SCIENTIFIC COMPUTING USING PYTHON

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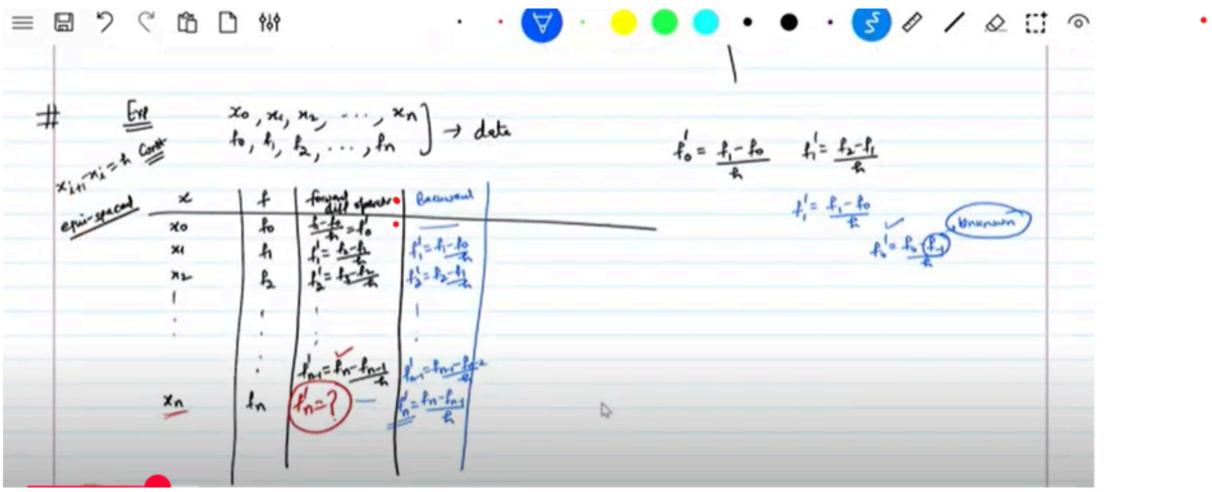
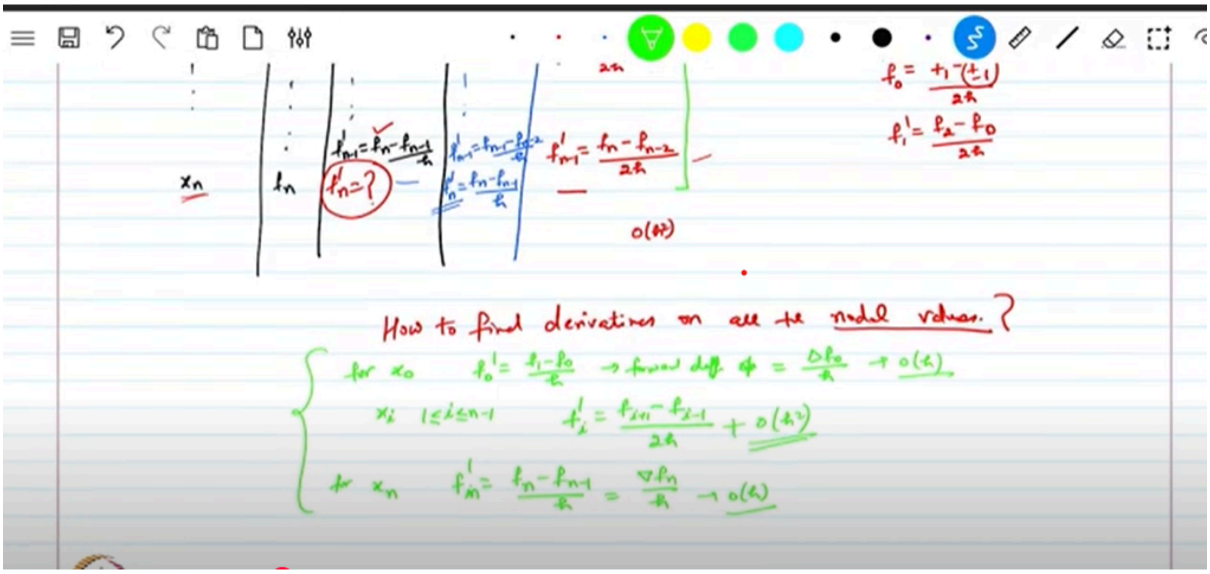
Lecture No. 29

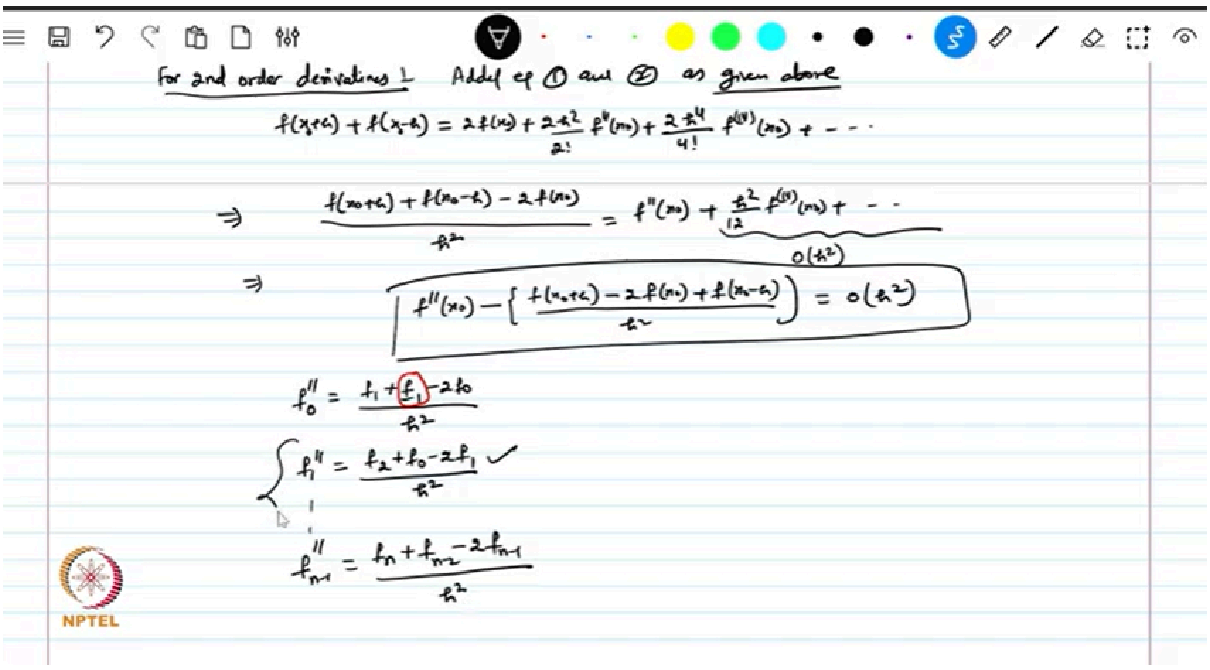
Welcome, everybody, to Scientific Computing Using Python. So when we started numerical differentiation, in the previous lecture we saw how we can define the derivative of the function at a point with the help of forward difference, backward difference, central difference operators. So today let us start from there. So let's get started. So we have seen in the previous lecture that if we go forward or backward then we have defined differentiation at a point, so it can be calculated like this and the order of accuracy will be of h. Similarly, we saw in the backward class. We have seen in the central that if we apply the central operator, apply the difference operator, then in that case, if we take the approximation, then it will be the order of h^2. Is it okay? So with its help we can calculate it. So today let us take an example of this, if we have data given to us then how do we find it out. Ok? So we have first data and we have equally spaced data also. So we have data. x0, x1, x2, ……x_(n + 1) data points. And on this, the values of our function f1, f2 are also given. So this is our data. And we have been told that we have to find its derivative. So, what do we do to find the derivative of this? Now, like we have this, this is xi. So x0, x1, x2,… xn is given to us. And this is what we have fi given. f0 f1 f2 fn. All these are given in the script. And we've assumed that x_(i + 1) - xi is a constant (h) . Ok? So what does constant mean ? That our xi is equispaced. They have uniform spacing. So now we're told to find the derivative of this. Ok? So now we have to see which operator we need to use to find the derivative. So we suppose we are finding the derivative of this. So I'm going from what is forward. I am using the forward difference operator. Now see what we have to do to use it. Now I have to find out the value of f at 0. So what happens in the forward ? This (f1 - f0) / h is correct , right? Because h is a constant, it will be included in it. If we want to see f1, then we have to go one step back and h, so we saw that if we put the value on f0 here, then it will come to (f1 - f0) / h, this will give us our f'0 approximate value of our derivative. Ok? Similarly, the value of our f1 will become (f2 - f1) / h or (f2 - f1) / h because we have only the values of the function fi. f2 so f2' will come to (f3 - f2) / h we will go on like this. In the last if you look, we have f' at n - 1. If we want to subtract it, it will come out to be (fn - fn – 1) / h, okay? Now we have used the last values we have here. See, I have used it here. Now if we were to find the derivative here. If we want to find f-n then we cannot find it in this case. Ok? So, if we have derivatives , we have to take them in such a way that we need all the derivatives on the left side and the last point of the point on the right side is the boundary point, even if we do not want the derivative on that point, it will be fine. So we will have to put a forward in it. Now, I will try this same work from backwards. So what is happening backward is that if we want to look backward, if we want to find out f1 d then what will that be? (f1 - f0) / h will move back one step. so what does that mean ? If I want to find f 0 I have to go (f0 - f – 1) / h. But these values are unknown. It is not known. Ok? So there is unknown value. So if this is an unknown value, it means we cannot find f- which is zero backward. But I can remove f1 here. So what will be f1? (f1 - f0) / h f2 What will this be? (f2 - f1) / h So if we continue like this then what will be f_(-n – 1)? This will be f_(n – 1) – f_(n – 2) / h. So in this case you will see that we can also find the last value. So this is f(- n), this will be fn - fn - 1 / h, so we have the derivatives, now look we have the derivative of fn also. But in this case, we will not get any derivative as it did not come in this. So what do we do if we don't need the derivatives on the left boundaries? I can apply this backwards. And the order of accuracy is the same in both. This is also the Order of H.



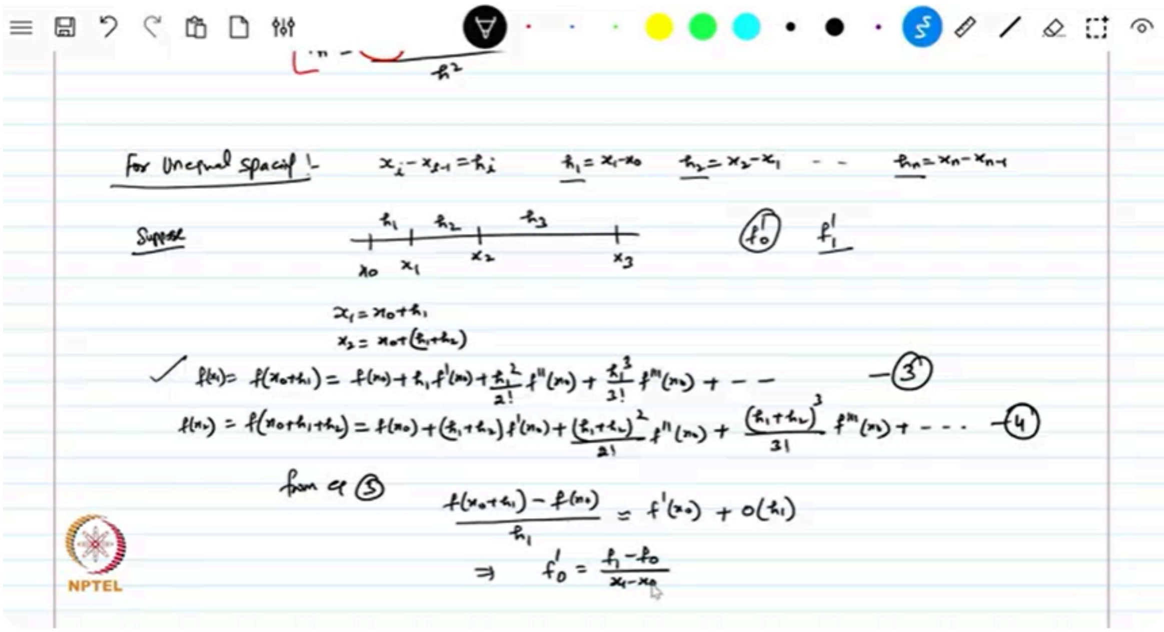
This is also the Order of H. Is it okay? So this differential operator which came forward to us, we took the derivative from it. taken out from the backward differential operator. Ok? So these are the values that came to us. Now we did the same work from Central. Now look what is here? This was central. What have we done so far? Applied the forward differential operator. Applied the backward differential operator. Now we'll do this here from central. So let me do this from central here. Ok? So what is the central differential operator doing ? Now look what is happening in central, if I want to remove f0 then what do I have to do? You will have to go one step further. Minus we'll have to go back one step and divide by 2x. Ok? So if we have to find the derivative from the central then you see, we have this unknown. We do not know this value. But if I have to find f1, then it will become f2 - f0 / 2h, which is the difference of two. So what that means is I can extract f here which is one. That will become f2, meaning this value - f0, this value divided by 2h will come out. Just like that I can find the derivative of f 2. This will become f3 - f1 / 2h and we will go on like this. So if we look at the last one, I can find out f_(n – 1) from here. What will happen to this? fn one step forward – f_(n – 2) one step back / 2h So see what we have in this case is that we cannot remove the fn. f was zero, we can't take that out. We just approximated the derivative on our interior data points, which were apart from the boundary points. And this derivative which we have approximated, we know that its order of approximation is the order of h^2. So this will help us in approximating the derivative better as compared to the forward differentiation operator and backward differentiation operator. But the problem here is that we are not able to calculate on the initial and final values. Ok? so what do we do now? So what do we generally do? How to find the derivative on all the nodal values. How do we extract all the nodal values ? So that our order should be more. So for that actually if we go in advance then we take ghost points but we are not doing that here. So now what I will do is that I will put this central on all the internal points. Ok? But I put together the data points that we had. for x0 which would be over here. So at x0 we know that only forward can be applied. So what did I do? I found out f0 and did it with f1 - f0 / h, which means we put it forward. Forward differential operator was installed. We can write it like this also. f0 / h ok? So, it came forward to us. Then I took xi and i from 1 to n. Ok? I can take the equals from one to n - 1. So whatever we calculated here, I will calculate it with this. So what will it be? which is central. Ok? So what will we do in Central? f which is i + 1 - fi - 1/2h right? So we will calculate from here. So we got this from Central. Now in the last for xi , xn the last boundary point for xn, what will we do for that, I will apply backward so xn because we know the backward value only, so I will do that, f at n, so this is fn - 1 by h, okay, so this which we have, has become backward, so in this way we can approximate all the derivatives on all the nodal points and also see the order of accuracy in it, its order of accuracy is order of h^2. Its order of accuracy is h and its order of accuracy is also h. So here mostly the points will be in the order of h and the boundary points will remain in the order of h. Ok? So in this way we can always find out all its differentials and derivatives.



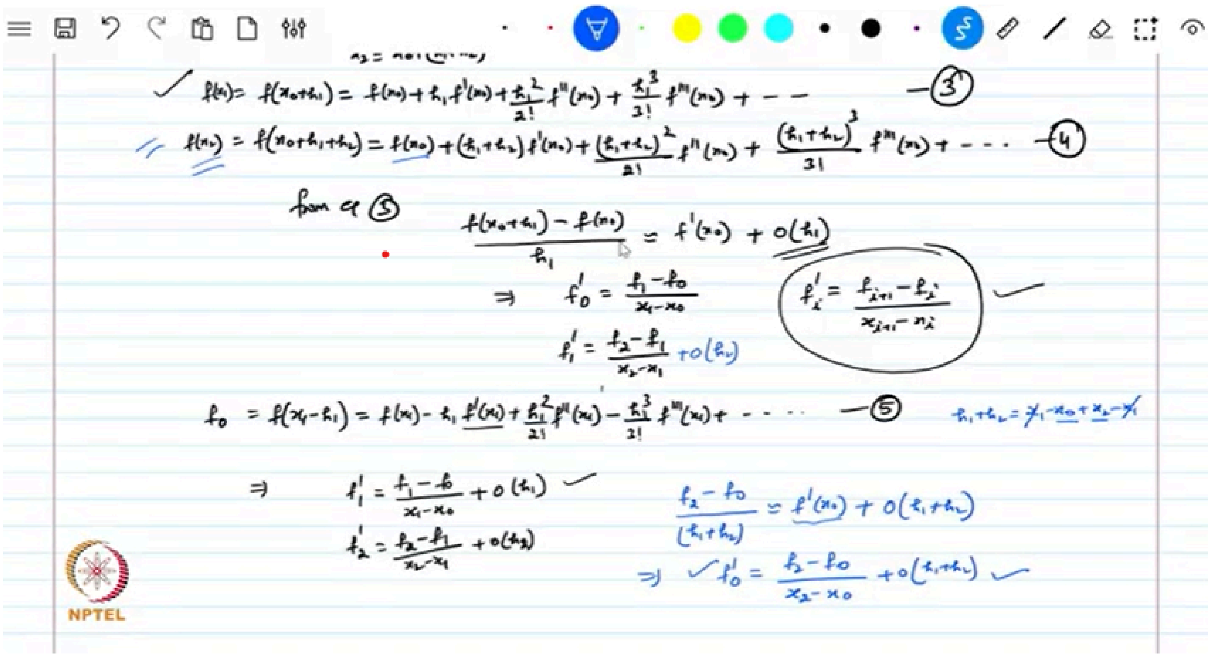
So just like we found this first derivative. Similarly, we can also find out the second derivative. Ok? So if we want to find out the second derivative, then we did this process. Ok? Ok? We had processed this. So suppose we are told to now find the second derivative from equation one and two. Ok? So if I want to find the second derivative of the one and two equation, what do I do? I will add both of them. Ok? So look what we are getting while adding. So I'll write this as a for the second order derivative. Because if we are given a position, then let us find the velocity from the first derivative. But if we want to find the acceleration also, then we will have to take the second derivative also. So what are we doing ? Adding equation one and two as given above. We looked up at what had just been given to us. Ok? So we will add it to us. So look, f (x + h) + f(x – h) this becomes the left side. What is on the right side ? 2 of f x okay? So, did I buy it at x not or did I buy it at x. It doesn't matter, it is x, not so let's take it as x not. So if you see this and this, then this and this will get cancelled. This and this will be cancelled. Only the even power of h will be left with us after adding both of them. Ok? So I did it like this. So if you see, we will write this x0. That becomes all the values plus $2h^2 / 2$ f' at x0 + 2h^4 / 4 f derivative at x0 and so on. It has come to us. Now I can write it like this. Ok? So this two to two will be cancelled. So here I am writing. See, f(x0) + h + f(x0)- h - 2 f(x0) divided by h because h^2 will now be common to all. I can write it like this. Now look at f' x0 plus h I took it common. So what we are left with here is h to the power squared / 4 so 4 is 24 so we get 12. fx0 plus this is how it comes to higher powers, okay, then h will become square, h will become like this, the values will keep increasing, so in this case, if we see, from here we can write that the second derivative will give an approximation of what order, this will give an approximation of the order of h^2. So if someone asks us, then we can say f-x0 minus its values which we approximated, the approximate values which we got, see that if we look here, then let me write it like this. x0 + h - 2x0 + f(x0) - h / h^2 This is the order of x^2. Right, once the order of x^2 comes, it will give us second-order accuracy. So if we have the data, then if we look at it, now I want to see that f double derivative at zero, if we take it, then we see what will come here, if we approach it, then it will come as f1 + f(-1) because -1 will go to -2f / h^2. So now these values are unknown to us. So what does it mean if there are unknown values ? That we will not be able to approximate f to zero or we find out its value by different method. Is it okay? So now we are not doing that here So this value that we have will come as unknown. But if I take the derivative of f1 and approximate it, what will I get ? f2 + f0 - f1 / h^2 can be calculated because all the values are known. So if we look at it like this, what will be fn - 1? What would that be? fn will be + f(n – 2) will be - 2f(n – 1) and by will be h. So this can also be calculated. So we have calculated from f1 to n - 1. Now if someone asks us to tell fn then what will be the fn? F(n + 1) + f (n – 1) - 2 times fn / h^2 This becomes f(n + 1) + f(n – 1) - 2 times fn / h^2. Is it okay? So this is the central difference.



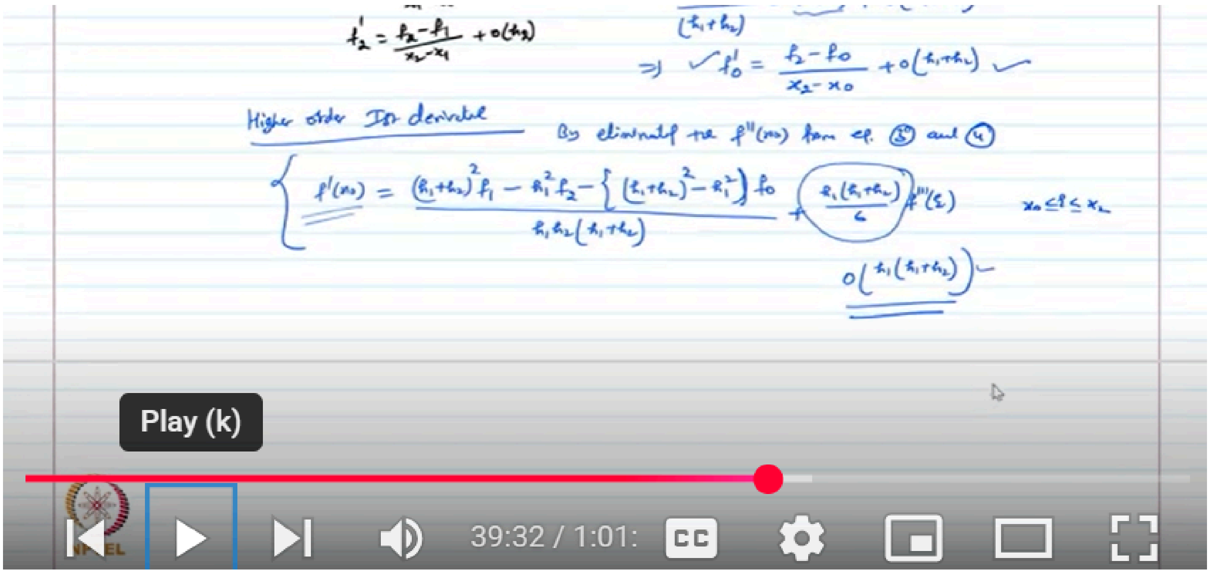
So in this case also the values we have are unknown. This is unknown. So we will not be able to calculate this also. Ok? So this one and this one, we will not be able to calculate. But the central which is the second order derivative for the central points, we can approximate it easily with the given data points. Ok? Ok? And the accuracy will also be good accuracy. The order of h^2 will arrive. So here what we have to do is on first pay and last pay, so here again we apply different methods. We can calculate what we'll do next using what we call the method of undetermined coefficients. Is it okay ? So, we will give him this work. So if you see, whatever things we did, we did it in such a way that the data points we had were in equispace. And what happens in equi space is that if we keep dividing by h or keep dividing by h^2 then we don't face any problem. But what happens in real life is that the data is not necessarily equally spaced. So if we have uneven space then how can we apply finite difference? So the For uncall spacing is okay? i and g spacing for uneven spacing. What does uneven spacing mean, what we have is xi - xi - 1, I will write it as hi. and hi so what does it mean? what will be h1? What is x1 - x0 h2? x2 - x1 will continue like this. So hn will become xn – x(n – 1). And these hi's are all different. Can also be equal. But it means that we are assuming that everything is different, there is different spacing. So in this case we have to see how to do this. Ok ? So let us try this, how to find this one. So we take the suppose or suppose we will then extend it. No problem. Suppose I take the points that we have here. This is some point we have x2. This is what we have x1 right? This is my x0. Here x is 3. Such are the points. So this value which will come is h1. This is h2. This is h3. All such values will come. Ok? So now we have to see how to calculate f-0 or how to calculate f1? So this is how we have to find it out. Now look, what is x1 in this case? x0 + h1 x1 arrives. what is x2? x0 + h1 + h2 Now we cannot take 2h. Now we will have to do h1 + h2 directly. Ok? So now let us see how this is calculated. So we'll do it the same way we were doing it. I wrote this x not + h1, I expanded it from Taylor. Basically, this is f x1. Ok. So, we have to calculate this. So, what will come of it? fx0 + h1 f-x0 + $h1^2$ / 2 f'x0 + h1 / 3 f and so on. Now we tried to calculate f x0 + h1 + h2. Ok? If we look at these values, this is our f at x2. So if we calculate this, it comes to fx0 + now here it will come to h1 + h2 f-x0 because I am using f'x0. This will come to h1 + h2 squared by 2 f'x0 + h1 + h2 to the power of 3 and this x0 goes on like this. Ok? So what do we do now? Have to take the first derivative. So first derivative if suppose I need it in this case from here if we need the first derivative. Ok? So now let's see what we are going to do from here. fx1 fx0 So if I look from here, let me name this one. Now let me give it a new name 3 and 4, is that okay? So from three and four, we can see that from equation three, what do I do? If I move this to the left side it will become f(x0) + h1 – f(x0) okay? We took this. And if we divide it by h1, then this will be our value, that is the approximate value, this one plus the order of accuracy will be h1 in this case. Is it okay? So this will become the order of h1. So look in this case, this is what it means, what we have seen from here, what has this become? f' at 0 what would that be? f1 - what was f0 / h1? is x1 - x0 okay? So it will become this. So like this, if I calculate this, I calculate f1. Ok? So here, instead of fx1, I should write x1 instead of x0. So what will it become? This becomes f2 - f1 by x2 - x1 so this value will come out. Ok ?



So if we see its meaning, if we take zero then one is coming and zero is coming. So this is basically going forward. We have done this as per the forward. Ok? So in the same way, if I take the derivative of any rectangle value, it will come out as f(i + 1) - fi / xi + 1 - xi. So, this is our derivative approximation. So accordingly we approximated it. Now if we want to do the same thing in that, now you will see that the values which we have calculated, these are the values which were taken. Now suppose we want to go backward, what will I have to do to go backward ? If I look at x2 I have to minus h2 to go backward. Then x1 will come. Is it okay? Or if I look at x1, I have to take h1 and then I get x0 back. Right? So for this work, if we have to do this then we can do it like this to go backward. So all of these have been forwarded by us. So we will take out all the forward ones which we have and it is obvious that we will not be able to take out the last one because we know that we will have to go one step ahead of the last one. We are not able to go there. Ok? So the last one, just like we did it in un space and equal space, it will happen here in the same way. Now we have to go backward, so what will we have to do in backward ? I suppose I'm writing f x1 - h1 ok? So this is x1. This point was x1 - h1 so x0 comes here. So this is our basically f0. So what will happen if we apply it on this ? f x1 - h1 f-x1 + $h1^2$ / 2 f'x1 okay? - h1 / 3 third derivative of at x1 will continue to have plus minus sign. So I give him fifth. Ok? So now in this, if you see that I have to calculate that f, so if we need its derivative, then from here you will see, if I write it here, I will write it directly, so this f- at one, this is correct, this will be equal to f1, this one - f0 divided by h1 and what was h1? x1 - x0 This was it. Ok? So this will come plus order of h1, we will take all those terms so this will come from here. So what happened? This is backwards. Ok? So we'll keep calculating on x1 like this. So if we know this then we will know what will be f- at x2 ? f2 - f1, x2 - x1 + order of h1 goes on like this. So according to this we will find it out by using the first order backward. Now we have done the same work, look, we have taken out fx2 here, so if I want to take out fx2, look, here if I have to take a higher order like I took it to the central, then what will happen is that now the forward and backward that have come to us, those orders of h1 have come. Ok? So it will keep changing like this. Ok? h1 will come in the values h1, h2, h3, h4. h1 came into this. h2 will be included in this because if you forward then changes will keep happening like this. Everything will change in this too. Now what we have is I'm going to use this one on the Fifth Fourth. Now let's see what happens. So see what is happening now in the fourth one. If you look carefully, I am writing f2 because it is equal to f2 - f0, this one divided by h1 + h2 and this will approximate our value and if you look carefully, here h1 + h2 will be common in all, so its order will be h1 + h2, like this, right? So, if we see its values, then this will come, so if we look here, then this is central. Why? Because we are calculating on the x node, we will get one point ahead, one point behind and the difference between the x coordinates when divided by them. Ok? Plus the order of this one. So if I want to write this, I can write it like this from here, what does f - 0 become? I can also write f2 - f0 x2 - x0 like this. Why? Because what will come from h1 + h2? x1 - x0 + x2 - x1 so this cancels out. So x2 - x0 will come out to be x2. And whatever its order is, this will happen. So its order is h1 + h2 order. Ok? So what did we do from here? Took out the centre. So in this way the values that we have will keep changing. Depending upon where we are? Here if you take h1 then it will be h1, if you take h2 then it will be h2. Now if we approximate it then the order of approximation here will be h2. Ok ? So what did we do? So did it backwards there. From here we did it via Central. So what was the previous order from Central ?

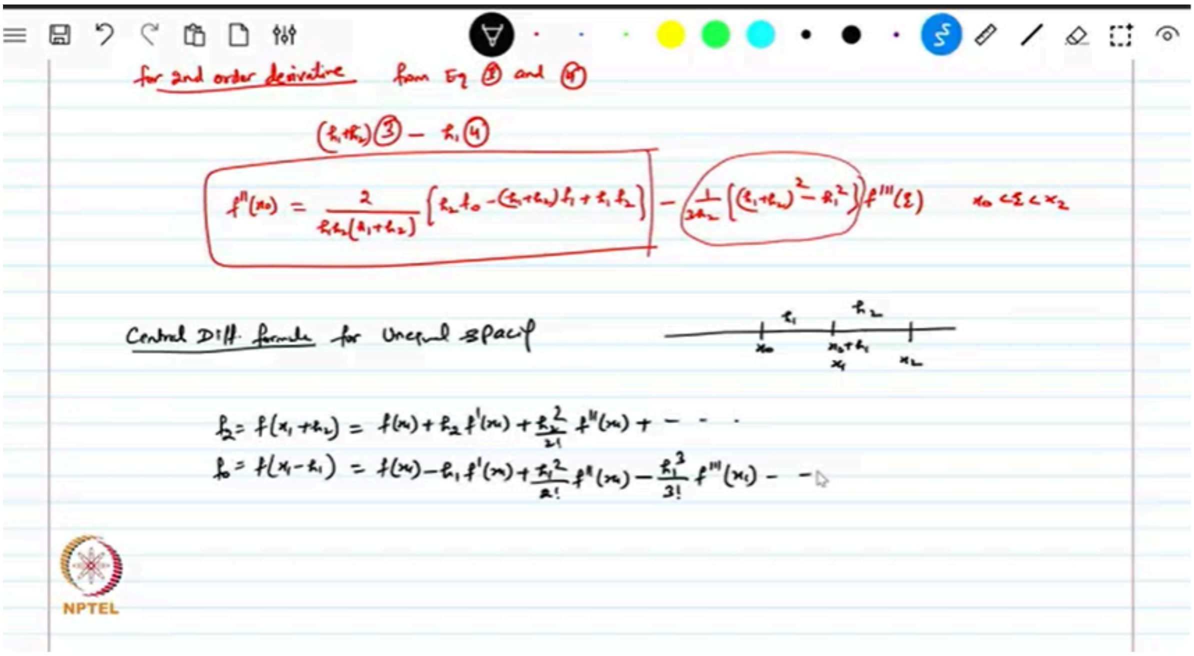


The Order of h was square. So what will happen in this case also? In this case, if we look at the values then this - this by this. So the values come from here. Ok? So the first derivative, as we did earlier, is ok, so what happened in that was this h^2, because when we added it, the sign got changed, we added it and after that we saw that all the terms, some terms were cut, some terms were correct, but this is not happening in this case. Why? Because the h's are coming out different. Ok? So in this case, the order that will come will be order h1 + h2 from here and we have taken these values like this. Ok? So if we have to do it from the centre then we will have to calculate accordingly. If the data points we have are not unique it means they are not equal spaces. So similarly, if we want to do this on higher order, then basically we can approximate the first derivative from higher order also. With the methods that we have, we can basically do any derivative of derivative from higher order. How? Like this is what we had, these values were 3 and four. Ok? So what if I get to three and four ? I want to eliminate this second derivative. Suppose I take this one and this one, what do I do? I eliminate the second derivative of x not from 3 and 4 so how is that possible? It is possible that I multiply three by the whole square of h1 + h2 and I multiply four by the square of h1 and subtract both. Well then. So if I do this, subtraction, then I can write here hi and order first derivative higher order approximation of first derivative this is what is happening. Ok? So in this case, if I apply higher order formula, if you do the higher order formula like this then it will become f-x0 and what has to be written is by eliminating the f'x0 from equation 3 and 4, so it has to be eliminated here. Ok ? So if I do this then we will get it. So, this will become h1 + h2 squared f1 for us because we had to multiply it by it. Minus h1 square f2 minus the whole square of h1 + h2 - h1 square which becomes f0 okay divided by h1h h2 h1 + h2 plus this will become h1 h1 + h2 / 6 and this will come to the third degree and I can take Z from X0 because in this it is going to X2 so I have taken X2. So in this case we will see that our higher order derivative has become this and its order of derivative approximation has become this. So it is of order, I can say this is h1 h1 + h2 of this order has come. Ok? So if I make h1 and h2 the same, here 2h1 2h and h then the order of h^2 will come. Ok? So look, we raised the first derivative to the order of h^2. Like we did in that uniform. But that thing was in uniform. If we take non-uniform then we have seen that the order of the derivatives has changed. Even in central it came to h1 + h2. Is it okay? So like this we can calculate it and we can also take the first derivative of higher order. It simply means that the more derivatives we take, the more we will observe, these values will keep getting used. Ok?

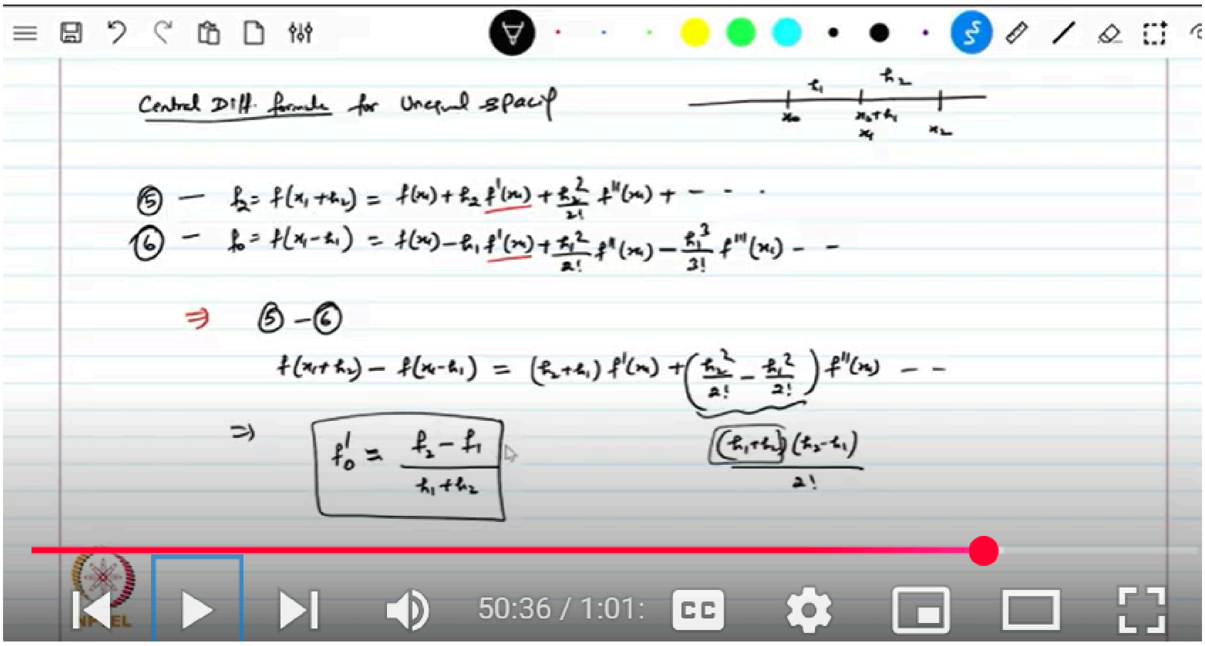


So if we take the first derivative of the second order of the higher order, then three values of the data will be used in it. F0, f1, f2. Ok? So what this means is that the forward values that are there in it are being used. f0 then f1 then f2 ok? So this is going forward. So we can calculate it like this. Now suppose I took this first derivative. Now, with its help, same with its help, with the help of three and four, I want to find the second derivative. So how do we do the second derivative? What will we do for this? He will eliminate f-x0. Ok? So if we want to eliminate f-x0, we will do this and this in the same manner. We will multiply this by h1 + h2. We will multiply this by h1. Were doing it from square one there. Now we won't do this to him. Ok? And what do we do from there? I will subtract it. So our second derivative will come out. So what am I doing now in this case? Similarly for second order derivative second order derivative look what to do from equation 3 and 4 which we defined above write this what do I do now? This has to do with the second derivative. Meaning, now we need f' so I have to remove f'. Ok? So what should I do with it? I will do three from h1 to multi h1 + h2.

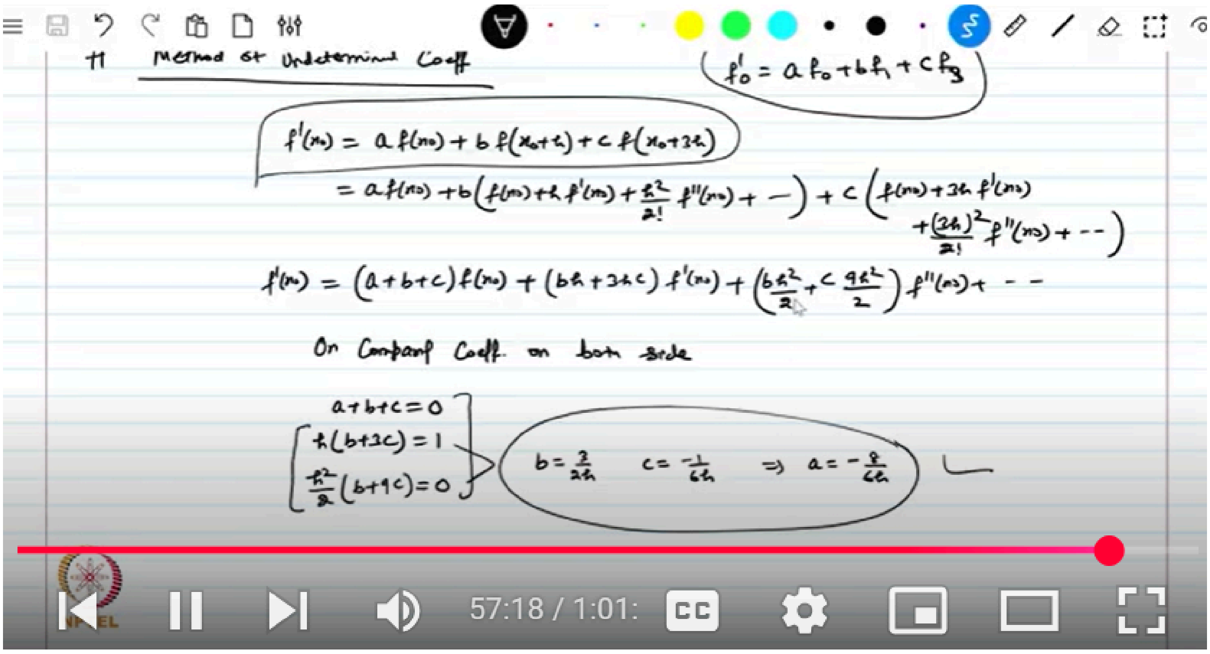
So here I am writing from equation 3 and 4 so what to do? Equation Three: Multiply it by h1 + h2 and then divide H1 by 4 and subtract it. So what happens now? The second derivative will come to us. On what? x not pay. So what will happen if you see it? This is multiplied by h1 + h2. This happened from h1. Ok? We did it like this on both sides. Then if we subtract from here, this h which is f1 d means the first derivative, it will be removed. We will be left with only the second derivative. So if I do it like this, then if we do a little calculation then we will get this value. So this comes 2 by h1 h2 h1 + h2 and this will come to h2 f(0) - h1 + h2 f1 + h1f2 this is what we have. Ok? So this derivative which has come and minus one more term will come that is 3 h2 h1 + h2 squared - whole square of h1 this will be left and we can take the third derivative and z that is it, I can take it between x0 to x2 so you will see that our second derivative has come. So from here I can say that this is what f ” . So this is approximately equal to this. And this is our order of error, what will happen, what order will the method be of ? So the order in this case will become this one. Ok? So if you see in this, if there is uniform spacing in h then it is very easy to do everything in it. But as soon as we have non-uniformity in the data then there is some problem in taking the derivative. Meaning, our calculations increase a little more. Our computer work will also increase considerably. But in this way we can approximate it. Ok? So this got forwarded. Like this we will have backwardness. Like this we will have central. Ok? So with its help we can work on any data. So this is what we have - backward, central, forward - we have done this. Ok? Now we have already done it in backward. So in the same way I can go to Central. So if you see in Central, then I will tell you a little bit about how to do it in Central. The difference formula is okay, for uneven spacing, there is nothing in it, the values that we have are given like this, this x is 0, suppose, okay? This is x0 h1 + h1. This is h2. So here's my x1. Here it is x2. So this is how we have to find out. So what will happen at Central? We can't withdraw it at 0. Ok? We will have to take it out to the forest. So from here I'm going to write f of x0 which is or instead of x0 I'm going to go here f at x1 + h2 and f at x1 - h1 okay? So this one of ours is f2 and this is f0. So if we calculate from here then it will come out that fx1 + h2 f-x1 + h2 square / 2 f'x1, we will calculate it like this. Now this is our fx1 - h1 f'x1 + $h1^2$ / 2 f'x1 then minus will come h1 / 3 x1 and go on like this.



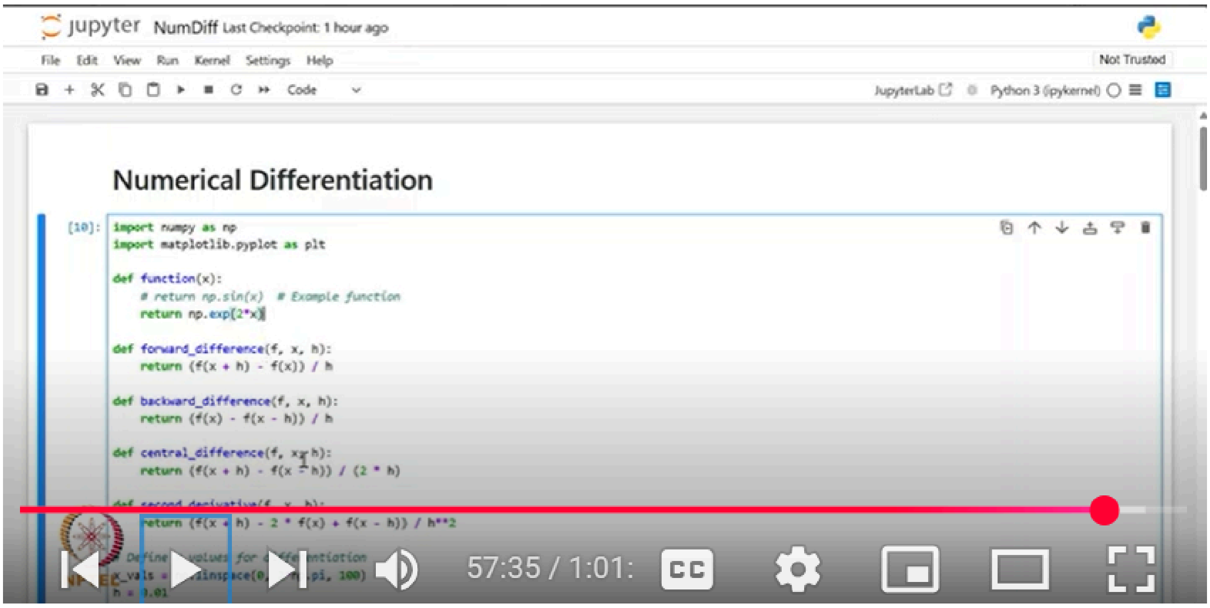
Now you will see that if I want to take the derivative of fx1 out of these two, then we have to calculate this one and this one, so how do we calculate it? Why? Because now here it is being multiplied by h2. Here it is being multiplied by h1. So if we eliminate this or if I want to see the first derivative of the first order, then now we have to see how to calculate this. Ok? So what do I do now? I will subtract from this. So let's look at the first derivative. Come on, here it comes. So, I will give it some name. 3 4 5 6 So what did I do? 5 - 6 were subtracted. So here we have f x1 + h2 - f x1 - h1 okay? Equal to, now both of these are cancelled. So this will come to h2 + h1 f' x1 this is it. Ok? + h2 square / 2 - h1 square / 2 f'x1 will do this for us. So now if we calculate from here then I will take h1 + h2 as common because here h1 is $h2^2$ - $h1^2$, so we can factorize it differently. So h2 + h1 and h2 - h1 will also come. So if we look from here, we will get this. So whatever they calculate, I will write it directly. So if we see, the derivative that we have f' at 0 will come to fx1 + h2, so f2 was - f1 / h1 + h2, right? So it will come from here, so if we see, this quantity will become h1 + h2 and h2 - h1 / 2, so h1 + h2 will come common from here. Remaining it will be saved. Ok? So this approximation will be of this type. - If we take h1 common then it is of order will become h2 - h1. But it is very difficult to get that common one. So whatever approximation we get, most likely this will be included in it. And we know that if we take the center, the order of x^2 would be there. But here we will have to do further calculation for this thing. Ok? So for uneven spacing, we'll have to do a little bit more work.



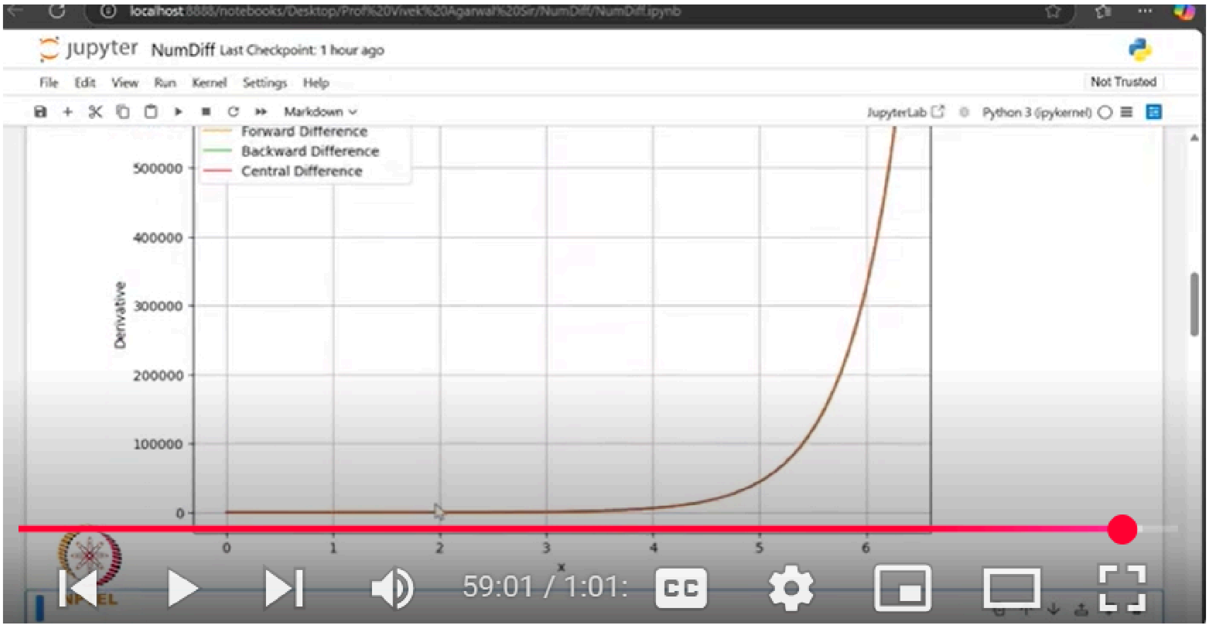
One thing we have is that Method of Indeterminate Coefficients is also one. So that is very important. So, let me take an example of what we have. Method of Indeterminate Coefficients. Ok? Method of undeterminant coefficients means we have coefficients but we don't know them. We have to calculate that. So, let me take an example. Just we are told to find f(x)0. In whose terms? in term of x0. B F in the form of x0 + h + C F in the form of x0 + 3h. What does it mean? We have been told to find out whatever F- we want to find at 0. Do A f0 + BF1 + CF4 here. 2 one came, then two, then three sorry three, so this three came, okay so f3 now this is called method of undeterminable coefficient because we have to find the values a b c, so now see how we can do this, I can write it like this a f x0, it will remain like this + b now if I approximate it with the Taylor expansion, if I do it then it will come to fx0 + h f'x0 + h^2 / 2 f'x0, okay, it will happen like this, plus c fx0 + 3h f'x0 + $3h^2$ / 2 f'x0 and go on like this. Now let us isolate the coefficient of fx0. So this will come to a + b + c if you see then fx0 has come. Plus now we're doing h the coefficient of f' of f'. So if we do the quotient of f' then this will come b h bh will come from here plus 3 h c f'x0 this has come, okay so bh will come from here 3ch will come from here plus now the next f double d will come from here bh^2 / 2 has come from here and the next one will come here c and this will come $9h^2$ / 2 f'x0 and go on like this. Ok? So if we do it like this then our f-x0 will come. Ok? So what do we do with it now? Which will compare the coefficients on both sides. So from here if we compare the coefficients on both sides then we can see what value we are getting. So on comparing coefficients which are on both sides. So from here we get a + b + c which is 0. f' whatever has come has come h you can take the common. b + 3c This one is here. Ok? Because whoever h is, he is coming here. Next whatever comes will be ours, h^2 by 2, I will take the common. Okay, so this will come b + 9c here comes this zero. Ok? So from here we will do this calculation. Now if we take both of them then we will get this from here. So if I calculate this then you will see what is coming from both of these here ? B so the value of b which will come to us, I have calculated it, so the value that we have got is B which is coming to 3 / 2h and c which is coming to us is coming to -1/6H. Okay, so c got -1/6h and b got 3 /2h and from here we will take out a, so the a which comes to us will come to - 8/ 6h, this has come to us, so from here the values that we have got, we have taken out the value of a b c, so if we put the values of a and c, then this will be our approximation, so this method is called Method of Undeterminate Coefficient. Is it okay ? So now we have created a small program for this.



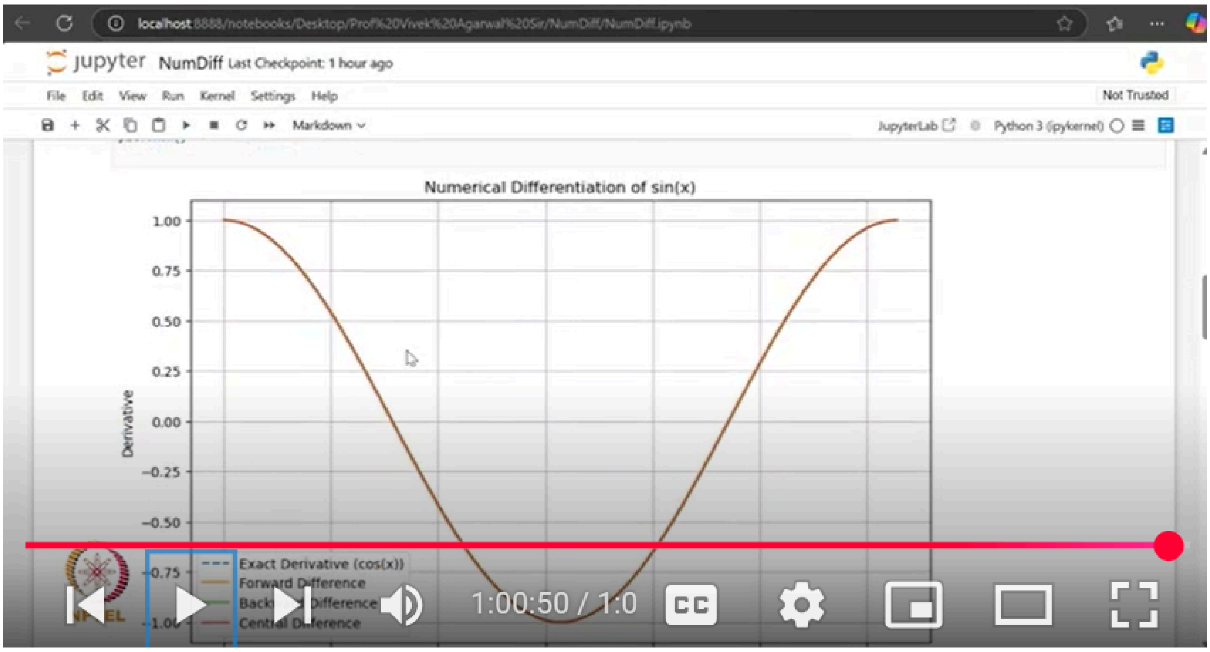
From that we can see how these codes run. So let us just look at this. Ah, this is our numerical differentiation code. Ok? So what did we do in this? We took some function. We approximated it by forward difference. Did it from the backward , did it from the centre. Ok? Looked up its second derivative. So we found the second derivative. So fx + h - 2fx + fx - h is the central part of the second derivative. This is the first derivative, this is the forward, this is the backward. Ok? So with its help we can calculate it. And suppose we took the function sin x or we took the exponential. Ok? We can take any function. So we calculated this. So here I took suppose numerical which is control C control V and I took this and here I took suppose exponential function. Ok? And I have taken the exponential function 2x, so I wrote 2x here.



Now I ran it and this is what we got, so numerical differentiation of the exponential 2x, so what did we do with the function? First we took any such function and saw its values, take 100 points between 0 to 2π, which is the uniform function, basically in this we calculated it, so see the approximation we got is this, so exact derivative cos x, so we do not have cos x in this, so we have 2 * e ^ x, so we will have to change all those places, we get the exact derivative, so here instead of exact derivative, I can write 2 * exponential x like this. Let's run it again. So the exponential came. Ok? e to the power 2 * e to the power 2x. So I will do x of 2, I will put it forward, I will put it backward, I will put it center. So we have calculated this. Ok? So similarly, we have another function, like earlier cos was written here. So what did we do? We did this with sin. Ok?



So let me define the function here, let's say, sin x. Ok? And the same formula will work for sin x also. Everything is the same. But it depends on what the derivative is. So here we will have to remove the derivative and take this one because the derivative of sin x is cos x. Ok? Here also I did cos x. I wrote this here. So I ran it. So look, we have got this and it is between 0 and 2π. So the derivative we take here becomes cos x. So we took it from the forward difference, from the backward , from the centre. Everything is overlapping absolutely exactly. So it means that if we are taking the derivative with the help of its forward difference, with the help of backward, then all these are giving us good results. Ok? Ok?



So like this we can calculate it for different values of functions. Ok? So now this code, we can extend this code for further processing. So we can take this function with the help of different functions and play it and see whether the derivatives that we have just learned are working or not. So we can do the same thing for non-uniform mesh points or nodal points also. So I hope you have understood this lecture and thank you for watching this lecture.