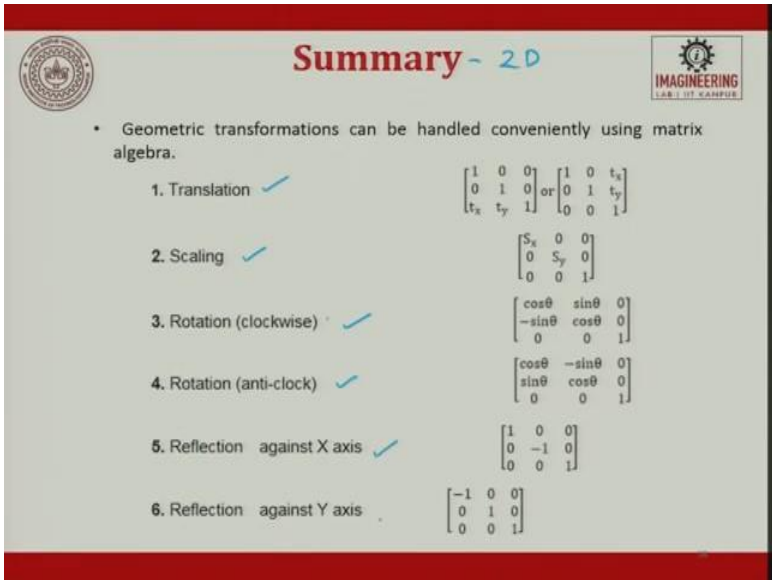

To summarize, these are the formulas which we saw in this lecture. So, translation, how do we do. Scaling, these are all for 2D. So, summary for 2D and rotation about clockwise, rotation about counter-clockwise, reflection about X and reflection about Y. You can also have reflection about a line.

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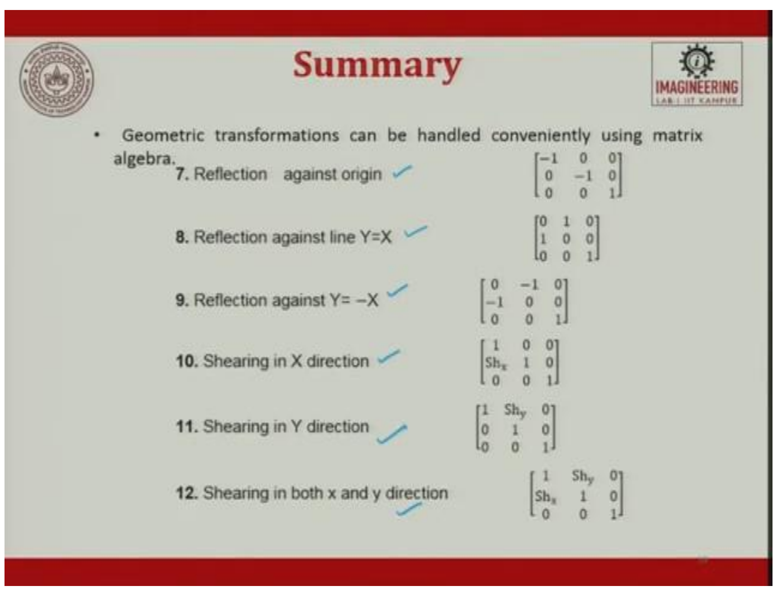

Then reflection about an origin, then reflection about against a line, you can have Y equal to X, Y equal to minus X. You can also have shearing, shearing is nothing but indifferent scaling shearing in X, shearing in Y, shearing both X and Y.

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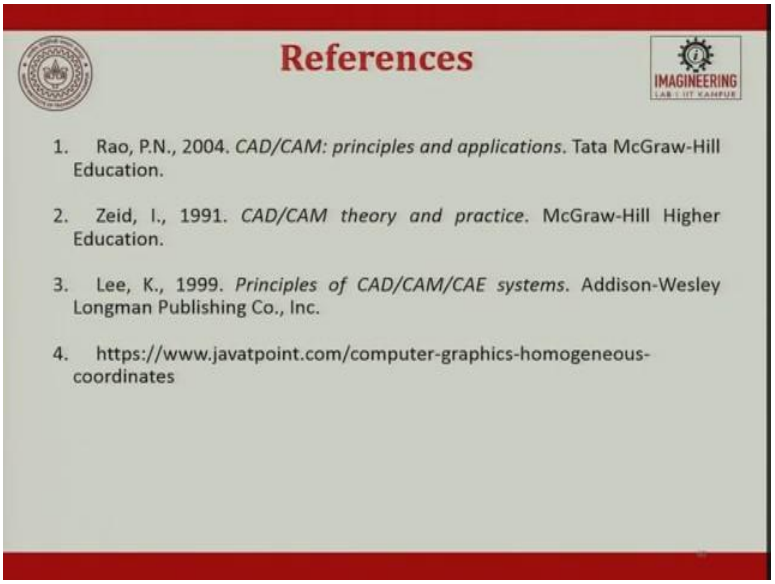

So, these are the reference.

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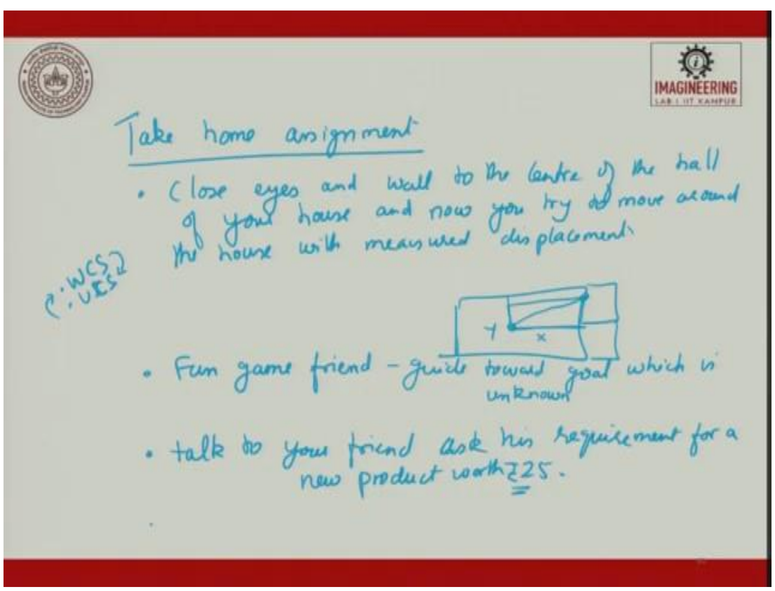

Let us take an assignment, take home assignment. And here, all the assignments what I give, you do not have to submit it to me. These assignments are only used for your better understanding and appreciating CAD.

Let us take the first thing, do it by yourself. First is, let us try to close your eyes and walk to the center of the hall of your house and now you try to move around the house with measured displacements. For example, you are in a hall, center of the hall, you have a room here, a room here, you know from here to here, this is the X and you know this is the Y and you have to enter it to this place. Now, you close your eyes and start going to this place and then try to approach the door.

So, when you do it, you will try to see multiple approaches, how does translation happen. Whether we move along here and then go here, or from here go here, or diagonally we go. So, this tries to teach you about translation. And try to also have a fun game with your friend, wherein which your friend keeps guiding you towards a goal which is unknown to you. A computer behaves the same way, all you have to do is you have to give him those information such that it goes.

The third thing is, so and also here you will try to appreciate the world coordinate system and user coordinate system. So, after entering into the room, still if you use this as a reference point, you will get confused. So, you will try to convert from world to user, user to world. So, the same thing when you do it on a computer, you will follow the same. Whatever I have said this, you will try to experience the same.

Next thing is, let us assume that you talk to your friend and ask his requirements for a new product worth 25 rupees. So, you should ask him, suppose I give you 25 rupees and I want you to buy a product or a part or a toy, what will you buy? So, or if you say that I am going to give you a toy worth of 25 rupees, now you tell me what is all expected from that toy.

So, if you do this, then you will understand how are the requirements from the customer are understood and the designer starts working. So, try to do these 2 assignments so that you will try to enjoy, along with the course what is dealt. Thank you very much.