Numerical Linear Algebra
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Lecture – 56
Power Method

Hello friends, welcome to my lecture on power method. We know that the eigen values of an n by n matrix are obtained by solving it is characteristic equation determinant of a minus lambda equal to 0. So, when we expand that determinant, we get the polynomial equation in lambda; lambda to the power n plus C n minus 1, lambda to the power n minus 1 plus Cn minus 2 lambda to the power n minus 2 and so on C naught equal to 0.

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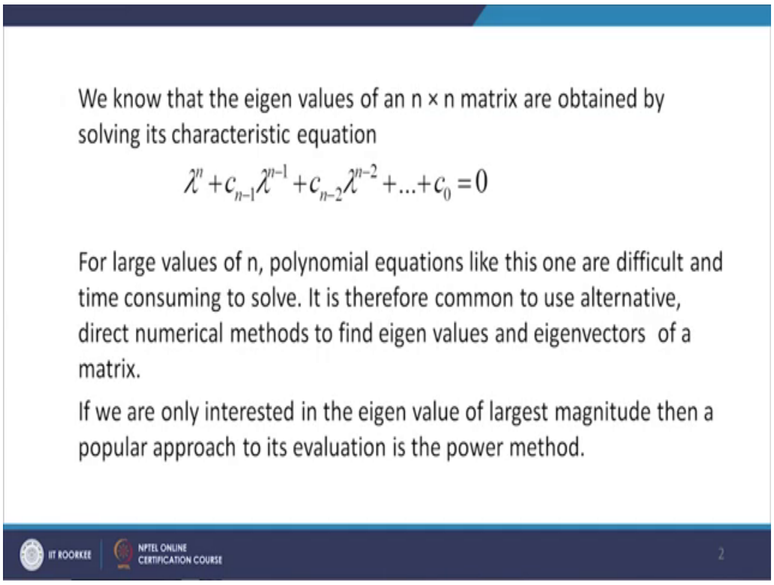

Now, for large values of n polynomial equations like this one are difficult and time consuming to solve. And the and it is therefore, common to use alternative direct numerical methods to find the eigen values and eigenvectors of a matrix. Now in many cases we need only the eigen value of largest magnitude. So, if we are only interested in the eigen value of largest magnitude, then a popular approach to it is evaluation is the power method.

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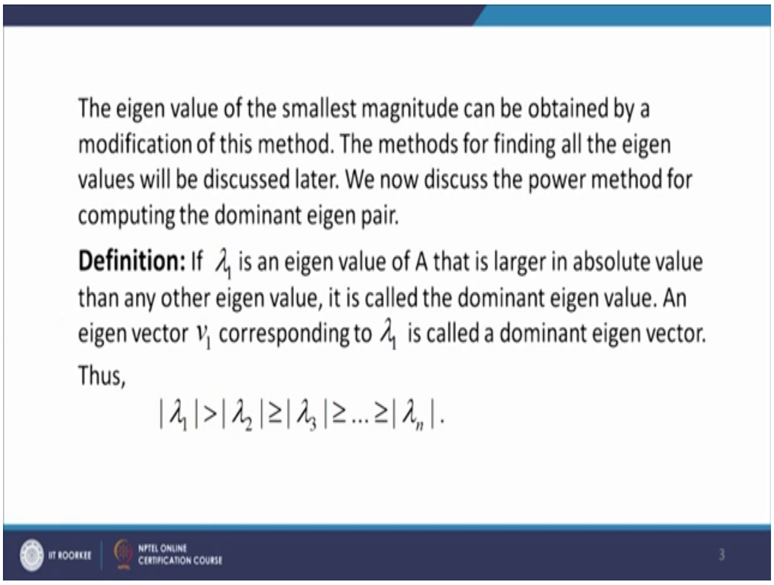

The eigen value of the smallest magnitude can be obtained by a slight modification of this method, which we shall be discussing after this power method. So, and then the methods for finding all the eigen values we shall be discussing later. First, we shall discuss the power method for computing the dominant eigen value and the corresponding eigenvector, which we call as the dominant eigen value together with the corresponding eigenvector is called as the dominant eigen pair.

So, we are going to discuss how we can compute that. So, if lambda was in an lambda 1 is an eigen value of A that is larger in absolute value, than any other eigen value it is called the dominant eigen value. And eigenvector v 1 corresponding to lambda 1 is called a dominant eigenvector. So, if lambda 1 is the dominant eigen value then mod of lambda 1 will be strictly greater than mod of lambda 2, and mod of lambda 2 may be equal to or greater than mod of lambda 3. Then lambda 3 maybe such that mod of lambda 3 is equal to mod of lambda 4 are greater than mod of lambda 4 and so on greater than or equal to mod of lambda n.

But you can notice that the lambda 1 eigen value such that it is modulus is strictly greater than the modulus of lambda 2.

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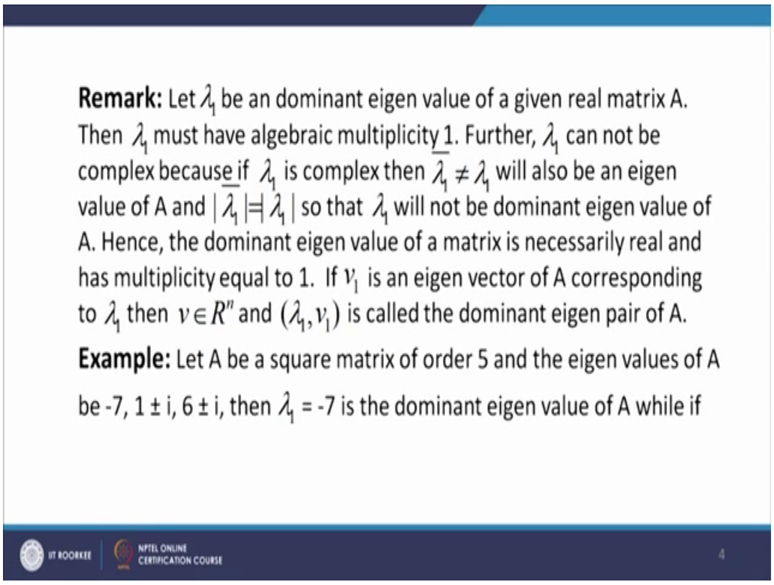

And because of this if lambda 1 is an dominant eigen value of a given real matrix then lambda 1 must have algebraic multiplicity equal to 1 because mod of lambda 1 is strictly greater than mod of lambda 2. Now further lambda 1 cannot be a complex number because if lambda 1 is complex then we are dealing with a real matrix. So, eigen values so the a polynomial equation in lambda will have real coefficients and therefore, if lambda 1 is a root of that and lambda 1 is complex then lambda 1 conjugate will also be a root of that.

So, then if lambda 1 is complex then lambda 1 conjugate is not equal to lambda 1, but their moduli are same ok. So, modulus of lambda 1 conjugate is equal to modulus of lambda 1 and therefore, lambda will not be a dominant eigen value. So, lambda 1 has algebraic multiplicity 1 and moreover; it is a real eigen value. So, hence the dominant eigen value of a matrix is necessarily real and has multiplicity equal to 1. If v 1 is an eigenvector of a corresponding to lambda 1 then v belongs to R to the power n and lambda 1 v 1 is called the dominant eigen pair of A.

For example, let us say a be a square matrix of order 5 and the eigen values of a are suppose minus 7 1 plus minus I 6 plus minus I then lambda 1 equal to minus 7 is the dominant eigen value of A because modulus of lambda 1 which is 7 is strictly greater than the modulus of all other eigen values.

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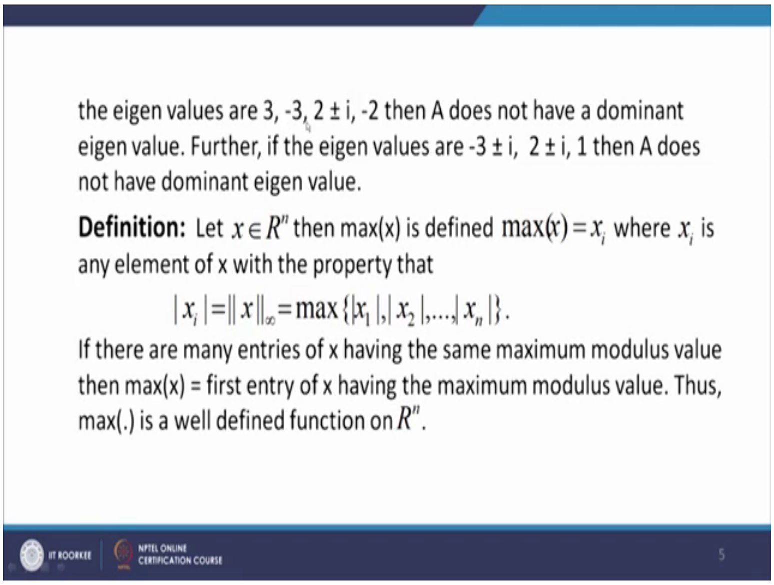

Now if the eigen values are 3 minus 3 2 plus minus I minus 2 then a does not have a dominant eigen value because there are 2 values whose moduli are same 3 and minus 3. Further if the eigen values are plus minus 3 plus minus I 2 plus minus I 1 then ok. Then since dominant eigen value must be real and there is a only 1 eigen value which is real and it is 1 and this eigen value is not strictly greater than the moduli of other eigen values therefore, a has does has a does not have a dominant eigen value.

Now, let us discuss how the maximum of a of a vector x belonging to Rn is defined here. So, maximum will be defined as a function here. So, let x belong to R to the power n, then maximum of x is defined maximum of x equal to xi where xi is any element of x with the property that modulus of xi is equal to norm of x infinity and which is maximum of mod of x 1 mod of x 2 mod of xn. So, the maximum of x is equal to xi such that modulus of xi equal to norm of x infinity. If there are many entries of of x having the same maximum modulus value then maximum at maximum of x is the first entry of x having the maximum modulus value.

Suppose there are 2 3 entries in x whose modulus is equal to norm of x infinity that is the maximum modulus modulus, then the first entry whose max whose modulus becomes norm of x infinity will be will defined the maximum of x. So, this way we can say that maximum is a well-defined function on R to the power n.

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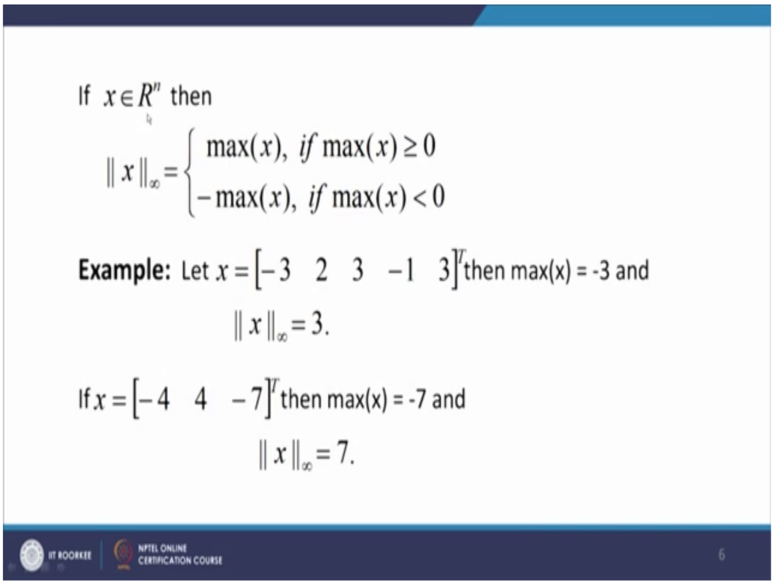

Thus, if x belongs to R to the power n we can say that norm of x infinity is equal to maximum of x, if maximum of x is greater than or equal to 0 and minus maximum of x if maximum of x is less than 0.

For example, suppose x is minus 3 2 3 minus 1 3 then you can see that absolute value of minus 3 is 3 this 3 is there this 3 is there. So, norm of x infinity is 3, but maximum of x will be minus 3, because the first entry in x whose modulus becomes norm of x infinity equal to norm of x infinity is minus 3. So, maximum of x is minus 3 and here in x equal to minus 4 4 minus 7, minus 7 is the value whose modulus becomes the maximum in all the 3 components.

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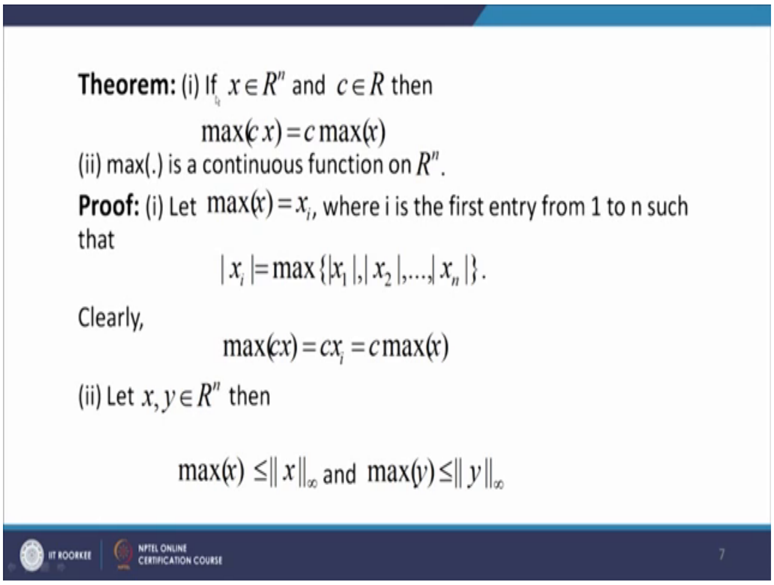

So, norm of x infinity is 7 and maximum of x is minus 7 ok. So, if x belongs to R to the power n let us discuss some properties of the maximum function if x belongs to R to the power n and c is the real number in R then maximum of cx is equal to c times maximum of x and the second observation is that maximum is a continuous function on R to the power n. Let us say maximum of the vector x is equal to xi, where I is the first entry from 1 to n ok, such that; modulus of xi is equal to modulus of x 1 modulus of x 2 modulus of xn. Then when you multiply the vector x by c maximum of cx will be equal to cxi and which is equal to c times maximum of x so this is very simple.

Now let us say xy belong to R to the power n ok. Then maximum of x is always less than or equal to norm of x infinity, by definition of norm of x infinity and the maximum function and my maximum of y is less than or equal to norm of y infinity.

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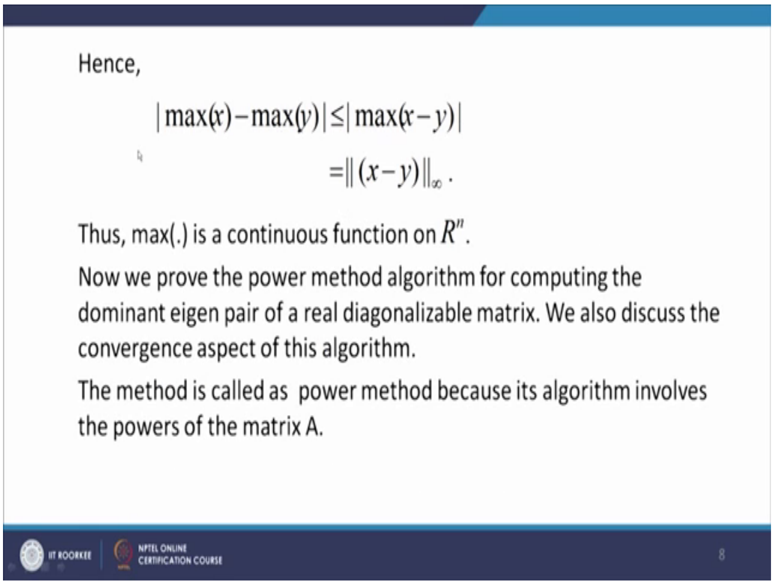

Now, it is it is clear that mod of maximum of x minus maximum of y by definition of maximum of x maximum mod of maximum of x minus maximum of y is less than or equal to mod of maximum of x minus y, but this modulus of maximum of x minus y is nothing but norm of x minus y infinity.

So, when norm of x minus y infinity is less than delta mod of maximum of x minus maximum of y can be made less than epsilon ok. And therefore, we can say that maximum of x is a continuous function in R to the power n. Now let us discuss the power method algorithm for computing the dominant eigen pair of a real diagonalizable matrix. So, we will also discuss the convergence aspect of this algorithm this x method is called as the power method because when we will see it is algorithm we will see that it involves the powers of the matrix A. So, we call it as power method ok.

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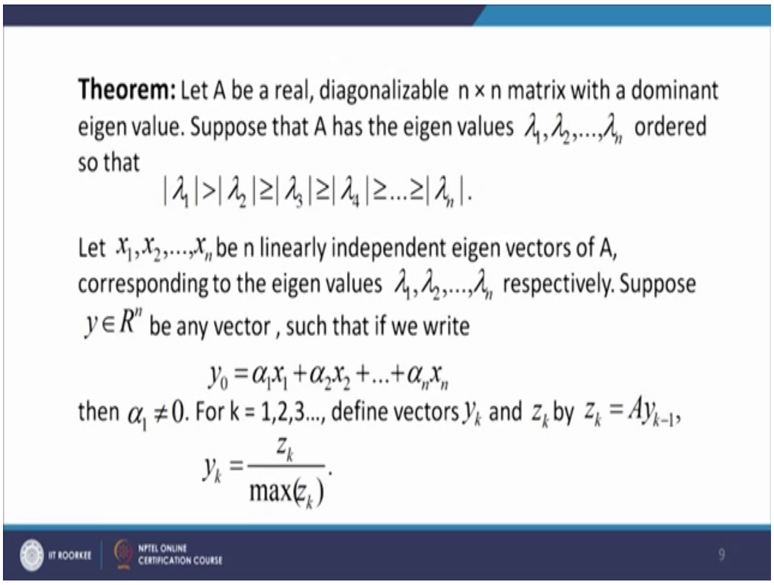

So, let us say a be a real diagonalizable n by n matrix with a dominant eigen value and suppose that the values eigen values of a are ordered. So, so that modulus of lambda 1 is greater than modulus of lambda 2 modulus of lambda 2 is greater than or equal to modulus of lambda 3 and so on greater than or equal to mod of lambda n. Let us say x 1 x 2 xn be n linearly independent eigenvectors of A we know that the matrix A is diagonalizable if and only if it has an linearly independent eigenvectors.

So, here we are assuming a is to be a diagonalizable matrix therefore, we can say that there exists n linearly independent eigenvectors. Let us take them as x 1 x 2 xn corresponding to the eigen values lambda 1 lambda 2 lambda n of A. Now suppose that y is any vector R belonging to R to the power n such that; if we write y naught equal to alpha 1 x 1 plus alpha 2 x 2 and so on alpha n xn we can consider we can take alpha 1 to be nonzero. Now for k equal to 1 2 3 we define the vectors yk and Z k as Z k is defined as a times yk minus 1 and yk is defined as Z k over maximum of Z k. Then we shall see that as k goes to infinity Z k goes to lambda 1 the dominant eigen value and k goes yk goes to c times x 1 x 1 is the eigenvector corresponding to lambda 1.

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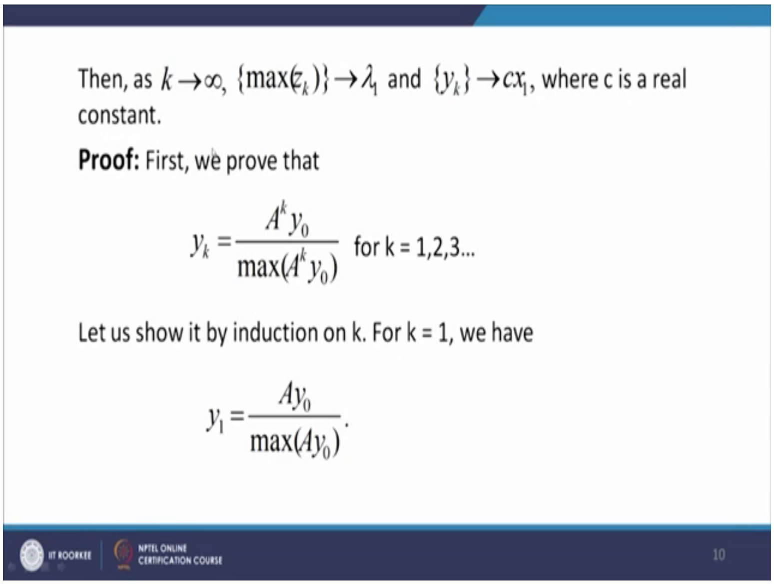

So, as k goes to infinity maximum of Z k goes to lambda 1 and yk goes to cx 1. So, where c is a real constant. Now first we prove that yk is equal to Aky naught over maximum of Aky naught for k equal to 1 2 3 and so on let us show it by induction on k.

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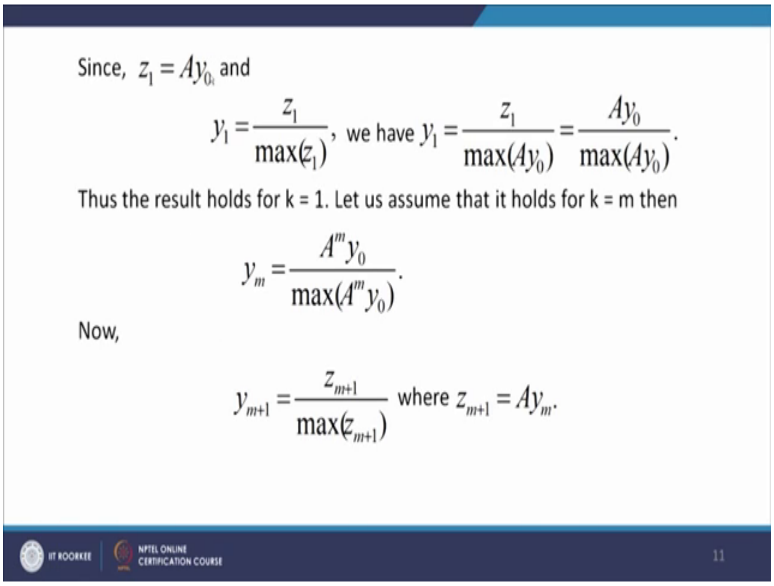

So, for k equal to 1 we have from this equation y 1 equal to Ay naught over maximum of Ay naught ok. Now we a have z z 1 equal to Ay naught we can see by definition of Z k equal to Ayk A A yk minus 1 z 1 equal to Ay naught. So, z 1 equal to Ay naught and y 1 equal to z 1 over maximum of z 1 y 1 equal to z 1 over maximum of z 1.

So, what do we have? So we have y 1 equal to z 1 over maximum of Ay naught and z 1 is Ay naught. So, Ay naught over maximum of Ay naught and therefore, we can say that the result holds for k equal to 1. Now let us assume that the result holds for k equal to m then by for hypothesis ym equal to am y naught over maximum of Amy naught now we have to show that ym plus 1 is equal to am plus y naught over maximum of am plus 1 y naught we go to the definition of ym plus 1. So, ym plus 1 by definition is Z m plus 1 divided by maximum of Z m plus 1.

But by definition Z m plus 1 is equal to Aym. So, let us replace the value here ok.

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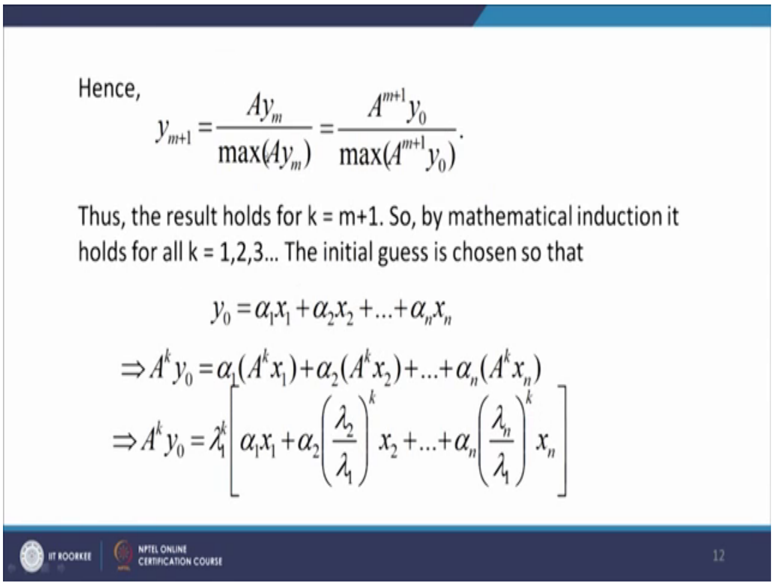

So, ym plus 1 equal to Aym over maximum of Aym now let us put the value of ym, ym is equal to ym is equal to am y naught over maximum of Amy naught. So, we can prove this easily we have ym plus 1 equal to Aym divided by maximum of Aym.

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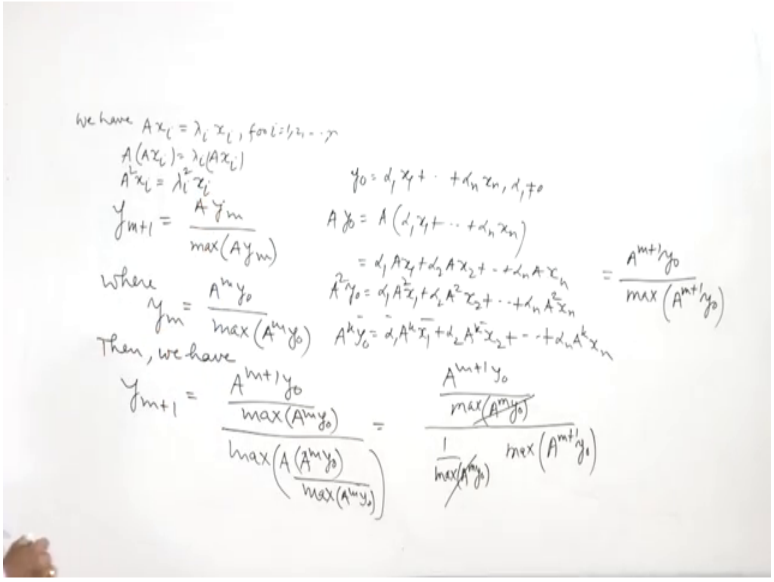

And ym ym equal to where ym equal to Am y naught over maximum of Amy naught. So, let us replace. So, then replacing the value of ym we have ym plus 1 equal to a ym ok. Aym then we operate by A on this equation Aym becomes Am plus 1 y naught divided by this is this is scalar value ok.

So, we get maximum of Amy naught ok. This ym plus 1 now divided by maximum of A ym ym is Amy naught. So, divided by maximum of; so, this is equal to Am plus 1 y naught divided by maximum of Amy naught divided by, now this becomes am plus 1 y naught and this is a scalar values. So, it will come outside the maximum. So, 1 upon this m maximum of am plus 1 y naught ok.

So, this is what we get. So, this since with this and we get am plus 1 y naught divided by maximum of am plus 1 y naught. So, this proves the result. So, the result holds for all k by induction ok. So, by induction it holds for all k. Now let us choose the initial guess y naught to be such that alpha 1 y 1 naught equal to alpha 1 x 1 plus alpha 2 x 2 and so on alpha n xn where alpha 1 is not equal to 0.then if you apply the operator a to the power k a to the power k y naught will be equal to alpha 1 y naught equal to alpha 1 x 1 plus alpha m xn alpha 1 is nonzero ok.

So, Ay naught if you apply Ay naught Ay naught will give you a of alpha 1 x 1 and so on alpha n xn ok. Now alpha 1 alpha 2 alpha n are scalars. So, alpha 1 Ax 1 alpha 2 Ax 2 and so on alpha n Axn we can again operate by A. So, we will get a square y naught we will get alpha 1 A square x 1 alpha 2 A square x 2 and so on alpha n A square xn and so on ok. Aky naught similarly we will give you alpha 1 Ak x 1 plus alpha 2 Ak x 2 and so on alpha n Ak xn ok.

now lambda 1 lambda 2 lambda n are eigen values of a x 1 x 2 xn are the corresponding eigenvectors of A. So, we have Axi equal to lambda I xi for I equal to 1 2 and so on up to n ok. So, when we operate again by a we get Axi equal to lambda I Axi because lambda I is a scalar I can write like this now this is our a square xi equal to lambda I and Axi is lambda I xi. So, we get lambda I square xi. So, if lambda I is an eigen value of a corresponding and the corresponding eigen vector is xi then lambda I square is the eigen value of a square and xi is the corresponding eigenvector.

So, we can do it again a k times if we do pre-multiplication by the matrix A we get a to the power k. So, this we can proceeding similarly we get a to the power k xi equal to lambda I to the power k xi for I equal to 1 2 and so on up to n.

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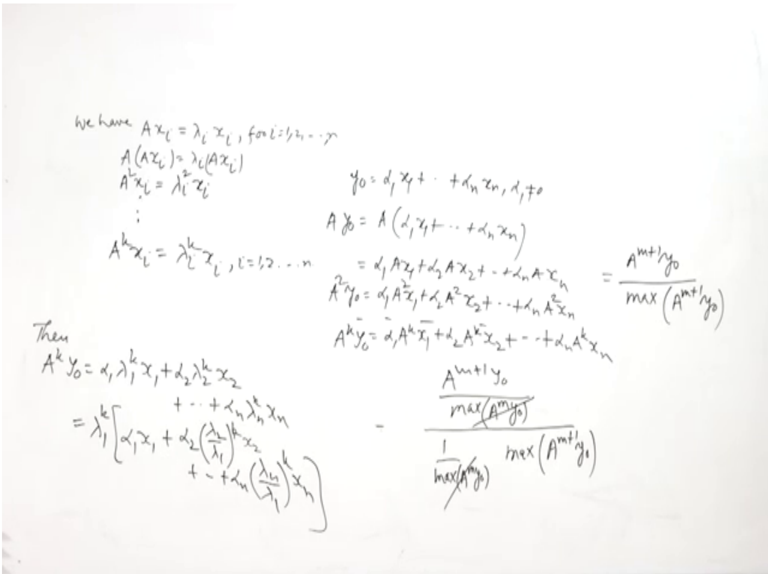

So, we can do this here so then the then Aky naught Aky naught will be equal to alpha 1 lambda 1 to the power k x 1 alpha 2 lambda 2 to the power k x 2 and so on alpha n lambda n to the power k x n ok.

Now, what we do we can write it as lambda 1 to the power k alpha 1 x 1 plus alpha 2 lambda 2 by lambda 1 raised to the power k and so on. We can write like this now, since lambda 1 is the dominant eigen value mod of lambda g over lambda 1 will be is strictly less than one for all j equal to 2 3 and so on up to n. And therefore, mod of lambda j over lambda 1 Goes to 0 as k goes to infinity for j equal to 2 3 and so on up to n.

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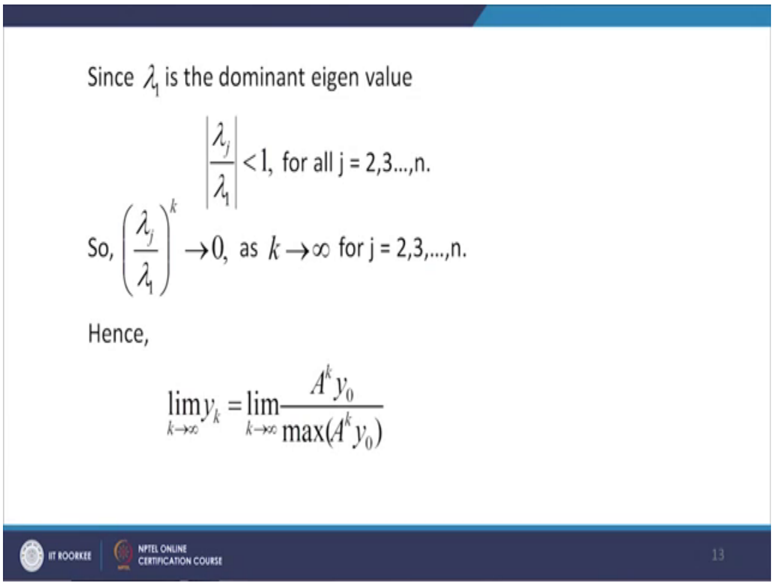

And therefore, we can say that limit k goes to infinity yk is equal to we have shown earlier that yk is equal to Aky naught over maximum of Aky naught ok.

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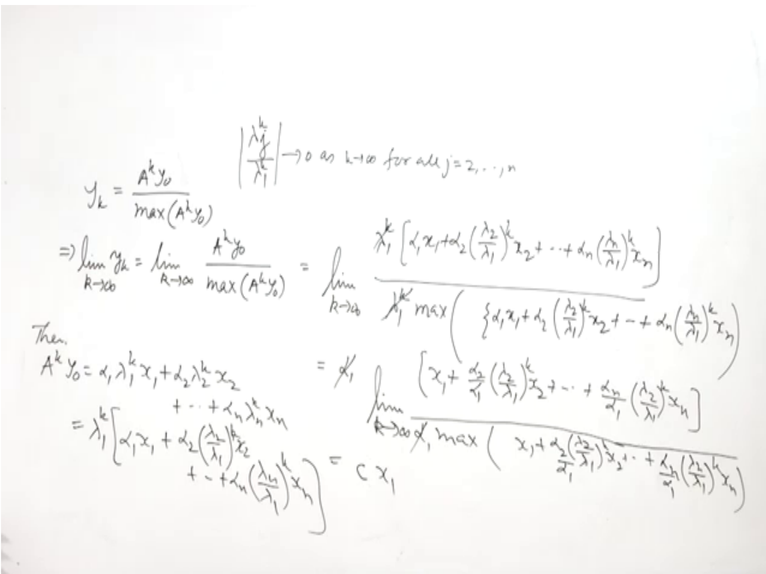

So, this implies limit k goes to infinity yk is equal to limit k goes to infinity. Ak y naught divided by maximum of Aky naught.

Now, so in the numerator, what we have? Aky naught we have seen just now this is Aky naught ok. Aky naught as k goes to infinity as k goes to infinity, what we have? we can say that this is equal to alpha 1 x 1 now when this this goes to 0 this goes to 0 lambda 1 to the power k and then we have here maximum of I may write like this; limit k goes to infinity see this is lambda 1 to the power k alpha 1 x 1 alpha 2 x lambda 2 by lambda 1 raised to the power k and then we have x 2 also here. So, x 2 and here we have alpha n lambda n by lambda 1 raised to the power k xn this divided by maximum of maximum of Aky naught.

So, lambda 1 to the power k lambda 1 to the power k then we have alpha 1 x 1 plus alpha 2 lambda 2 by lambda 1 to the power k x 2 and so on. Alpha n lambda n by lambda 1 raised to the power k xn. And then we have limit k goes to infinity ok. So, this can be written as we can take lambda 1 to the power k outside here because it is a scalar. I can write it here and then this lambda 1 to the power k lambda 1 to the power k can be cancelled alpha 1 is nonzero.

So, I can take alpha 1 outside and then we have alpha 1 times limit k goes to infinity. I can take outside so we get x 1 plus alpha 2 over alpha 1 lambda 2 by lambda 1 raised to the power k x 2 and so on. Alpha n by alpha 1 lambda 2 by lambda 1 raised to the power k and we have xn here, and divided by divided by maximum of alpha 1 x 1 plus alpha 2 lambda 2 by lambda 1 to the power k x 2 and so on alpha n by sorry alpha and lambda n by lambda 1 to the power k xn.

So, when k goes to infinity lambda j by lambda 1, this goes to 0 as k goes to infinity. This to be power k goes to 0 as k goes to infinity for all j from 2 to n ok. So, these terms go to 0 and we get x 1 here ok. And then this quantity this quantity gives you a maximum 1 upon maximum of this when k goes to infinity this defines a constant. So, we get alpha 1 here we will have alpha 1 be connected alpha 1 outside here also then also alpha 1 is a constant ok. So, this alpha 1 can be cancelled with this alpha 1 and we will have some c times x 1 ok. This x 1 this quantity is tend to 0 and here we have these quantity tend to 0. So, maximum of this quantity we can consider.

So, 1 upon maximum of that can define a constant c. So, we have cx 1. So, by k as k goes to infinity gives us c times x 1 and where alpha 1 is not equal to 0.

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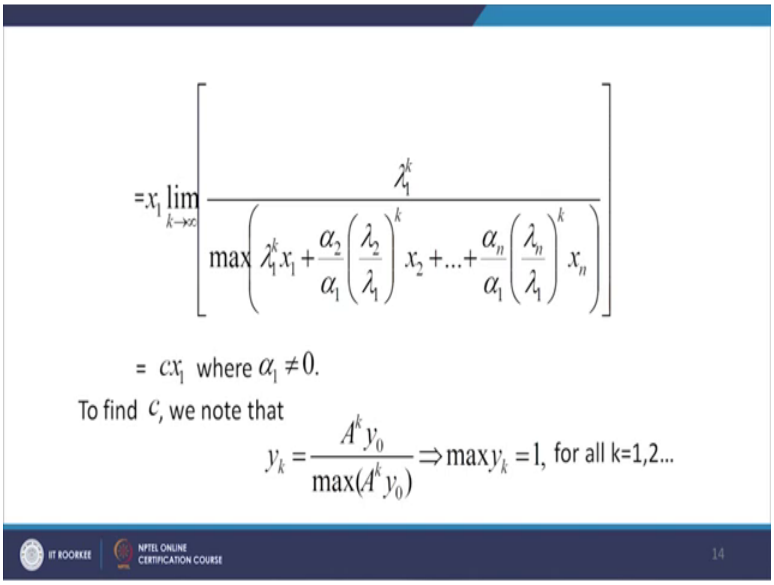

Now we need to find the value of the constant c. So, for this we need we note that yk is equal to Aky naught over maximum of Aky naught. So, when we will find maximum value of k, maximum value of k will be maximum of Aky naught divided by maximum of Aky naught yk is equal to Aky naught divided by maximum of Aky naught.

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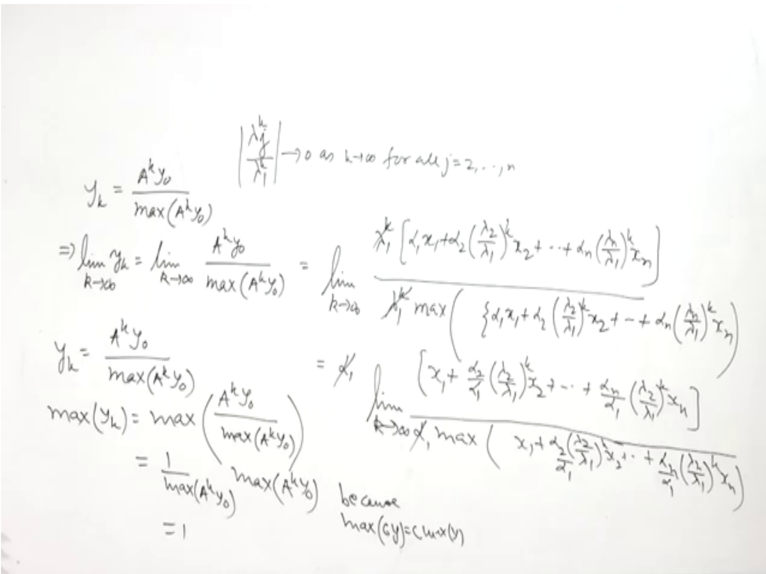

So, when we find maximum here maximum of yk. Then this is this one maximum of Aky naught is a constant ok.

So, we shall have this 1 upon maximum of Aky naught and then maximum of Aky naught this is equal to 1; this is because maximum of a constant c times y equal to c times maximum of y we have earlier scene where c is any constant. So, because of that, maximum of yk is equal to 1 for all k equal to 1 2 3 and so on.

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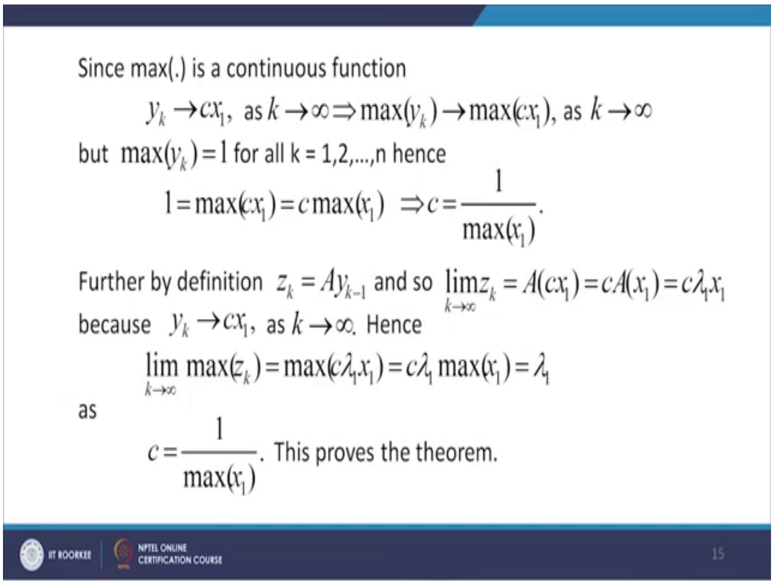

Now yk max we have seen that maximum is a continuous function. So, then yk goes to cx 1 as k goes to infinity maximum of yk will go to maximum of cx 1 and maximum of cx 1 will be equal to c times maximum of x 1 ok.

So, we will have c equal to 1 upon maximum of x 1 let us notice that limit k goes to infinity yk equal to c of x 1 ok.

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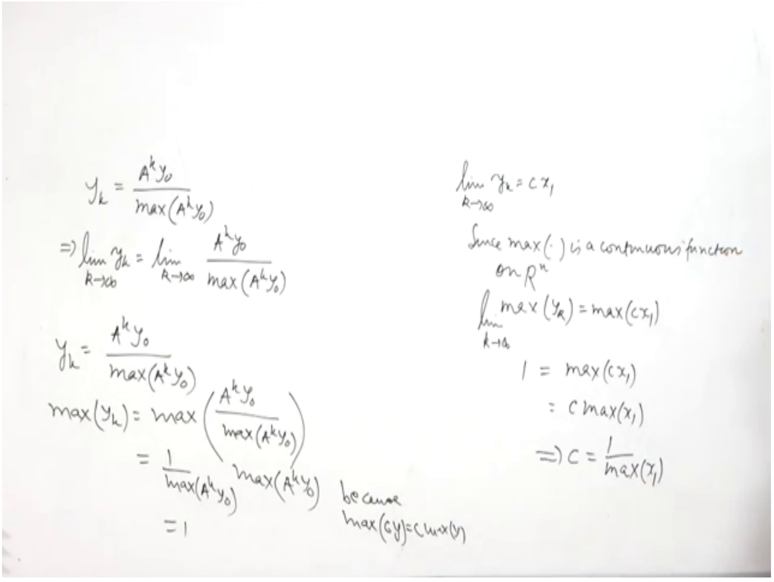

Since, maximum of maximum is a continuous function in Rn it is a continuous function on Rn. Maximum of yk as k goes to infinity will be equal to maximum of cx 1. So now, maximum of yk we have seen maximum of yk equal to 1 for all k ok. So, 1 will be equal to maximum of cx 1 which is equal to c times maximum of x 1 and this gives us c equal to 1 upon maximum of x 1 ok.

So, this is how we obtain the constant c c is equal to 1 upon maximum of x 1. Now by definition Z k equal to Ayk minus 1. So, as k goes to infinity Z k goes to limit of Ayk minus 1 yk goes to cx 1. So, yk minus 1 also goes to cx 1. So, A of cx 1 we have A of cx 1 is equal to c times Ax 1 we can see here Z k equal to A yk minus 1. So, limit k goes to infinity Z k equal to limit by limit k goes to infinity A of yk minus 1. So, yk minus 1 will also go to cx 1. So, Acx 1 which is equal to c times Ax 1 and Ax 1 is equal to lambda 1 x 1 because lambda 1 is an eigen value of a and x 1 is the corresponding eigenvector.

So, this is c lambda 1 x 1. Now hence limit k goes to infinity maximum of Z k ok. Maximum is again let us recall that maximum is a continuous function. So, when Z k goes to c lambda 1 x 1 maximum of Z k will go to maximum of c lambda 1 x 1 c lambda 1 is a constant. So, c lambda 1 times maximum of x 1, but maximum of x 1 into c is equal to 1 because c is equal to 1 upon maximum of x 1. So, this is equal to lambda 1 and therefore, as k goes to infinity maximum of Z k goes to lambda 1. So, this proves the theorem.

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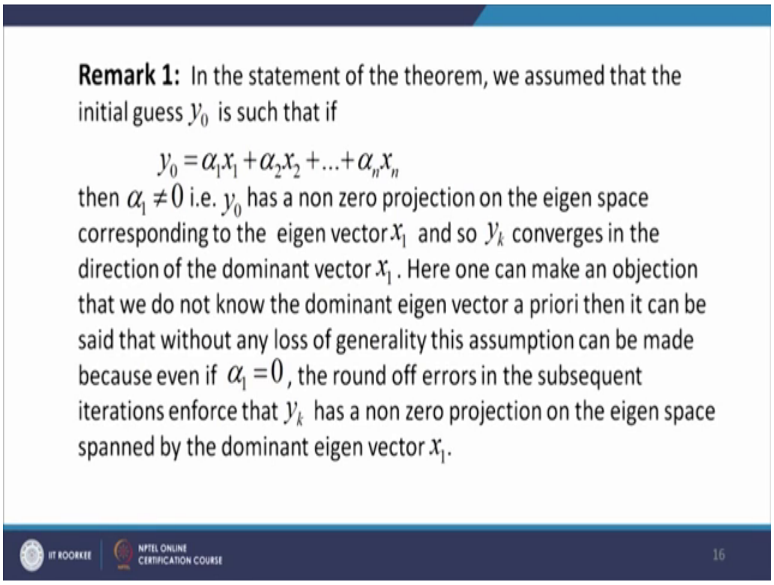

Now let us look at some on the remark one in a this statement of the theorem we assumed that the initial guess y naught is such that; y naught is equal to alpha 1 x 1 plus alpha 2 x 2 and so on alpha n xn and where alpha 1 is not equal to 0 that is y naught has a nonzero projection on the eigen space corresponding to the eigenvector x 1. And so yk converges in the direction of the dominant vector x 1.

Now, here one can raise an objection that we do not know the dominator eigenvector a priori then it can be said that without any loss of generality this assumption can be made because even if alpha 1 is equal to 0 the round of errors in the subsequent iterations will enforce that yk has a nonzero projection on the eigen space spanned by the a dominant eigenvector x 1.

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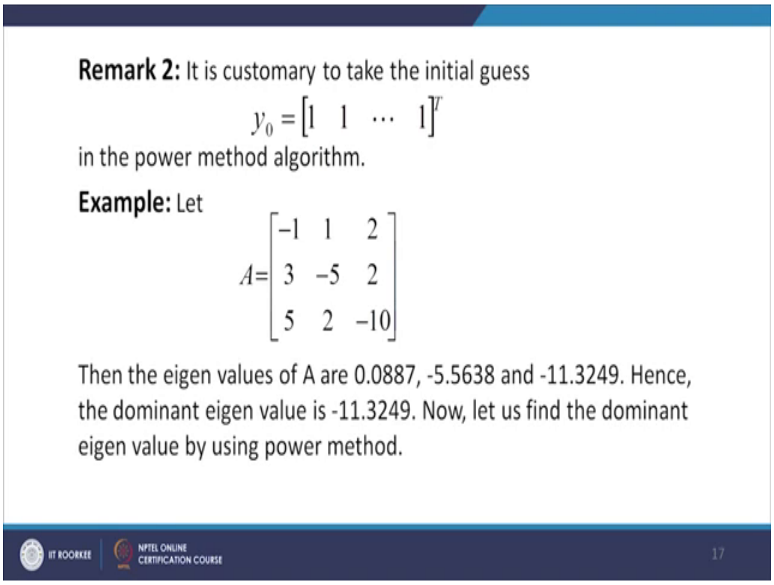

Now it is called when we solve a any matrix for the dominant eigen value and the corresponding eigenvector it is, conventional to take the initial guess as a 1111 ok.

So, we take y naught equal to 1111 in the power method algorithm. Now let us take an example suppose we take a 3 by 3 matrix. So, a equal to minus 1 1 2 3 minus 5 2 5 2 minus 10 ok. Then the eigen values of a are 0.0887 minus 5.5638 and minus 11.3249. Here is the dominant eigen value is minus 11.3249 ok. You can see among all these 3 eigen values the greatest one the whose absolute value is the greatest is minus 11.3249.

Now, let us find the dominant eigen value. So, this is the dominant eigen value let us find this dominant eigen value by the power method let y naught equal to 1 1 1 transpose.

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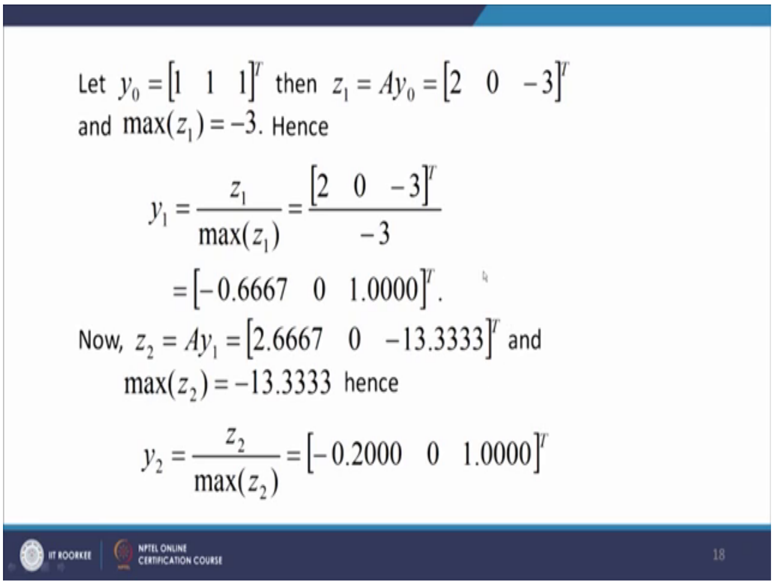

As we said in it is conventional to take y naught as 111 be here it is 3 by 3 matrix should we take y naught equal to 1 1 1 a maybe 3 components. Then z 1 we find Z k equal to Ayk minus 1 ok. So, for k equal to 1 k equal to 1 means; we are doing the first iteration ok. So, Z k equal to Ayk minus 1 becomes z 1 equal to Ay naught. And when you multiply the matrix A the given matrix A by the y naught vector 1 1 1 we will get a column vector 2 0 minus 3 ok.

And from 2 0 minus 3 we can say that maximum of z 1 is minus 3 by definition of maximum, we know that maximum of z 1 is the value the value the value the component which has the maximum absolute value ok. So, maximum absolute value in 2 0 minus 3 is 3. So, we have maximum of z 1 equal to minus 3 hence, y 1 is equal to by definition y 1 equal to yk equal to Z k over maximum of Z k by that definition. So, y 1 equal to z 1 over maximum of z 1 z 1 is 2 0 minus 3 transpose divided by maximum of z 1 minus 3.

So, we get column vector minus 0.66670 1.0000. Now we do the second iteration. So, z 2 equal to Ay 1 ok. A matrix is multiplied by the y 1 vector this 1 minus 0.66670 1.0000 transpose. So, by this vector when we multiply the matrix A we get 2.66670 minus 13.3333. Now minus 13.3333 is the a component which has which has the maximum absolute value.

So, we maximum of z 2 is equal to minus 13.3333 hence y 2 equal to z 2 over maximum of z 2. So, z 2 vector this vector z 2 vector is divided by a maximum of maximum of z 2 value and what we get is? Minus 0.20000 1.0000 transpose.

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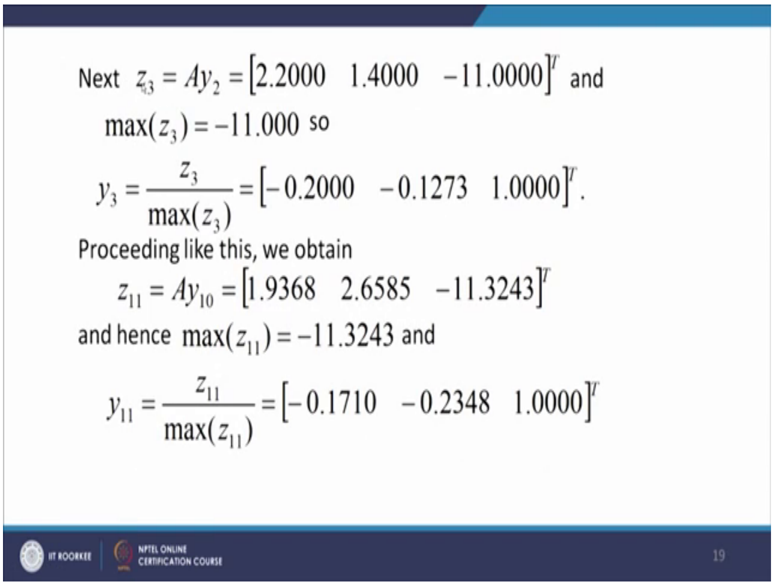

Now we for k equal to 3 we get z 3 equal to Ay 2. So, z 3 comes out to be 2.2000 1.4000 minus 11.0000 transpose again maximum z 3 is minus 11.0000.

So, here we find y 3 by z 3 upon maximum of z 3 and we get minus 0.2000 minus 0.1273 and 1.0000 ok. Proceeding in this manner ok, proceeding in this manner we can find z 11 ok. So, when we take the 11 in the 11th iteration we will get z 11 z 11 equal to Ay 10 which will be 1.9368 2.655 minus 11.3243 to the transpose.

So, here maximum of z 1 is minus 11.3243 ok. And y 11 which is z 1 1 z 1 1 divided by maximum of z 1 1 it will be minus 0.1710 minus 0.2 3 4 8 and then 1.0000 transpose. Now let us see.

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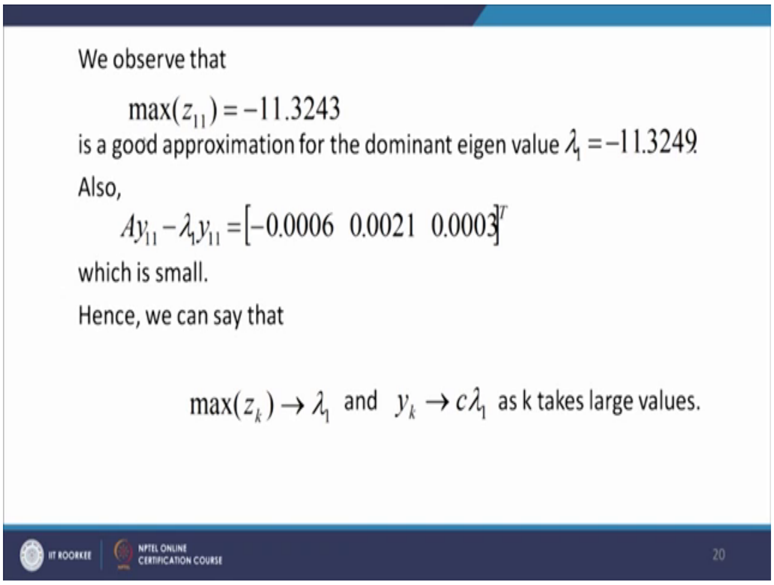

So, here maximum of z 1 is minus 11.3243 and which you can see is a good approximation for the dominant eigen value lambda 1 equal to minus 11.3249.

So, we can say that maximum of Z k approaches to the dominant eigen value lambda 1 and furthermore we also notice that Ay 11 minus lambda 1 by 11 ok. Lambda 1 we have found to be minus 11.3243 z maximum of z 1 we are taking as minus 11.32 minus 11.3243 as the dominant eigen value. So, this eigen value we are multiplying by y 11 y 11 we have found earlier this is y 11. So, y 11 is this and lambda 1 is this ok.

So, we multiply here and subtract from Ay 1.1 and when you do that what you get is this vector minus 0.0006 0.0021 0.000 transpose, and which is very small you can see hence we can say that maximum of Z k converges to lambda 1 and yk the yk vector goes to c times lambda 1 as k takes large values ok.

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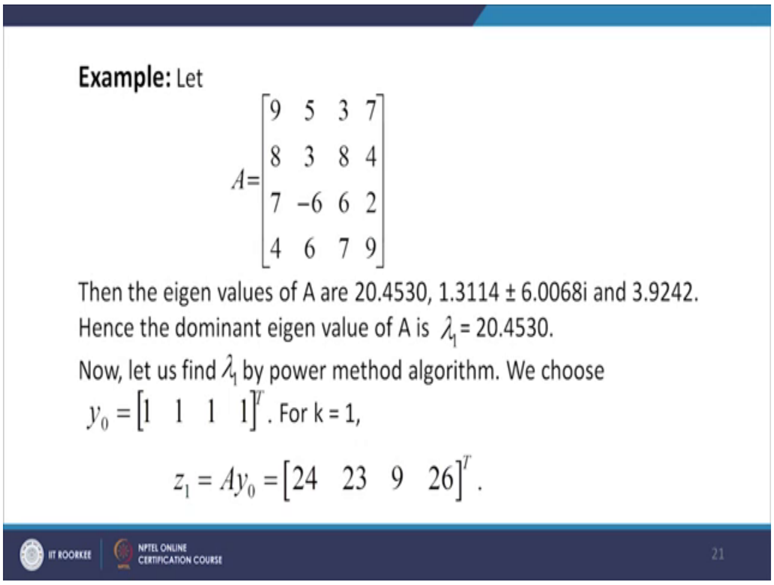

So, let us now take the next example A equal to 9 5 3 7 8 3 8 4 7 minus 6 6 2 4 6 7 9.

So, here we are taking 4 by 4 matrix ok. The eigen value of the matrix A are 20.4530 1.3114 plus minus 6.008 into iota and then 30 3.9242. So, the absolute my maximum value here is 20.4345 20.4530 and therefore, lambda 1 the dominant eigen value of a is 20.4530. Now it is a 4 by 4 matrix we will choose initial guess y naught to be 1 1 1 1 ok.

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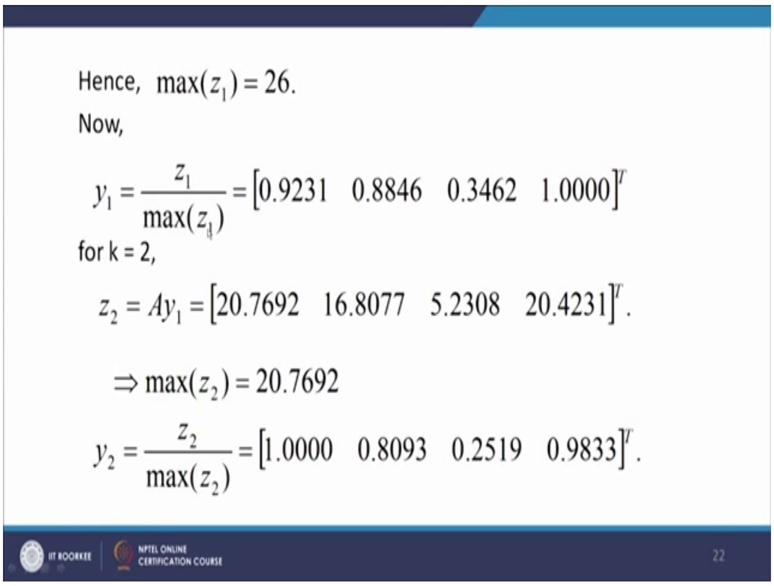

For k equal to 1, we will find z 1 z 1 equal to Ay naught now here you can see maximum of z 1 we can find ah. So now, y 1 equal to a z 1 upon maximum of z 1 and we will get 0.9231 0.8846 0.3462 and 1.0000. Now for k equal to 2 z 2 equal to Ay 1. So, we can find Ay 1 into n it will come out to be 20.7692 16.8077 5.2308 20.4231 transpose and maximum of z 2 here you can see is 20.7692.

So, we can find y 2 here y 2 equal to z 2 upon maximum of z 2 it is 1.0000 0.8093 0.2519 0.9833 and the maximum absolute maximum value here is that you get is 0.999833.

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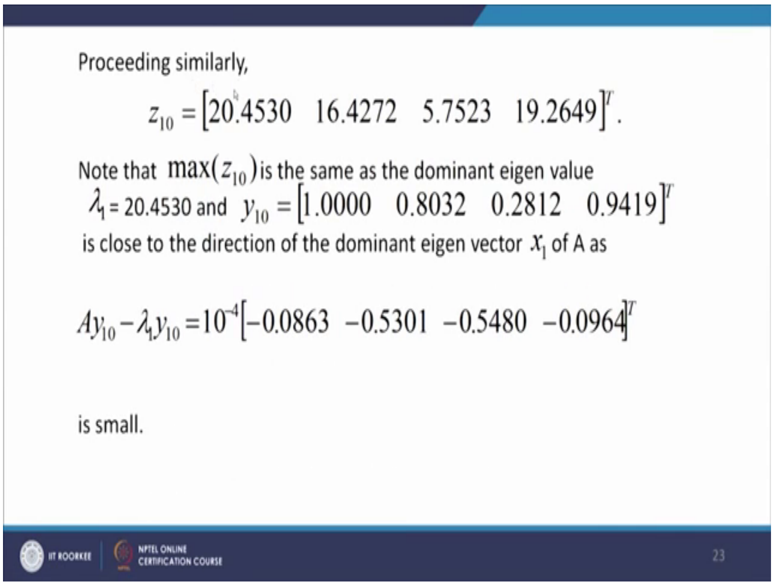

So, we we will get proceeding similarly z table z 10 comes out to be 20.4530 16.4272 5.7523 19.2649 transpose. And we note that maximum of z 10 maximum of z 10 here is 20.4530 which is same as the dominant eigen value lambda 1 equal to 20.4530, y 10 you can see y 10 it is y 10 oh sorry y 10 y 10 comes out to be this 1.0000.

So, y 10 is z 10 divided by maximum of z 10 ok. We have found z 10 maximum of z 10 is 20.453 0. So, when we divide z 10 by 20.4530 that is that is 1.0000 0.8032 0.2812 0.9419. Now here you can see the actual dominant eigen value was a dominant eigen value was this 20.4530 and we could get this value 20 4 ah.4 5 3 0 after 10 iterations ok.

So, maximum of z 10 is exactly same as the dominant eigen value 20.453 and the eigen vector y 1 y 10 y 10 is this one ok. So, when you multiply this y 10 by the lambda 1 20.4530 then Ay 10 minus lambda 1 y 10 you can see comes out to be 10.10 to the power minus 4 minus 0.0863 minus 0.5301 minus 0.5480 minus 0.0964 a transpose which is very small. So, we can say that as k goes to infinity maximum of Z k converges to lambda 1 and yk convergence to c times x 1 that is, yk converges to x 1 in the direction of x 1. So, the so the, this is because Ay 10 minus lambda 1 by 10 is small, with that i would like to conclude my lecture.

Thank you very much for your attention.