SCIENTIFIC COMPUTING USING PYTHON

Professor Vivek Kumar Aggarwal & Professor Mani Mehra

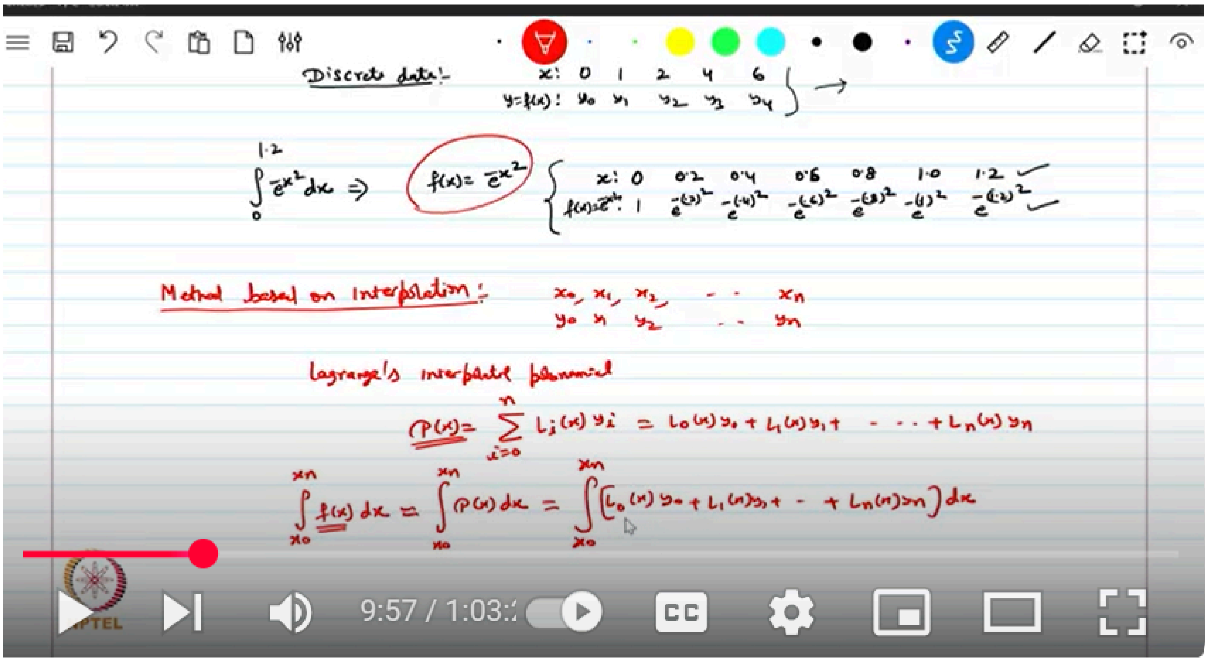
Department of Mathematics

Indian Institute of Technology, Delhi

Lecture No. 30

Welcome, everybody, to Scientific Computer English Python. So today we are going to start a new topic and it is called numerical integration. So let's get started. So our topic today is as we discussed in the previous lecture, numerical differentiation. So today is our numerical integration. So if we look at numerical integration, if we have any numerical integration, we have to do the integration of a function and that function is quite complicated. So we know that we will not be able to integrate it analytically. For example, suppose I have to do integration from 0 to 1 and I take the function e^- x^2 dx, then there is a lot of problem in solving it analytically. We cannot solve this. So if we are unable to solve this function, this integration, then what do we do ? Then we turn to numerical integration. There is some other function, it is a complicated function. Ok? f(x) dx so what do we do with this? Then we will move towards numerical integration. So this is the case when the function we have is given and the integration is quite complicated. So such complicated integration happens in many places when we do analysis or data science, so sometimes we have functions in many places. If we are finding the length of the curve, then you know that the integral becomes quite complicated. So it becomes a little difficult to solve it. Or do we have this for the function or what do we have? There is data. There is discrete data. Ok? What is discrete data? We have values. The values of x are given by us. Suppose 0 1 2 4 6 this pay value is given. And here we have y = even function. His values are given. So here we have given values y0, y1, y2, y3, y4. The values can be anything. Ok? So we have to integrate this type of data. So, during integration, we cannot integrate edge truth which is discrete data. So what should we do? We will integrate this numerically. And in this we are going to discuss the methods that we have for doing numerical integration. Ok? So we have two cases: there is a function but its integration is very complicated to find out, or there is discrete data and we have to integrate it, so how do we integrate it? So these two things will be possible through numerical integrations. So what happens in numerical integration is that our main purpose is to convert the data into discrete form and then calculate it like we found out by interpolating polynomials or calculate method of indeterminate coefficients from it. As if someone is telling us that brother, integrate this from 0 to the power of e^- x^2 dx. Ok? It could be one, it could be 1.2, it could be 1.3. So I suppose I take it to 1.2. So we have to do numerical integration of this. So that is not analytically possible for us. So what will we do here? So what should I do? My function is f(x) which is e to the power of - x^2. Ok? So suppose x is my zero. If I want to go up to 1.2, I will divide it as 0.4 0.6 0.81 1.2. Is my x domain okay? I divided it. And here I will calculate the values that I have of the function f(x) - x^2. Ok? So what will happen at x? It became a forest. This one's going to come here, this square of e to the power of -2. I will calculate this. e to the power of -4 squared is e to the power of - 6^2, okay? Whole square to the power of -.8 to e. So we can calculate this with a calculator. This is how I will calculate the whole square of one and e to the power of -1.2. So what we have are the nodal values and these will be the values of the data points above them. So the data that we have now, the functions that we used, we converted them into a discrete data form.

Now we have to see how to solve this. I mean, what methods should we use so that we get the solution that we have. So now let us see in what way we can solve this. So basically if we see, we have one method which we can always use and that is method. Based on interpolation, we can always do this, so what did we do in interpolation, just like we did it in method based on interpolation differentiation, similarly we will do it in integration, so what do we have to do in this, now we have this data, we have data points like x 0 x1 x2. Ok? So we have these data points and suppose y0 y1 y2 up to yn which is the value of this function is also given. So what do we do now? Suppose I did it by mistake. Ok? I solved it using interpolating polynomials. We can certainly do that. So suppose the polynomial that came to us is px. So px we know what that would be? It will be even, we had lix which was the fundamental polynomial of take the leg and here we have y i which starts from zero to n. What does it mean? L0x y1 y0 OK. L1 x y1 + Lnx yn so this will be our summation. Now what we have to do is that we had the data like this. Ok? What did we do based on this data? Found this polynomial out. which was the interpolating polynomial. What do we do now ? We have to find out integration. This is how this function has to be integrated. What does it mean? So we suppose I take xn from x0 somewhere. and there is some function f(x). I have to integrate it. So what did we do? We approximate this function by this polynomial, the Lagrange polynomial, so what do we mean by this? This will be the integration of its function which we can approximate to px dx. I can do this. And what is this thing ? This we have from x0 to xn and here we have l0x y0, okay, so this l0x is a polynomial, we know, and we can always find the integration of a polynomial very easily because polynomials are the simplest functions that we can integrate easily. So, we integrated it. Plus so what shall we do? Instead of writing the whole thing again and again, I will write it directly. Ok ? L1x y1 like this we have dx for all of them, now we will integrate all of them. Now we know that integration is a linear function. So this will get separated and we will have its solution. Ok? So we can do this work on the basis of Legendre, sorry Lagrange, which is the interpolating polynomial. Similarly, we can do Newton divided difference. Ok? Forward operator, Backward operator.



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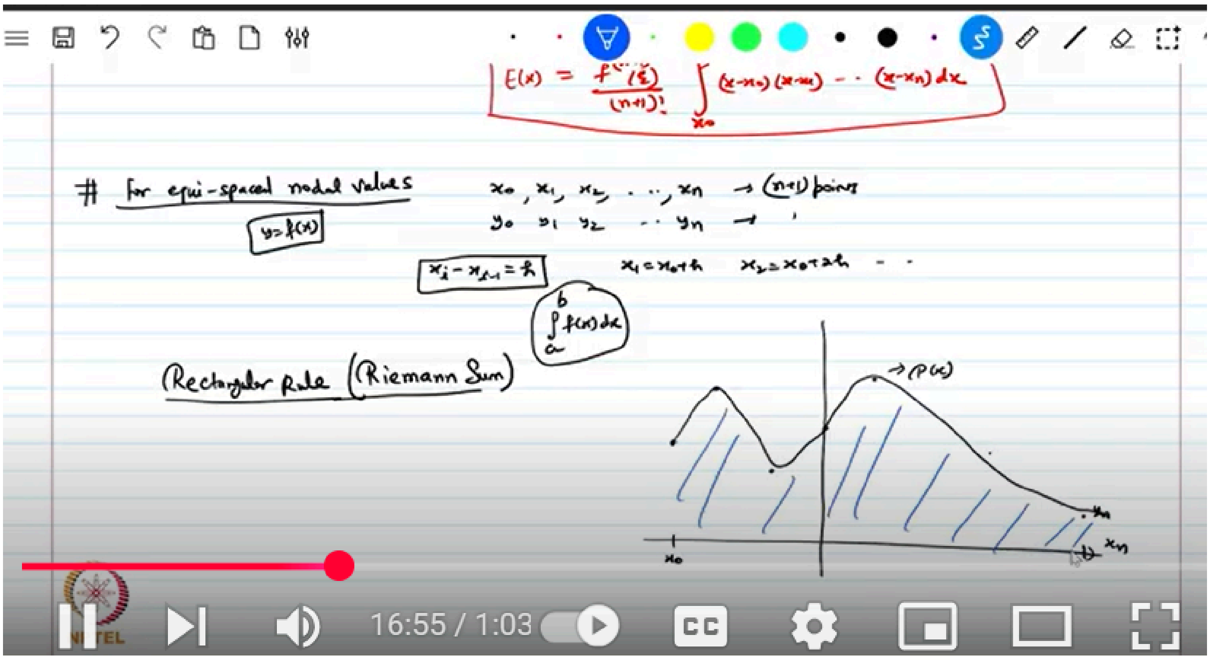
This is the uneven spacing that we have , we can do it in equal spacing also. So we can calculate and find out any interpolating polynomial that we have and then integrate it. So the method will be based on interpolation. So in this case, the error that we will get, the error that we will have, the error in this will become x0 (x - x1) (x – xn) f” / n + 1 dx. Ok? So this is a constant function. So we will calculate it like this. We will take n + 1 out because firstly we will get the constant value and then we will simply integrate n + 1. Is it okay? So we will calculate it like this and whatever error we have in this integration, we will calculate it and find it out. So these methods, we are not calculating them again. Look at interpolating polynomials for the given data and then integrating. So, we are assuming that we can do this work very easily. So what do we do now? So now see, we have taken the single integration. Such second integration and third integration can also be taken. So now we are discussing only the first single integration. So now these methods that we have are based on interpolating. But now we always find the interpolation polynomial first and then integrate it, so it is a very time taking process. So what we do for this is that we try to find out the methods directly. Ok? So that we have the data points and how can we find them out by integrating them. So what we have is what we're doing in this case is for equal space. Nodal Values This is what we're doing. Ok? First, what does it mean? We have the values x0, x1, x2, xn. Ok? So this is n + 1 points. And these values are given to us y0, y1, y2, yn okay? All these values are given and y is our function fx. If there was any function y that is equal to fx, then these values are given to us. Ok? Now we're going to assume that xi - xi - 1 is h. is constant. What does it mean? The x1 is x0 + h. Which is x2 is x0 + 2h. Is it okay? So this is how we can find it out. So the first method that we have, we will use those methods which we used when we learnt integration, double definite integral, at that time. Ok? So this type of thing is called rectangular rule. Ok? And if we see this then we also call it Riemann sum. So the easiest one for us is the rectangular rule. We can apply that. Ok? So what do we do in this? What is the meaning of rectangular rule? Have to divide it into rectangles. Now suppose we have some values given. So I have a value given. Ok? The second value suppose this is given. Then the next value I have is this. Then next is this. Then next from that is this. Next is this. It will go on like this. So suppose this last one is xn and suppose this is x0, so now what do we have to do with this, if we see, what is the integration, if we have the function that we have, let me not take it now, I will not take the negative value, I will take the positive one only, so suppose my last value is this, sorry xn is this and the value that is yn is this, the value that is there somewhere is y. Ok? So now if we look at this, then what is our integration, the integration of a function? Area under that curve. Like we have to take an integration we have a over b fx dx okay ? And if fx is positive then what will it give? This will give the area under that curve. Ok? So now we have the values that we have given to it, we have the discrete values that we have given to it.



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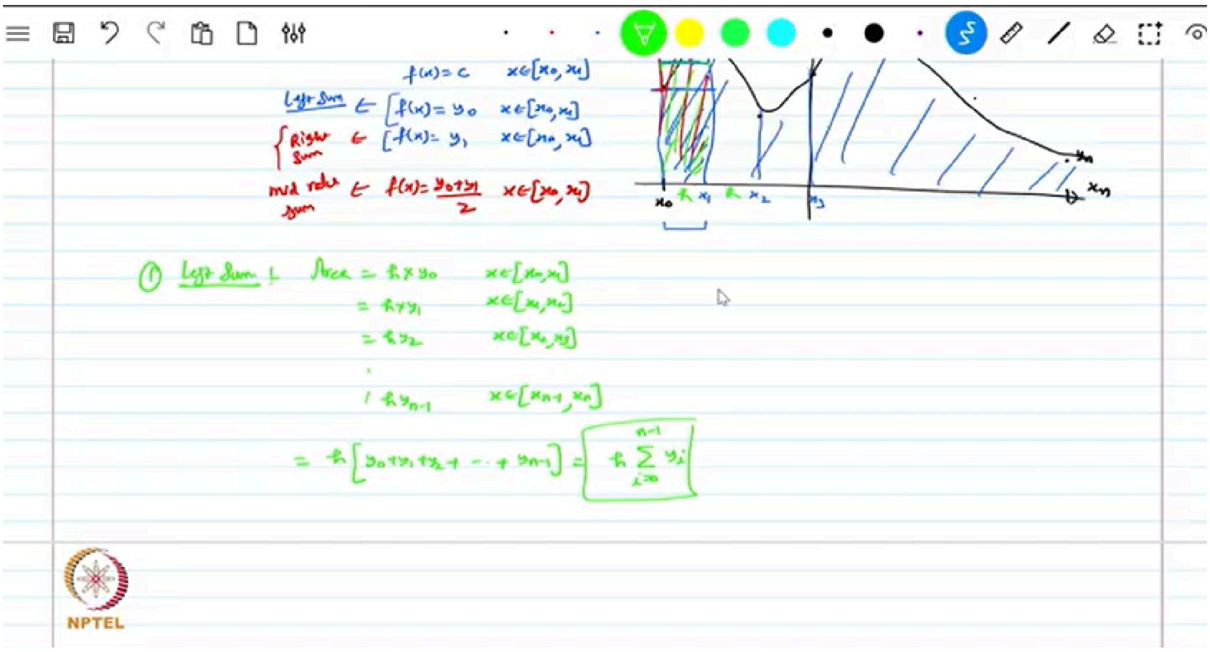
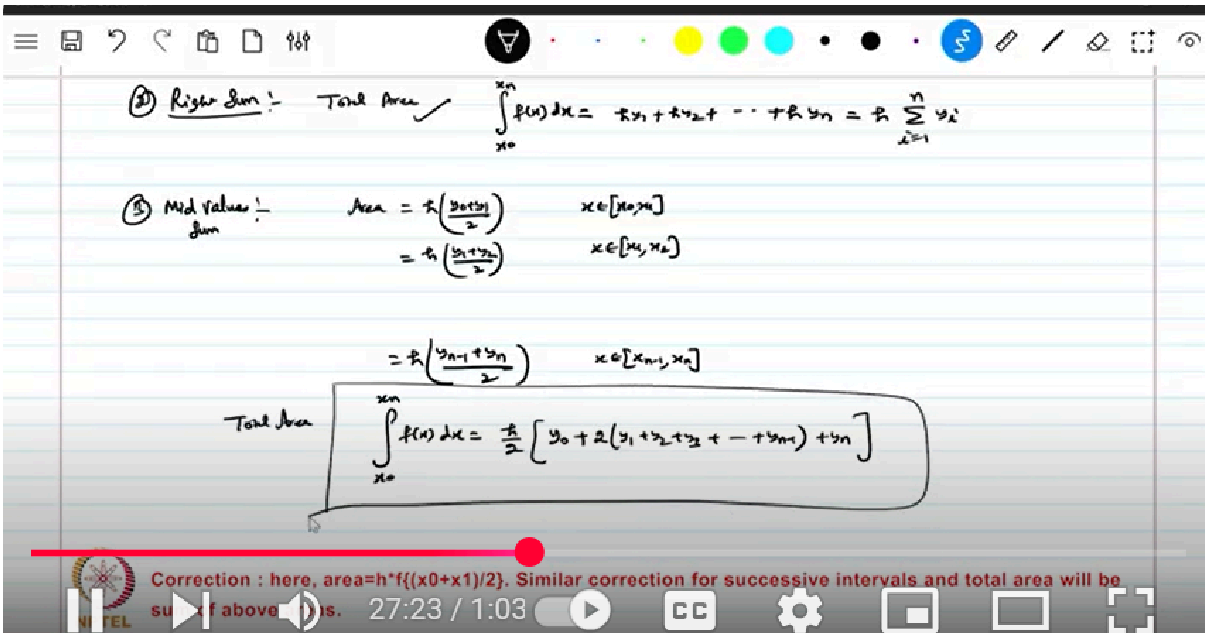
Now he has to see that if interpolating is done then what will happen in interpolating? This way you will get an interpolating polynomial. Whatever it is, it is going away. And we approximate that function f(x) by this. This is my supposition px. Ok? And then we integrated it. So this is the area we have. So this is what we did when we had to find out the interpolating polynomial. Second, what are they? The easiest way is the rectangular rule. What do we do in rectangular rule? We have taken these points. The second point is this. The third one is this. The fourth one is this. There are such points. So what did we do now? Now look, first I will go from x0 to x1. Then I will go from x1 to x2, x2 to x3 like this. So if we calculate in the first one. First of all, what we do in this is that in the rectangular rule, we consider the value of the function as a constant. Suppose there was a function in this, but we assumed that the values of the function f(x) are equal to c constant when x is from 0 to x1. Now what can this constant value be ? I can do this by taking the value of the function f(x) to be y0. I can also take the value of the function to be y1. I am watching the first one. We will probably do it like this in all of them but we are looking at the first one. So if we look here in the first one, what did I do? Let the value of the function be y0. Meaning, let's accept this one. So in that entire x, which is our first x not to x1 interval, these same values will come, so if we look at our interval, then this has become a rectangle in it and we will find out the area of this rectangle. Ok? But what happens is that while calculating this, this error will occur. Because if this curve had area, then the entire curve would be area. But we are assuming this here. Is it okay? So we call this left sum. The left sum is the lower bound of the area. Ok? Now what did we do? What did I do in the second case? I accepted this. The values on the right hand side of this interval were considered. If I assume this value of its right hand side then you can see it has become rectangular. So we have this complete rectangle. This is okay? And we can easily find the area of this rectangle. Length * Breadth. So we will find its area. So whatever comes, we will call it right some. Ok? Now he was told to take the left sum. So the error that we have, the area that is coming is very less as compared to the original area, let's take the right sum. So it depends on what the function is, whether it is decreasing or increasing. So if we do the right sum, we will get an error in that also. So now what did we say, let's take the mean, so what did we do in the mean, let's assume fx to be y0 + y1 /2, so we called this the mid value. Ok ? So what to do in mid value? If we look at this, this was our y0 and this is y1, so it will come somewhere here. So this got here somewhere. So now look at the area that we have, the area of that rectangle will come, this will come. So this area is a little bigger than the left one but smaller than the right one. So this is because we have taken the mid value. Ok? So in this, this mid value means that it will give us better value in integration. So but these three rules that we made are rectangular rules. Because in this, instead of dealing with the function, we are calculating its values according to the minimum value i.e. left sum, right sum or middle sum. So now let's see what will happen in it. So the first method we are taking is based on left sum. So look what we did in the left sum. Now if we calculate from x0 to x1, we will have the area. So look what are you doing in the left sum? AREA What is that coming ? h is between x0 and x1. This is all h. So what is length * breadth and width? y0 comes in the first one when x belongs to x0 to x1 then here comes h * y1 when x belongs to x1 to x2. Then this comes h y2 when x belongs to x2 to x3, okay it will go on like this and in the last we have the x which we are taking on the left, so this y n - 1 will come when x is lying in the last interval, so it means our left sum becomes oh I take all the commons so what does this become y0 + y1 + y2 yn - 1 the last yn will not be included in this. So what does this mean? We calculated it easily. I made them all equal. To take i from 0 to n - 1. is this ok? So what did we do? Very easily, I have to sum the values that were given to us, the y that came, just leave the last one, okay, and multiply it by h. We have taken h in it because it is uniform, so the one that we have got will be left sum.



Okay, we want to do the second one, right sum, so right sum. See, we will have to start from x1 instead of x0. So look at the total area, what do we have? This becomes x0 to xn fx dx whatever is the sum of this will become equal to this. So in this case we have x0 to xn fx dx what does it become? fx means yx. Keep in mind that y is fx because generally when a function is given, we write it as fx. But when its values are discrete then we write it like y 0 y1 y2. Ok? So now look in this case, what will the area of the first rectangle become? We will take the right value. So this will come in the right value. so its length * breadth means y1 right? So this will become h * y1 + h * y2 and in the last h * yn so what does it mean, y will not come in this so this becomes h summation i from 1 to n will come yi so we will add this and its sum will come out so this will come out to be right sum, right, so there will be a lot of error in this and the third one is the mid values, okay, mid value all, what will be that now we have to find out the total area, so you will see what will be the area when x belongs to x0 to x1 in the first one, so the area will be h y0 + y1 / 2 this is it. Ok? What is x belongs to x1 x2? That will be h y1 + y2 / 2. This is done. Ok? So this value will keep on going throughout and in the last x belongs to xn - 1 xn which is the nth interval, in that sub interval it will become yi n - 1 yn / 2 so this will become even for us. If we add all of these together, what will be the total area of the function from x0 to xn ? Now we will make everyone equal. Now we will treat everyone equally. Look, y1 which is the connected nodal values, this one which is interior, these are the values on this, in this case you will see that y1 is coming here as well and it is coming here as well. y2 will come here and will come in the next also. So if we look at it, what is happening ? h/2 I take the common one from all. Is it okay ? I took h/2 as common.



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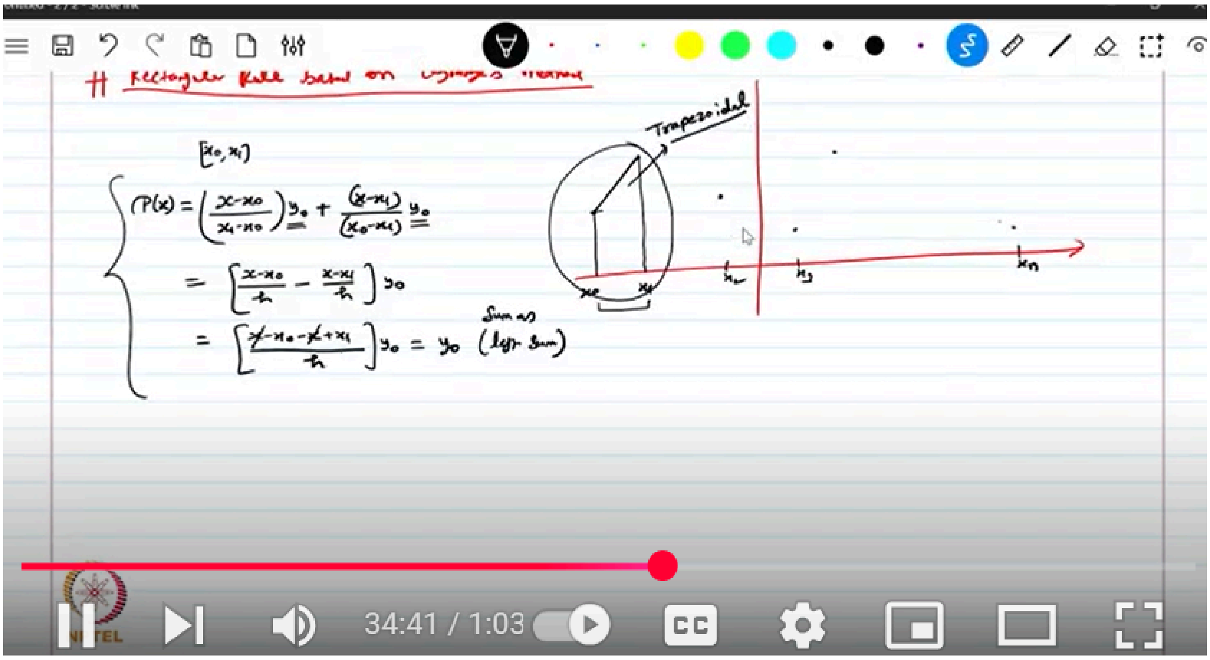
So this becomes y0 + h/2 h/2 I took the common one. y0 I got it here. So y0 is now y1 is coming here as well and y1 is coming here as well. So 2 times of y1 then y2 then y3 and go on like this. And in the end it will come up to yn - 1 here and what will be left in the end? yn will be saved. Ok? So, this area which has come, this area has come from our mid value. So like this we can calculate it very easily. So these areas are based on the rectangle, it is based on its base. So, we have already read about this thing which is similar to Riemann. So we can always use it. The next thing that comes is this. So in this case, look, whatever H you are, it depends on what the value of h is. If h is small then you will get better results. Is it okay? Edge compared to the bigger H. So we would always like to take the value of h which should be the minimum value. And the lower the value of h, the more data points will come and the more data points come, the more calculations will have to be done. So this method that we have is based on rectangle. Now, whatever other methods we have, we have made it remand sum. Now we want to see if there is any other method that we want to discuss. Ok? So we're going to deal with rectangular methods next. Is it okay? But let's see what happens from our numerical point of view. So this remand sum is done now. Next thing that comes to us is this rectangular rule. Ok? Now we will do it based on the rectangular rule, let's do it with Lager, let's see the Lagrange method or formula. Ok? So what did we do now? Now we have to use these rectangular rules. But let's look at Lagrange methods, which were our Lagrange interpolating polynomials. We saw that the Lagrange interpolating polynomial, if you remember, was discussed only when we had unequal spaced nodal points. Is it okay? So if there are parallel node space nodal points then we will have to do Lagrange or Newton division. But in this case we are assuming that the nodal values are in equal space. Ok? So the Lagrange is robust. He can work for both unlike and equal. So now we see what will be the rectangular rule based on Lagrange methods? Ok ? So now we have achieved our main purpose, that is now we have data values. Ok? So we have a value supposition like this. The second value is this. The third value is fluctuating. The value is going up in 4 or 5 such ways. So what did we do now? x0 and this x1 right? This is y0 y1. x2 will continue like this. x3 and lastly our x1. Now see what we do with it. If we look at it, if we have two points, if we have two points x0 and x1, let's talk about the first one. Ok? So if in this we have two values, we have x0 and x1, then we know that the polynomial that we will get is the interpolating polynomial which is the Lagrange, we can discuss a linear interpolating polynomial. Ok? But what we have to do in this case is to see how the rectangular rule will be formed. So what will we do to create a rectangular rule ? We will write x - x0 / x1 - x0 like this. Plus x - x1 / x0 - x1 I will not write y1 here. I will just write y0 here. So what does this mean? The polynomial that we will get will come in this type. So, I have taken the same value of y. Ok? So if we calculate this, then I can now calculate it like this. So if we look at this, what has happened? x - x0 / h - x - x1 / h y0 And this will come to us x - x0 - x + x1 / h y0 So how much is this? This comes to us x1 - x2 so this comes to us not y. so what does that mean? The polynomial inside this, we had to find out the linear interpolation polynomial, we considered only y0. Similarly, I can take y1 also. So if you see this case which has come, this left sum has come. Or what will you do in right sum? We will take the value y1. Ok? So if we do y0 then we get the same as left sum. Okay, so we have approximated this polynomial with the left sum. If I take y1 then it will become right sum and if I take its mid value then it will become mid value sum. So if we want to do rectangular rule based on Lagrange. Ok? So if we do it like this then we will have rectangular methods. But now we have seen that the rectangular method is good in many ways but it involves a lot of errors. so what do we do now? Let us see, if I use this method as I think it would be, then suppose I approximate this function to a linear line, what would happen to it? So if I see this thing then will you see what happened? This is not a rectangle. Is it okay? So if you look carefully, this line, this figure which has been formed by us, we call it trapezoidal.



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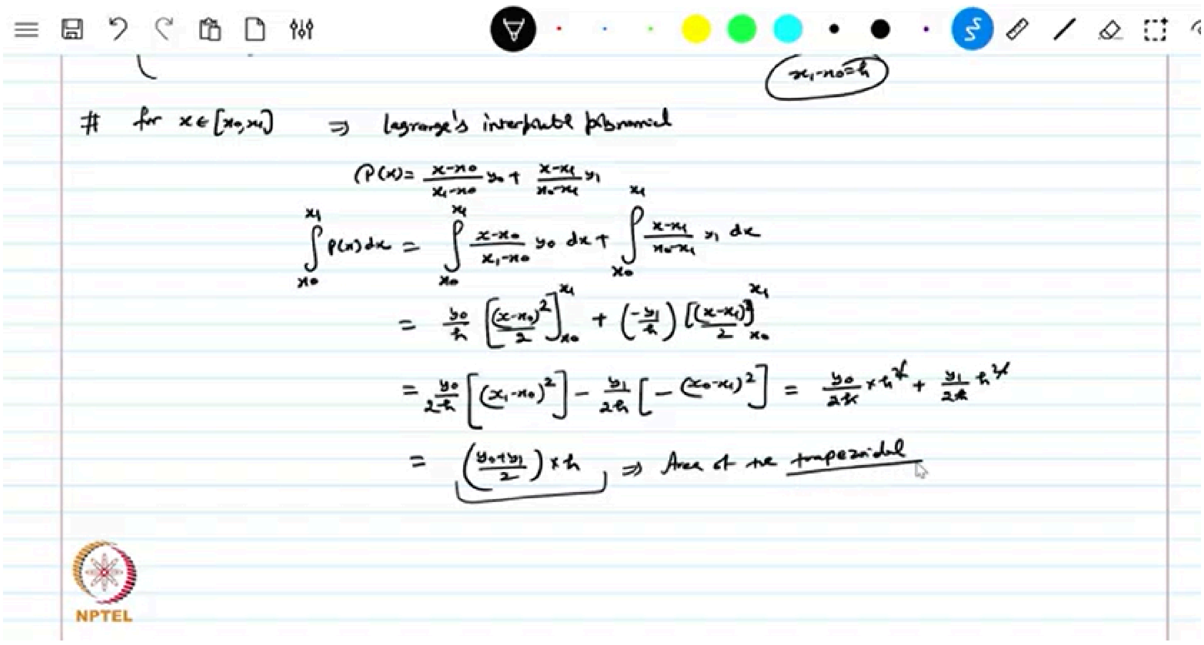
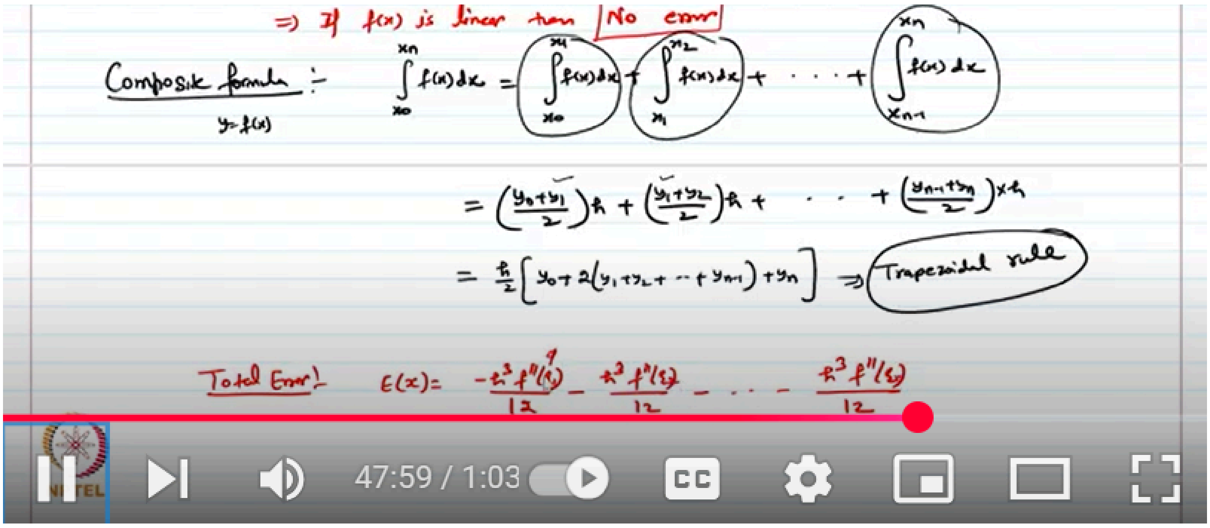
If it is trapezoidal then what do we do? Let's try to discuss the trapezoid rule. Ok? So what did we do now? What did I do in this case? Lagrange applied. So for x belongs to x0 x1 that's what I'm doing. Ok? what did we do? We apply the Lagrange interpolating polynomial. So let's see what comes. So let's see, we know that it is px and what is that ? (x - x0)/(x1 - x0) y0 + (x - x1)/(x0 - x1) y1 this is what we have. When x comes to x0 and x1 because only two points are given to us. Now we have to calculate this. px dx The area under this curve between x0 and x1. So that becomes we have x0 to x1 right? This becomes (x - x0)/(x1 - x0) y0 dx + (x0 - x1)/(x - x1) (x0 - x1) y1 dx. Now we know what x1 - x0 is. It is h because I am taking it for uniform, so it will become H. Ok? So if I look at this, this becomes y not / h and this will come to x - x0^2 / 2 / 2 x0 x1 this is the integration plus y now this is x0 - x1 so this becomes - h so this is okay so this will become - y1 / h and this will come to x - x1 squared by 2 x0 times x1 this is what we have. Ok? So now we will put limits on its values. So here we have y0 / h, okay? Too I can take it out. So I took two out. I took it from this also. Now what we have is x1 - x0^2 this is left over and it's just going to be minus 0. Ok ? minus y1 okay ? bye h 2 i took it out. So then let me write this separately. Square. Ok? So firstly I am writing this minus y1/ 2h now this will come to x1 - x1 so it becomes 0. So this will be saved –(x0 - x1)^2 this will be saved. And what is this? What is x1 - x0? it is. So if we look at this, we have y^2 / 2h * h^2 plus minus minus plus y1 / 2h and this is square. No problem. So h^2 has arrived. Is it okay? So here we have it, if we look at this, it is cancelled by h. What is this left with us? If I write it like this y0 + y1 / 2 * h, this is what we are left with. So if we see, this value has come. And what is this thing? If we look at this trapezoidal, this trapezoidal, this is its area. Ok? What is its area? Which makes both of them equal, which are parallel lines. Ok? Let us take their meaning. Multiply by, which means we multiply it by the distance between the parallel lines. So I took the mean of the lines y0 and y1 and multiplied it by the distance between them. So if we look at this, we will say that it is of the trapezoidal which is made in our first interval. So what this means is that whatever we can do directly, if we do it like this and calculate its area, we will get it. But this trapezoid of ours has come from x0 to x1. So now I can do the same work like this also. So whatever value was coming from here, suppose

I did linear interpolation in between it. Then after this I did it here. Is it okay? It happened here. Then it was done from here. So what can I do like this? Our linear approximation for the entire interval, if we remember that we have calculated it as it is called piece wise linear interpolating polynomial, so what will happen from that, now we can calculate it completely in that, so whatever is its name, now see, we have calculated it, right? So now if we want to see the ah error in it, then we know what the error will become first in that and what will be the error of our ex, it will become x0 to x1 x - x0 x - x1 / 2 dx and y is we know that our line between x0 to x1, okay, so if we calculate it, then we will get a constant value and I have calculated it as x0 to x1 dx. Ok?



So if we calculate this. Ok? So let us integrate this. Let us substitute the limits. So whatever values will be created, they will come to us - h / 12 will come. If we calculate this then this is what we will get. Ok? So this value has come, it has come. So this means that the error is coming in the second derivative. This means that if our function is linear then there is no error. Ok? So this implies that if fx is linear then there is no error. Ok? So for linear, because its second derivative will become zero. Ok ? But if we have a non-linear function then we will get an error. Ok? So for linear it is clear that if we apply trapezoidal rule then there is no error. So what do we do with it now? I calculated this in an interval. So in the same way we will calculate it using a composite formula. What will be the composite formula that we will keep adding all the intervals that we have, calculate their area and keep adding them. So what is the composite formula ? Now look what we have? Composite formula means that we have to calculate the whole from x0 to xn and our function f. So what will we do with this? We divided it. x0 to x1 fx dx + x1 to x2 fx dx, like this I divided it. I divided this. Now we have to see that we have approximated this using the trapezoid rule. I approximated this also using the trapezoidal rule. I approximated this also using the trapezoid rule. So if we see, this will become what we have, the value I am taking above. where is A? This is y0 + y1 / 2 * h right? So we have the values y = fx given to us. So those values that we have got will be plus y1 + y2 / 2 * h and this will become yn - 1 + yn / 2 * h okay? H Whatever is there is uniform so there will be no problem. In this, if it is non- uniform then it will have to be written separately. Ok? But we are assuming that it is the same but it will work the same in non-uniform also. There will be no problem. The only thing is that the h is different, the values of both of them, I will keep writing them instead of h like this x1 - x0 x2 - x1. So, we have calculated this. Now let me simplify this a little. So if you see in simplify, I will take it a little bit as h/2 common. ok i took h/2 common. took h and took y2. So if you look here, you will get y0. Plus now y1 will be found from here and y1 will be found from here. Ok? So from here I can write 2 times of y1 similarly y2 will become y2 + y2 and in the end we will have yn - 1 which will become + yn. Ok? So this formula of ours, we call it trapezoid rule composite formula, so we can calculate it easily. Ok? So this is basically the Trapezoid rule. Now the error which we had found in one will also get added to it. Is it okay ? This is the error. So what does this mean ? Now what will be the total error? The total error which is the truncation error, basically, so what will it be in our case, now we have n intervals, all the sub intervals will come in n intervals, so firstly the total error is ex, I am writing what will be the total - h f'e / 12 - h, like this we will keep adding and take its maximum. Ok? So in the last also h^3, now this size, I can take it like this 1 2 n. And it depends on where she is bringing this from. Ok? So now I can combine all this and write it like this. - n h /12 will now be between x0 and xn. Ok? So we combined whatever we had and took the maximum error. So these values that we have got, now I want to write them like this and we also know that nh is b - a. Ok?



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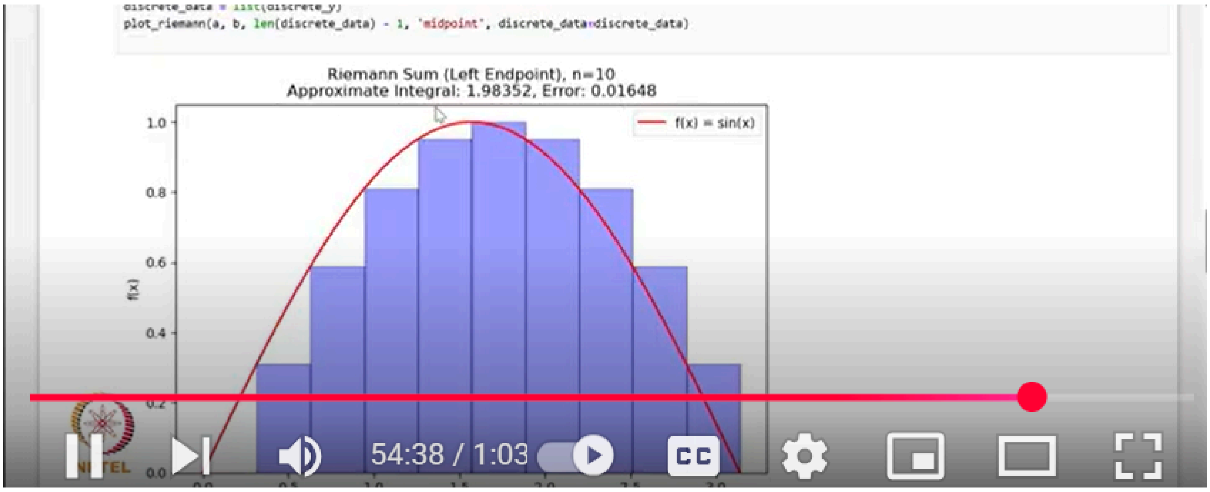
So the error that we have created here is our total truncation error, I can also write it like this - b - a which is nh h^2 by 12, so it will become this. So this error is in the form of h and this b - a, whatever will be our interval, we can take it like this and we will get that error. So this error will be basically our total truncation error over the entire interval. If we integrate that function using the trapeze order rule. Ok? So now suppose I want to do the trapezoidal rule using a calculator, I want to do it using code, I want to write Python code for this, so first of all we saw that we tried to calculate it using Riemann sum. Ok? So first we have taken two functions for remand sum. One is a continuous function and one is discrete data. So what are you doing in Continuous? We get sin x, so I'm trying to integrate sin x. So now I am taking methods which are different. Left method, right method, mid point method we have discussed all three. From there I defined a function which is remand sum and a which is the left interval

from a to b is the value of a. This is b. How many number of intervals do we have, we need them. We have to take them. And the method has to be defined and that's it, okay? Not for discrete data. So that's just what I'm doing right now. Ok? So we took the function. After that, we created Lean Space. Ok? Basically, we have to solve that function discretely only. But the function given to us is sin x, so we will keep putting the value of x in it. And the discrete values that we have in our data will keep coming out. And dx is what our h is, how much is it? b - a by the length - 1 so these n numbers of sub intervals we will have. If the method is left then we will take this value of x points. Is it okay? The x points are our nodal values. All nodal values between a and b. So the x not is a and the x n is b. So we took this left value. Ok? On that we found the y values for the given data. This way I took the right value. On that, we extracted the data points. Took the mid value. We extracted data points from that. Ok? So now it depends on which formula we apply to get our sum. So we knew the sum of the y values we had was * h. So I wrote the sum here. And we returned the same values. Ok? We have returned all the dx y values of the rectangular shape. Ok? So this x that we have is the link space. dx This is ours. We have defined these methods. Left hand point. If this is it then left hand right if this is it and if this is the mid point then this is it. Ok? So we have got the sum value. We have calculated the exact value. So we know that the integration of sin x is cos x. - cos x So, we have to calculate the values of - cos x on that x. So look at this - np cos + np cos a, so we have calculated these values. If we do direct integration from a to b sin x dx, then the solution that will come out will be the exact value and we have calculated the error, the absolute error. So now we will plot it. Here, we will plot the data that we have and make a figure. What is there in the figure, because this is rectangular data, we want the rectangles to be visible. Ok? So, what do we need to do about it? The visualization should be good. So, we found all this out for the purpose of visualization. Ok? So, now we have given the values. We divided it into 1000 points. y = fx. Ok? I wrote this figure sin x. sin is for x. Ok? And we plotted the Riemann Sam. Now look, I have given the parameters. The value of a 0 b π this is our interval. Number of rectangles: We are assuming that we will divide it by 10. Plot the Different Methods. So we did the plotting. Ok? So we called for that. Function Plot Riemann All these values, we wanted to see whether it works for discrete or not. So we will calculate its values next.

[49:56]

So you see, I found the left side sum of n = 10 and the integral we got of sin x between 0 to π is 1.98 and the error is 0.016, so it's a very big error. Is it okay? Because the function is a very easy function. It is a simple function. So we calculated this by using the end point that we have on the left. Now we did the same with the right hand point. Ok? In the right hand side points we take the next value and remand sum. So its value is also the same. 1.98352 is okay? And this is the error. But what are we doing now? Taking the mid point values. So look at the mid point values, the value we have has changed. 2.00 has arrived and the errors have been reduced significantly. Is it okay? So it is giving the exact value till the two digits. So this is exact, we have got exact accuracy till the second digit. So, we told you that we always benefit from the mid point. So we did this from the mid point. Ok? And if we had discrete values instead of sin x, we would have this data. It is the same function but without giving its values, we have calculated the discrete values. Ok?



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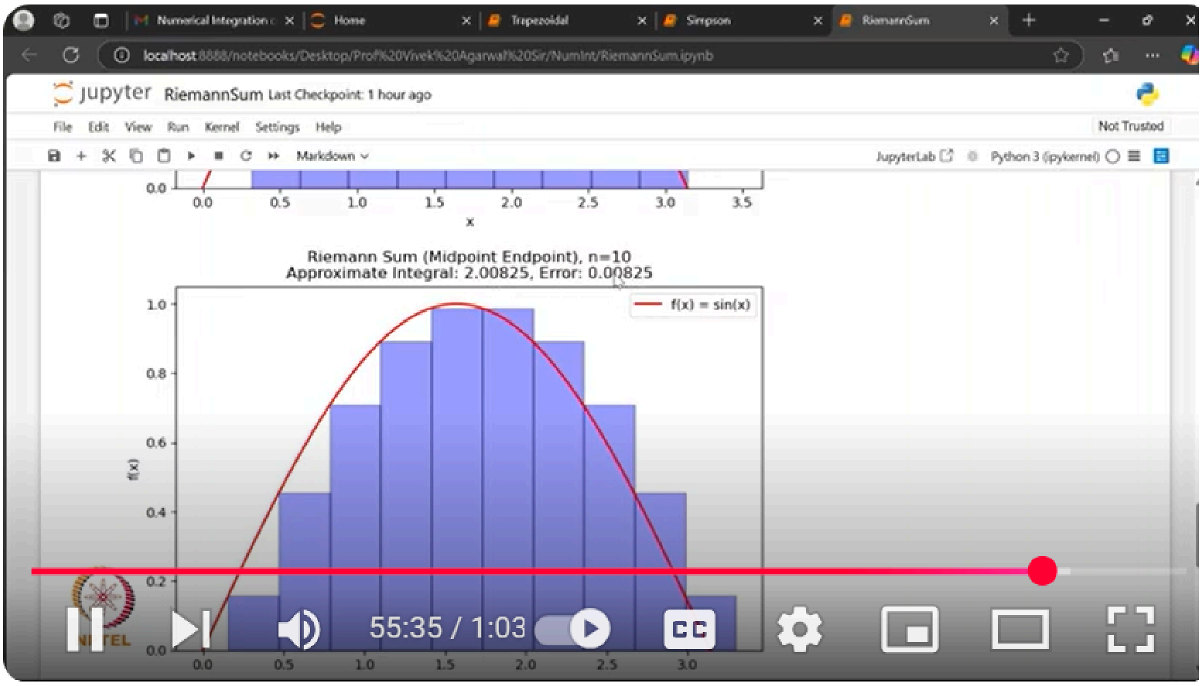
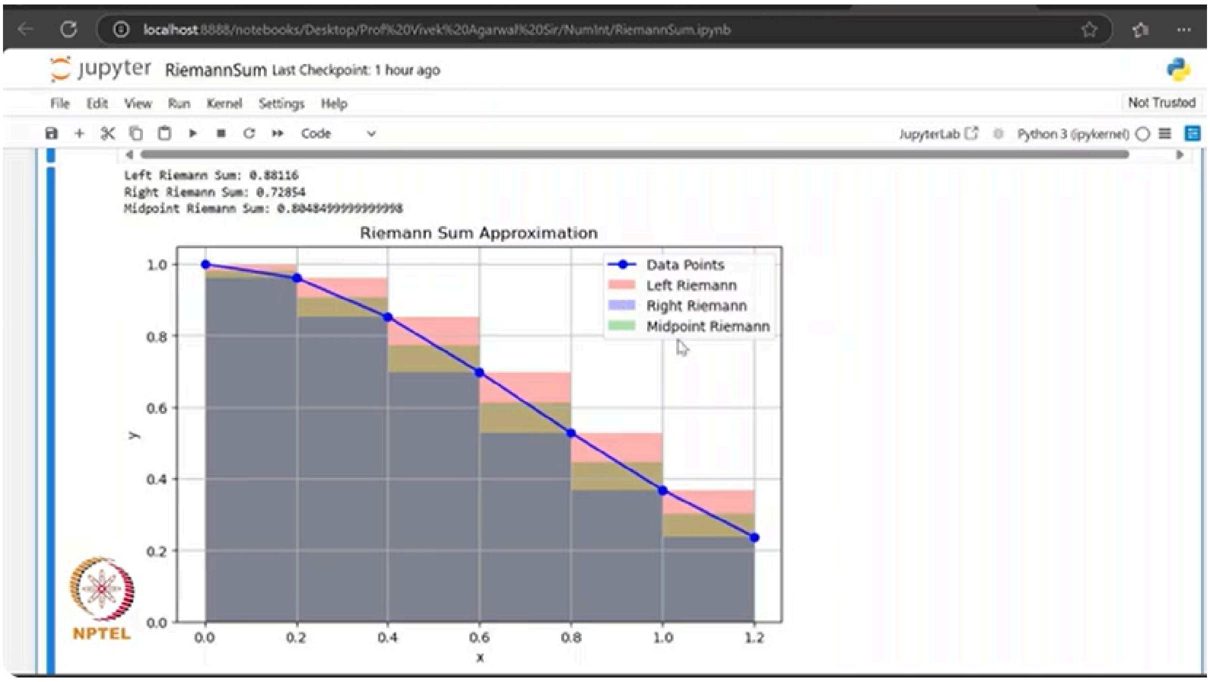
So above, we saw these values, those are discrete values, we had defined discrete x, right? So now 15 after that we took the discrete y. After that the discrete data came to us and we called the function. So if we did not have the function and only its values were given then we would have this. So this discrete remand sum came from the mid point and its value was 1.9690. See, for exact, two is coming but for this, 1.9690 is coming. So we did it. So what did we have in it? The function was given to us. Once a function is given, we can always calculate it in discrete data. But what happens now is that the data we have is discrete. Only we don't know what the function is. Ok? So what do we do in this? That riemann sum has to be defined. Is it left, is it right , is it mid point, what is it? But in this case, what we have to do now is that since the length is -1, which is n number of sub intervals will come because we will have n + 1 points, so n sub intervals will come. So that is why the length, this ln, will give the length - if we make it 1, it will become n and if it has to be defined initially, then the value of remand sum is zero. Now more will be added to it. So what do we have? Riemann has been defined like this. Ok? So left some right some depending upon what we are taking ? Taking mid value. So we have defined mid. Now what did we do? It is done for discreet. So suppose I now have this data value 0.2.4.6.81.2 and on this the values of y are given to me 1 0.9608 0.8521 so we have discrete values given. Now we have to see that we have to find the integration using the function and its values. So now we are not given the function. His values are just given. We don't know what that function is. So we calculated this. We have calculated the left sum, right sum and mid point values. Is it okay? Print left remand sum right remand sum And what did we do for visualization? This is what we plotted using Mat Plot. Plot dot bar means bar will give us. Ok? We calculated the mid values and ran it. So you see, in this case the left remand sum is 0.8, the right remand sum is 0.72 and the mid point remand sum is 0.8048 and we have calculated this. So this is ours. So these were our data points. Ok? We have found the left remand sum. I have used the right remand sum and it has been done from the mid point. So the one with the mid point will be in the middle. This is the one on the left. The right one is in grey colour. The one below will be the right one. Ok ?



[55:35]

The one on the left is the left one. So by using this we approximated the Riemann sum. So now, see, the values that we have got are 0.88116 0.72

so at the mid point we took 0.8048. So if we see, as per what we just discussed, the values that we have taken are basically the values of e to the power - x^2 between 0 to 1.2. So its integration was not possible but when we calculated it, we came to know that its values are approximately this much. So we have calculated this. Similarly I can take other data. I have got this data. Ok? So it took 0 1 2 3 4 5 6 7. 2 3 5 7 11 13 17 19 We have calculated this. After that I ran it. Ok? So, we conducted this function like this. So the left sum is 58. Right sum is 75. The mid point is 66. So we already said that the left sum gives the lower bound of these areas. Right sum gives the upper bound and the mid value will be somewhere in between. So when we calculated this, our function was an increasing function. In the previous one, the function was decreasing because it was a Gaussian function. If we notice, that is a Gaussian function e to the power - x^2 but this is our increasing function. So in this we will see, we have calculated the left sum, right sum and mid point and from here we got our riemann sum. So just keep giving us any data points like this. We will keep inputting it from here or if it is in any Excel file then we can input it from there also. So from there, we did the input. Ok? We will calculate it using the formula and show it to you that what we have is the Riemann sum. So now we will calculate for different types of data and solve it. Ok?



So this is our calculation, this is for the remand sum which we did in the beginning. Which we saw rectangular because Riemann Sum is also giving the values for rectangular.

[1:01:21]

So we have calculated the remand amount. Ok? So in this we have discussed the left values, right values and mid point values and we have shown that if the function we have is given or its data values, which are the nodal values, are given.

So how can we find out the integration. So I hope you understood this lecture and thank you for watching it. Hello