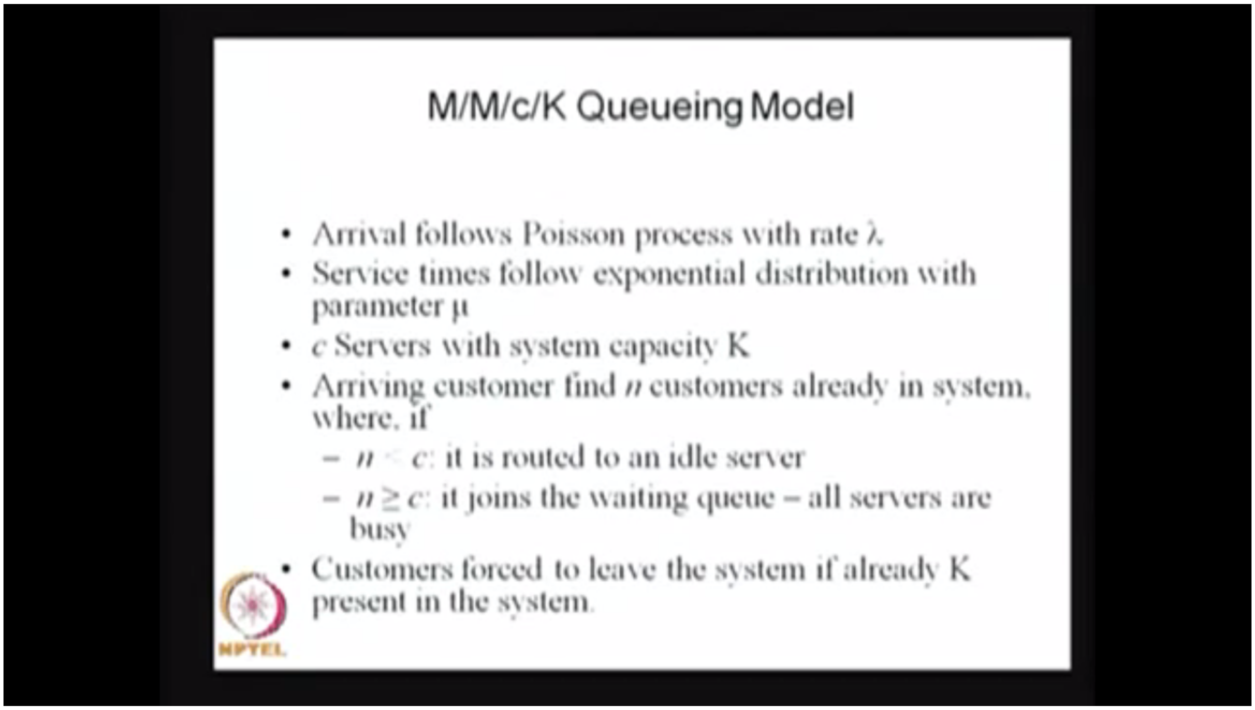
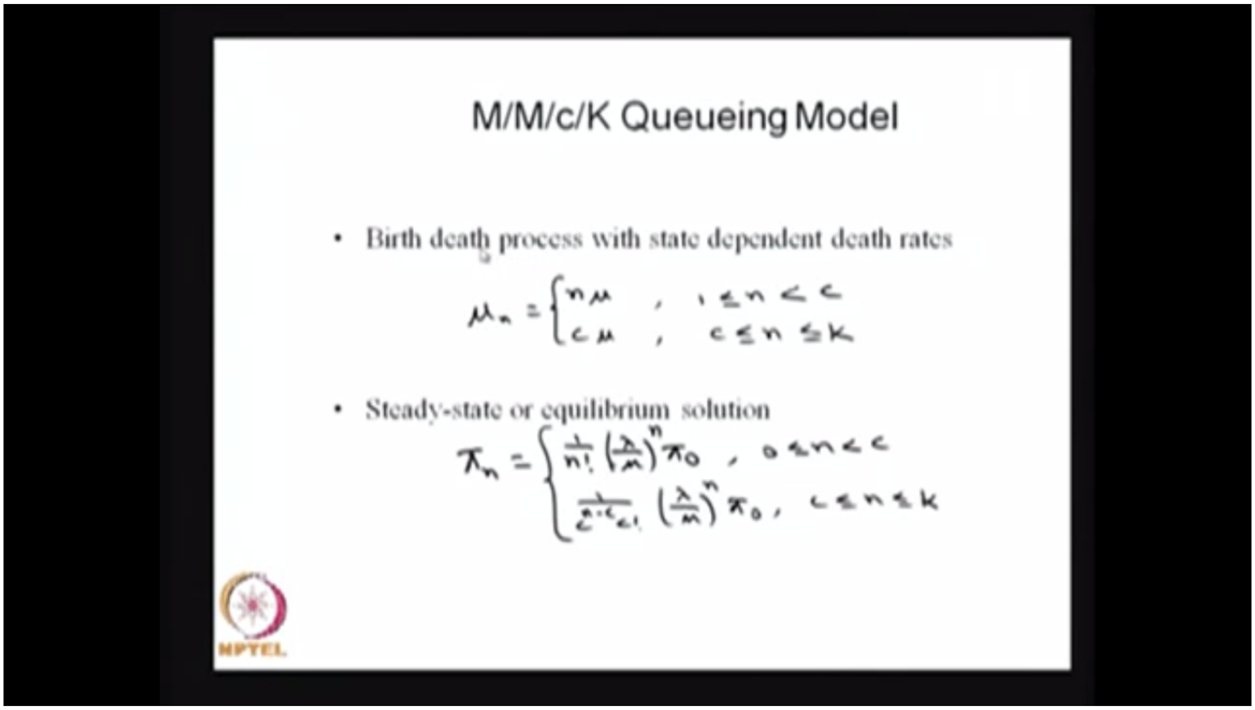
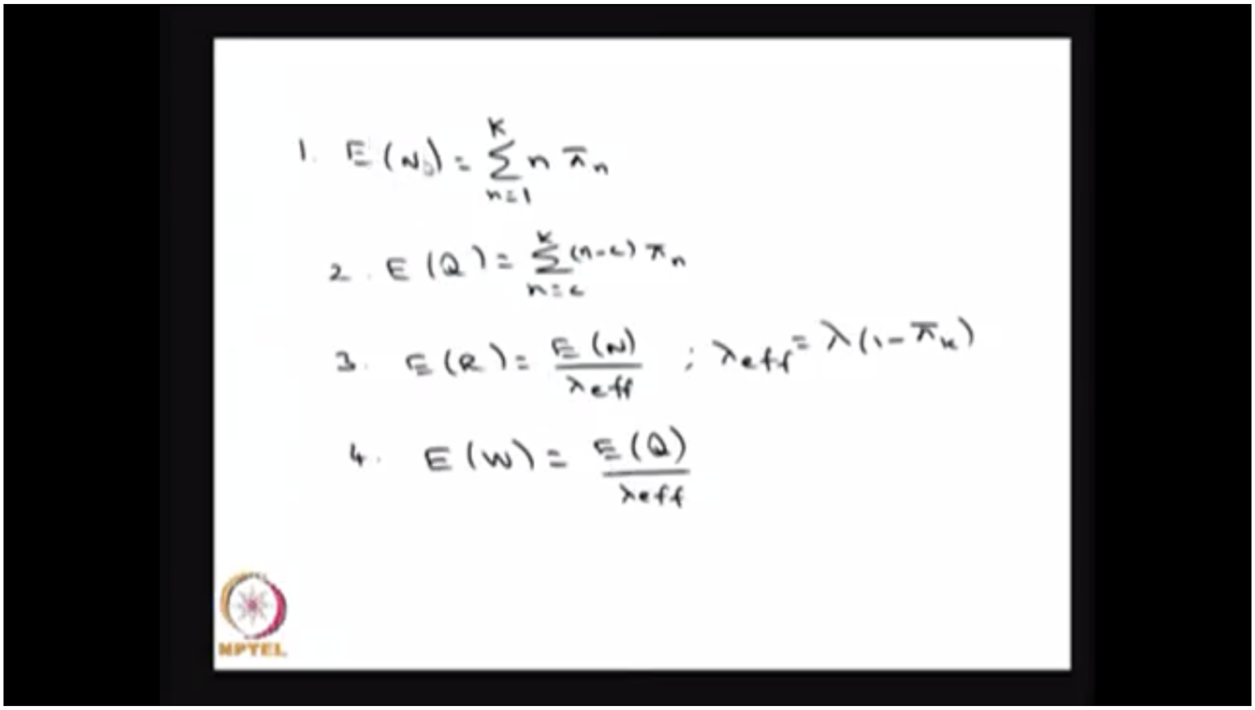


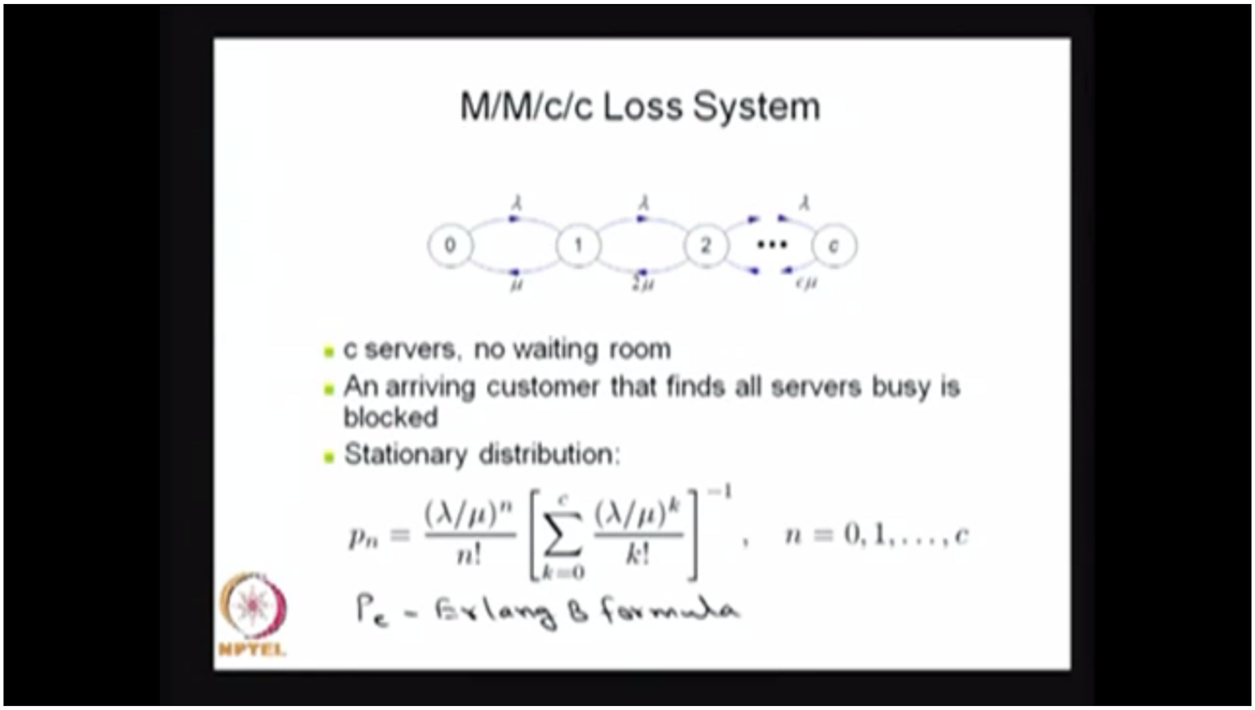
Now I'm moving into MMcK model queuing model. So here the change is instead of one server in the MM1 model you have a more than one servers C and you have a finite capacity that is capital K, capacity of the system so the arrival follows a Poisson process service is exponential. We have c identical servers, the capacity is a capital K and this is a scenario in which a whenever the system size is less than c it will be routed into the ideal server if it is greater or equal to c that means all the servers are busy that means the customer has to wait but if the system size is full that means as c customers are under service and K minus c customers are waiting in the queue for the service then whoever comes it will be rejected, forced to leave the system. Therefore you have a waiting as well as a blocking because it's a finite capacity there is a blocking and since you have always we choose K such that it is K is always greater or equal to c if K is equal to c then it is a last system. If the K is greater than c then K minus c customer maximum can wait in the system in the queue.



Therefore the underlying stochastic process here the stochastic process is again number of customers in the system at any time t. Therefore this stochastic process is also going to be a continuous time Markov chain because of these assumptions inter-arrivals are exponential distributions service, each service by each server is exponentially distributed and all are independent and so on so in these assumptions in this stochastic process is a continuous time Markov chain and at anytime only one forward or only one backward the system can move therefore it is going to be a birth-death process also and the birth rates are lambda because it's a infinite source population. So all the lambda n's are going to be lambda whereas the death rates are state dependent that is going to be n times mu lies between 1 to ceph. From c to K onwards it is going to be cmu. So I have not drawn the state transition diagram for MMcK but you can visualize the way we have a MM1N and MMc model so the combination of that that is going to be the state transition diagram.



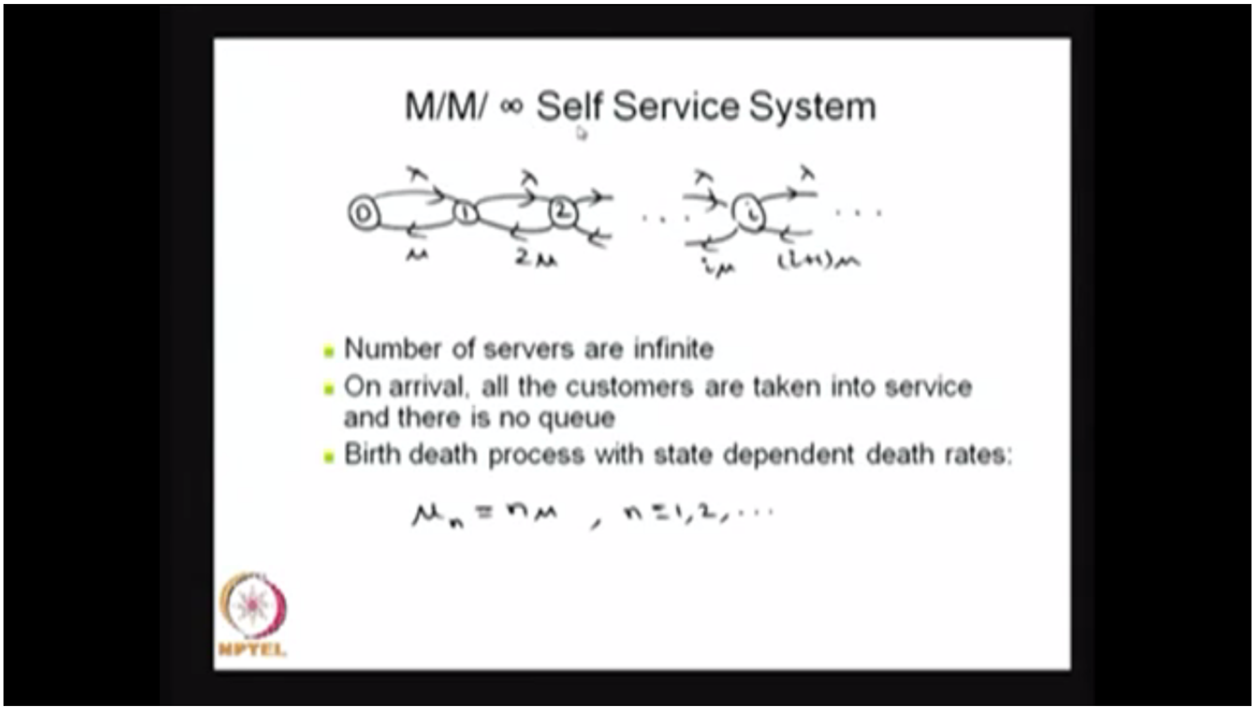
Since it is a finite capacity model it is easy to get the steady state and the equilibrium solution so first you solve Pieq is equal to 0 that means you write the Pien's in terms of Pie naught and use a normalizing constant summation of Pi's is equal to 1 using that you get Pie naught. So I have not written here so you use the normalizing constant the summation of Pi is equal to 1 get the Pie naught then substitute Pie naught here therefore you will get Pien in terms of Pien complete. After that you can get all other average measures the way I have explained MM1N and the MMc Infinity the combination of that you can get the average number of customers in the system, average number of customers in the queue that is a N minus c times Pien the combination, the summation goes from c to K and the average your time spent in the system since it is a finite capacity you have to find out the lambda effective. Effective arrival rate and that is 1 minus it's a capacity is capital K therefore 1 minus Piek that is the probability that the system is not full so the effective arrival rate is lambda times 1 minus Piek substitute here and get the average time spent in the system. Similarly can find out the average time spent in the Q also using the Little's formula.



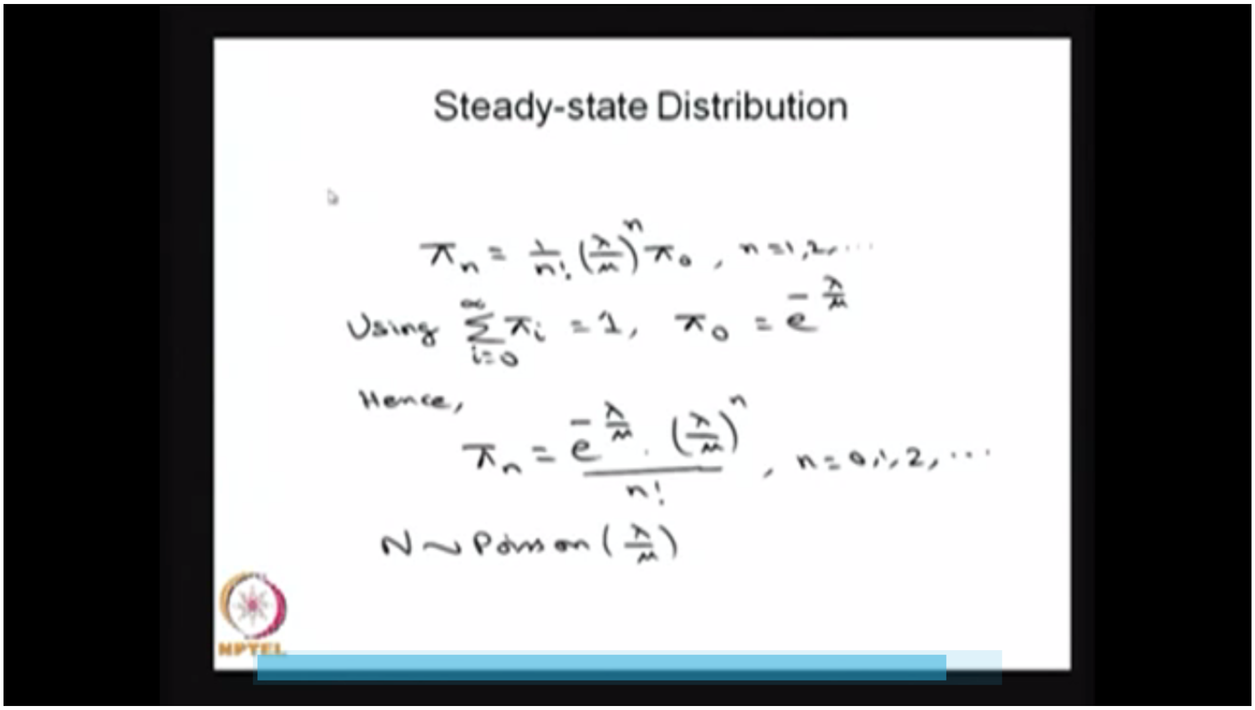
Now I am moving into the fourth simple Markovian queuing model. First I started with a MMc infinity MM1N. Then I did MMcK and now I am going to K is equal to c that is last. It's not a queuing system because you have c servers and the capacity of the system is also c. Example is you can think of a parking lot which has the some c parking lots and the cars coming into the system that is if you make the assumption is inter-arrival time is exponentially distributed and the car spending time in each parking lot that is exponentially distributed then the parking lot problem can be visualized as the MMc loss system.

So here we have a c identical service, no waiting room. So since it is a c capacity and the c waiting room you can think of a self-service with the capacity c that also you can visualize. So the inter – here the inter-arrival times are exponentially distributed and the service by each server that is exponentially distributed with the parameter mu. Therefore, the system goes from 2 to 1, 1 to 0 and so on. It is going to be how many customers in the system and completing the service therefore the time is exponentially distributed with the sum of those parameters accordingly therefore it is going to be 1mu, 2mu till cmu.

Since it's a finite capacity and so on it is a reducible model positive recurrent therefore the steady-state probability exists, limiting probe this also exists and that is same as the equilibrium probabilities also therefore by using a Pq is equal to 0 and the summation of Pn is equal to 1 you can get the steady state or equilibrium probabilities that is Pn's. The piece of Xc that is nothing but the probability that the system is full and that is same known as the Erlang's B formula. So this is also useful to design the system for a given or what is the optimal c such a way that you can minimize the probability that the system is full. For that you need this formula therefore to do the optimization problem over the c and here we denote a piece of Xc that is a Erlang's B formula. Whereas Erlang's C formula comes from the MMcK model for the last system we get the Erlang B formula.



The fifth model that is MM infinity it's not a queuing model because the servers are infinite. Unlimited servers in the system therefore the customer who ever enter he will get the immediately service. The service will be started immediately and as the service time is exponentially distributed with the parameter mu by the each server all the servers are identical. The number of servers are infinite here. Therefore you will have the underlying stochastic process for the system size that is a birth-death process with the birth rates are lambda because the population is from the infinite source, the death rates are 1mu, 2mu and so on because the number of servers are infinite.



So the model which I have discussed in today's lecture all the five models are the underlying stochastic process is the birth-death process. This is simplest Markovian queuing models. We can get the steady state distribution use the same theory of birth-death process and if you observe this steady state probabilities is of the same Poisson this of the form that is a probability mass function of a Poisson distribution. Therefore you can conclude in a steady state a number that is Poisson distributed with the parameter lambda by mu because the probability mass function for the Pien is the same as the probability mass function of exponentially distributed random variable with the parameter lambda by mu.