Numerical Linear Algebra
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Lecture – 57
Rate of Convergence of Power Method

Hello friends, welcome to my lecture on Rate of Convergence of Power Method. In our previous lecture we studied how to determine the dominant eigenvalue of a real diagonalizable square matrix of order n, by using the power method algorithm.

Now, we are going to determine the rate of convergence of the power method algorithm. So, we shall study a theorem on the rate of convergence of the power method algorithm, which we know is used to determine the dominant eigenvalue of A real diagonalizable square matrix A, A of order n. If we assume that the eigenvalues of Ar orders in such a way that let us say if the eigenvalues of A A r lambda 1 lambda 2 lambda n, then mod of lambda 1 is greater than mod of lambda 2, greater than or equal to mod of lambda 3 and so on greater than or equal to mod of lambda n. So, lambda 1 is the dominant eigenvalue of A.

So, let us see how what will be period of convergence of the power method algorithm if a real diagonalizable a square matrix of order n that the eigenvalues lambda 1 lambda 2 and so on lambda n where lambda lambda 1 is the dominant eigenvalue.

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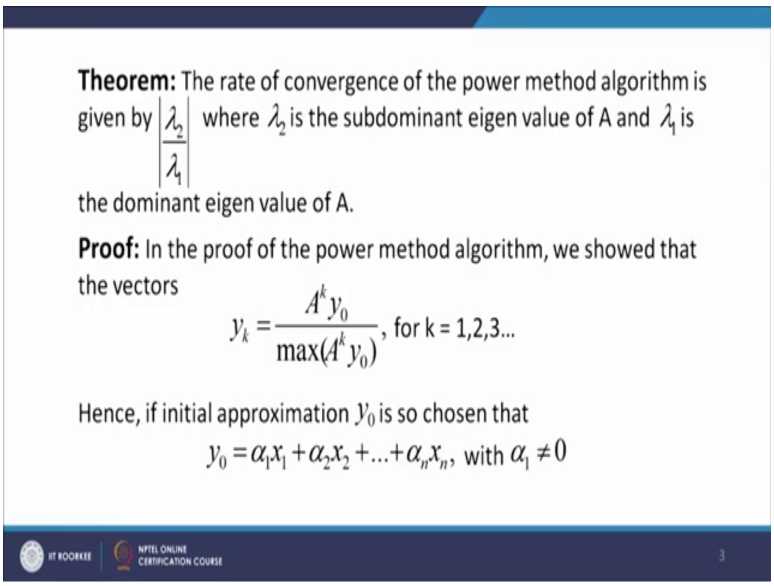

The theorem says that the rate of convergence of the power method, algorithm is given by mod of lambda 2 over lambda 1 where lambda 2 is the sub dominant eigenvalue of A and lambda 1 is the dominant eigenvalue of A sub dominant eigenvalue means it is the next dominant eigenvalue of A. Lambda 1 is the dominant eigenvalue, and lambda 2 is the next dominant eigenvalue are sub dominant eigenvalue of A.

Now in the proof of the power method algorithm if you recall, we proved that the vectors y k are given by A k y naught over maximum of A k y naught for k equal to 1 to 3 and so on by using the induction process on mathematical induction on k. Now if the initial approximation y naught here is chosen in such a way that y naught is equal to alpha 1 x 1 plus alpha 2 x 2 and so on, alpha x n where be assumed that alpha 1 is not equal to 0.

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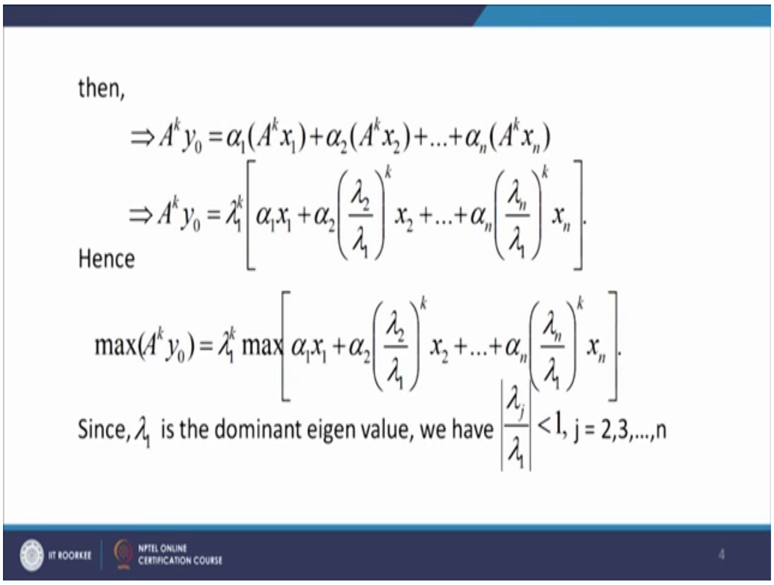

Then A k y naught you can apply A k operator on this equation you get A k y naught equal to alpha 1 A k x 1 plus alpha 2 A k x 2 and so on alpha n A k x n.

Now if lambda 1 is the eigenvalue of A, and x 1 is the corresponding eigenvector, then we know that lambda 1 to the power k is an eigenvalue of a to the power k and x 1 is the corresponding eigenvector. So, here A k x 1 is lambda 1 to the power k into x 1 and A k x 2 is lambda to the 2 to the power k into x 2, and A k x n is lambda n to the power k into x n. And there so, what we do is we write A k y naught equal to lambda 1 to the power k times alpha 1 x 1 plus alpha 2 times lambda 2 over lambda 1 to the power k and so on.

Obviously, lambda 1 is not equal to 0, because mod of lambda 1 is greater than mod of lambda 2 and mod of lambda greater than or equal to mod lambda 3 and so on.

So, lambda 1 in non-0 and therefore, we can write A k y naught in this manner. And so, maximum of A k y naught, maximum of a k minus lambda 1 to the power k into maximum of this expression in the inside the brackets, because we have seen that maximum of c into x, maximum of c into x is equal to c times maximum of x.

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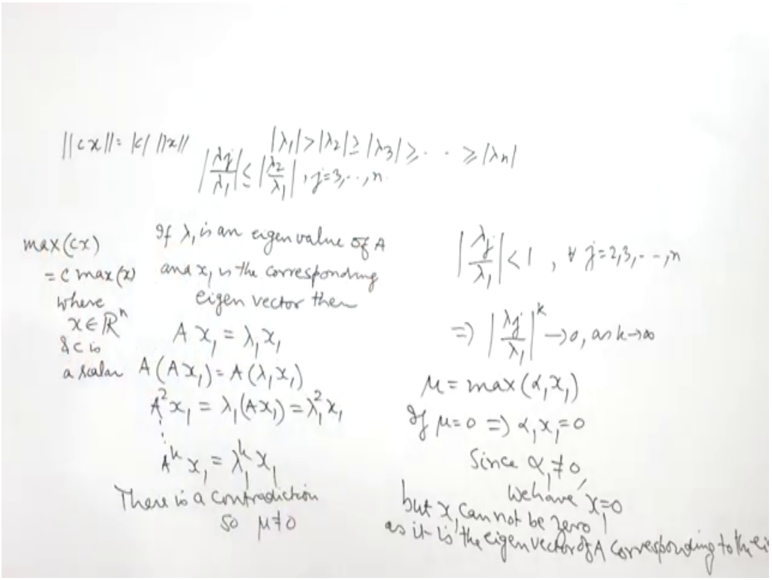

Where x is the vector belonging to R to the power n, and c is a scalar.

So, maximum so, lambda 1 to the power k is a scalar. So, maximum of a k y naught is equal to lambda 1 to the power k into maximum of this expression inside the bracket now lambda 1 is the dominant eigenvalue. So, modulus of lambda j over lambda 1 will be less than 1 for j equal to 2 3 4 and so on up to n.

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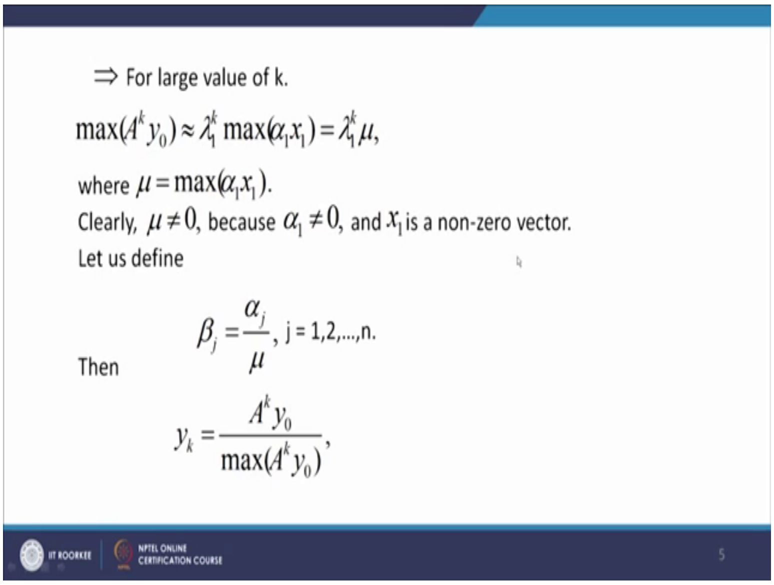

And because of this for large value of k mod of lambda j divided by lambda 1 is less than 1 for all j is equal to 2 3 and so on up to n.

So, what will happen as lambda so, this implies that mod of lambda j over lambda 1 to the power k is goes to 0 as k goes to infinity. And therefore, for large value of n for large value of k, we can say that maximum of A k y naught, maximum of A k y naught is equal to lambda 1 to the power k into maximum of alpha 1 x 1 because the remaining terms will be very, very small. So, approximately we can say that maximum of the expression inside the brackets is equal to maximum of alpha 1 x 1. So, we have a maximum of A k y naught equal to lambda 1 times lambda 1 to the power k into maximum of alpha 1 x 1 approximately. And let us denote maximum of alpha 1 x 1 by mu 1. So, we get lambda 1 to the power k into mu. Now here, so mu is here maximum of alpha 1 x 1.

Now, let us note that mu is not equal to 0, why because mu is equal to mu is equal to alpha 1 x 1. So, if mu is equal to 0, if mu is equal to 0, then maximum of alpha 1 x 1 is equal to 0 means that alpha 1 x 1 equal to 0. Now we have assumed while writing the equation y naught equal to alpha 1 x 1 plus alpha 2 x 2 and so on, alpha n x n we have assumed that y naught is expressed in such a way that in terms of x 1 x 2 x n in terms of x 1 x 2 x n it is expressed in such a way that alpha 1 is not equal to 0. So, since alpha 1 is not 0 we have x equal to 0. But x x 1 equal to 0, but x 1 is an eigenvector of the matrix A corresponding to the eigenvalue lambda 1, so, x 1 cannot be 0 ok. But x 1 cannot be 0, as it is the eigenvector of A is the eigenvector corresponding to the eigenvalue lambda 1, corresponding to the eigenvalue lambda 1. So, so, x 1 cannot be 0 and therefore, there is a contradiction. So, o, mu is not equal to 0.

Now, so, let us define new constants beta j beta j equal to alpha j over mu for j equal to 1, 2, 3 and so on up to n. Then, y k we can write as then since y k is equal to A k y naught over maximum of A k y naught ok, what do we get y k is approximately alpha 1 x 1 plus alpha 2 x 2 lambda 2 lambda by one to the power k x 2 and so on, alpha n lambda n by lambda 1 to the power k x n.

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$$y_k \approx \frac{\left(\alpha_1 x_1 + \alpha_2 \left(\frac{\lambda_2}{\lambda_1}\right)^k x_2 + \ldots + \alpha_n \left(\frac{\lambda_n}{\lambda_1}\right)^k x_n\right)}{\mu}$$

$$= \frac{\alpha_1}{\mu} x_1 + \frac{\alpha_2}{\mu}\left(\frac{\lambda_2}{\lambda_1}\right)^k x_2 + \ldots + \frac{\alpha_n}{\mu}\left(\frac{\lambda_n}{\lambda_1}\right)^k x_n$$

$$= \beta_1 x_1 + \beta_2 \left(\frac{\lambda_2}{\lambda_1}\right)^k x_2 + \ldots + \beta_n \left(\frac{\lambda_n}{\lambda_1}\right)^k x_n.$$

See the numerator here we have written for A k y naught, A k y naught is what A k y naught is this one, lambda 1 to the power k into this bracketed expression. So, that we have written for A k y naught in the numerator here ok, and maximum of A k y naught maximum of A k y naught is approximately lambda 1 to the power k over mu. So, we have used this approximate value in the denominator. So, we get and lambda 1 to the power k then we canceled so, we get y k approximately equal to this, ok. Now we have alpha 1 by mu into x 1 plus alpha 2 by mu into lambda 2 by lambda 1 to the power k into x 2, and so on alpha n by mu into lambda n by lambda 1 to the power k into x n.

Now by our notation, we have said that we have denoted alpha g over mu by beta j for j equal to 1 2 3 and so on up to n. So, we write alpha 1 over mu as beta 1. So, we get beta 1 x 1 then alpha 2 over mu at beta 2. So, we get beta 2 lambda 2 by lambda 1 to the power k into x 2 and so on, alpha n by mu is beta n should be get beta n lambda n by lambda 1 to the power k into x n. So, this is approximate value of the vector y k, ok.

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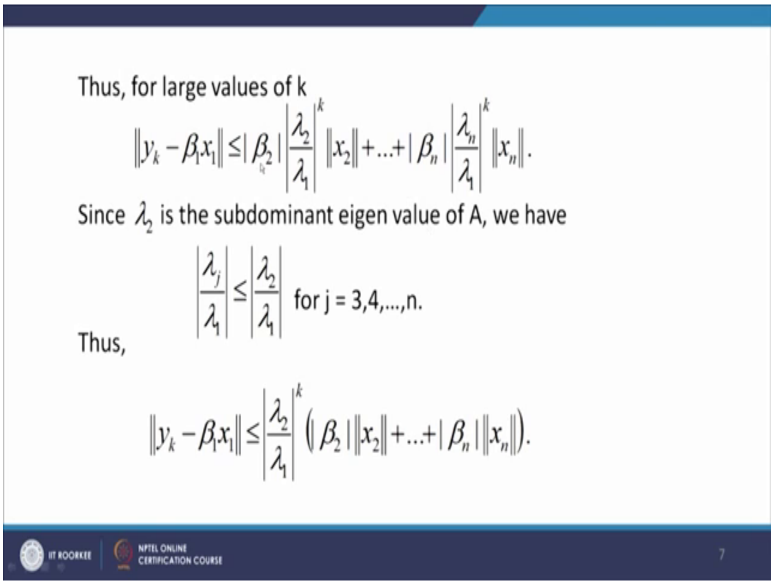

Thus far large values of k, because for large values of p only maximum of A k y naught was approximately equal to lambda 1 with the power came to mu. So far large values of k what do we get norm of y k minus beta 1 x 1. So, let us see this is y k y k equal to beta 1 x 1 into this beta 1 x 1 plus beta 2 upon into lambda 2 upon lambda 1 to the power k into x 2 and so on. So, let us bring beta 1 x 1 to the left side and take the norm. So, norm of y k minus beta 1 x 1 is less than or equal to norm of the norm of beta 2 into lambda 2 by lambda 1 to the power k 2 x 2 and so on, norm of beta and into lambda n by lambda 1 to the power k into x n.

Now, using the properties of norm, what we have? Mod of beta 2 into mod of lambda 2 by lambda 1 to the power k into norm of x 2 and so on mod of beta n into mod of lambda n by lambda 1 to the power k into norm of x n. Because you know that when we take the properties of norm is that c times x is equal to norm of cx is equal to mod of c into norm of x. Wherein x is in where c is a scalar and x is an element of the norm linear space, ok.

So, here now again lambda 2 is the sub dominant eigenvalue of A therefore, mod of lambda j over lambda 1, is less than or equal to mod of lambda 2 over lambda 1. You can see you see we have assumed that this is what we have assumed ok. So, so, dominant eigenvalue is one sub dominant eigenvalue is mod of lambda 2. So, mod of lambda 2 over lambda 1 mod of lambda j over lambda 1, is less than or equal to mod of lambda 2 over lambda 1. Or you can say that mod of lambda 2 is greater than or equal to mod of lambda j for j equal to 3 4 and so on up to n ok. So, it is clear? From here it is clear?

So, mod of lambda j one divided by lambda 1 less than or equal to mod of lambda 2 over lambda 1 and therefore, what we can do in this equation I can write mod of norm of y k minus beta 1 x 1 less than or equal to modulus of lambda 2 over lambda 1 to the power k, inside we shall have mod of beta 2 into norm of x 2, because in every term mod of lambda j over lambda 1 to the power k will be less than or equal to mod of lambda 2 by lambda 1 to the power k. So, we can write mod of beta 2 norm of x 2 and so on mod of beta n into norm of x n.

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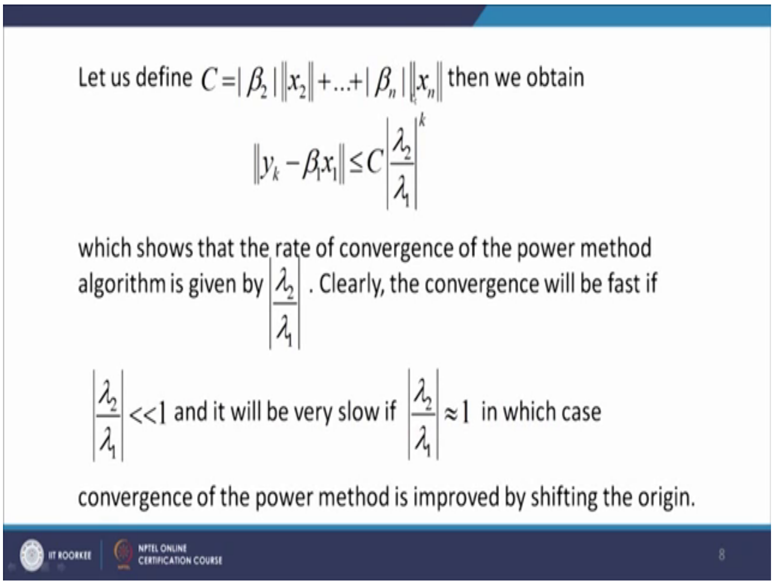

Now let us define this mod of beta 2 into norm of x 2 and so on mod of beta n into norm of x n by a constant say c, then we obtain norm of y k minus beta 1 x 1 less than or equal to some constant c times modulus of lambda 2 over lambda 1 to the power k. And which gives us the rate of convergence of the power method algorithm. And a it follows that mod of lambda 1 over lam lambda 2 or lambda 1 is the rate of convergence of the power method algorithm.

Now here you can see that the convergence will be fast if mod of lambda 2 what lambda 1 is very, very less than 1, because then it be having a mod of lambda 2 over lambda 1 to the power k, as k goes to infinity as k takes larger and larger values. If mod of lambda 2 over lambda 1 is very, very less than 1, then y k will be approximately equal to beta 1 k 1 x 1 so, the convergence will be fast. And the y k will tend to beta 1 x 1 very slowly if mod of lambda 2 over lambda 1 is nearly 1 in which case the convergence of the power method algorithm will be slow.

Now, so, when the modulus of lambda 2 over lambda 1 is nearly 1 what we do to accelerate the rate of convergence of the power method algorithm. So, there is a method by which we can speed up the rate of convergence of the power method, in the case where mod of lambda 2 over lambda 1 is nearly one. And that method is the method of power method with shift. So, let us see how we apply that method to accelerate the rate of convergence.

. So, when the again I repeat when we have mod of lambda 2 lambda 1 nearly equal to 1, we apply power method with a shift to improve the rate of convergence.

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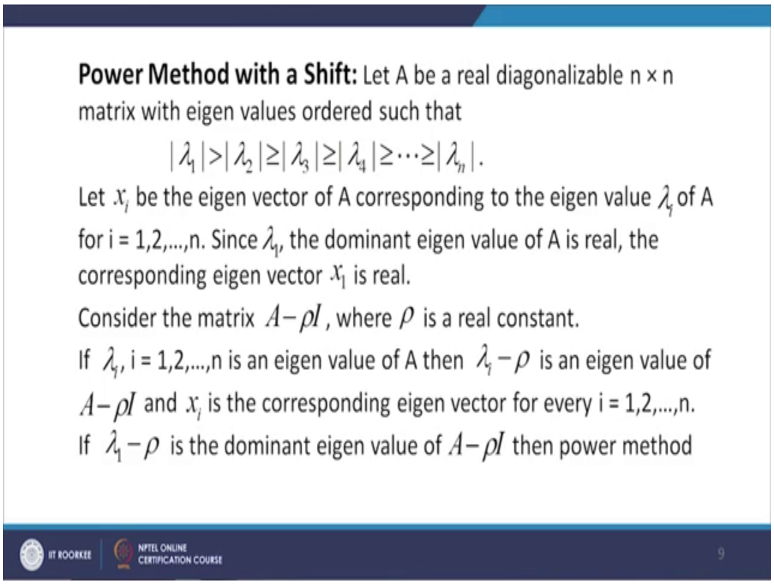

So, let us discuss power method with a shift. Let A be a real diagonalizable n by n matrix with eigenvalues ordered such that mod of lambda 1 is greater than mod lambda 2 greater than or equal to mod lambda 3 and so on, mod greater than or equal to mod lambda n. So, again lambda 1 is the dominant eigenvalue here and lambda is the sub dominant eigenvalue of A.

Let us say x i be the eigenvector of A corresponding to the eigenvalue lambda i of A for i equal to 1 to 3 and so on up to one. And so, lambda 1 lambda 2 and lambda n are eigenvalues of A, and x 1 x 2 x n are the corresponding eigenvectors of A. Since lambda one is the dominant eigenvalue of A, and we have seen in the previous lecture that if lambda 1 is the dominant eigenvalue of A, then it is algebra is algebraic multiplicity is one, and moreover it is real, this we have seen. So, since lambda 1 the dominant eigenvalue of A is real the corresponding eigenvector x one is also real. You see, we have suppose lambda 1 is the dominant eigenvalue of A and x 1 is the corresponding eigenvector then what do we have?

So, so, let us say lambda 1 be the eigenvalue of A, and x 1 be the corresponding eigenvector.

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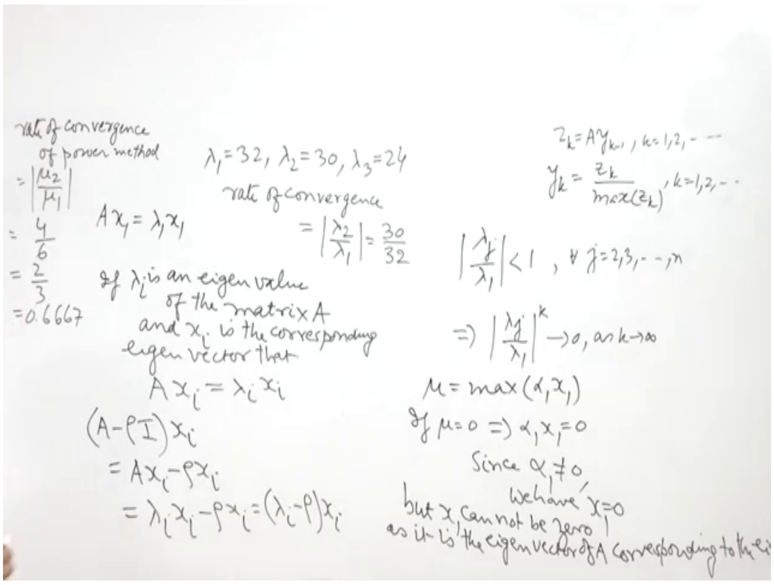

Then we have a x 1 equal to lambda 1 x 1. Lambda 1 is real; A is a real matrix ok. So, x 1 will have to be real eigenvector. So, the corresponding eigenvector x 1 is real.

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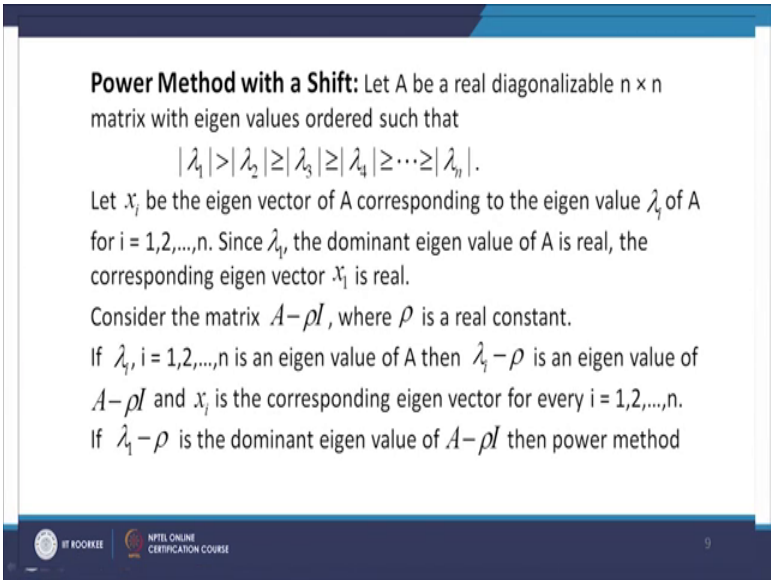

Now, let us consider the matrix A minus rho I where rho is a real constant. If lambda i, i equal to 1 to n is an eigenvalue of A then lambda i minus rho i, lambda i minus rho is an eigenvalue of A minus rho I. Y because let us say if lambda is an eigenvalue of lambda i is an eigenvalue of the matrix A, and xi is the corresponding eigenvector, then Ai Axi is equal to lambda i xi ok, we have this matrix equation.

Now, let us say let us consider A minus rho I matrix. A is a n by n matrix, this A matrix rho is a constant. I is unit matrix of order n. So, A minus rho I let us multiply it y xi. So, what do we get? Axi minus rho times Ixi that is xi, ok. Axi is equal to lambda Ixi so, what do we get? A minus rho I into xi equal to lambda i minus rho into x i. So, if x i if lambda is an eigenvalue of A corresponding and xi is the corresponding eigenvector, then lambda i minus rho is the corresponding eigenvalue of A minus rho I. So, if lambda is an eigenvalue of A then lambda i minus rho is an eigenvalue of A minus rho I, and xi is the corresponding eigenvector. Eigenvector does not change ok, for every i.

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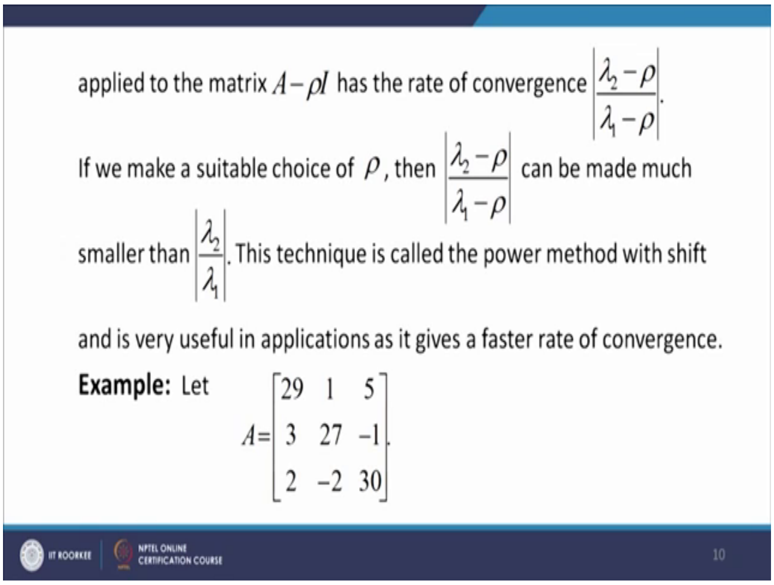

Now, if lambda i minus rho is the dominant eigenvalue of A minus rho I ok, then power method applied to the matrix A minus rho i. So, let us assume that lambda lambda 1 minus rho is the dominant eigenvalue ok, lambda 1 minus rho is the dominant eigenvalue of A minus rho I then the power method applied to the matrix A minus rho I has the rate of convergence mod of lambda 2 minus rho upon lambda 1 minus rho, ok.

. So, now if we choose this rho in a suitable manner then mod of lambda 2 minus rho upon lambda 1 minus rho can be made much smaller than mod of lambda 2 over lambda 1, which is nearly one we are assuming. So, mod of lambda 1 minus rho lambda 2 minus rho lambda 1 minus rho can be made much smaller than 1 and therefore, the by shifting the origin the rate of convergence of the power method improves. This technique is called the power method with shift and it is very useful in applications as it gives the faster rate of convergence.

Let us illustrate this by an example. Let us consider the matrix A equal to 29 1 5 327 minus 1 2 minus 230.

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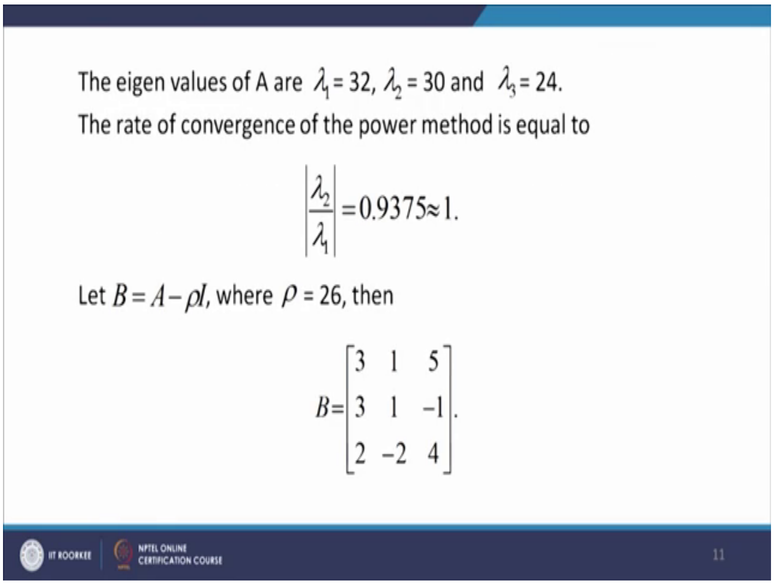

Then the you then if you calculate the eigenvalues of this 3 by 3 matrix which will not difficult to find ok, what do we get? Lambda 1 equal to 32 lambda 2 equal to 30 and lambda 3 equal to 24. The rate of convergence of the power method here, now you see lambda 1 equal to 32, lambda 2 equal to 30, lambda 3 equal to 24, ok.

So, we can see that, lambda 1 is the dominant eigenvalue here, because lambda 1 is greater than lambda 2, lambda 2 is greater than 3. So, the rate of convergence of the power method will be mod of lambda 2 lambda 1, which is equal to 30 over 32, ok. Lambda 2 is the subdominant eigenvalue which is 30 and lambda 1 is 32. So, 30 over 32 if you determine it comes out to be 0.9375 which is approximately equal to 1.

So, what we do? if you apply power method algorithm here, then the rate of convergence will be very slow because mod of lambda 2 over lambda 1 is not very, very small smaller than 1, very, very is less than 1. So, what we do is, we apply power method with the shift let us consider the matrix A minus rho I. We call this matrix A minus rho I as B ok, and choose rho as 26. So, when you choose rho as 26, A minus rho I A minus rho I is the matrix where the diagonal entries of A are subtracted by rho ok. So, the diagonal entries of a are subtracted by 26 and when we do that it comes out to be c we diagonal entries of A are subtracted by 26. So, they will become 3 1, and here 4 and we get the matrix B, B is the matrix 3 1 5 3 1 minus 1 2 minus 2 4 ok.

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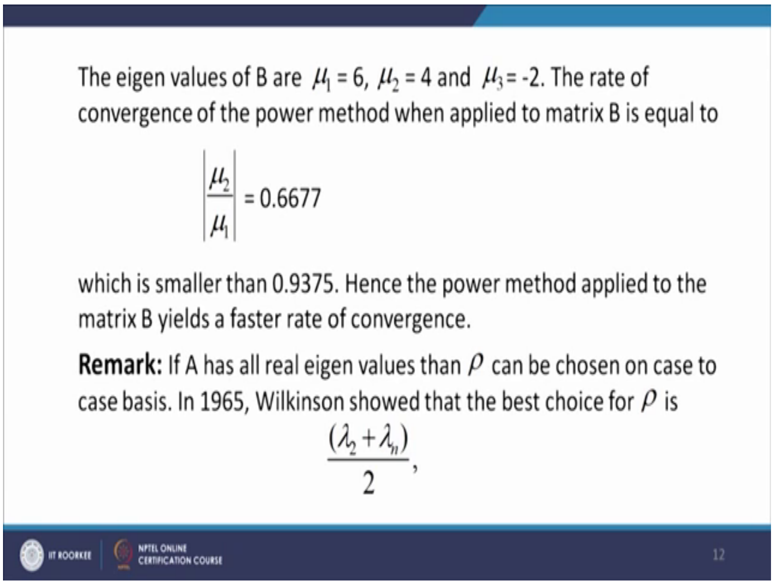

And the eigenvalues of v if they are determined, they come out to be let us denote them by mu 1 mu 2 mu 3, then they come out to be mu 1 equal to 6 mu 2 equal to 4, and 3 equal to minus 2.

And you so, if you see the dominant eigenvalue here. It is 6 sub dominant eigenvalue is 4. So, what do we get rate of convergence of this matrix, rate of convergence of the power method here is mu 2 over mu 1, ok. Mod of mu 2 over mu 1 and mu 2 is 4 mu 1 is 6, ok. So, we get 2 by 3 so, which is equal to 0.667, ok. So, which is much smaller than 1 ok, and n which is smaller than much smaller than 0.9375, which was the rate of convergence of the power method before using the shifting.

So, the power method after we applied shifting, improves the rate of convergence of the power method improves ok. So, hence the power method applied to the matrix B yields a faster rate of convergence. Because here the ratio of the dominance of dominant eigenvalue, and the dominant eigenvalue is much smaller than 1, it is 0.6677.

Now, if a has all real eigenvalue, ok. If it so happen that the matrix A has all real eigenvalues, then rho can be chosen the on case to case basis we have to see the problem, and there we have to choose a rho accordingly. In 1965 Wilkinson showed that the best choice for rho is lambda 2 plus lambda n by 2 lambda 2 plus lambda n by 2.

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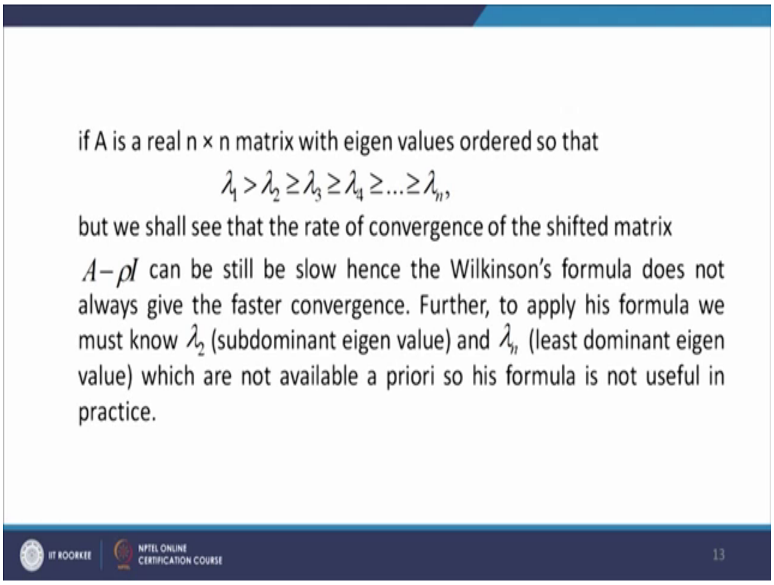

But we notice that f is a real n by n matrix with the eigenvalues ordered so that lambda 1 is greater than lambda 2 greater than or equal to lambda 3 and so on greater than or equal to lambda n; that means, we are if A is a real matrix diagonalizable matrix whose eigenvalues are all real such that they are ordered that lambda 1 greater than lambda 2 greater than or equal to lambda 2 lambda 3 and so on, greater than or equal to lambda n then do will Wilkinson said that the base choice for rho is lambda 2 plus lambda n by 2, ok.

But then we see that there exist examples where it is not true. So, we shall see that the rate of convergence of the shifted matrix A minus rho I, can we still can still be slow? A if be if the will be apply the we will concern formula, ok. Wilkinson formula so, Wilkinson formula does not always give the faster convergence, ok. And moreover, to apply the Wilkinson's formula one has to know the sub dominant eigenvalue lambda 2, and the least dominant eigenvalue lambda n, beta not available a priori ok. So, his formula is not useful in practice we can say.

Let us illustrate that by an example that the Wilkinson's formula does not always give a good choice of rho.

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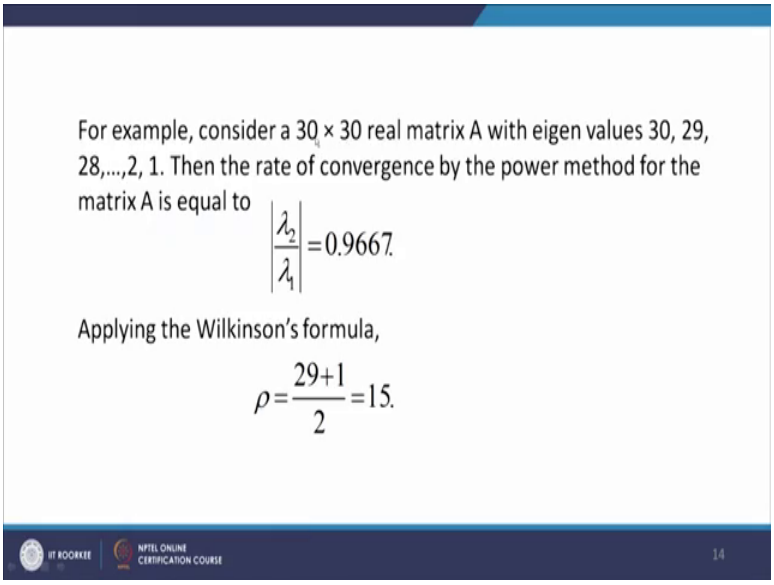

So, let us consider a matrix real matrix 30 by 30 real matrix. Suppose whose the eigenvalues of this 30 by 3, 30 matrix Are 13 329 28 27 and so on, 2 1, ok. Then the rate of the convergence by the power method for the matrix A is equal to mod of lambda 2 over lambda 1, lambda 2 will be equal to 29, and lambda 1 will be equal to 30. Because 30 is the dominant eigenvalue and chop dominate eigenvalue is 29. So, 29 by 30 will be equal to 0.9667, which is approximately one.

Now, let us find the value of rho by using Wilkinson's formula. So, rho gives is equal to lambda 2 plus lambda and lambda 2 is equal to 29, lambda n equal to 1 ok.

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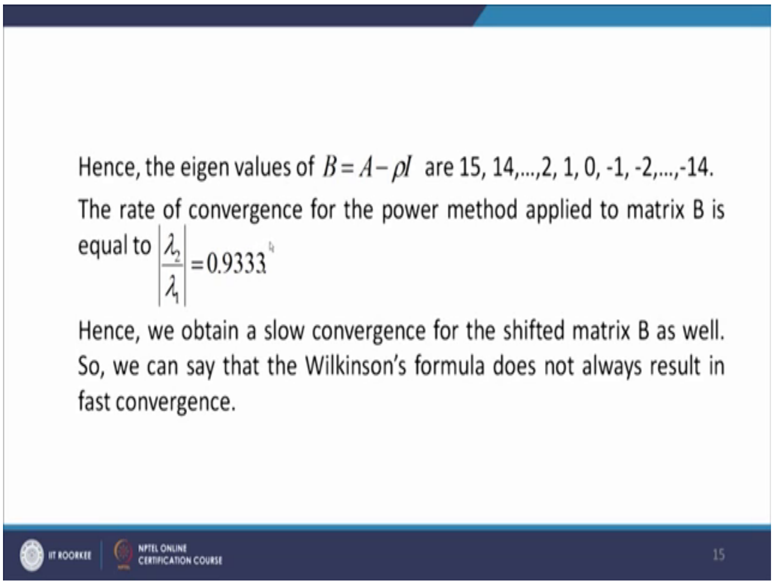

So, lambda 2 plus lambda 1 divided by 2 is 29 plus 1 by 2 which is equal to 15. So, by Wilkinson formula, the value of rho comes out to be 15. So, if you take this value of rho, then what do we get? Then the matrix B, which is equal to A minus rho I will have again values lambda i minus rho lambda is an ith eigenvalue of A. So, eigenvalue B will now become 15 14 2 1 0 minus 1 minus 2 and so on minus 14 earlier there were 329 now they will all be subtracted by 15.

. So, the rate of convergence of the power method algorithm applied to the matrix B will be then equal to again mod of lambda 2 over lambda 1 now lambda 2 is equal to 14 and lambda one is equal to 15. So, 14 by 15 which is 0.9333. So, we can see that we do not get a faster convergence, by making a choice of rho as per the formula given by Wilkinson mod of lambda 2 by lambda 1 here is not very, very less than mod of lambda 2 by lambda 1, before using this value of rho, ok.

So, hence we obtain a slow convergence, for the shifted matrix B as well. So, we can say that the Wilkinson's formula does not always result in faster convergence. So, we have to when we have to take a value of for a as per our problem. Let us see how we will choose the value of rho.

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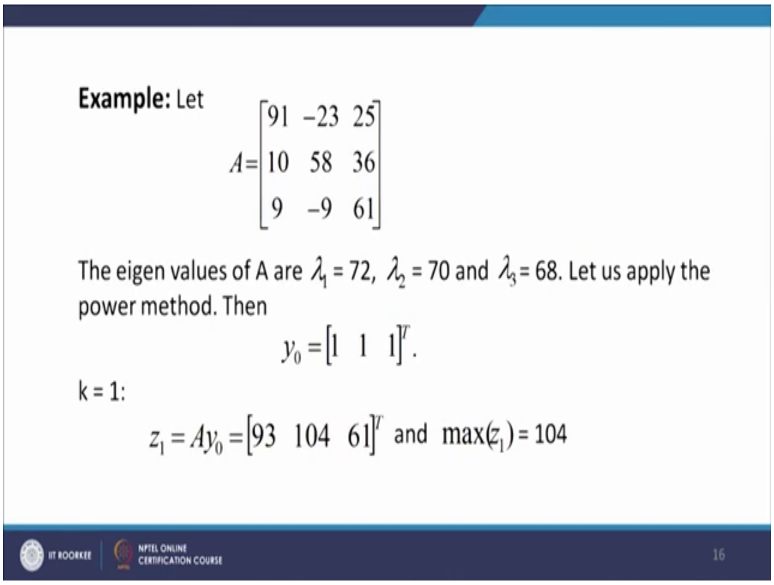

So, let us say let us take this problem A equal to a 91, let us take this 3 by 3 matrix A equal to 91 minus 23 25, 10, 58, 36, 9, 9, 61 then the eigenvalues of this matrix are 72, 70, 68. So, if you apply power method to this matrix, then the rate of convergence will be lambda 2 over lambda 1, mod of lambda over lambda 1 which is 70 over 72 which is approximately equal to 1. So, we shall see that the rate of convergence of the power method algorithm applied to the matrix A is very slow ok, let us illustrate this.

. So, what we will do and in the previous lecture be it is convention to start with the initial vector y naught as 1 1 1. So, here we take y naught as 1 1 1 transpose, and k denotes k equal to 1 denotes the first iteration. So, in the first iteration we find we apply power method algorithm. So, z z k equal to Ayk minus 1, in the power method algorithm zk equal to Ayk minus 1, where k is equal to 1, 2, 3 and so on.

So, z 1 is equal to Ay naught the matrix A which is this here this matrix. It is multiplied y naught the column vector 1 1 1 ok. And then what we get is the column vector 93 1 0 4 61. And maximum of these 3 component of z 1 is equal to 1 0 4 maximum, z 1 is 1 0 4. So, then we find y 1, y 1 is z 1 zk, y k is equal to y k equal to zk upon maximum of zk k, k equal to 1 2 3 and so on so, y 1 is equal to z 1 over maximum of z 1.

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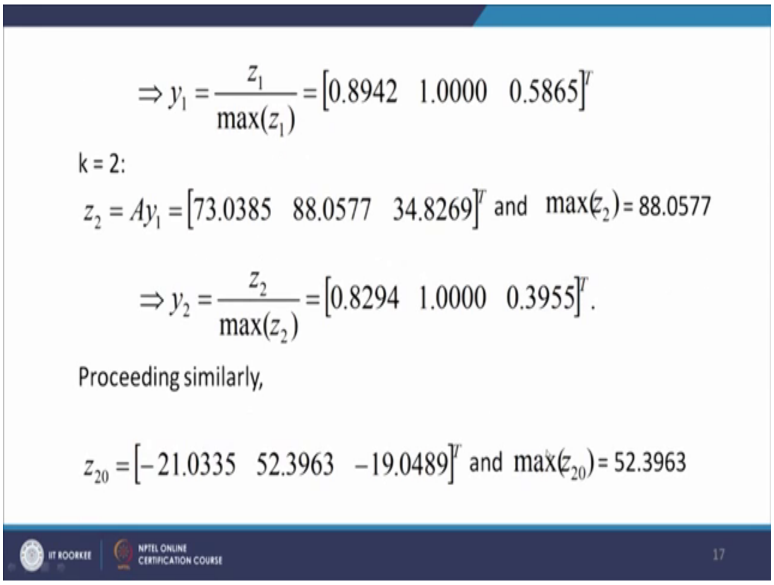

So, the z 1 vector which is this column vector 93, 1, 0, 4, 61 it is divided by 1 0 4; that is, each component of z 1 is divided by 1 0 4 and that we get is this vector, 0.8942 to 1.0000 0.5856 this column vector we get, ok.

Now, let us take these second iteration k equal to 2. So, z 2 is equal to A by 1. Multiplying matrix, A by this y 1 vector this column vector and we arrive at z 2, which is 73.0385, 88.0577, 34.8269, this vector, ok, and maximum of z 2 here you can see is 88.0577 ok.

. So now, we can determine y 2 vector, y 2 vector is z 2 over maximum of z 2 divide the z 2 vector by the maximum value of z 2; that is, 88.0577 you get y 2 vector which is 0.8294 1.0000 0.3955; this column vector. We go on finding out zk in this manner. In this manner when we proceed in the twentieth iteration, ok. In the twentieth iteration that is when k is equal to 20, z 20 comes out to be minus 21.0335 52.3963 minus 19.0489, and maximum of z naught therefore, is equal to maximum of z 20 maximum of z 20, therefore, is 52.3963.

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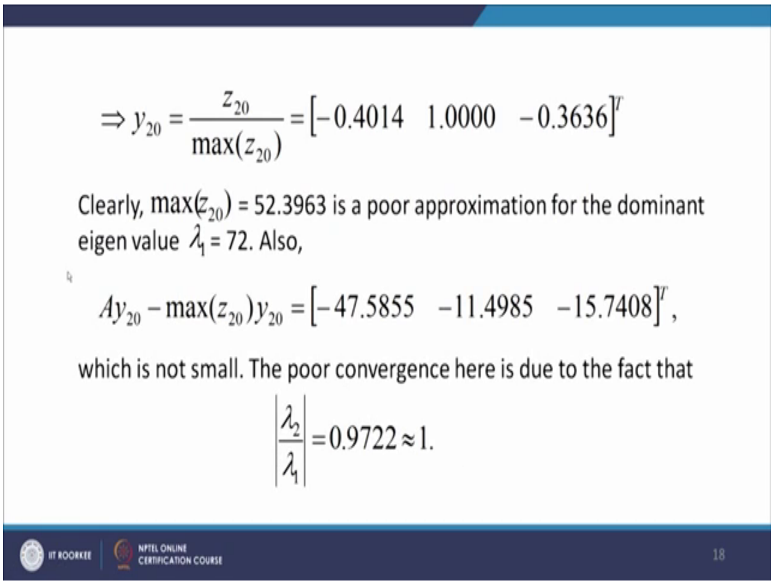

Now, so, now let us find y 20, y 20 is z 20 divided by maximum of z 20, and it is minus 0.4014, 1.0000, minus 0.3636, ok. Now we can see that maximum of z 20 is 52.3963. And y 20 is this vector so, maximum of zk ok, we have shown earlier that maximum of zk when k goes to infinity goes to lambda 1 ok.

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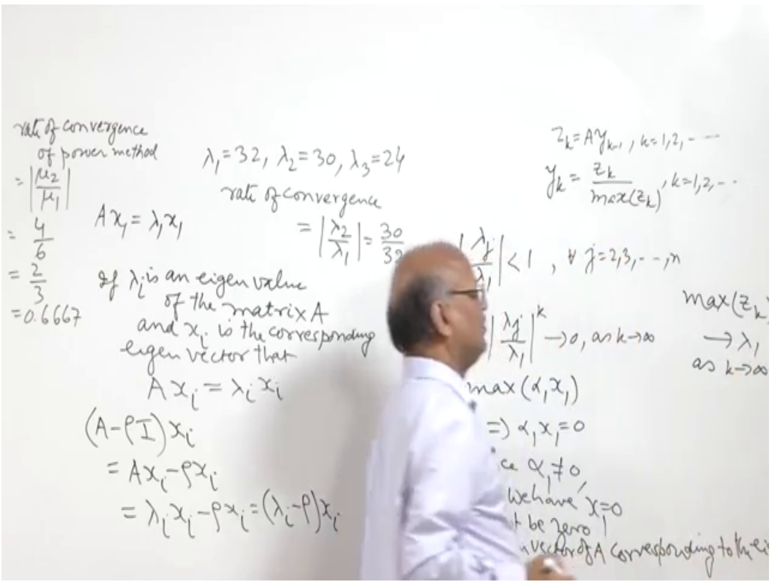

As k goes to infinity so, maximum of zk goes to lambda 1 as k goes to infinity. So, here we can say that here we can see that even after 20 iterations the value of maximum value of z 20 is 52 5 3 9 6 3, which is a poor approximation to the dominant eigenvalue lambda

1 equal to 72, because it is quite far away lambda 1 is from in 72, while maximum of z 20 is 52.3963.

So, we go on finding we go on iterating, maybe we have to be very large after that become very near to lambda 1 equal to 72. So, we can say that when we apply power method algorithm without shifting ok, then the rate of convergence is very slow. And moreover, you can see that Ay 20, the matrix A multiplied by this y 20 vector the column vector. This column vector is the eigenvector corresponding to the eigenvalue 52.3963. So, Ay 20 minus maximum of z 20 into y 20 is the equal to minus 47.5855, minus 11.4985 and minus 15.7408.

So, Ay 20 is not very good approximation to maximum of z 20 into y 20, that is difference of Ay 20, and maximum of z 20 in 2 y 20 is not very small, ok. Because if by 20 is to be the eigenvector corresponding to the eigenvalue maximum z 20, then a y 20 minus maximum z 20 into y 20 must be very small that; that means, the this right side column vector the components of the column vector must be nearly I mean nearly 0.

So, the so, here the poor convergence is due to defect that mod of lambda 2 over lambda 1 is equal to 0.9722. As I said in the beginning of this example that is lambda 2 is 70 here lambda 1 is 72 lambda 2 lambda 2 is 72 and lambda 2 is 71 here. So, that is the that comes out to be 0.9722 which is very near to 1, and therefore, the poor convergence is due to that in the of the power method algorithm, ok.

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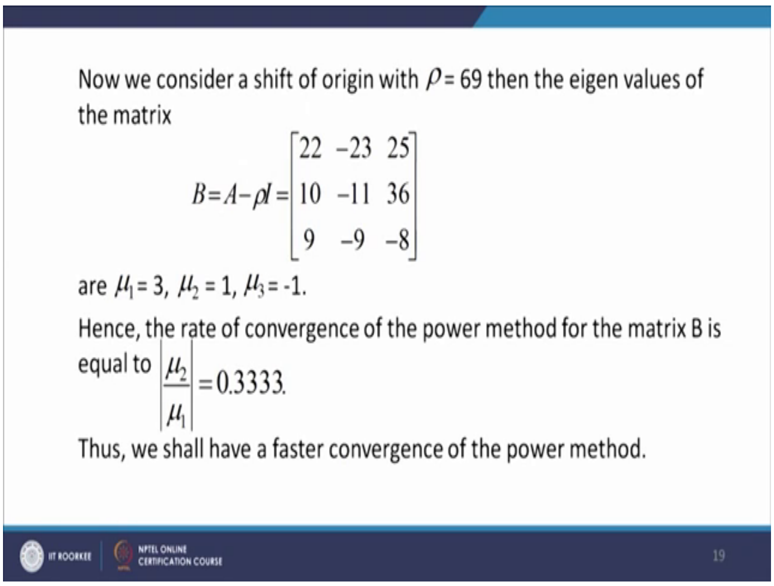

Now, we will consider a shift of origin. So, let us choose the rho to be 69. And then the eigenvalue of so then the matrix B will be equal to A minus rho I, the diagonal l in entries of a will be subtracted by rho that is 69, and what the matrix B comes out to be B is comes out to be 22 minus 23 25 10 11 36 9 minus 9 minus 8. And the eigenvalues of the matrix then will be equal to the eigenvalues of A will be subtracted by 69. So, dominant eigenvalues of 72 when subtracted with 69 gives you mu 1 equal to 3 then sub dominant eigenvalue lambda 2 was 70 which when B subtract by 69 we get mu 2 equal to 1. And the third eigenvalue was lambda 3 equal to 68. So, when we subtract lambda 3 by 69 we get mu 3 which is equal to minus 1.

So, now you can see. So, here the rate of convergence will be mod of mu 2 over mu 1, ok. And mu 2 is one while mu mu 1 is equal to 3 so, 1 over 3; that means, 0.3333 ok, which is very much less than 1. And therefore, if we use this rho value of rho and if you we use the power method with the shift, then this method will converge very, very fast. So, thus we shall have a faster convergence of the power method.

Let us see how what convergence rate here we get.

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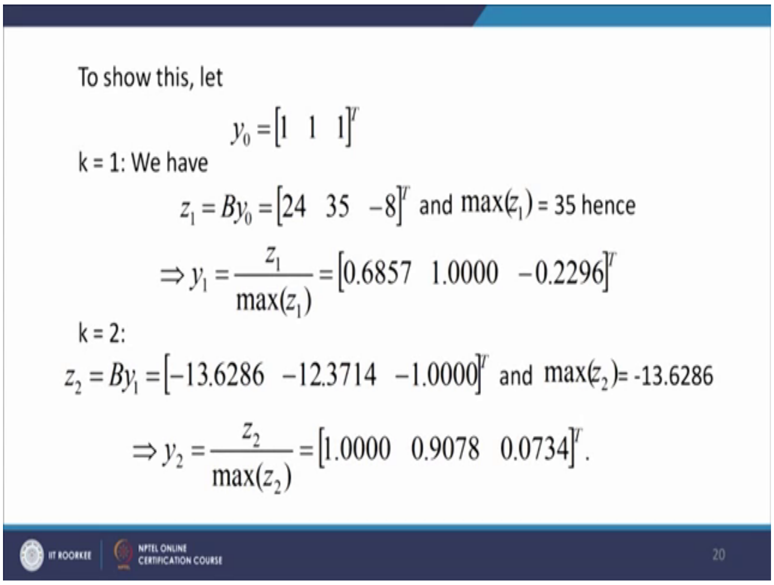

So, again we start with y naught initial approximation y naught equal to 1 1 1, we begin with be first iteration k equal to 1. Now so, we will apply the power method algorithm to the matrix B should you add 1 equal to B y naught, B why not gives you 24 35 minus 8 the column vector 24 35 minus 8. And maximum of z 1 is equal to 35 so, we can find y 1-year y 1 comes out to be 0.6857 1.0000 minus 0.2296.

Now, in this second iteration you see, z 2 equal to by one gives minus 13.6286 minus 12.3714 minus 1 point 0 0 0 0, and maximum z 2 comes out to be minus 13.6286, and y 2 comes out to be this 1.00000, 0.9078 0.0734, ok.

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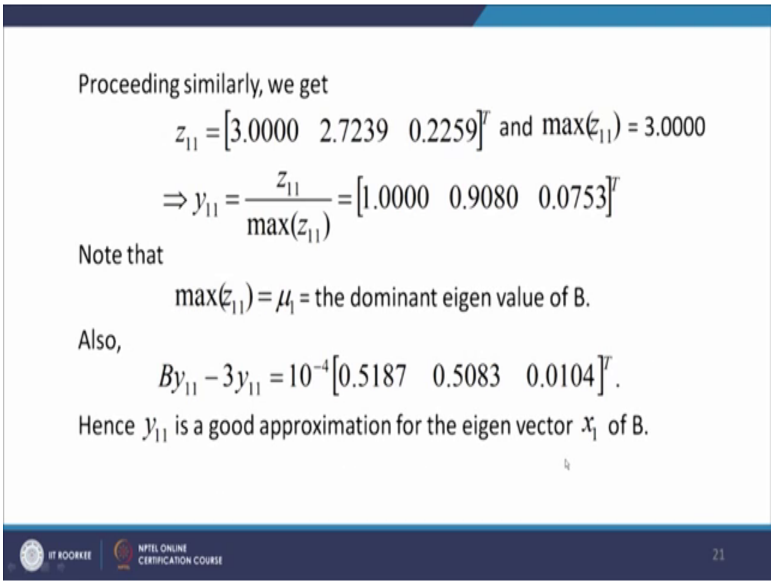

Now proceed in a similar manner, in a in the 11th iteration, z 11 comes out to be 3.0000, 2.7239, 0.2259. And so, the maximum value of z 1 is 3.0000. And if we determine y 1 1, y 1 1 comes out to be z 1 1 over maximum of z 1 1 ok. And so, it is 1.00000, 0.9080 and 0.0753 maximum of z 1 1 here. You can see maximum of z 1 1 is 3 and the dominant eigenvalue of the matrix B was also 3 ok.

So, maximum of z 1 1 is equal to the dominant eigenvalue of the matrix e which is mu 1 ok, they are equal. And moreover, that be y 1 1 minus maximum of z 11 which is 3 into y 11, comes out to be 10 to the power minus 4 into this column vector 0.5187 0.5083 0.114, ok, these components of the vector is are multiplied by 10 to the power minus 4. So, each component is very near to 0. So, B y 11 minus 3 y 11 is approximately equal to the 0 vector, ok. And therefore, we can say that, y 11 is a good approximation for the eigenvector, x 1 of B corresponding to the eigenvalue lambda 1 of mu 1 of B.

So, you can see by applying the power method with a shift, we can get the the convergence very fast, ok. So, this is what I have to say in this lecture.

Thank you very much for your attention.