Scientific Computing Using Python

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Lecture No. 13

Welcome all of you to Scientific Computing Using Python. So, in the last lecture, we saw that we discussed about norms. So, today we will see how we can solve a system of linear equations. Let's get started. Today's topic is system of equations.

So, we are talking about system of equations, linear equations. So, all of you must have solved this type of system till now. Ax equal to b,in general we represent it like this: Ax equal to b. Like, if we have a system 2 x1 plus 4 x2 plus x3 minus x4 is equal to some value, suppose 5 comes. Okay. In this way, we have x1 minus x2 plus 3 x3 minus x4 comes 1. 3 x1 plus 2 x2 plus 3 x3 plus 4 x4 is equal to 0.

And let's take another equation: x1 + x2 + x3 + x4 = 1. So, if we see, this is a system of equations, 4 by 4.

So, if this is a 4 by 4 system of equations, then if we have to solve it by the methods that we have been using till date, then if we see, what will we do in the methods that we have been using till date? What will we do? What we have been doing till now?

What will we do? Write it in the form of A x = b, and after writing it, we will see that if the matrix A is non-singular — if it is non-singular, non-singular means if its determinant is non-zero — then what will we do?

I will write x like this: A inverse B, and we can solve it. Or we can do it by Cramer's rule. But if we see, this system is four cross four, so the matrix in it is four cross four.

So, to find its inverse, first check whether this matrix is non-singular or not. Then, after that, finding its inverse will take a lot of time, and there will be a lot of tedious job right now.

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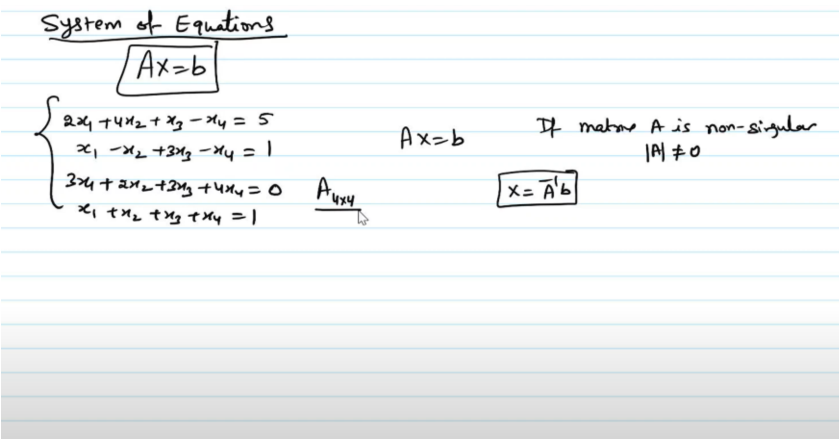

This is 4 by 4. If I do 5 by 5, 6 by 6, 10 by 10, then it will be very difficult with pen and paper. Even if we solve it with a computer, then finding the inverse — which is, for a computer — becomes very difficult to find the determinant.

So, the real-life matrices have very high dimension. So, if it is a very large matrix, and if we have to solve a system, then to solve the system, we need computational methods.

So, the computational methods are not inverse-based because finding the inverse is very difficult. So, what happens for that?

So, numerically, we will do some methods with the help of which we can solve this system of linear equations. You can also write it here as system of linear equations.

So, now we know that we have a system. Let me take a system of linear equations. So, I wrote here a system. We have defined A, n, cross, n. The vector that will come will be X, n cross 1. And this vector on the right-hand side is b, n cross 1.

So, first of all, we check in this. Let me name the system one. First of all, check when this system is consistent. So, we can say that the solution of this system will exist. So, for this, we know what we do. Let's create an augmented matrix. Augmented matrix means to add it. So, we generally represent an augmented matrix like this. And what is it? Here, we will write the matrix, and in the end, we will write another column in which we will add b. This will form a matrix. Now, its dimension will become A, n cross n+1, and because we have increased one column, what do we check?

Let's see that if the rank of the matrix — augmented matrix — if it is the same, the rank of the matrix, then we say that the system is consistent. Consistent means the solution exists.

Now, this solution can be either infinite or finite. So, if we see and write that if the rank of this matrix is equal to this, then it will be consistent. And if the rank of A becomes A, then we will get a unique solution. And if the rank becomes A, then the infinite many solution is okay.

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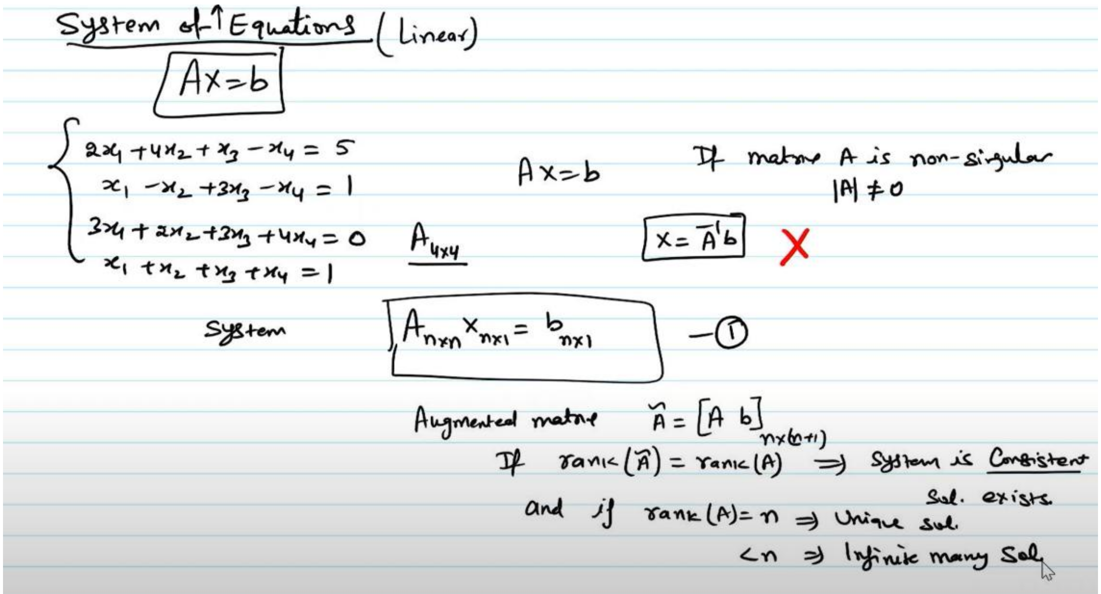

So, this is what we do to check whether the system which we are solving is suitable or not, whether its solution exists or not. So, we know these things from our previous knowledge.

What will we do after that? Now, we have to see that we have to solve the system. So, if I give you a system and this system is a Ax = b, and I say that the matrix A is diagonal, so if the matrix A is our diagonal matrix, then we will come to know that this is a system — we can easily find its solution, okay?

If the matrix we have is diagonal, similarly if the matrix — then we can take the first case. We can take the second case. I will say A is upper triangular or lower triangular. If it is upper triangular and lower triangular, then also the system can be easily solvable. It may take a little more time as compared to diagonal matrix, but this too will be solved very easily. This too will be solved easily.

So, what is our purpose now? What is our purpose? The matrix that we have is A, n cross n matrix. We are talking about square matrix right now. So, if we have this matrix, what will we do with it? We will convert it into a new matrix.

So, the new matrix that will come — if I name it A, then suppose I name it B — then this matrix B, if we transform A into B, then the matrix B can either be diagonal, or it can be upper triangular, or it can be lower triangular. And after that, we will solve the system.

So, we have to do such work. And if we look at this work, we have done it in 9^{th}, 10th, 11th, 12th also. So, we will convert it into a new matrix. We will do it according to the algorithm so that we can program it in the computer as well. If we find a solution in the computer, then how does the computer compute it? We need to see that. So, we have the method for that. We are discussing what happens in the methods. So, our main purpose is to solve this A x = b.

There are two ways to solve this in the computer. One is the direct method, and the other is the iterative method. So, we will use both to solve the system.

So, what happens in the direct method? Direct method — as the name suggests — is direct. It does not mean that the answer is straight. It means that the solution is obtained in a finite number of steps, and no initial guessing is to be taken. So, in this, we will get the solution in a finite number of steps. No initial guess is needed. So, we will call it the direct method. So, in the direct method we are going to use, we are going to use Gauss elimination. Gauss elimination, after that LUD decomposition, and after that LUD decomposition, we will do it in two ways. And we will do the different parts of LUD decomposition. So, what do we do in this method? The solution — which is the initial solution — the guess we are taking, we will improve it over the iterations, and after that we will stop depending upon how much tolerance we have. So, what happens in this case is initial — you already know that the initial guess is needed — so what will happen in this is that the solution will keep improving on each iteration. And depending upon how much accuracy we need, we will get the solution.

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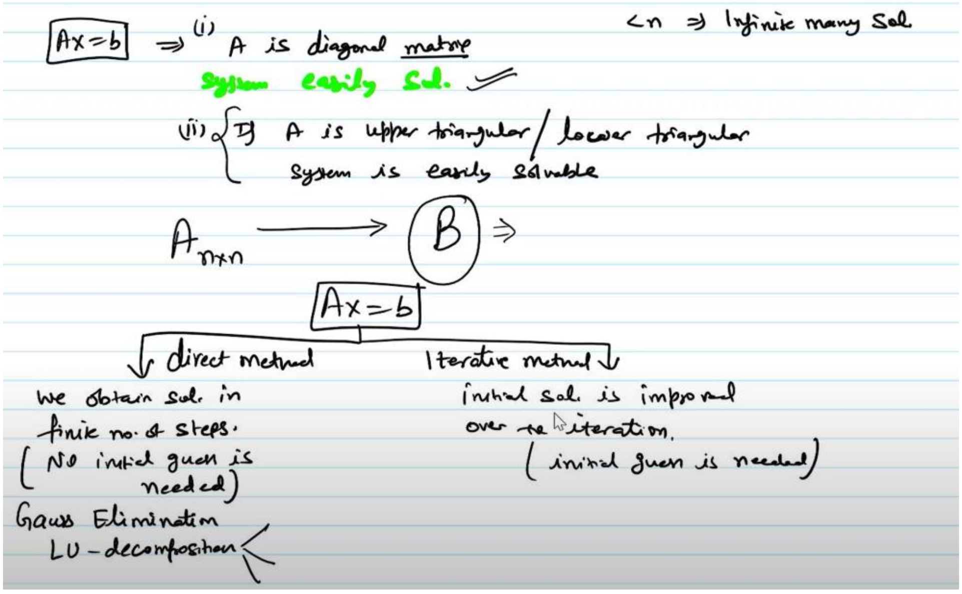

So, these two types of methods are used to solve the system of linear equations. So, now our first task is the Gauss elimination method — that we will solve a system with the help of Gauss elimination.

So, from here, the first method that we are doing is Gauss elimination. So now, with the help of this, we define what is this Gauss elimination, how are we going to solve this, and the spelling of this is Gauss.

So, in Gauss elimination, our purpose will be that we have a system of matrix Ax = B, okay? And A is a square matrix, a n cross n matrix. So, what is our main purpose? In this case, we will shift A — we will convert it into an upper triangular matrix — and after that, using back substitution, we will get the solution, okay? So, with back substitution, we will get its solution.

So now, we have to see how do we solve this? How are we able to apply this method? So, if we take any one — I will take that — our main purpose in this is that the elementary matrices are being used, okay? So, what we have to use in this is elementary. We will use the matrix to find out how we do elementary transform. So, we have to use it in this.

Now, suppose I have a matrix or a vector. So, suppose I have a vector. Inside it, there are values — 2,3, 4 . And I want to convert this vector so that here remains 2; all the values below it become zero. So, what do I do? I take a matrix, okay? L1 means I am taking the lower triangular matrix with 3, means need to take 3 by 3 matrix. So, I am taking its diagonal elements. So, we will call it unit elementary lower triangular matrix. So, we have named it.

Now, I have to see what to put here. Okay, here also zero will come because it is a vector. So, what do we do? What do I do here? See, if I want to make it zero, what will I do? The first element is 2 — I will divide it by 2 and multiply it by minus 3 and add it — then it will become zero.

What does it mean? Here, the multiplier that I have to take is minus 3 by 2, okay? Here, I have to take the multiplier is minus 4 over 2, okay? 4 by 2 means -2. So, what will we do?

Here, I have to convert this element. So, I will divide it by 2 and multiply it by minus 3 and add it to it — then it will become zero. I will divide it by minus 2 and multiply it by minus 4. If I add it to it, then it will become zero.

So, these things that have come — we call them multipliers. So, with the use of these multipliers, we can see what happens to this matrix. I can write it as this directly minus 2. So, this matrix has come.

Now, we will see what happens to L1v. So, let us see — 1, 0, 0, minus 3 by 2, 1 , 0, -2, 0, 1 — and I multiplied it by this vector. So now, what happens to this? In this, by multiplying, we see what will come out. So, I multiplied it by two — so it will come here only. Now, I multiplied it by this — so how much did it come out? -3 by 2 into 2, so, -3 and plus, 3 became zero. I did this -4, and at the last plus 4 also became zero.

So, you see, if we leave only the first component of this vector, then all the values which were below it — all those values became zero with this transformation. So, we will use such a transformation of this type, and we call it unit. Unit means all its diagonal elements are unit. And this is the lower triangular matrix.

Lower triangular matrix means that the elements of the main diagonal will be non-zero, and only below it or below it. All the values above it will be zero. This is called lower triangular matrix, okay?

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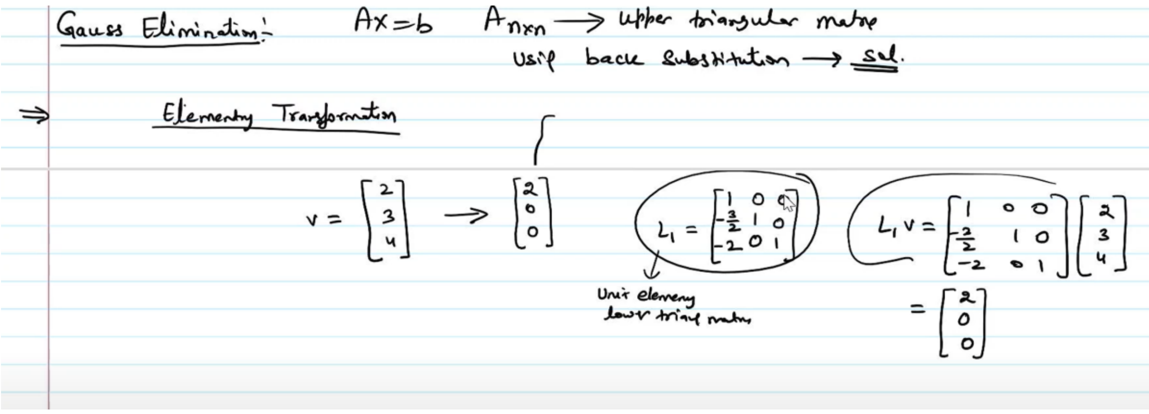

So, we will make such a lower triangular matrix with the help of which we will do all this Gauss elimination. This is a process — we will do it — and the values that we have defined here, the values that I have written here, this is what we call multiplier, this multiplier.

So now, our main purpose will be to solve this. So, let's do that. So, let's start the process of Gauss elimination. So, suppose we have — what I do — I take a four cross four matrix or three cross three, and after that we convert it and see how it will be formed. But we need such a matrix.

So now, see, let me take an example in which I take four cross four elements. So, the equations that we have in the Gauss elimination system — what is it — I have to take four cross four. So, what did I do? I took it like this. So, let's take it like this: a11 x1 plus a12 x2 plus a13 x3 plus a14 x4 = b1 — so the first equation is, okay?

Like this: a21 x1 plus a22 x2 plus a23 x3 plus a24 x4 = b2, such that a31 x1 plus a32 x2 a33, 34, b3, and a41 x1, a42 x2 , a43 x3 plus a44 x4 = b4. So, we have this.

Now, what do we have to do with this? Now, if I want to solve this — so what do I do? A matrix will be formed. I have A whose elements are a11, a12 — sorry — a21, a31, a41, okay? Like this: a12, a13 a14, a22, a 23, a24, a32, a33, a34, a42, a43, a44. This is formed — we have, okay?

Now, the right-hand side vector is b. We have written it as b1, b2, b3, b4, okay? And our x is the vector x1, x2, x3, x4. So, this is what we have to find — the solution. And for this system, we can write this system as Ax = b.

Now, what we have to do is we have to convert this matrix, which is A, into the form of an upper triangular matrix. So, if we convert it into the form of an upper triangular matrix, then we have to go through one step, and this right-hand side vector will also keep on changing along with it.

So now, see what we have to do. In the first step, what we have to do in step one is that we have to make all the non-zero elements — and the elements below it — zero. So, in the first step, we have to do the same thing: that the first non-zero entry in the first column, which is our first element, we have to make all the entries below it zero.

So, for this, what I do is I construct an L1 matrix, that we did earlier also whose diagonal elements will all be one one, and it is lower triangular. So, that is why it should be of four cross four. So, we have to focus only on the first column.

So, what will I do now? See, if I have to make a21 zero, then what will I do with the first one? Okay, which is of whatever thing — the elements which we use are a11 — because we have to make it zero by using a11. So, we name the a11 as pivot, it is very important. And the row in which this pivot is coming, we call it pivot row.

Now, we have to make these elements zero by using the first row. So, what will I do to make this second a21 zero? What will I do in the first row? I will divide it by a11 and multiply it by a21 and put a minus sign and add it to the second one — then it will become zero here. It will also become zero.

Similarly, the third one will become zero. We will do it in the fourth one. The fourth will become zero. So, if we see, I give it a name — multiplier m21. So, I write m21 here, just like we defined it above. So, I wrote it here, like this.

I will get m31, and it will come minus a31, a11. So, this came m31. I will define it in a m41, minus a41 divided by a11 is m41. So, we have placed this multiplier here,L1.

So, what will we do? I will multiply the matrix by L1 from the left side. So, it is left multiplication, and L1b, I will also multiply b by L1 because both sides have to be done. What does it mean? That our system is one. I am multiplying it by the lower triangular L1, which is called left multiplication or pre-multiplication. So, there will be no effect on x. So, the effect will be on this matrix, and it will be on the right-side vector.

So, if we apply this, we will get a new matrix. So, after applying this, I write L1A. From here, I get a new matrix. I name it one, meaning the matrix that I get after step one, and this will be a matrix. And what kind of matrix it will be will be like this. In this way, a11, a12, a13, and a14 will remain the same. It became zero here as well. It became zero here as well.

Now, the main thing is that this too will change. So, this is a22. So, I will apply superscript to it like this: a23, a24. I named it like this because we got new elements: a 32, a 33, a 34, a 42, a 43, a 44. So, our matrix — I gave it a new name, A1.

Now, what do we have to do in this matrix? Now leave this one and this one first — just forget the first row and first column. So now, we get the 3 by 3 matrix. So now, I have to repeat the same process on this matrix.

So now, what do we have to do? We have to make these elements zero — leave this same and make these zero, okay?

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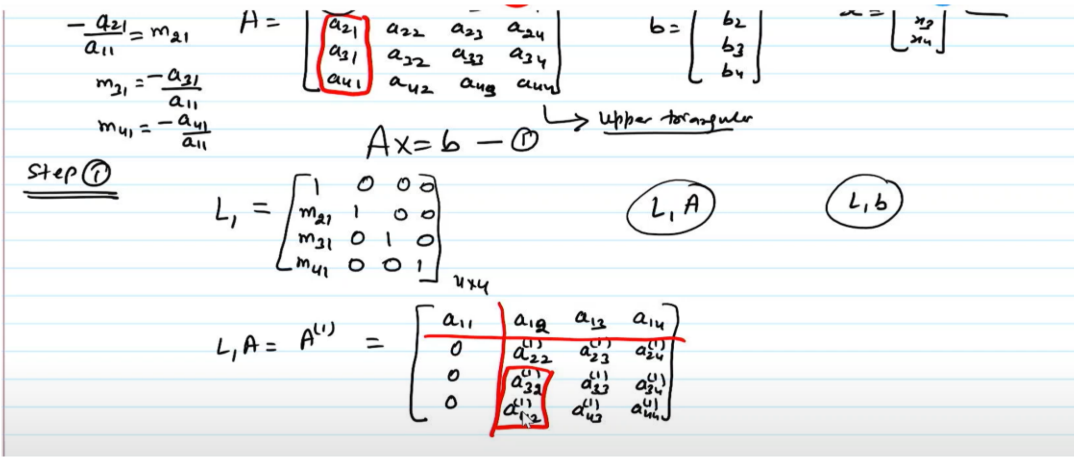

So, what to do to make it zero? Now we will have to find out a new matrix. I will name it L2, okay? So, if I name it L2, then what does L2 mean? Now see, the matrix will be four cross four only, right? So, if we have to define it, okay, then if I have to write it here — so I will write this one one one. It will always be one one only. Okay? Zero here also, zero here also, zero here also.

So, what do we have to do here now? We will get multipliers. How will the multipliers be? m32, m42 — because now we have to put zero here. We only need zero here, at this place, at this place. So, we will get a multiplier of 32 and a multiplier of 42.

Now, we have to define how these multipliers will be. What will be m32? So, what will be m32? If we have to divide by this element, then it will come below a1 22 — and above it, multiply it by a1 32 and put a minus sign in front of it.

Like this, we get m42. So, what will be m42? Minus a1 42 divided by a1 22. So, we get these multipliers.

So, from here we have L2, which is the new matrix — which is our unit element lower triangular matrix. We got that.

So now, our next target here is — this is step two. So, what do we have to do in step two? On this matrix that we have got, we have to pre-multiply A1 by L2. And similarly, I have to pre-multiply it by L2. So, this one that was there — this one — we have named as b1. So, we pre-multiplied it by b, and I have named this matrix. Now, I have given it a new name.

This matrix has been named as A2, and this one has been named as b2. So now, if we look at this matrix, what do we get? So, how will the matrix that we have, A2, be like? Now the matrix will look like — now we have a11, a12,, a13, a14, 0, 0, 0. Here, we have a22, a23. There will be no change in this a24. There will be no change here. It will become zero here. A new element will come here. So, we will write a33. I will put two here, a34. Then I put two here as well. Here I put a2 43 and a2 44. This new matrix has come, and our b2 — what has it become? What has it become? What we had is b1, b2, b3, b4 — because we have taken four elements of b — so b1, b2, b3, b4. So, we have A2.

Now look, we have A2. In this way, now we are converting it into upper triangular. So, if I convert it into upper triangular, then this element, okay, will have to be made zero below it. So now, our next purpose is to make it zero. So now, our job is to make it zero. So, what will we do? We have forgotten all this. We have forgotten all this. Just focus on this 2 by 2 matrix.

So, now the next step that we have is — in step three — we will define a new matrix L3.

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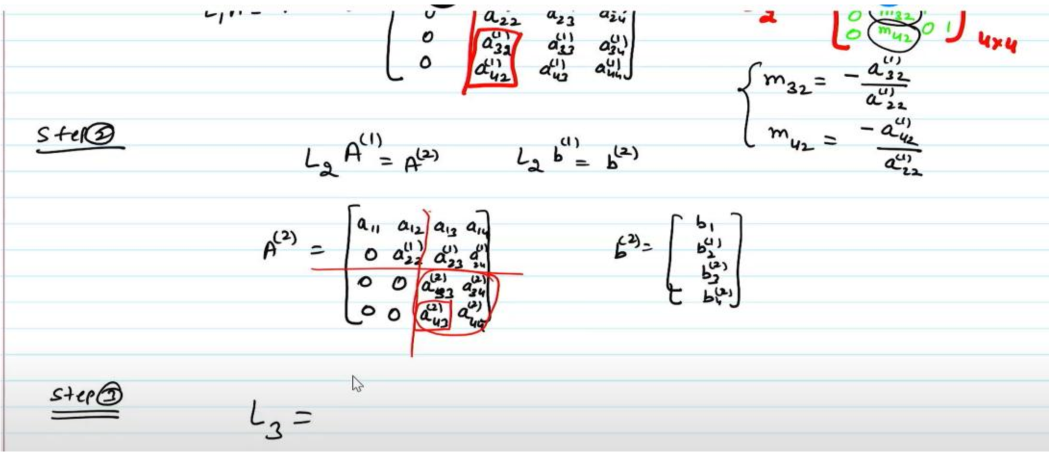

I will define a new matrix whose diagonal elements are all one. It is lower triangular. There is zero here as well. There is zero here as well. If we want this multiplier, then I will name it m43 and what will be the multiplier, which is m43. We have to apply minus then divide by a33 and a43 come on top. Now we have a new multiplier, so we have to calculate this multiplier.

Now what we have to do is — we have to multiply L3 with A2, pre-multiply, we will get A3. In this way, I have to apply it on L3b. So, the new vector that we get — I will name it b3. So now see, what will be the new element that we have now? The new matrix that we have will become A3 in this case. Now a11, a12, a13, a14, 0, 0, 0, a22, a23, a24. Now here we have 0, 0. Now here we have a33, which was in the second step. It became zero, a34, and here comes a44, which will come to us in the third step. And b3 becomes b1, b2, b3, and b4 after the third step.

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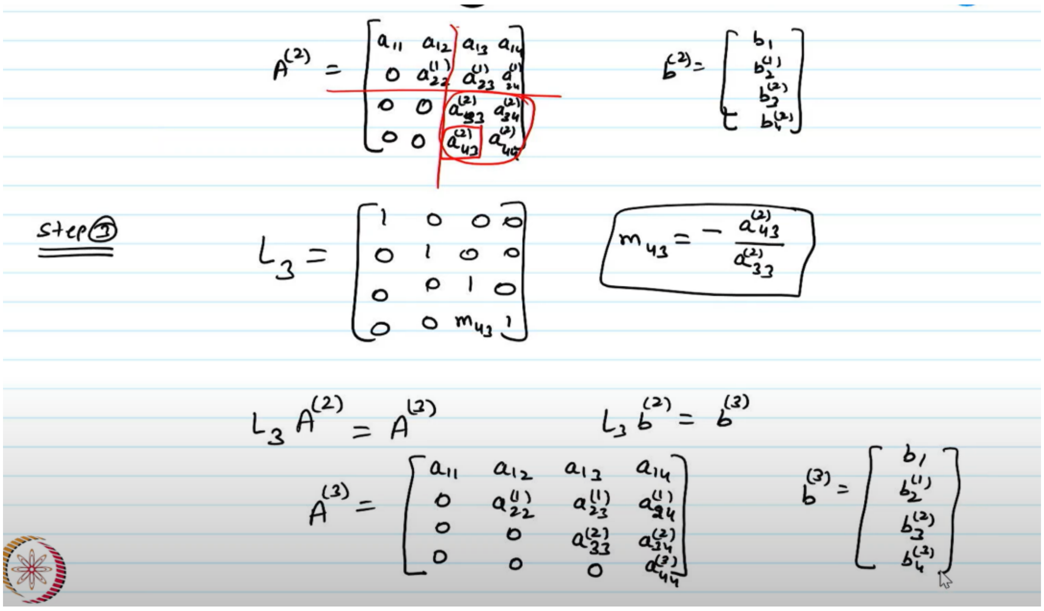

So look, the matrix that we have is this matrix. Now, if we see, what do we have? This is an upper triangular matrix. So, we have an upper triangular matrix. Now, how do we solve this?

Now see, it will be solved easily. Now, what will I do? A3 x =b3 will solve this. Now see, if I do this, in the end, we will get an equation — a33 44. This element, I will multiply it by x4, and it will come on the right side — b4, which is this element. From here, we got x4. We got this.

Now, I will use x4 in this. From there, we have a — see, what did we get from x44? Now, if we go above this and look in this row, it will come to us — a2 33 x3 plus a34 x4. So now, we have got x4. So, from here, we will get x3. Then, in the same way, we will get x2. Then, we will get x1.

So, what did we do? First, we took out x4. Then, we took out x3. Then, we took out x2. Then, we took out x1. So, we followed this process from the bottom. So, we call this process back substitution. Back substitution — because we are first finding x4, then using it we are finding x3, then using them we will find x2, then we will find x1, and we will get the solution.

So, this process which we can do for any n dimensional matrix — it is not necessary that we have only 4 by 4 matrix. Now, we can have 10 by 10, 20 by 20, anything — and very easily, we can calculate it according to this algorithm.

So, if we see, what have we done here? See — we took the matrix A, and on it we applied L1, then applied L2, then applied L3 in this case. And what is here? And from here, we got U. And U, we got is upper triangular matrix, right? And y, which is L1, L2, L3 — all these three matrices are lower triangular and with unit element at the diagonal elements.

So, if you see, l3, L2, L3 — which.

This is a lower triangular matrix, so we know that if we want to find the determinant of this matrix, then it is the product of diagonal elements, right? So if I want to find the determinant of its L3 here, then the result will be one. The product of diagonal elements is one. So the product of diagonal elements is one. So the whole is non-singular. And we know that the product of two lower triangular matrices, their product is also a lower triangular matrix. So if I do L1 into L2 like this, the product of both will also be a lower triangular matrix. So the lower triangular matrix will be one. Their diagonal will also be one. So in this case, the whole lower triangular matrix is non-singular and their product is also lower triangular.

So if we see from here, the matrix that we get is also a unit lower triangular matrix whose dimension is 4 cross 4. So we can solve it like this and we will get the solution of the system. So we will use this method. It is called Gauss elimination method and you have used this type of method a lot before, and we can solve it with the help of computer also. So we will also make a Python code for it that shows how to solve it.

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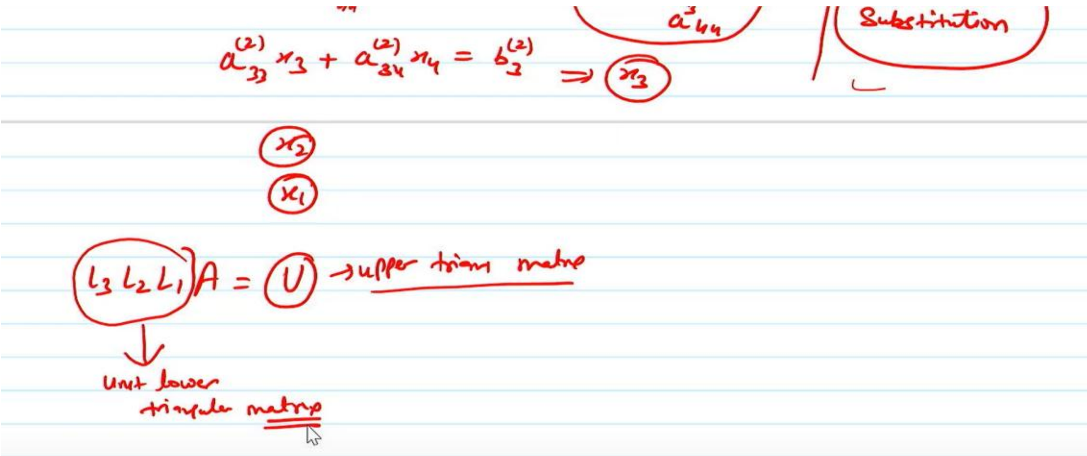

So now let's take an example. From this example, we will get to know how to do it. So now we have a matrix and we have solved it. Let's take an example. Now what did I do? I took a system. This is x1+ 2 x2+ 3 x3+ 4 x4 = 10. In, after that, I took 5 x1+ 6 x2+ 7 x3+ 8 x4=26, after that I take x1+ x2+ 3 x3+ 3 x4= 8. And in the last, I took 2 x1+ x2+ x3+ x4 = 5.

This is okay. So we have defined a system. Now what do we have to do with this? We have to solve it using Gauss elimination. So we will solve it using Gauss elimination. So what did we do? Let's do it quickly. Let's apply step one.

First, what is our work in step one? So let's first write the matrix. So our matrix is 1, 2, 3, 4, 5, 6, 7, 8,1, 1, 3, 3, 2, 1, 1, 1. And the vector on the right side is 10, 26, 8, and 5.

Now what do we have to do? The first step is to define L1. So let's see how to define it. 1, 1, 1. Now what do we have to do? We have to apply a multiplier here. So what do we do? This is already 1. This element is okay, so we will have to divide it by 1, so no problem. And we will have to multiply it by minus 5. So its multiplier here, -5 is coming, here -1 is coming, and here -2 is coming, okay?

So this is our matrix L1. We got this. It means our multiplier was m21 is -5 . m31 is -1. M41 is -2. So this multiplier is here. So we will use this multiplier.

So what did we do? L1 A. So what we have is this. We have it. And the A we have, this is it. So let us write it one more time because by writing it again and again, this and it comes here: 1, 2, 3, 4 then 5, 6, 7, 8, 1, 1, 3, 3, 2, 1, 1, 1. If we solve this, then it will come 1, 0, 0, 0.

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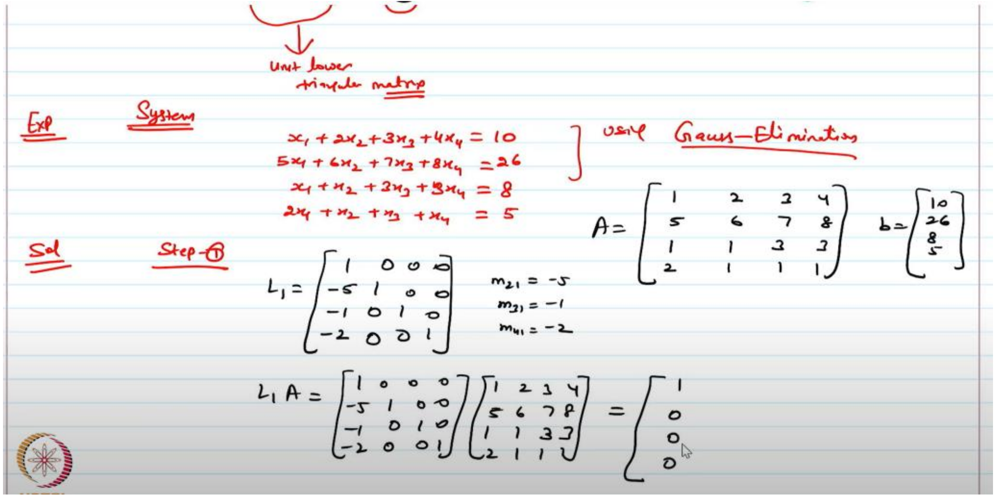

See, the first one has come because we will multiply it by this. We will multiply it by this, so all these values are zero and the same. So this first row will come same: a, 1, 2, 3, 4. This is it.

If I do this from the first column to the second row, then see, it will come: 1 - 5 + 5 = 0. I did this with -1 + 1 , -2 + 2. So all the values that we have are like this and comes to be zero. Now let us see what will this come, two two, if we multiply the second one by the second, then minus 2 minus 10 plus 6 to come minus 4. How much will it come? Minus 15 and plus 7 ,minus 8 and -20 and 8 will be - 12. So this is what we have. Okay?

So if we keep solving it like this, then it will come to us in this case: -1, 0, -1, -3, -5, -7. This came to us and I named it, A1. And now we should also see what will be the L1 b.

So the L1 b, which is, we will write it now like this: -5, 1, 0, 0; -1, 0, 1, 0; -2, 0, 0, 1. And b came: this, 10, 26, 8, 5. This came. So if we calculated it, then the first one will remain the same: 10.

Now we will do it here. See how much is coming. Minus 50 plus 26, -24 comes. Then we did it with minus 10 plus 8 = minus 2. And from the last one: minus 20 plus 5 = -15. So this is what we have, and I named it b1. Okay?

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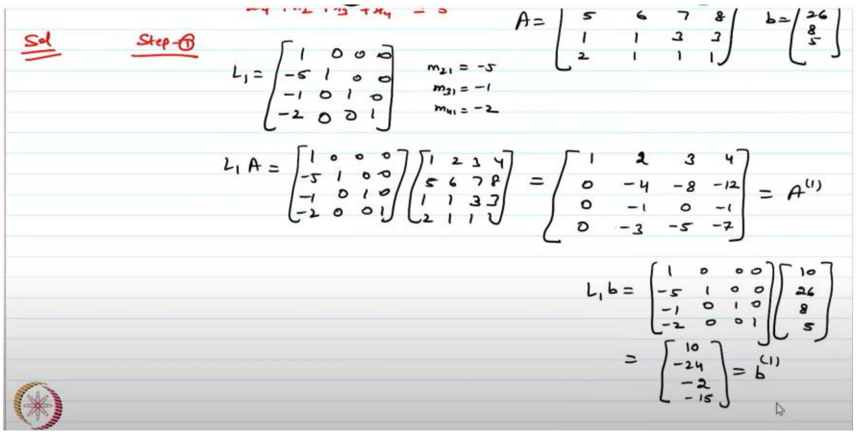

So then we have to do the next step. The next step we can directly write: step two. In this case, so L2 will be ours. Now what do we have to do in this matrix which has come? If we see, we have to make these zero. Okay? So what will happen to our L2? , 1,1,1,1. All of these will be zero. This will also be zero. This will also be zero. It will just come here.

Now see, if I have to make it zero, then first I will divide this row by minus 4, okay, and multiply it by minus one, whatever element comes, and take its negative. So if I want to take that negative, I can write here, okay? Then it will come to us here, minus 1 by 4 will come, okay, and here minus 3 by 4 will come. We have L2.

So as soon as I write L2, now what do we have to calculate? L2 A1, okay. So L2 A1, if I calculate, here is L2 and A1 is this, then we will pre-multiply it. So if I want to do this, then I will get the value of A2. This will come. I will directly write: 1, 2, 3, 4, 0, -4, -8, -12, 0, 0, 0, 0. Zero will come here and it will become 2, 2, 1, 2. This we have it, okay. So this L2A1 and this has come to us, A2.

Like this, we will get b2. If we look at b2, what are we doing? We will multiply this b1 by it. Now let's see, 10 will remain the same. -24 will also remain the same. We will multiply it by this. So here it will come. This multiplied minus 1 by 4, so it came to 6, and -2 came to be 4. And what happened here, this gave 18, -53 came. So we have b3. So we took out b3 from here.

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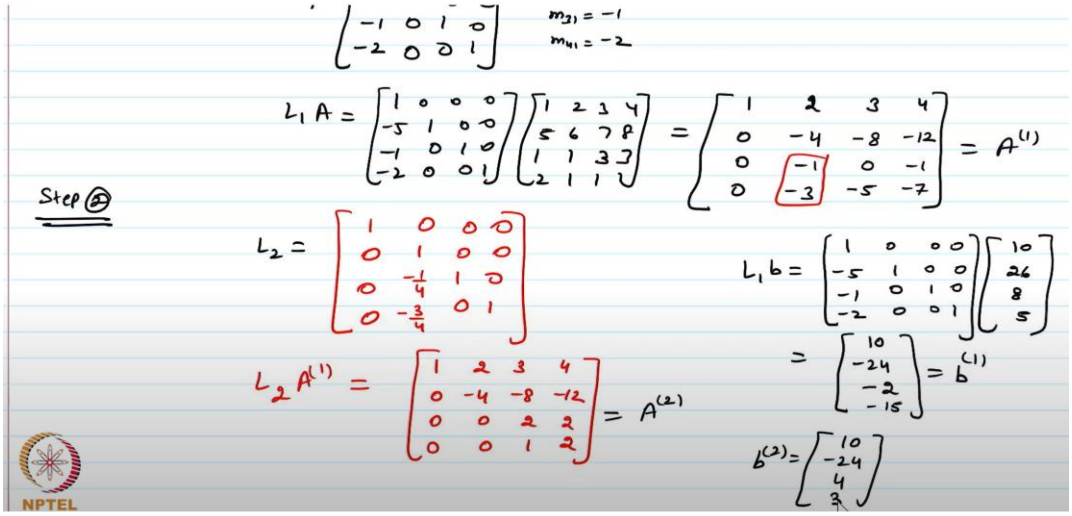

Now the last step is left. So in this step, you see, we have to make this zero here. In this case, L3 will become simple in this case, 1,1,1,1, zero will come everywhere. We just need a multiplier here. So what are we doing? We will divide it by 2 and from minus 1. So minus half, okay. So what will come here? Minus half will come and I will pre-multiply it by A2.

So what we have in the final is: 1, 2, 3, 4, 0, -4, -8, -12, 0, 0, 2, 2, 0, 0, 0, 1. It will come one here, okay. And the b3 that is left with us on the right-hand side, what will come: 10, -24, 4. Now I will pre-multiply this by L3, okay. So what will come here? Multiply 4 by minus half, so we will get -2 + 3 = -1, okay. One comes, okay. -2 plus 3 is 1. So this is what we have. So now on this, we have come to know that our matrix, this matrix has become upper triangular. Now, if we solve it, then what will we get after solving it.

So let me name it U. So now this has come. Suppose we have U, so this will come: 1, 2, 3, 4, 0, -4, -8, -12, 0, 0, 2, 2, 0, 0, 0, 1, x1, x2, x3, x4. And on the right-hand side, the new vector b3 has come: 10, -24, 4, 1.

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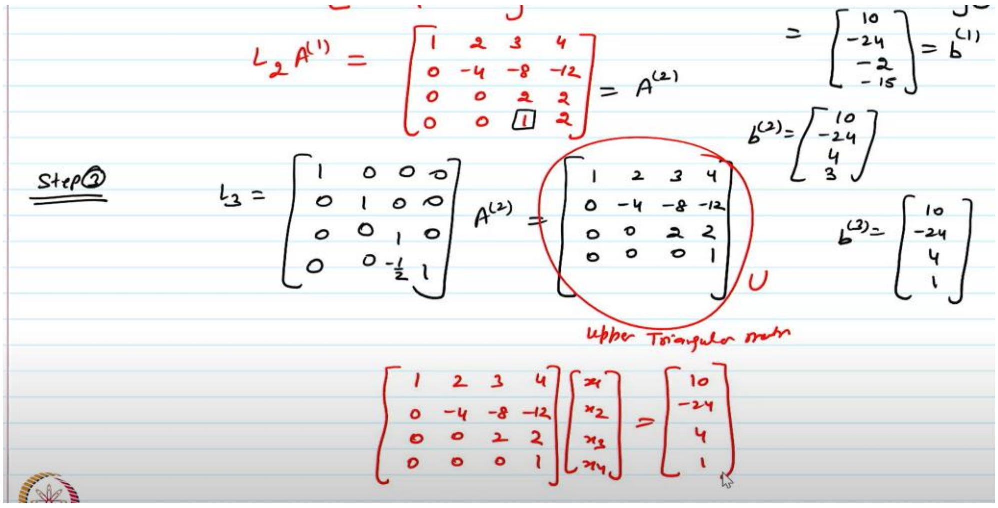

So you see, from this we will solve it from the last one. What came? The x4 that came is 1. From the second last, what came? From the second last, we have 2 x3 + 2 x4 = 4. Now x4 is one, so from here, the x3 which came to us also became one. Similarly, if we did it in the above one, then what will come in this? If we can take four as common, then it will come from here: x1 + 2 x2 + 3 x3 = 6. This came. This one put, we put. This one put from here—sorry—x2, x3, x4. From here, x2 also became one. In the last, when we calculated, then x1 also became one. It means our solution came: 1, 1, 1, 1.

Okay, so from here, you have seen how easily we solved it. So this was the method of Gauss elimination method by applying back substitution. Because in this, we did back substitution, so after applying back substitution, we solved it and the result we got.

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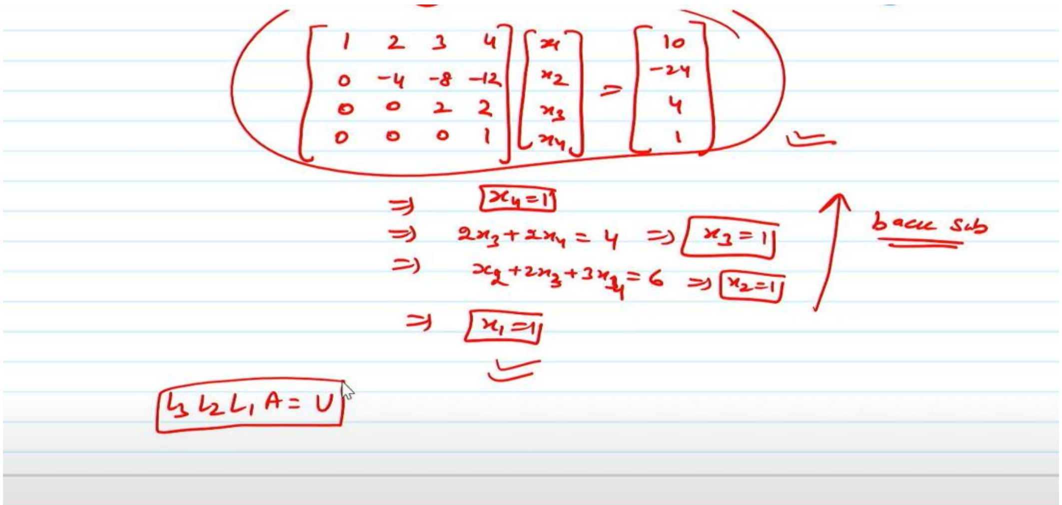

So what has happened in this? Which were matrix: L1, L2 and L3. We applied it on A and we got U, did this, and we will see all the matrices—their determinants are coming out to be 1.

And all the non-singular matrices are non-singular matrices. And the answer we get from our pre-multiplication is upper triangular matrix. That is coming out to be U.

After that, we solved it and our solution came out from there. So this type of thing. So now you see that we applied steps to it. It is a 4 by 4 matrix, so we applied step 3 to it, okay. So if we get a general matrix now—suppose we have to write its code in the computer and our matrix is n cross n, okay—so we will have to do this.

So what will we have to do in this? We will get step 1. In step one, we will do L1 A and from there we will get a11 and L1 b, from there we will get b1, okay. Step two, then we will apply L2 on it. So from there we will get A2, and if we apply L2 b, then from there we will get b2. If you see the step, it will go up to n-1 steps. Because 4 by 4, so it was going up to 3 steps. So ultimately, we will get L. In this, we will apply n-1, A n-2 on it, and this A will become A n-1, and here we will apply L n-1, b n-2, and from there we will get b n-1.

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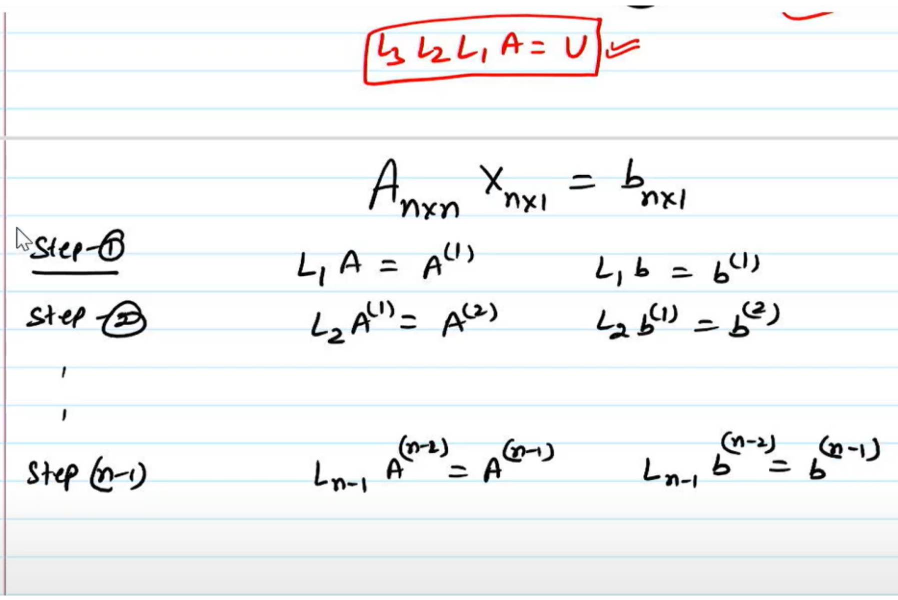

So these many steps will involved in the computer. And then we will get an upper triangular—this UX equals to b—which is our new, which is new. We can write it like this. We will apply it and we will find its solution by back substitution.

So in this case, if we have to do coding in the computer, then we have to do coding according to it so that we can calculate its solution very easily. And this algorithm is fine. Algorithm is needed for programming in the computer. So we do not have to change the algorithm. According to this, if we have to keep doing calculations, then we will keep doing calculations. And we will also write its computer code.

But today we have done it, so let's finish till here, okay? From here, today we did the basis, which was the Gauss elimination method. And we saw how the unit lower triangular matrix is being used on converting that system into the upper triangular form. And then, very easily, we are able to solve by back substitution. So this is the work that we have done. Now, in the next lecture, we will go further from this and will also discuss its Python code.

So I hope you have understood this lecture. Thank you.