Numerical Analysis
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Lecture No 40
Solution of Linear Systems of Equations minus3
Gauss Elimination Method

Good morning everyone, in the previous class we discussed decomposition methods which belong to the class of direct methods for solving a system of linear algebraic equations and ex plained how we can solve a system A x equal to b by Dolittle Method and Crout's method. In Dolittle Method, the decomposition is in the form A equal to L into U, where L is a unit lower triangular matrix and U is any upper triangular matrix. Whereas, in Crout's method the factorization of A in the form L U consists of unit upper triangular matrix and any lower triangular matrix and we also mentioned about Choleky's method wherein we stated the result under which Cholesky's decomposition is possible.

So we said that if A is a real symmetric positive definite matrix then A has a unique factorisation of the form L into L transpose, where L is a lower triangular matrix with positive diagonal entries. Here is a square matrix, so we shall see how we can solve a system of equations using Cholesky's decomposition method. So let us consider the problem solving the systems A x equal to b, so let us look at the matrix A.

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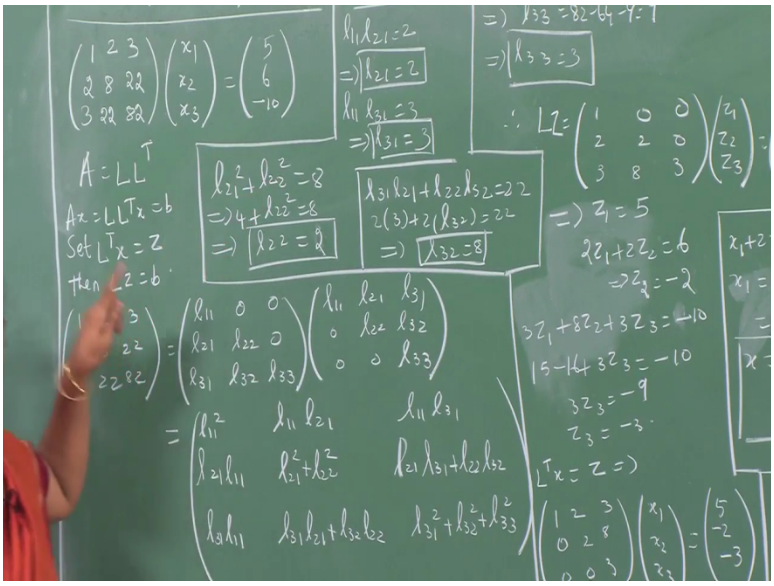

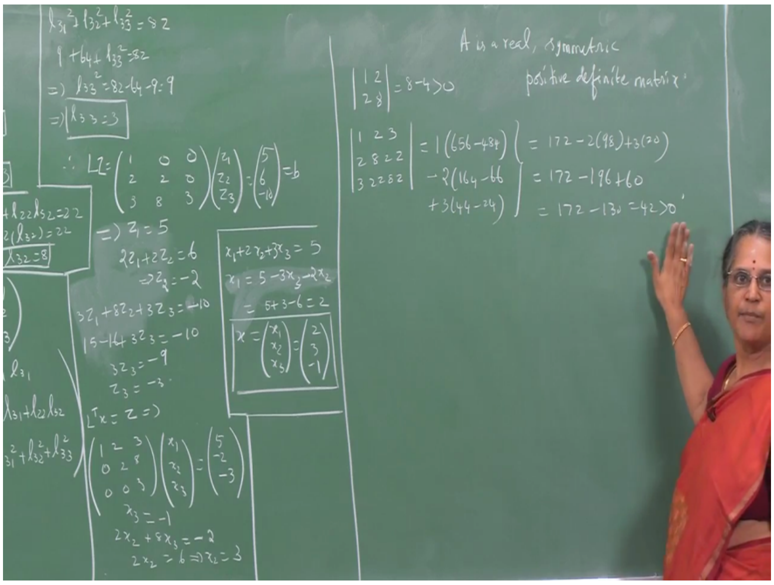

We observe that A is a real matrix it has real entries, it is a symmetric matrix because we see that a 12 is a 21, a 13 is a 31, a 23 is a 32 and so it is a symmetric matrix . In addition we find all the leading principle miners, the leading miners of order 1 is 1 and it is positive, a leading miner of order 2 is determinant 1, 2, 2, 8 and that is 4 that is again positive and the leading principle miner of order 3 is determinant of A. And when we compute determinant of A it turns out to be 42 which is again positive, so all the leading principle miners are positive and therefore, A is a real symmetric positive definite matrix and therefore, matrix A has a unique factorisation of the form L into L transpose where L is a lower triangular matrix with positive diagonal entries.

So we consider A in the form L into L transpose, we have to solve the system A x equal to b, so L L transpose x equal to b, so if we set L transpose x as z then the system becomes L z equal to b. So we observe that when we set L transpose x equal to b, L transpose is an upper triangular matrix so we have to solve a system of equations, where the coefficient matrix is upper triangular matrix and therefore, we solve this system by backward substitution. So, on the other hand when we have this system L z equal to b then L is a lower triangular matrix and therefore, we solve this system by forward substitution.

So 1st solve L z equal to b by forward substitution, determine the components of L z, use that information here and solve L transpose x equal to z and obtain the unknown vectors x and that will give you the solution of the system using Cholesky's decomposition method. So let us now see how we can factorise A in the form L into L transpose; what is the property of this matrix L? L is a lower triangular matrix with positive diagonal entries so keep this in mind, so let us take L to be a lower triangular matrix in the form l 11, l 22, l 33, appearing along the diagonals and other entries are written below the diagonal, its transpose is going to be an upper triangular matrix so we write down L transpose so the column will become the first row entries, the 2nd column entries will be the 2nd row entries, the 3rd column entries will be the 3rd row entries.

So we multiply the 2, so the 1st row into the 1st column will give you l 11 square, 2nd row into 1st column will give you l 21 into l 11 plus 0 plus 0 so it is l 21 l 11, then the 3rd row into the 1st column will give you a l 31 into l 11 and the rest of the 2 terms will be 0, then we consider 2nd row into the 1st column that will give you l 21 l 11 plus 0 plus 0 so that is what is written here. And then the 2nd row into the 2nd column et cetera, you compute the products of the 2 matrices L and L transpose and you will know that the resulting matrix is this and this is to be equal to the matrix A and therefore the corresponding entries are equal. So we equate the corresponding entries and try to determine the l i j which appears as unknowns in the matrix l. So we start with 1st column entries in the resulting matrix.

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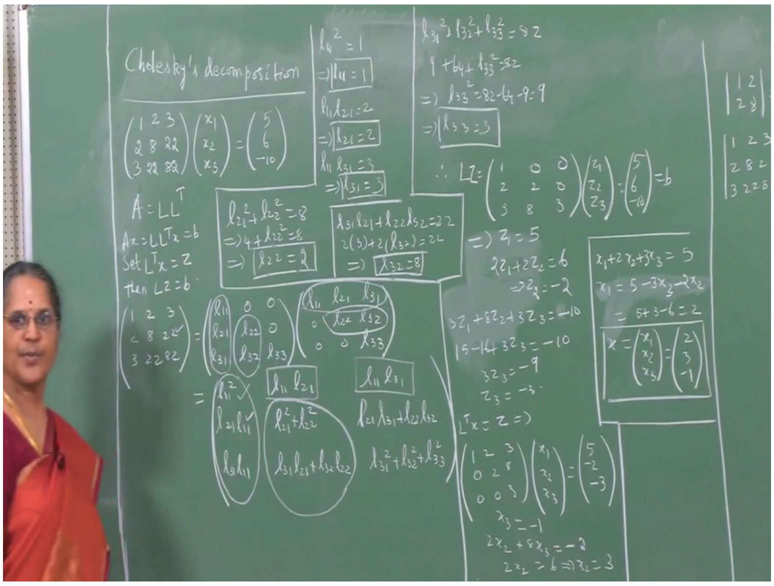

So l 11 square must be equal to 1 so l 11 equal to 1, why do I take only the positive root? What is l 11? it is the 1st row 1st column element, it appears as a diagonal element in the 1st row and therefore, when we wrote down the result under which matrix A can be decomposed in the form L into L transpose, where A satisfies the conditions that it is a real symmetric positive definite matrix, we said that the entries on the diagonal on the matrix L are all positive and so we choose the positive root here so l 11 is 1. Then we move to the element l 21 l 11 and that equal to the corresponding entry here which is 2, we have just now computed l 11 so that gives you l 21 to be 2.

And then the entry here which is l 31 l 11 and that must be equal to 3, so use the fact that l 11 is one that gives you l 31 equal to 3 so you have been able to determine l 11, l 21, l 31, namely you have been able to determine the entries in the 1st column of L namely the entries in the 1st row of L transpose. Now if you move over to the entries where you have already determine l 11, l 21, you have already determine l 31, l 11 because the entries which appears here are the same as the entry which appear here, so we need to make use of the values which corresponds to the values of these entries in the matrix a so I consider l 21 square plus l 22 square and equate to 8, l 21 is already known which is 2 and so l 21 square is 4 and so this immediately gives you l 22 square to be equal to 4 so l 22 is 2, again we take the positive square root because l 22 appears as the diagonal entry in the 2nd row.

Once l 22 is determined, we move over to the next entry here which is l 31 l 21 plus l 32 l 22 and that equal to the corresponding entry l 22. All the others are unknown except l 32 so when we substitute and solve for l 32 we get l 32 to be equal to 8 so what is it that we have got at this step by equating the entries in the 2nd column below the 1st row to the corresponding entries in a b ended up with the entries l 22 l 32 in the 2nd column of L and therefore on the 2nd row of L transpose. The only entry that remains to be obtained is l 33 and so we make use of the information here which says, l 31 square plus l 32 square plus l 33 square equal to 82 and therefore substitute for l 31, l 32, and that gives you l 33 square to be 9 and hence l 33 equal to 3.

Here again we take the positive root because l 33 appears on the diagonal in the 3rd row and we want to determine L in such a way that its entries allow the diagonal are positive and therefore we choose the positive square root when we get equation for l 11, l 22 or l 33 because they appear on the diagonal. so we have determined all the unknowns which appear either in L or in L transpose and hence we write down the matrix L and we 1st have to solve the system l z equal to b so I consider l z equal to b, I write down l namely l 11, l 21, l 31, l 22, l 32, and l 33 and z has components z 1, z 2, z 3 and the right answer in the vector b is given here. So when we have a triangular coefficient matrix in a system of equation then we make use of forward substitution and obtain the solution.

So the 1st equation gives you 1 into z 1 equal to 5, the 2nd equation gives you 2 z 1 plus 2 z 2 is 6, z 1 is already known so this gives you z 2 as minus 2. Then we move over to the third equation which gives 3 z 1 plus 8 z 2 plus 3 z 3 equal to 10 and z 1 and z 2 are known so from here we solve for z 3 and that turns out to be minus 3 so the unknown vector z is now determined so we have to now solve the system l transpose x equal to z, here z is known because we already have solved for it, x is an unknown vector which has to be obtained, right.

The coefficient matrix for this system is L transpose so solve the system L transpose x equal to z so write down L transpose, what is it? It have entries l 11, l 12, l 13, then0, l 22, l 23, 0, 0, l 33 multiplied by x 1, x 2, x 3 equal to the vector z that you determine namely 5, minus 2, minus 3. and you observe that in this case the system has coefficient matrix to be an upper triangular matrix and therefore, you have to solve this system using backward substitution procedure so you start from the last equation that gives you 3 x 3 is minus 3 so x 3 is minus 1.

The last but one equation gives 2 times x 2 plus 8 times x 3 is minus 2, you just now computed x 3, use that information that gives you 2 x 2 equal to 6 so x 2 equal to 3 and you move over to the 1^{st} equation which gives you x 1 plus 2 x 2 plus 3 x 3 is 5 so use the values of x 2 and x 3 in this that gives you x 1 equal to 2 so your unknown vector or the solution to the system of equations A x equal to b is given by x equal to x 1, x 2, x 3 and they are given by 2, 3 and minus 1. If you substitute these values in the given system of equations, you will see that they are identically satisfied, it is just a check on your computations, so we have obtained the solution of the system A x equal to b using Cholesky's decomposition method.

We had ensured that A satisfies the condition namely it is a real symmetric positive definite matrix and therefore it processes unique factorisation of the form L L transpose and in determining the matrix L, we have taken care to see that the diagonal entries and L are positive. So we now know how to solve a system of equations by decomposition methods namely factoring A in the form L into U with appropriate L and U depending upon what decomposition method we want to use to solve the system. And these 3 methods namely Dolittle method, Crout method and Cholesky's method, they belong to the class of direct method of solving a system of equations A x equal to b, where A is an n cross n matrix .

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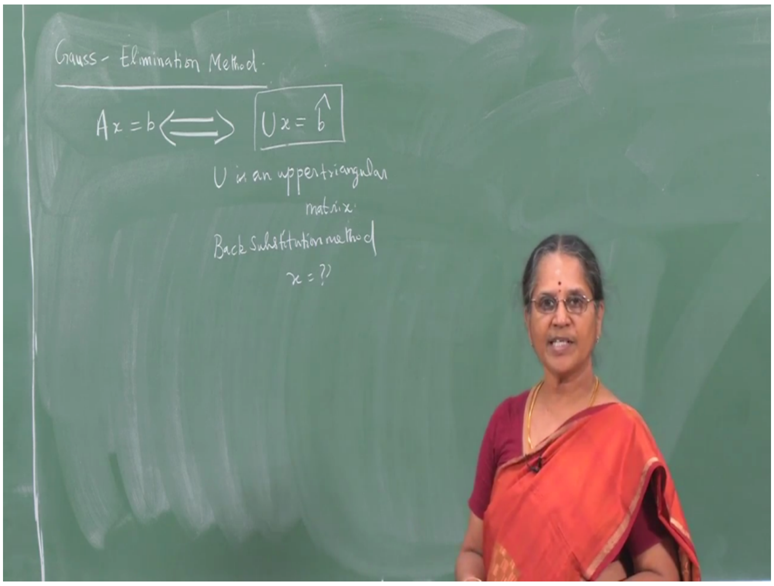

So we now move onto another type of direct methods, which is referred to as Gauss elimination method so we shall try to understand browser elimination method and see how we can solve a system of equations by this method. So we consider Gauss elimination method, what does this method do? Given a system of equation A x equal to b, it implies elementary transformations and reduces this matrix A to an equivalent system which is U into x equal to say some b cap, where U is an upper triangular matrix. So you start with the system A x equal to b and apply elementary row operations on the rows on the matrix A and reduce the matrix A to an upper triangular matrix U, so you end up with an equivalent system U x equal to b cap.

And you know how to solve this system because the coefficient matrix is an upper triangular matrix and so you employ back substitution procedure and solve this system and determine what x is. So Gauss elimination method involves 2 steps, the 1st step is reducing a given matrix A to an upper triangular matrix , the 2nd step is employ back substitution method and solve the system U x equal to b cap. So let us try to understand browser elimination method and see how a matrix A can be reduced to an upper triangular matrix using elementary row operations. So we shall illustrate the details by taking a system of 3 equations and 3 unknowns, it is easier to understand and the method works in the same way for a system of n equations in n unknowns.

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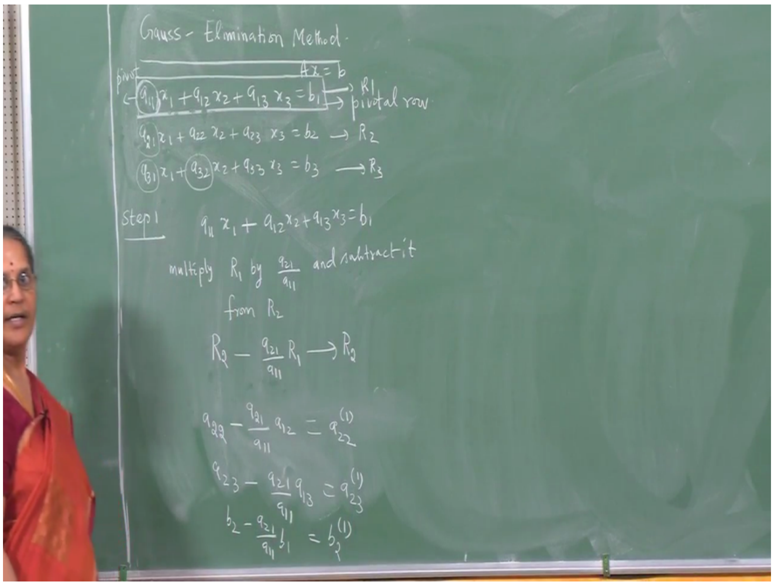

Let us consider a system of 3 equations with 3 unknowns given by a 11 x 1 plus a 12 x 2 plus a 13 x 3 equal to b 1, a 21 x 1 plus a 22 x 2 plus a 23 x 3 is b 2, a 31 x 1 plus a 32 x 2 plus a 33 x 3 equal to b 3, which can be written in the form A x equal to b, so what is it that we do in reducing this matrix to an upper triangular matrix? What does that mean? If we have to make this into an upper triangular matrix then I must have the entry here which is the coefficient of x 1 to be 0. And also the entries which are coefficient of x 1 and x 2 in the 3rd row must be equal to 0. So I must perform elementary row operations in such a way that the entries which are rounded here, they must be made into 0s, so let us do the following. The 1st step is the following, namely I keep the 1st row as it is, I called this row as pivotal row at this step and the coefficient of x 1 is what is called the pivot.

So I take the 1st equation for the unknown x 1 and therefore I call its coefficient as the pivot and called the 1st equation as the pivotal equation. So I keep the 1st equation as it is and right down a 11 x 1 plus a 12 x 2 plus a 13 x 3 equal to b 1, what do I want? I want to convert this system to an upper triangular system. As I already remarked, I want the entry which is the coefficient of x 1 in both the equation namely the 2nd and the 3rd equations to be 0, so I perform the following operation. What is it that I do? I multiply the 1st equation maybe I call this as row 1, this is row 2 and this is row 3, so I shall multiply the 1st equation R 1 by a 21 by a 11 and subtract it from the 2nd equation R 2. So my new R 2 is such that it is the old R 2 minus a 21 by a 11 times R 1 so my new R 2 replaces the old R 2 so this is one of the elementary transformations.

What is it? Multiplying an equation by a scalar and adding it to another equation, so I am adding it to another equation and taking the coefficient as negative so I say I am subtracting is from the 2nd equation. So I have performed an elementary row operation and replace the 2nd equation by the following; namely it is better R 2 minus a 21 by a 11 into R 1. Let us see what happens when the multiply the 1st equation by a 21 by a 11, so this becomes a 21 by a 11 into a 11 so here we will have a 21 x 1, we are subtracting it from a 21 x 1 so the coefficient of x 1 will be 0 in the new 2nd equation at this step. Let us look at this, what are we multiplying? We are multiplying the 1st equation by a 21 by a 11 so a 21 by a 11 into a 12, we are subtracting that from a 22 and so let us call this as a 22 at the 1st step.

So the coefficient of x 2 now will be a 22 minus a 21 by a 11 into a 12 and I called that as a 22 1 that is the coefficient of x 2. Let us go to this, so I will subtract sum a 23 a 21 by a 11 into a 13 let us call that as a 23 1 and that is going to be the coefficient of x 3 in the 2nd equation, what about this? This will be b 2 minus a 21 by a 11 into b 1 call that as b 2 1.

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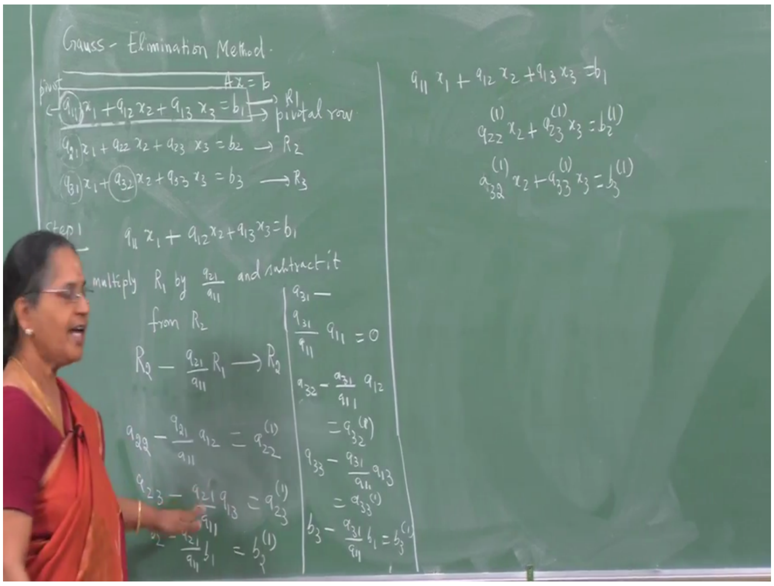

And therefore when we perform this I will have my equation to be as follows; the 1st equation remains as it is, it is (())(24:12) equation so I take it as it is and I have performed an elementary row operation and as a result the coefficient of x 1 in the 2nd equation is 0, the coefficient of x 2 is a 22 1 so I write that here and the coefficient of x 3 is a 23 1 and the rightminushand side is b 2 1 so this is my new 2nd equation. I do the same thing with the 3rd equation namely, I have to make this entry 0 coefficient of x 1 in the 3rd equation to be 0 so what do I do? I multiply equation 1 by a 31 by a 11 and subtract it from the 3rd equation, so I perform another elementary row operation involving the 3rd row and the 1st row.

So what do I do? I take the 1st equation, multiply that by a 31 by a 11 and subtract it from the 3rd equation, so the 1st term namely the coefficient of x 1 will be such that it is a 31 minus this and therefore, it will give 0 and so coefficient of x 1 will be 0 in the 3rd equation. What happens to the coefficient of x 2 in the new 3rd equation? It will be a 32 minus a 31 by a 11 into a 12, and then call this as a 32 1 and what about the coefficient of x 3? You will subtract from a 33 the product a 31 by a 11 into a 13 and called that as a 33 1. What about the rightminushand side? It will be b 3 minus a 31 by a 11 into b 1 and call that as b 31. So at this step your new 3rd equation will be a 32 1 into x 2 plus a 33 1 into x 3 and that equal to b 3 1.

So you have completed your step 1 where you have used 2 elementary operations, one involving row 2 and row 1, the other involving row 3 and row 1 and the resulting system is something like this and you see that the coefficient of x 1 in the 2nd and the 3rd equations are made 0. If you have more equations say if you have n number of equations in n number of unknowns, what is step 1? Step 1 is to use elementary operations in such a way that the coefficient of x 1 in all the rows below the pivotal row are made into 0s that is what we have done.

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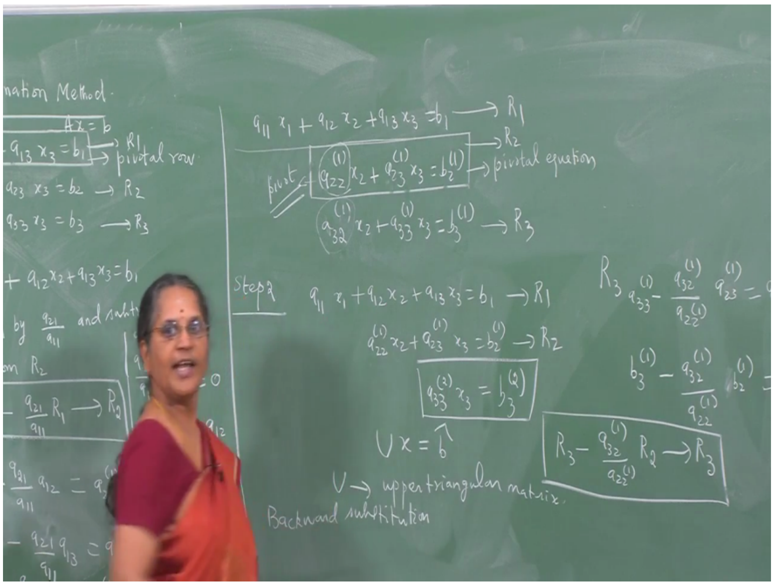

Now we move over to step 2, what is step 2? At this stage we do not touch the 1st equation, we keep it as it is and regard the 2nd equation as the pivotal equation and we call this coefficient which is the coefficient of x 2 the 2nd unknown where the 2nd equation is the pivotal equation at this step as the pivot at this step. And then what do I want to do? Keeping this as the pivotal equation it should make the provision of x 2 in all the other equations which lie below this equation into 0s making use of elementary row operations. Here I observe that I have 1 equation and the coefficient of x 2 in the 3rd equation is a 32 1 so I must make this entry into 0 by an elementary row operation.

So when I do that, I will have at the 2nd step my equation to be a 11 x 1 plus a 12 x 2 plus a 13 x 3 equal to b 1 and my 2nd pivotal equation will remain as it is and then my 3rd equation will be such that I have to make this entry into 0. So what should I do? I must do an elementary row operation, what is it? I must take so here this will be my 2nd equation, this is the 1st equation and this is the 3rd equation so I keep the 1st equation as it is, the 2nd equation as it is but when I come to the 3rd equation, what do you do? My new 3rd equation will be such that it will be obtained by the following... Multiply the 2nd equation by a 32 1 by a 22 and subtract it from the 3rd equation, so when I multiply this by a 32 1 by a 22 1, I will have a 32 1 and I subtract it from the 3rd equation which has coefficient a 32 1, it will be 0.

And therefore what happens to the coefficient of x 3? That will be a 33 1 minus a 32 1 by a 22 1 into a 23 1 and I call this as a 33 2. So my equation here will be a 33 2 into x 3 will be equal to... So I have to do the same operation on the rightminushand side, so I will get what will I get? b 3 1 minus I am multiplying and the 2nd row by a 32 1 by a 22 1 and so this multiplied by b 2 1 and that is the new b 3 2 so I end up with b 3 2 and my 3rd equation is going to be this, so what is the elementary operation? R 3 minus a 32 1 by a 22 1 of R 2 gives me the new 3rd row namely R 3. So it is very important that at every step you indicate what is your elementary row operation, namely R 1 goes to R 1, R 2 goes to this and R 3 again goes to the following which we have indicated here.

So indicate what is the row operation that you perform to arrive at the system of equation at the next step, so what happens here? R 1 remains as it is, R 2 remains as it is, the new R 3 is obtained by performing the elementary operation which is old R 3 minus old R 2 multiplied by this factor gives me the 3rd equation in step 2. Now look at this system, we observe that it is of the form matrix U multiplied by an unknown vector equal to the rightminushand side vector b cap where U is an upper triangular matrix and therefore you can solve this system of equations by backward substitution. We already know how we can obtain the solution of system of equations when the coefficient matrix is upper triangular matrix, so from the last equation you can get what x 3 is.

Substitute that x 3 in the last equation, determine what is x 2, knowing x 2 and x 3 from the 1st equation you determine what x 1 is, so you have the unknown vector x having components x 1, x 2, x 3, which are determined using backward substitution procedure, so Gauss elimination procedure essentially involves the following 2 steps namely reducing the coefficient matrix A in the system A x equal to b to an upper triangular matrix using elementary row operations and then the resulting upper triangular system will be solved by backward substitution procedure, so let us illustrate this method by some examples.

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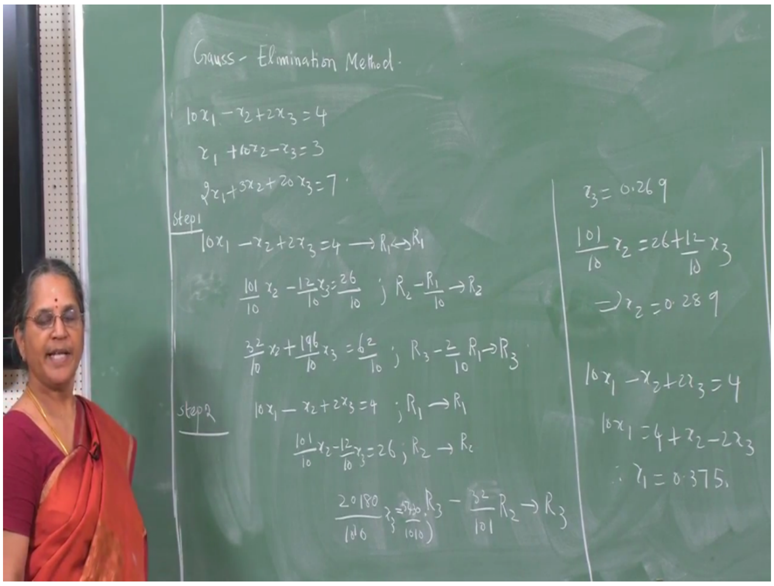

Let us consider the system of equations given by this and let us try to solve the system by Gauss elimination method and we know that this method involves 2 steps namely reduce the coefficient matrix A to an upper triangular matrix and then use backward substitution and solve for the unknown vector. So in step 1, I take the 1st equation as it is and that is going to be my pivotal equation at this step and the coefficient of x 1 is the pivot at this step. And now I have to make the coefficient of x 1 in this equation and that of x 1 in the 3rd equation to be 0s, so what do I do? I take the new 2nd equation to be as follows; namely I multiply the 1st equation by 1 by 10 and subtract it from the 2nd equation, so my old R 2 minus 1 by 10 R 1 will give me the new 2nd equation.

So if I multiply this by 1 by 10 and subtract it from 1 times x 1 that will give me 0 so I do not have coefficient of x 1 here in the 2nd equation. Then here I multiply this by 1 by 10, so minus 1 by 10 and subtract that from this coefficient and therefore, I will get the new coefficient to be 10 minus of minus 1 by 10, so 101 by 10 into x 2, then this coefficient will be 2 by 10 so minus 1 minus 2 by 10 that will be minus 12 by 10 x 3 and then 3 minus 4 into 1 by 10 so 3 minus 4 by 10 and therefore that gives me 26 by 10 so that is my 2nd equation. Then in the 3rd equation I have to make the coefficient of x 1 which is 2 to be 0, so multiply the pivotal equation by 2 by 10 and subtract it from the 3rd equation. So my new 3rd equation will be obtained as follows, the older 3rd equation minus 2 by 10 times the 1st equation, which is the pivotal equation, so I get 2 minus 2 by 10 that is 0.

Then here 3 minus of minus 2 by 10 and so that is 32 by 1 x 2, then 20 minus 2 by 10 into 2 and that will give me the coefficient as 196 by 10 into x 3 and 7 minus 2 by 10 into 4 that is 62 by 10, so I obtain the new 3rd equation as follows namely the old 3rd equation minus 2 by 10 the 1st equation which is the pivotal equation. So I have in the 2nd step a system of equations to be given by the following namely, the 2nd and the 3rd equations do not have the coefficient of x 1 in these 2 equations as 0s. And now I move to the next step, what do I do? I keep the 1st equation as this and write down the 2nd equation and treat this 2nd equation as the pivotal equation at this step and the coefficient of x 2 is going to be the pivot at this step.

What do I do? I will have to make the coefficient of x 2 in all the equations which lie below the pivotal equation at this step to be 0. I have only one equation between namely this equation from which I have to make the coefficient of x 2 to be 0, what should I do? I should take the 3rd equation and then subtract from the 3rd equation 32 by 10 by 101 by 10 namely, which appears in the 2nd equation and that is what we have written. The new 3rd equation is obtained by subtracting 32 by 101 of the 2nd equation and you will end up with the 3rd equation in which the coefficient of x 2 will be 0 and that is what you see there. Similarly, take 196 by 10 minus 32 by 101 of minus 12 by 10 and that gives you this coefficient multiplied by x 3.

The right hand side will be 62 by 10 minus 32 by 101 times 26 by 10 and that turns out to be, so you observe that at this step you have a system of equations such that its coefficient matrix is an upper triangular matrix . So step one namely 1st step that is involved in Gauss elimination method is completed namely, reducing the coefficient matrix A in the given system A x equal to b to an upper triangular matrix and the next step is to apply backward substitution procedure and then solve this system. So the last equation gives you x 3 as 0.269, the last but one equation gives you 101 by 10 x 2 is 26 plus 12 by 10 x 3, so substitute for x 3 and simplify and get what x 2 is. And the 1st equation tells you 10 x 1 minus x 2 plus 2 x 3 is 4, we already have computed x 2 and x 3, substitute and solve for x 1 that gives you 0.375.

So you have been able to obtain the solution to the system of equations by Gauss elimination procedure. It is a powerful procedure and you will see how it can be very efficiently used in obtaining solutions which are very accurate. Let us consider another example because you may think that Gauss elimination method will work always, we will give an example where it fails to work then we will see how we can overcome this difficulty by what is called a pivoting strategy, so explain what is a pivoting strategy and then employ what is known as partial pivoting strategy and we will overcome the difficulty that we may come across while applying Gauss elimination method and then solve a system of equations by Gauss elimination procedure with partial pivoting, so let us ex plain this idea in details in what follows.

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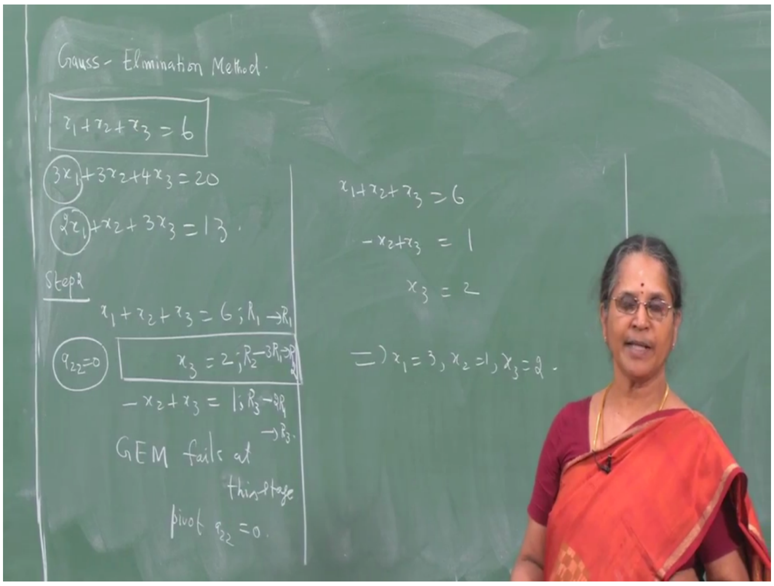

Let us now consider this example, so solve a system of equations given by this by Gauss elimination method, now that the method is clear let us quickly apply steps which are involved there. So I retain the pivotal equation as it is and then I have to make the coefficient of x 1 in the equation which appear below the pivotal equations to be 0, so what should I do? I must multiply row 1 by 3 by 1 and subtract it from row 2, so 3 x 1 minus 3 x 1 that will be 0 then row by 3 and subtract it from the 2nd equation so 3 x 2 minus 3 x 2 that is also 0, then 3 x 3 must be subtracted from 4 x 3 that gives me x 3, then 3 into 6, 18 must be subtracted from 20 so I get 2, so what did we do? We obtain the new 2nd equation as follows, R 2 minus 3 times R 1 gives us the new 2nd equation, R 1 is taken as it is.

Now let us move to the 3rd equation, I want to make coefficient of x 1, 0 so what should I do? Multiply the pivotal equation by 2 by 1 and subtract it from the 3rd equation then you will get 2 x 1 minus 2 x 1 that is 0, then x 2 minus 2 x 2 so minus x 2, 3 x 3 minus 2 x 3 so x 3, 13 minus 2 into 6, 12 so that is 1, so what did you do to obtain the new equation? It is R 3 minus 2 times R 1 and that gives you the new R 3 namely the 3rd equation. So at step 2 you have obtained a system in this form, so let us continue Gauss elimination procedure, what do we have to do? We have to take the 2nd equation at this step to be the pivotal equation that is what Gauss elimination method tells.

And what is the pivot? The pivot here in this step is the coefficient of x 2 in the 2nd equation which is the pivotal equation at this step. But I observe that there is no x 2 term here because the coefficient a 22 is 0 in this case is the coefficient of x 2, so the pivot in the 2nd equation which should be taken as the pivotal equation turns out to be 0 when we apply Gauss elimination procedure to the given system. So what should I do in Gauss elimination method for the next step in reducing this system to an upper triangular matrix ? I must if I have the position of x 2 as some a 22 which is nonzero then I must take that pivotal equation and multiply it by minus 1 by that coefficient and subtract it from this equation. But the coefficient there is 0 so I simply cannot perform this step and therefore I observe that Gauss elimination method fails at this stage.

Why does it happen? Because the pivot at this stage namely a 22 turns out to be equal to 0 and therefore, if at any stage of Gauss elimination procedure if you observe that the pivotal element at that stage in the pivotal equation turns out to be 0 then you cannot proceed further and apply Gauss elimination method, and Gauss elimination method fails in that case so what is it that we can do in order to solve this system? So let us look at this example and see what is it that we can do, right. We observe that we have a 3rd equation in which coefficient of x 2 is nonzero so this suggests that I can interchange row 2 and row 3 and take the 3rd equation as the 2nd equation and consider that as the pivotal equation at this step. Yes, I am performing an elementary row operation namely; interchange of equations is one of the elementary row operations.

So here I interchange 3rd row with the 2nd row namely; bring the 3rd equation to the 2nd equation and the 2nd equation to the 3rd equation, so I consider a system of equations now in which the 1st equation remains as it is, I have interchanged the 3rd equation with the 2nd equation so brought the 3rd equation as the 2nd equation so it is minus x 2 plus x 3 is 1 and the 2nd equation comes here as the 3rd equation, so x 3 equal to 2. Now I look at the system, I observe that the system has its coefficient matrix to be an upper triangular matrix and so I can solve this system by backward substitution procedure so I get x 3 as 2 and so from here x 2 as 1 and from this equation x 1 equal to 3, so the solution of the system has been obtained.

But we did encounter some difficulty at our 2nd step, where in the pivotal equation the pivot turned out to be 0 and therefore we could not proceed further with Gauss elimination procedure and observe that Gauss elimination method fails at this stage and this can happen when you apply Gauss elimination procedure to a given system of equations. When does it happen? When the pivot in the pivotal equation at that stage turns out to be 0 or the pivot is a very small quantity so that when we divide by that pivot large round of errors might occur, in all these cases we say that Gauss elimination method fails at that particular stage and employ what is called a pivoting strategy, so we shall explain this strategy as follows.