Scientific Computing Using Python

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Lecture No. 18

Welcome everyone to Scientific Computing Using Python. So, in the last lecture, we used the Power Series Power Method. How can we use it to find out eigenvalues? We found the dominant eigenvalue and the smallest eigenvalue. So, let's see a little about that today — how can we find out the in-between eigenvalues?

So, let's get started.

In the last lecture, we discussed everything about how power methods are used, how the inverse power method is used. There is only one place where, if we are taking the inverse, then we are discussing it one by. Now, I am discussing this code. Suppose we discussed this code yesterday. So, what do I do in this? Let's take a matrix whose eigenvalue we know.

So, I take this type of matrix. So, look, I have taken the matrix diagonal dominant. The matrix that I have taken is a lower triangular matrix. What is in the lower triangular matrix? 2.5, 0, 0 is the first row, 2, 4, 0 is the second row, and -4, 1, 0.5 is the third row.

So, we know that the eigenvalues of this will be the eigenvalues — in fact, the diagonal elements. So, this means that the maximum eigenvalue of this is 4, minimum 0.5, and in between 2.5. So, I use this and see whether all these things are happening through the power method or not. And I take the initial solution, the condition, the initial solution, 1, 1 ,1 only I took. So I ran this. On running it, see, 4 came.

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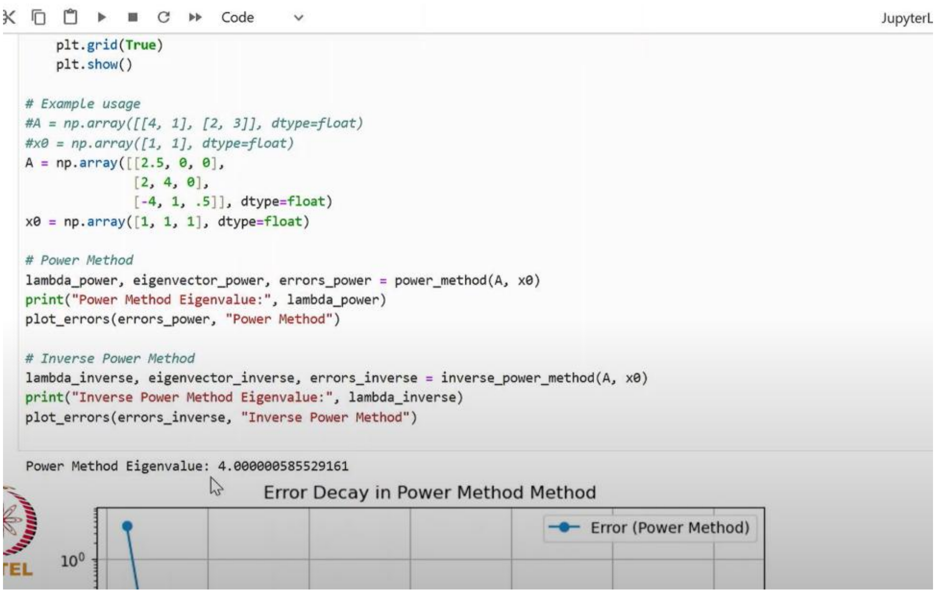

Our power method gave us this 4 value, which was the largest. Now, what did we get from the inverse? We got 2 from the inverse. So, what does it mean to get 2 from the inverse? Now, the value that we got is 2. So, now we want to find the lowest eigenvalue. So, what does it mean? This eigenvalue is the eigenvalue of A inverse. So, the value of A inverse that is largest is 2, and we have it. So, we have to find out smallest eigenvalue of A that is 1 by 2 that is 0.5. So, whatever value we get here, because we have run both the codes in the same code, whatever value we get, we will do the reciprocal procedure one by that, and the original value we have will come here. So, we were getting 2 there. So, 2 is its largest eigenvalue of the this, of A inverse and if A inverse has largest eigenvalue 2, then what will be the eigenvalue of A? 1 by 2, then it will become the smallest eigenvalue.

So now, what do we have? We have found 4. We have found 0.5. Now we have a range — that our lambda should fall between 0.5 and 4. So now, if we see the eigenvalue of this sign, then its sign value is 2.5. So, if I want to find the eigenvalue of 2.5 with the help of this method, then what will I do?

In this, I know that it is 2.5, so I will look at the in-between. So, what will I do? I will shift it — A minus 2I. I will check this matrix, okay? So, I will try to find out its eigenvalue, because we know that its eigenvalue will just be 2 shifted and the same eigenvector will remain.

So, what do we have to do now? Now we have to see the value near it. Now, if we look near it, the matrix that we had — our matrix that was 2.5 — then here it will become 0.5, okay? If it was four, then it will become 2. If it was 0.5 here, then - 1.5 will come, okay? And the value here will remain the same.

So, what happened in this case? Now, what do we have to do? Now, see how we are using it. So, if any of our eigenvalue is around 2, then it means the eigenvalue of A minus 2I will be that — is a very smallest eigenvalue. So, if we have to find out the smallest eigenvalue, then what do we do?

Find the eigenvalue of A minus 2I, inverse, and then divide it one by, and the value that we get — so now we can check it with the same code, that what is happening in this code.

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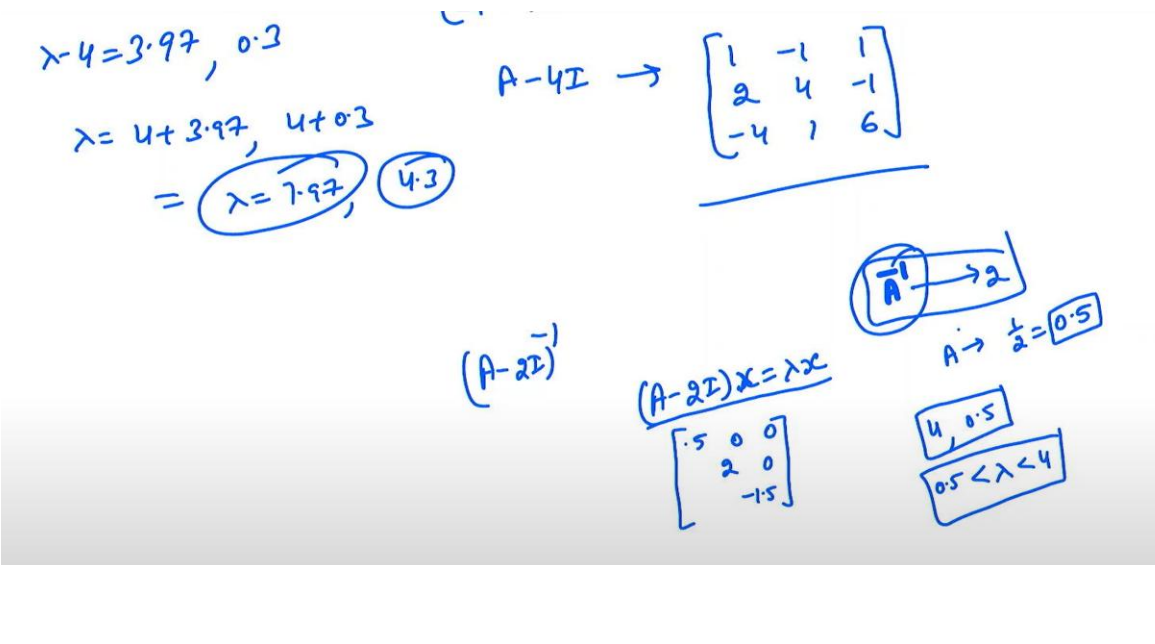

Now, what do I do is I shifted it by 2. So, here it came to me 0.5 , 2 came to me here, and -1.5 came here. Okay, I shifted it. Now, I want to see the A inverse, so the A inverse is this . Okay. So, the largest eigenvalue of inverse of A-2I has come this, 2. So now, if it comes 2, then what will we do now?

Okay, so the largest event value of A-2I is 2. So, it means we wanted the smallest eigenvalue of A-2I. It means 1 by 2, 0.5. It means that we got to know that its eigenvalue was — if we had assumed mu, then mu or lambda — let us write mu here. So, we write it down the same way as we did in the previous example. This is mu.

So now, see, λ-2 = mu, in this case, mu s 0.5. So, how much will the λ be? 2.5. So, we use our in-between values like this. So, we have to keep in mind that our largest eigenvalue from the A inverse — divide that number by doing one by — only then will we get the smallest eigenvalue in the matrix.

So now, what do we have? We can find out the largest and the smallest, and we can also find out all the in-between with the help of the shifting method. Okay, so now if we do not know that from where to where are our eigenvalues. For that we have a method and we call it, the method is called Gershgorin.

So, today we will discuss the Gershgorin Theorem and its applications. So, what does Gershgorin say?

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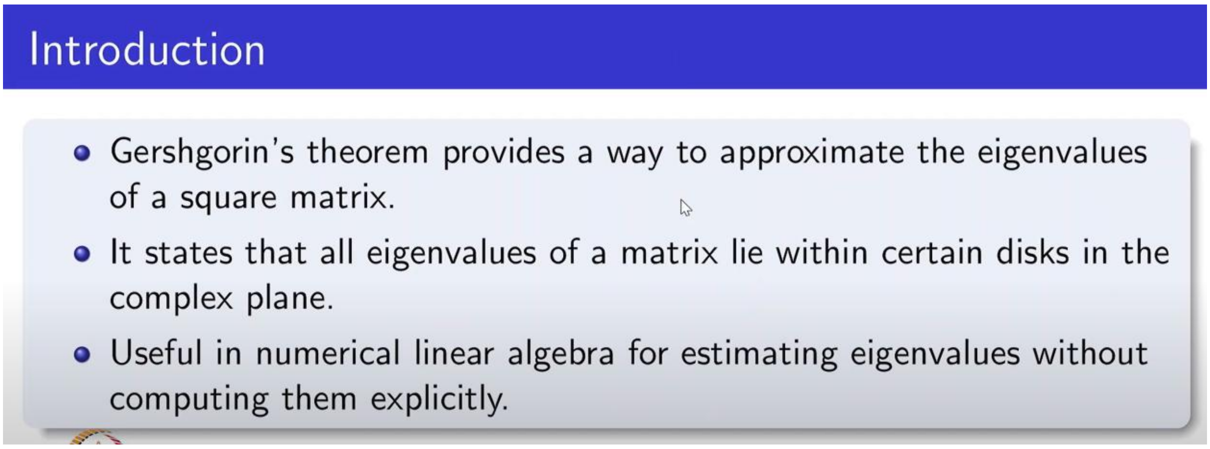

It will tell us that our eigenvalue — and up to what range it can go. In which range can it be found? Okay, so the Gershgorin Theorem provides us a way to approximate the eigenvalues of a square matrix. So it tells us that eigenvalues will approximately lie here. It states that all eigenvalues of a matrix lie within certain disks in the complex plane. And what is its use? It is useful in numerical algebra, because in our numeric answers, we need the highest, largest eigenvalue. And if we come to know that this largest eigenvalue is coming in such a disk whose value will be less than one, then we can very easily use it. We don't even need to find it out by the power method. Okay?

So, this value is — so we will get to know that disk through the Gershgorin Theorem. How is it? Okay, so now we have to see how to find it, how to find out the disk. So now, like this theorem — the Gershgorin Theorem says that if A matrix is n cross n, then it will be a square matrix. Only then can we find its — we can talk about the eigenvalue. If the matrix is not square, then we will talk about its singular value. That is a different thing.

So, if we have a matrix with complex or can be real Gershgorin disks, we have defined it like this: Di is equal to the z belong to the complex number such that Z-Ai that are its elements less than equal to Ri, some radius. And the Ri, what is it?— its row-wise element or column-wise element — depends on how we are taking it. So, what does it mean? That every eigenvalue of A lies within at least one of the disk Di. So, we will have many disks. It also depends on i. Now, like if there is n, then I, 1,2, 3,4 will go up to n. So, we will have n number of disks.

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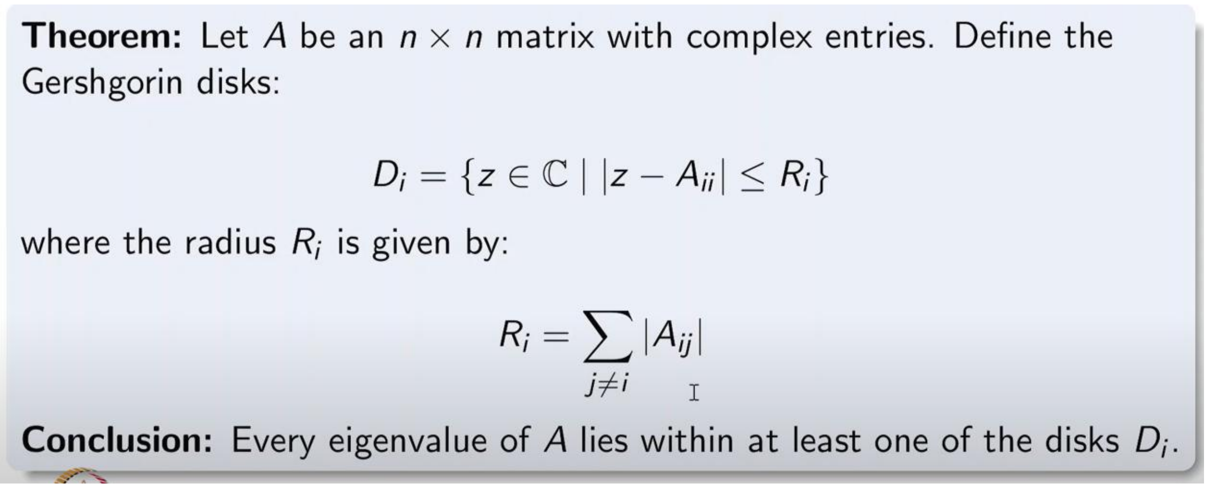

As soon as we have n number of disks, then after that we will come to know that everyeigenvalue lies in some disk or the other. And if we take their union, then all the eigenvalues will lie in it. So, this thing is — that we can do it with the help of this theorem. So, how to do it?

If we see in this, then — I mean, we can prove it. If we can do it, then how to prove it? So, a different meaning — let's prove this. Now, what we have to do is try to prove it. By the way, one of the ways is like this also, but we prove it. See, what we have to do is to take the eigenvalue.

So, the eigenvalue — we know we have A matrix. And a matrix is n cross n. So, n cross n matrix, and here there is a vector — and this λx. So, we write like this to find eigenvalue. Now, see, the x vector is this. Now, we are talking about ith eigenvalue. So, if we talk about ith, then we will write it: i1, i2, xin — write it like this.

So now, what do we do? Like this is the matrix. I am writing it like this. Look at the matrix — how am I writing x i1? And here I am writing a11, a21, an1. I have written like this — plus x i2, a12, a22, an2— this second element means second column of the matrix. Okay? I will keep writing like this.

And in the end, we have x I n a1n, a2n, and ann. I can write it like this — this system equal to λ, and here there is a vector whose elements are i1, i2, x i. So, what did I do? I wrote this system in this form — if we see, then okay. So, I name it one.

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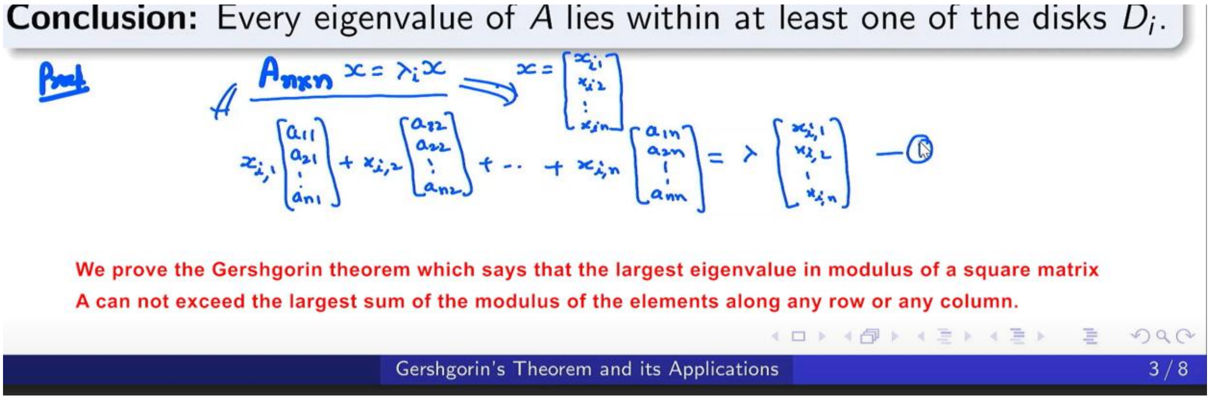

Now, what do we have to do, here is ith. Now, see, from where our i will move 1, 2, 3, so from that we will get all the eigenvalues, and we will get the eigenvector. Okay, so if we take $\lambda 1, \lambda 2, \lambda 3$ corresponding our vectors, eigenvectors we will get — so I assume that, let now — there is a vector in which there will be some element which will be its largest element.

So, let me assume — in the largest element, okay? So, I write this — let x I k is the largest element in which, in this vector, there is some i where there is some kth element which is the largest element of that vector. So now, what do I do? Correspondingly I write a row. Corresponding to the kth, so kth will come somewhere here, x i k, it can be 1, it can be 2, it can be n, it can be anything.

So I have written this. Suppose I have taken k, so the rows that are made corresponding to k, that we have will become this. Now I will write it. Look, x i1 will remain the same, a, 1, 2, 3, n is coming, so here it will come k1 plus x i2, a k2, x in, a kn is equal to λi xi k here.

So this is how we have written it. Now look, what do I do from here? I will bring this quantity here.

So look, from here I am writing, λ1 which will become this. It will become ak1 element. Okay, x i1 divided by x ik , plus a k2, x i2 divided by x ik, like this. We will also get kth element that will come. If I write it, then I will write it. What will come here, a kk will come because it will be a diagonal element, so one will come to us in that row. If we see and its coefficient will be cut, then one will come, plus and at the last we will have a kn, x i n divided by x ik. Okay, so we have written it like this.

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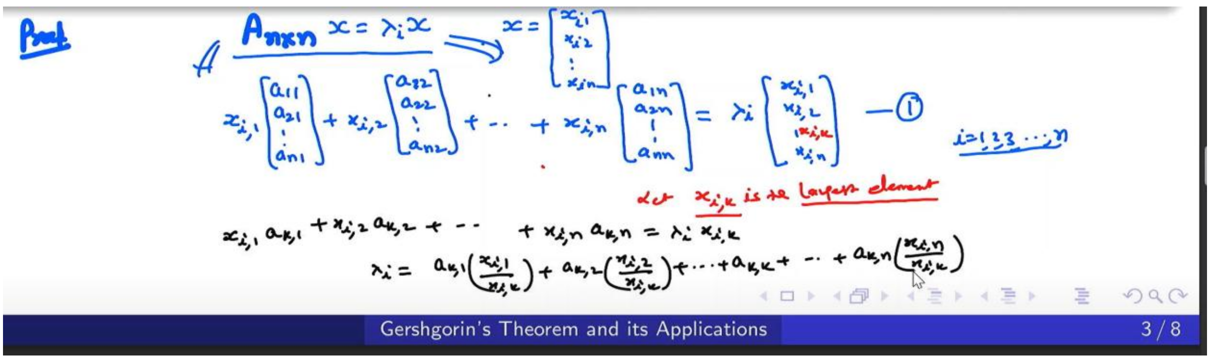

Now let's see further. Let's see the matrix example now. We have seen its proof, so it is okay. So now it has come like this. Now what will we do? See, the value we had chosen was the largest. So if it was the largest value, then if we see, this was the largest value. This value, this value — so the magnitude of this will be less than one. It may be negative, but the magnitude will basically be less than one, that's for sure, because we had the largest value of x i k in that vector, so we divided it by that, as we do in normalization. Basically, we normalized it.

So what happened after that? These largest values — all these values will come below one, and the one value was to come only here; it is one here. Otherwise, all these values are less than one. Okay, so now this thing, I will write it like this. So if we see, I can write this from here in the case that — let's take the magnitude of λi the magnitude which is, it will always be less than a k1 plus a k2, a kk , a kn. This will always happen. If we take the magnification of both the sides and apply inequality, then we will get this. Right?

Now since this is true for k — so what is k? An arbitrary. We had chosen an arbitrary. And if we look at this, then what is this thing? What is this? k1, k2 means kth row basically. So from here, we get that the λi in this case will always be the sum of all the elements k i i from 1 to n — so in the kth row the elements, if we take there magnitude and sum all of them, it will be less than that. And our k is arbitrary. Right?

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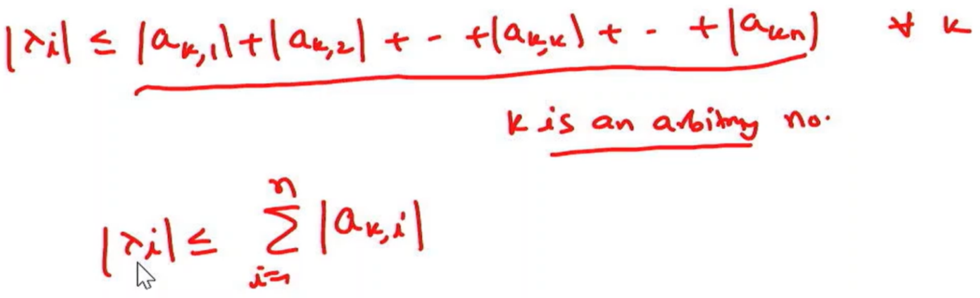

So what will happen because of it being arbitrariness? We can assume that any λ will always be less than that. So we just take the sum of all of them. So what can I do? I take it to the maximum. It is certain that this will always be less than the maximum. Where this is a, I take it as i j. Okay. Over the so took i as one and shifted the whole row, then took i as two, then shifted it completely and take j from one to the n. Okay, so this quantity is i. If I take i1, then the first row will come. If I take i2, then the second row will come, then the third row. Like this, we have all the rows. So we saw that any eigenvalue will always be smaller than this.

So this is the quantity that we have. If we take the modulus of the elements of all the rows and sum them, then we have this value. Okay, so if we see that we have rows and this was this quantity, so if we see, then this is the proof of our Gershgorin theorem. Similarly, this is the value of rows. Similarly, we know that A transpose has same set of eigenvalues. Okay, so in that case, we can say that the one which comes is the largest, so we can also say that the eigenvalue is λ is always less than the maximum of j, summation a i j, i from 1 to n.

So what do you do? I take I as 1, 2, 3. So if we are doing the sum column-wise, the sum will be less than this. So what does it mean? That the sum will be less than the column and also less than the row. So if we come to know, then this is our thing that the Gershgorin theorem tells — that the values that we have will always be less than this. Okay? So this disk will be created with us. So now what do we have to do? That, like this, we have — so now we have to see that — and we can do this in a different way also. Inside, we do that simply. We will get to know from here also, but we can do it like this. We know that this Ax = λx. What do we do? I take the norm on both sides. Okay, so what do we get from here? We can do it like this.

I wrote it opposite. I wrote it here, and I did it like this. And we know that this is always less than equal to the norm of the matrix and the norm of the vector. Now this norm and this norm are the same. So I let me cancel it from here. So from here, I can tell you that the norm of the λ is always less than the norm of this matrix. So we should always keep this in mind. The eigenvalue of any matrix that we take cannot be greater than the norm of the matrix.

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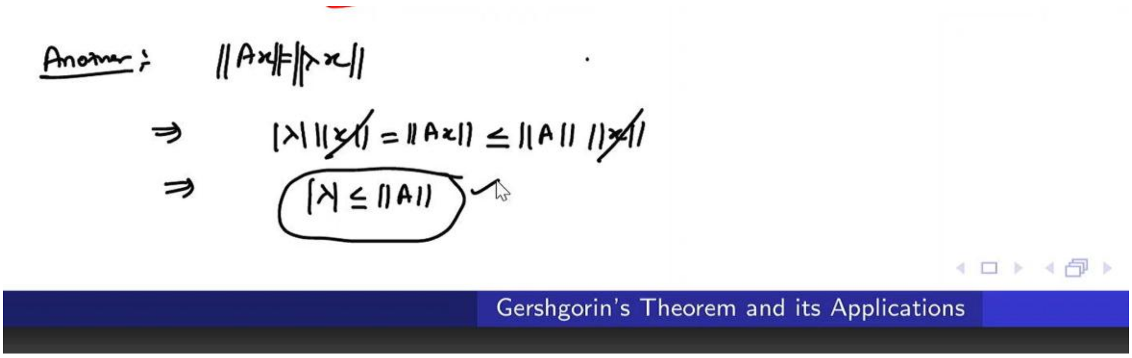

So we cannot get a value greater than this norm. Now, if we take it as infinity norm, then the maximum row sum will be come. If we take one norm, then the maximum column sum will be come. And this is our row sum and column sum. So from here we can say that this λ is always less than one norm and infinity norm. This is a big disk about the origin. Its radius will be this. So here we have the upper bound of that eigenvalue. We will always find the eigenvalue inside it.

So now we have another way to calculate the disk — how to find the disk. So to find the disk, we have this. This is the way we will find out. Now, how can we calculate the disk?

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Consider the matrix:

$$A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Compute the Gershgorin disks:

$$D_1 = \{z \mid |z-4| \leq |-1| + |1| = 2\}$$
$$D_2 = \{z \mid |z-3| \leq |-1| + |-1| = 2\}$$
$$D_3 = \{z \mid |z-3| \leq |2| + |-1| = 3\}$$

If we want to find out, then to find out the disk, we have four. So what do we do? Basically, what do we have to do here? We write A - λI. So let's take it magnitude and from there we get the characteristic equation and from there we find out the eigenvalues.

So what am I doing in this? In this I am doing 4 - λ. We will see that the less than equal to magnitude of -1 plus magnitude of 1, so this will come out to be 2. Like this, 3 - λ less minus one minus one. So minus 1, just leave the diagonal element, it comes. And the third one also comes , 3 - λ, is less than, 2 plus -1. So this comes. So from here we have three disks.

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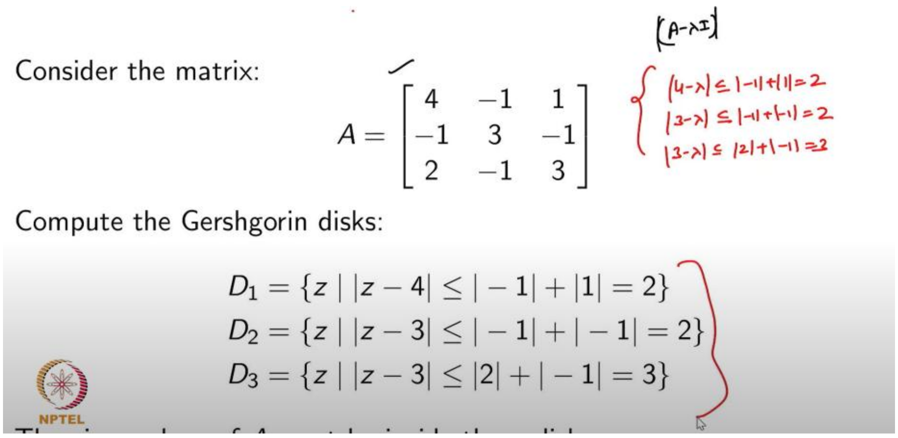

Okay? We call it a Gershgorin disk. But how can we find out this disk and how is it showing? So we will very easily discuss it. And this was the disk which we found out. So we will do it through theorem.

The Gershgorin theorem disk is one by the Brauer's theorem. So, the Brauer's theorem says that if we have a matrix A , n cross n matrix, so what will happen in this case is the matrix that we have a11, a12 and so on a1n; a21, a22, a2n; an1, an2, an3 is a square matrix.

So, what did Brauer say? That we write it like this: A - λI. We know that to find this out, we have to find the eigenvalue so we write it like this: A x = λ x. I can write it in this form. So, going ahead, I can write it like this: a11 - λ, a12, a1n okay, a21, a22 - λ, a2n, an1, an2 and this ann - λ can be taken like this, multiply by this vector x1, x2, xn, and this is our 0,0,0.

So, we wrote it like this. Now, we have this matrix, basically. So now, if we do this with the Gershgorin as we did above, okay, we worked in Gershgorin. So, if we do the same, if we do the process, then we can write it like this: that any a kk - λ. Okay, what did we have there? λ If we see, I take it up — see, this was λ — we have λ.

So, what did we do? This one which is there, I take it to the left side, okay. So, λi – a kk — or take it here — the remaining one, if I bring it to the left side, then this will happen. So, if we look at this star, then what do I do? I use it and write it in it, I here, okay.

So, from here, if you see, what will we get? Because it is this side of a kk, then I can write λ – a kk or a kk - λ. Now, what will happen if we see it according to that? Then it will be always be less than. Now, the diagonal element has come on this side, so we will leave it and write the remaining element, okay. So, from here I can write that always less than equal to a. Now, so we take i. Now, any one also, whatever values will be there, can we write that k i where i is not equal to k, and i will always start with 1, 1 to n.

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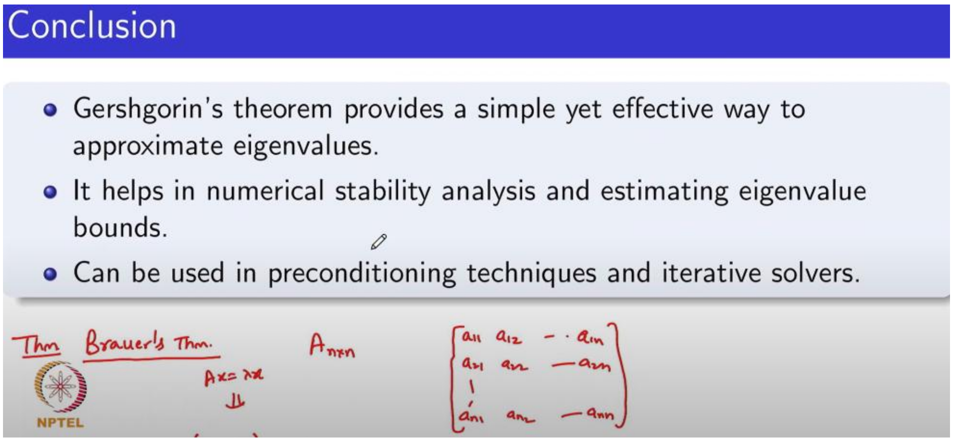

So, what does it mean? That the remaining elements will be left. Leave the diagonal element in each row and take the value of the one that is left. So, from here we will get a disk, right?

So now, let us see how to do it. For example, like we have taken any example now, so I took suppose matrix a 2, 1, -1, okay. Here I took three, here I took -2, here I took 4, 0, 1, 2. I took any matrix. Now, what do I have to do? I have to show its disk, has to find out the Gershgorin disk.

So, how will we find it? See, now for this, we will get a disk. So, what will I do? We have to do $A - \lambda I$ okay, and $x = 0$. So, now what do we have to do is define $A - \lambda I$. So what will happen: $2 - \lambda$, 1, -1, 0, $3 - \lambda$, 4,1, 2, $-2-\lambda$. So we have, with the help of the Brauer's theorem the Gershgorin disk that will be formed $2 - \lambda$ less than equals 1 and 1 so 1 plus minus 1, so it becomes 2 , and this $3 - \lambda$ — this will become 0 +4, 4 and this $-2 - \lambda$ will become 1 +2, 3. Okay.

So, what do we have? This three has now become a disk, and the three disks we have now come to know. That if we plot this also, now see, $\lambda-2$ — we can also write it like this — λ minus 2, this less than equal to 2. So, if I do this somewhere here, so if there is 2 somewhere, suppose there is 2 here, then about it I will take a circle of radius 2. This is it, okay. So, suppose we have 2, and this was the circle $\lambda-2$ less than equal to 2 . So this is the disk. So, this is the disk, like this we have. So, there is three somewhere here, because this will be four. So, about this 3, I have taken 3, 4, 5, 6. If it is like this, then I have to take about 3 whose radius is 4. Then it will go up to 7.

So, what did I do? A big disk will come out like this. Okay. Similarly, I can write it as $\lambda + 2$. So, we will take about - 2, okay. $\lambda - 2$ is one here, -2 is here. About this, I will take a disk of radius 3. So, now we have this disk. So, we can say that all its eigenvalues will come in some disk or the other.

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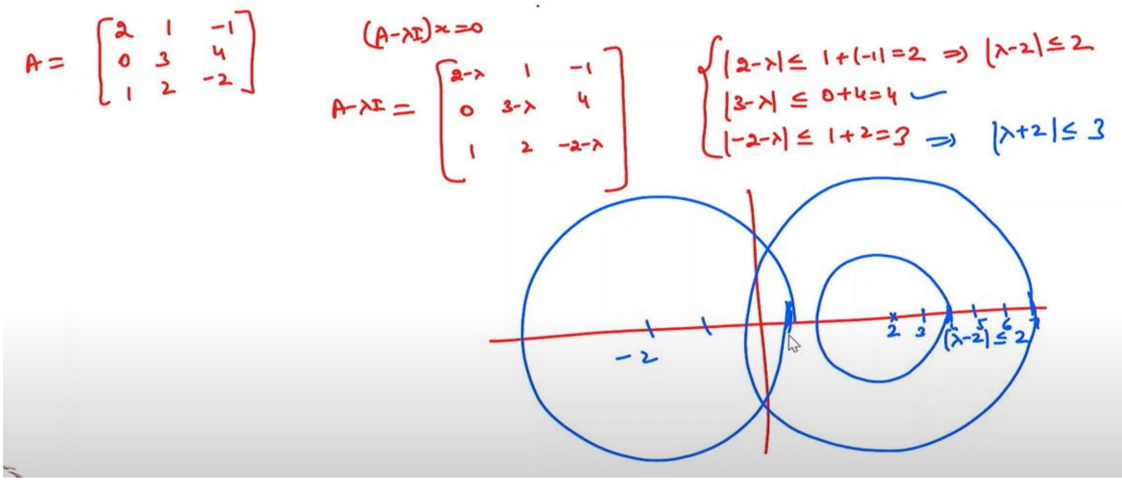

It is possible that any eigenvalue is here. If some eigenvalue is complex then it will be found somewhere in the complex, if real then found in real, if imaginary then imaginary. Like this when now we have the disk so all the eigenvalues lie in union of all these disks then this gives us the bound, okay. It gives us the bound that till where maximum our eigenvalue can go. In which bound will it come? So, we use this a lot. And from this, we also came to know that any eigenvalue cannot be bigger than the norm of the matrix, okay.

So, this thing also has to be always kept in mind, that if we have to find any estimate — by chance, if someone asks us to tell us what would be the eigenvalue of any matrix — then we can tell that any eigenvalue would always be less than the matrix norm.

So, if we look at it, we will have a disk whose radius is — and we can assume that this is the upper bound basically — and the disk that is coming to us is giving a slightly confined radius. So, we can also calculate it with the help of this. Okay. How will we calculate it? We have discussed it.

So, now we can calculate it as a circle, Gershgorin circle. This is the Python code we have.

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So, with the help of this code, we will verify it and see where the eigenvalues lie. So, we can take any matrix. So, like I have taken this matrix 0, -1, -2, 1, 1, -2, 2, 1, 2. Now if we see, what is there in it? It will become λ - 0. Its radius will become less than 3. Okay, because it is zero. Then, about one 0 we will get a disk. About one 1 we will get a disk. About one 2 we will get a disk. Now the radius of the disk we get will be 3. Okay, so if we see, what will happen in this case? So, if we plot it — I plotted it — then we got this disk.

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Gerschgorian Theorem

```
import numpy as np
import matplotlib.pyplot as plt

def gershgorin_disks(A):
    """ Computes the Gershgorin disks for a square matrix A. """
    n = A.shape[0]
    centers = np.diag(A)  # Diagonal elements
    radii = np.sum(np.abs(A), axis=1) - np.abs(centers)  # Row sum excluding diagonal
    return centers, radii

def verify_gershgorin(A):
    """ Verifies if the eigenvalues of A lie within the Gershgorin disks. """
    centers, radii = gershgorin_disks(A)
    eigenvalues = np.linalg.eigvals(A)

    print("Eigenvalues of A:", eigenvalues)
    for i, eig in enumerate(eigenvalues):
        is_inside = any(abs(eig - centers[j]) <= radii[j] for j in range(len(centers)))
        print(f"Eigenvalue {eig}: {'Inside' if is_inside else 'Outside'} Gershgorin Disks")

def plot_gershgorin(A):
    """ Plots the Gershgorin disks along with the eigenvalues of A. """
```

Okay, the disk has come. Its eigenvalues are the complex, let’s reduce its width from 8 to 3 so it can be visible. So we solved it and these are its eigenvalues 1 + 2, 1 + 2.2i, and 1- 2.2i, this we know that whenever complex eigenvalues come they always come in conjugate pair.So, if this is there, then this will also come. The conjugate will also come, and the third one is our one whose imaginary part is zero. So what did we do? We calculated the disk. See, one disk is about one, one disk is about origin, and the third disk is about 2. And we saw where the eigenvalues are bringing. So, one eigenvalue is the complex; its conjugate will come here just opposite, and the third one is coming here. So from here, we came to know that all the three eigenvalues of this matrix are bringing here. So some disk is definitely doing it. Brother, see, one is doing one in it. If we see, one is doing one in it — in the smaller one — and in the second one, which is bigger, both are doing it.

So, if we take the union of all these, then we will come to know that all the eigenvalues will bring in that union. And if we do the intersection, then it is not necessary that all the eigenvalues will bring in the intersection. Because if we take the intersection of all the three, then this circle will appear, and in this circle, only one is coming. So, in this way, we will get eigenvalue.

If I change it — I changed this matrix — I wrote two here. I wrote zero here and suppose one here. Now let's see what will happen. Now see, the eigenvalue that is coming is also complex, so its eigenvalue is 1.2+2.2i, 1.2-1.2i, and the third one that is a 3 by 3 matrix. So we know that the eigenvalue that will come can only be real. In this case, one eigenvalue will be real . So we solved it. So see, this is our result. Our disk has been changed. These eigenvalues are complex.

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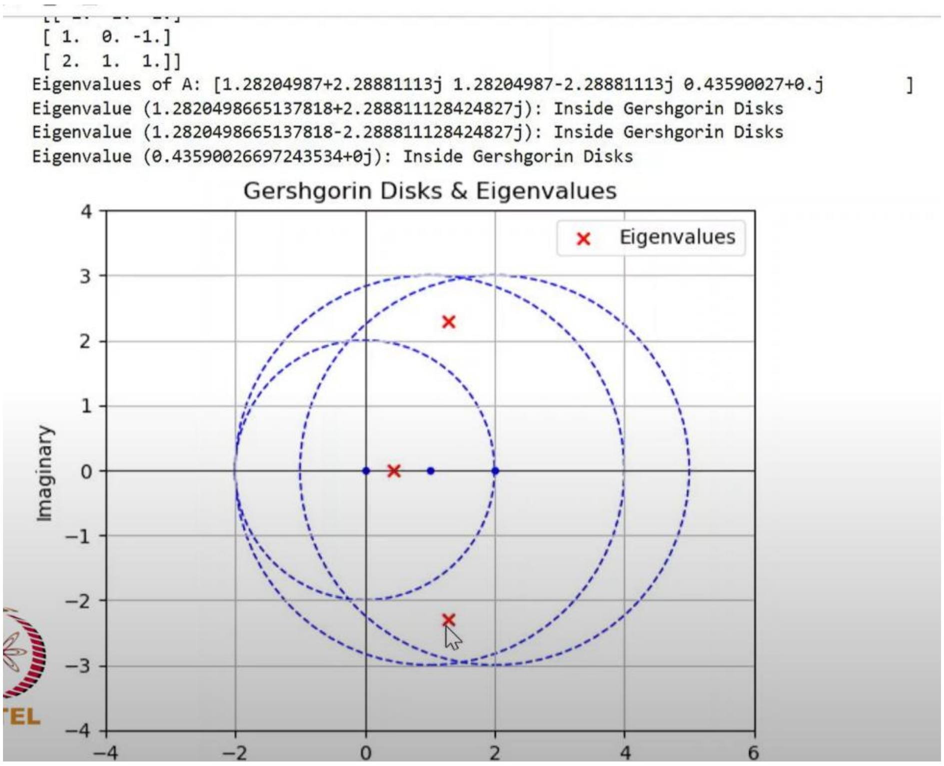

So this I took a 3 by 3 matrix, now I can write it as 4 by 4 matrix, so I can comment it out, and I took this matrix as, suppose, 4 by 4. In this, we take 0, 2, 0, and -1 . So this has become 4 by 4 matrix.

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    ax.axhline(0, color='black', linewidth=0.5)
    ax.axvline(0, color='black', linewidth=0.5)
    ax.set_title('Gershgorin Disks & Eigenvalues')
    ax.set_xlim(min(centers) - max(radii) - 1, max(centers) + max(radii) + 1)
    ax.set_ylim(-max(radii) - 1, max(radii) + 1)
    ax.set_aspect('equal')
    plt.legend()
    plt.grid(True)
    plt.show()

# Example usage
#A = np.array([[2, -1, -2],
#              [1, 0, -1],
#              [2, 1, 1]], dtype=float)
A = np.array([[2, -1, -2, 0],
              [1, 0, -1, 1],
              [2, 1, 1, 0], [0, 2, 0, -1]], dtype=float)

print("Matrix A:\n", A)
verify_gershgorin(A)
plot_gershgorin(A)

Matrix A:
 [[ 2. -1. -2.]
 [ 1.  0. -1.]
 [ 2.  1.  1.]]
Eigenvalues of A: [1.28204987+2.28881113j 1.28204987-2.28881113j 0.43590027+0.j        ]
```

Now let's see. This is a matrix. All four of its eigenvalues are complex, -1.8 no this is real. -1.865 and this 1.129 are coming real, and two are coming complex. 2 eigenvalues are coming real and 2 are complex. So, in this now we have formed 4 disks. And disk is formed about, one about 2, one about 0, one about 1 and one about -1. So about 0 this is formed, the bigger one. This is about 0. This is about -1. This one, about -1. Okay, so the four eigenvalues that we have come are these. There are two real eigenvalues — one this and one this — and two complex eigenvalue.

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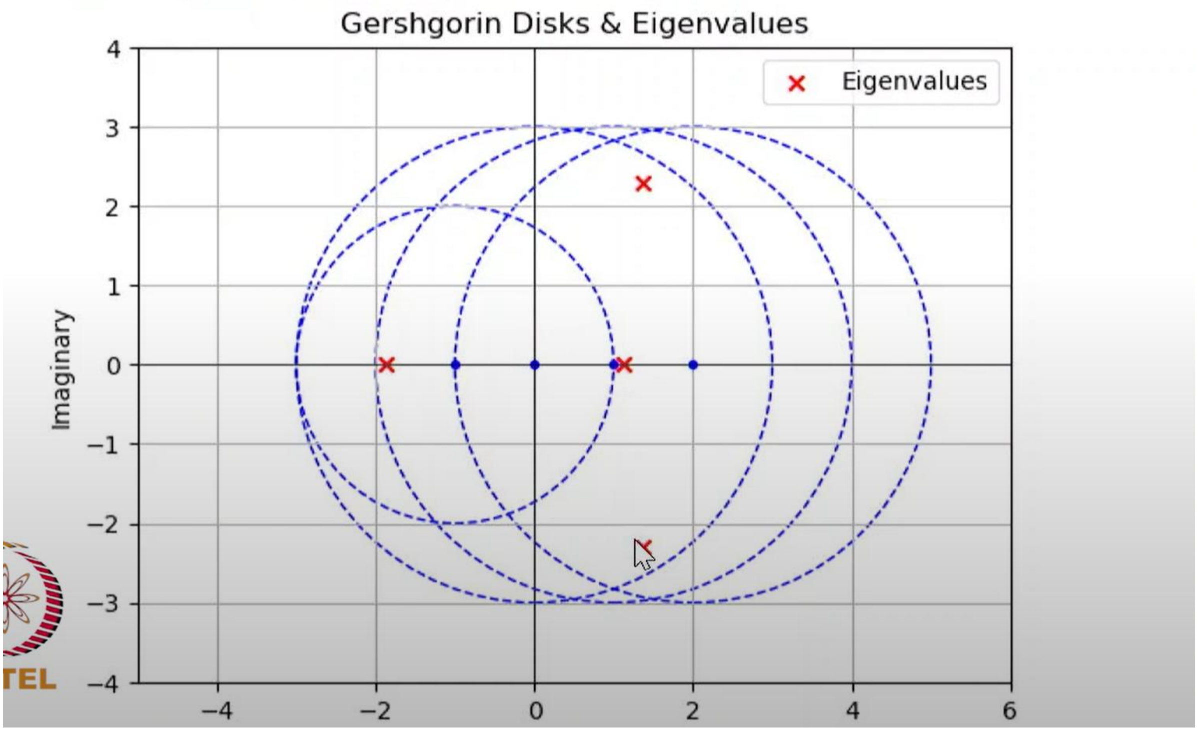
```
Eigenvalues of A: [-1.86518073+0.j        1.36775965+2.29091325j  1.36775965-2.29091325j
  1.12966142+0.j        ]
Eigenvalue (-1.8651807277794505+0j): Inside Gershgorin Disks
Eigenvalue (1.3677596524734918+2.290913247089965j): Inside Gershgorin Disks
Eigenvalue (1.3677596524734918-2.290913247089965j): Inside Gershgorin Disks
Eigenvalue (1.1296614228324708+0j): Inside Gershgorin Disks
```



Now see if we take union of this then all our eigenvalues lie here. So, in the by Gershgorin disk we come know the range and bound of our eigenvalues. So with the help of this, we will get it.

If I take this case, suppose here, then if we see in Gershgorin, if we have taken this matrix, then in this matrix we have got this disk. If I want to do this with the help of norm, that is, what will happen with its help?

So, if I take the help of this case, then we know that it will be less than this. It will be less than infinity norm. Okay, so if we take this, then what will come in it? So now we can take the λ. So one norm is 2 plus 1, 3, so the maximum3, 6, 7 so this 7 will come and this will come maximum row sum. In row 4, 7, 4 so this will also came 7. So we can say from here, this is always less than this.

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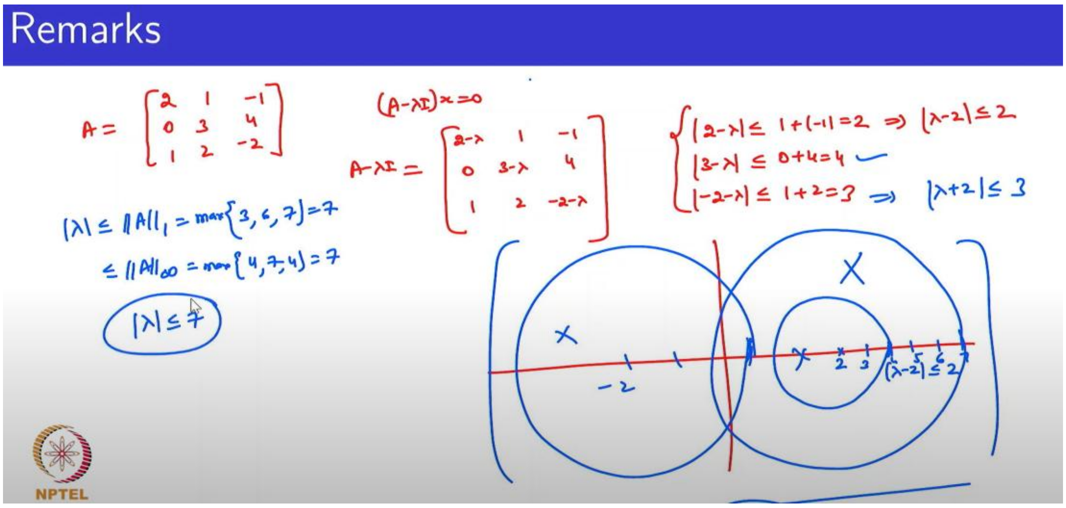

This means that if we take the value about origin — if I take the value about origin here — then we will have a circle of radius 7. All the eigenvalues will be found in it. All the eigenvalues will be placed inside it. And if we see, these three disks that we have created are coming inside it. So we got an upper bound, and we got another disk.

So if we see from the union of those disk, then we will get all the eigenvalues. So with the help of this, we can find out all the eigenvalues. So this was our process related to eigenvalue. Almost, if we want to find out the eigenvalue, we can find it by power method. If we want its estimated value, that is, from where we need the bounds, then we can use Gershgorin method. Okay?

Because the eigenvalue will be useful to see the convergence of Jacobi, Gauss Seidel methods. If we want to see the convergence, then we know that the convergence matrix of it is its spectrum. We can find out the radius with the help of which we will come to know from the spectrum radius whether the method is converging or not. All these things we have used for the system of equations. We have used the ill conditioned system.

So, this chapter related to the system of equations, linear equations, is now over. So now our next topic is — so we have almost finished this system of linear equations, and we have come to know how we can find out their solution by direct method, by iterative method. Okay, how can we find their eigenvalues. So we have discussed all this in this unit.

So now what we are going to start next will be related to the interpolation. So, I hope you have understood how to solve this system of linear equations. And to watch this lecture, THANK YOU.