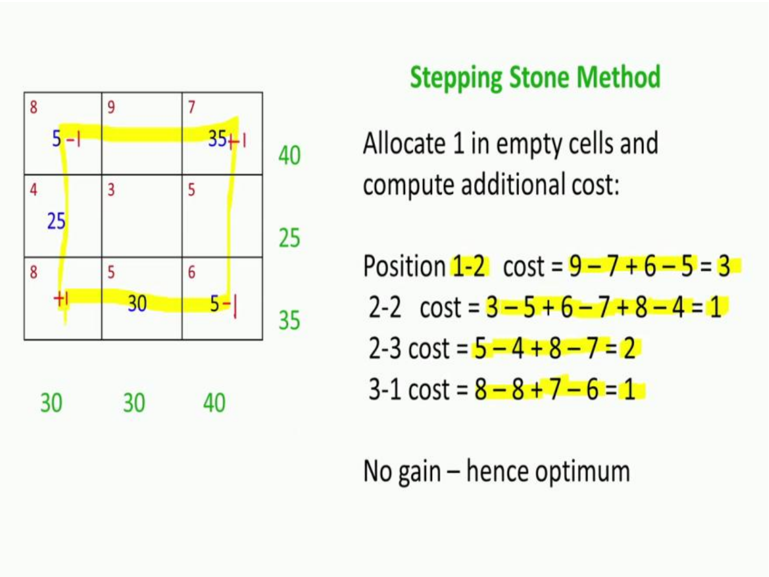
Introduction to Operations Research
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Module - 06
Transportation Problem
Lecture - 04
Stepping Stone Method and Modified Distribution Method

In this class, we will look at methods that provide us the best or the optimal solution. We would also look at methods that tell us how to identify that the best or optimum solution has been reached.

(Refer Slide Time: 00:37)



So, we take one of the starting solutions, this is the solution that is given by the penalty cost method or the Vogel's approximation method. So, this was the solution that we saw in the last class, so this is from the penalty cost or Vogel's approximation. So, let us take the same solution to analyze it further. Now, this is a solution that has been given by the penalty cost method with the cost of 565. Now, only two things can happen, this could be the optimum solution, this is not the optimum solution.

But, at the moment we do not know whether this is the optimum solution or not, but if this is not the optimum solution, then some other solution has to be optimum and that some other solution should have an allocation in at least one of the un allocated positions. Otherwise, given these five allocated positions, this is the best and only allocation we can get. Therefore, if this solution is not optimum then it should have at least one allocation in an unallocated position. So, we will now see whether it is profitable to put something in an unallocated position.

Now, there are four unallocated positions, now the first position is this position which is called position 1 2. Now, let us take this position 1 2 and let say that I want to put something in this, if I put something in this let say I put a plus 1 in this, now I realize that the supply is now 41 instead of 40. So, I have to remove 1 from either this 5 or from this 35, so at the moment I will remove from this 35 so I get this minus 1, so that this has become 40.

Now, what is happened to the demand of the third? I have taken away 1 from this 35, therefore this has become 34 and the demand is only 39. So, I have to add a 1 here, so that I get 40, but then if I add a 1 here then this only 35 is available, but 36 have been allocated. So, I have to take away one from this, so this becomes 29 and plus 1 becomes 30. So, if I put a plus 1 in an unallocated position I have to adjust from the existing allocations such that I balance it out.

Therefore, when I put this plus 1 you would realize that I had a choice of either reducing from this 35 or reducing from this 5 and I choose to reduce from this 35. And when I reduce from this 35 I started from this, I went to this, I went to this, I went to this and I came back. So, I completed a loop, the first one plus 1 I put in unallocated position, rest of the adjustments, where only from allocated positions whereas, if I try this 5 I would not be able to get such a loop.

Now, getting such a loop comes out of working out some number of problems, so to begin with let us take it that this is the loop when we put a plus 1 here. So, when I put a plus 1 here to balance it, I have to put a minus 1 plus 1 minus 1. Now, what is the effect of doing this? When I put a plus 1 here my cost increased by 9, when I took away 1 my cost reduced by 7, when I put a plus 1 the cost increased by 6, when I took away 1 the cost reduced by 5, now that is shown as 9 minus 7 plus 6 minus 5.

So, 9 minus 7 is 2 plus 6 minus 5 is 1 it is 3, so the net cost of putting something in this position is 3. So, if I put a plus 1 in this position my cost is going to increase by 3, and therefore it is not profitable for me to put something in this position. So, now let me look at the second unallocated position which is here. So, I put a plus 1 here I can balance it with the minus 1 from this, again I balance it with the plus 1 from this. Observe that I am making these adjustments only from the allocated positions, I can put a minus 1 from this, I can put a plus 1 minus 1 from this and I can put a minus 1 from this.

Now, it balances out, now you realize that my loop started from here went to this, went to this, went ahead came down, went to this and it has come back, that is my loop. So, the ability to draw the correct loop will come as we solve more number of problems. So, to begin with we will say this is the loop and the effect is an increase of 3 which is shown here, reduces by 5 which is shown here, increases by 6 which is shown here, reduces by 7 because of minus 1 that is shown here, increases by 8 which is shown here and reduces by 4 which is shown here.

So, either way you can move around, you can say the loop is from this or the loop is from the other way it means one at the same, so plus 3 minus 5 plus 6 minus 7 and so on. So, the net result of this is a plus 1 which means if I put 1 unit in this position, my cost will increased by plus 1, therefore it is not profitable for me to put it in this position. Now, let us look at the third position here which is this position, so I put a plus 1 here. So, it becomes a minus 1 here, a plus 1 here and a minus 1 here and the loop is balanced.

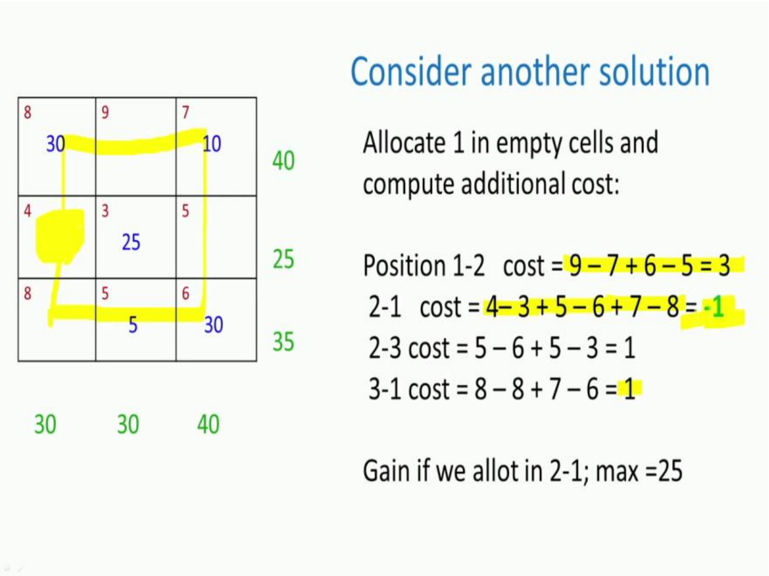
So, the loops started here it went to this, it went to this, it went to this and it came back. Now, what is that effect? This plus 1 as increased the cost by 5, this minus 1 has reduced it by 4, increased by 8 and reduced by 7 and the net cost is 2. So, if I put a plus 1 in this unallocated position my net cost is going to increase. I cannot take away one from this to decrease the cost, because there is nothing in that position, therefore I cannot consider that position.

So, again I go back and I look at the last position here, this is the last position. So, I try to put a plus 1 here, so I try to put a plus 1 here, so I get a minus 1 here, I get a plus 1 here, I get a minus 1 here and it balances. Now, the loop here will be starting here, it goes to this, it goes to this, it comes back. Always remember, the plus 1 is put in the unallocated position and all other adjustments are done in the allocated positions and it has to balance out.

So, the net cost is a plus 8, because of this plus 1 minus 8 because of this minus 1 plus 7 because of this plus 1 and minus 6 because of this position, the net cost is 1. Again it is not profitable to put anything in this position, because it will only increase the cost. So, all the four unallocated positions we have observed that it is not profitable to put one there, therefore the algorithm will now say the present solution is indeed optimum.

Because, if it were not optimum then at least one unallocated position it should be profitable for us to allocate and that did not happen, therefore this solution with 565 is the best solution, now let us move further.

(Refer Slide Time: 09:43)



Now, let us go back and look at, so whatever I have explained is now shown once again, so now, look at another solution. Now, let us look at a solution which is given here 30, 10, 25, 5 and 30 let us check whether this is the solution from the min cost method. So, this had 30, 10, 25, 5 and 30 with cost equal to 590. So, this is the solution that we obtained using the min cost method, now this had a total cost of 590, because we already found out that the optimum is 565 we know that this is not the optimum solution.

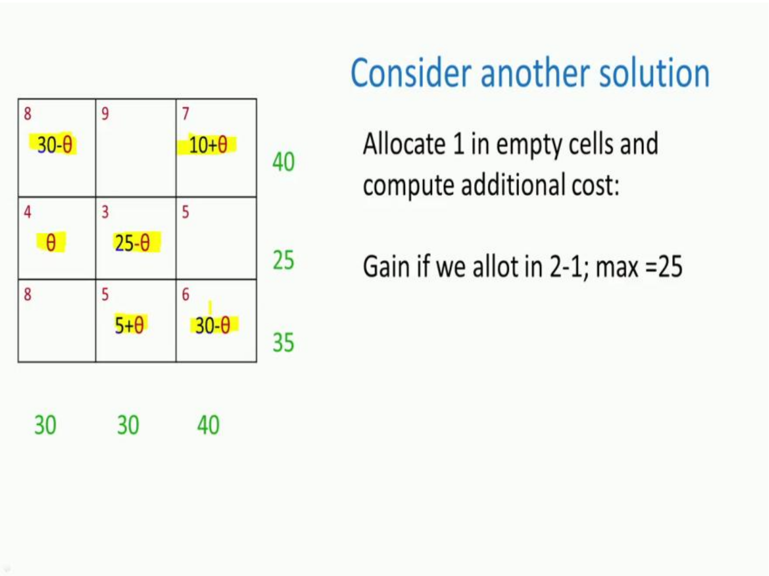
Now, we apply the same idea, if this is not the optimum solution then putting a plus 1 in an unallocated position should give us a gain, so we try to do that. So, we start by looking at this position, we put a plus 1 here, so this becomes minus 1, this becomes plus 1 and this becomes minus 1. So, the net effect of this plus is 9 minus 7 because of the minus 1 plus 6 minus 5 which happens to be 3.

Now, we try the next position which is here I put a plus 1, so when I put a plus 1 I get a minus 1, I get a plus 1, I get a minus 1, I get a plus 1 and I get a minus 1 to balance it. Now, the net effect of this is plus 4 which comes from here, minus 3 which comes from here, plus 5 which comes from here, this 5 minus 6 because of the minus 1 which comes from here, plus 7 which comes from here and minus 8 which goes from here. So, 4 minus 3 1, 1 plus 5 6, 6 minus 6 0, 0 plus 7 7, 7 minus 8 is minus 1.

So, we observe that putting a plus 1 here can actually give us a gain and can reduce the cost. Now, we try other positions to see whether we can get a further reduction, so to do that we come back. So, if we try in this position the third position, if we put a plus 1 in this position, the minus 1 will come here, the plus 1 will come here, the minus 1 will come and we observe that the cost are 5 minus 6 plus 5 minus 3 the net cost is a plus 1, therefore it is not profitable.

And if you put in the last position, we also observe that if you put last position this will be our loop. So, it will become a plus 8 minus 8 plus 7 minus 6 which is again plus 1. So, out of the 4 unallocated positions, we now observe that putting something here can give us a benefit and that benefit is this minus 1.

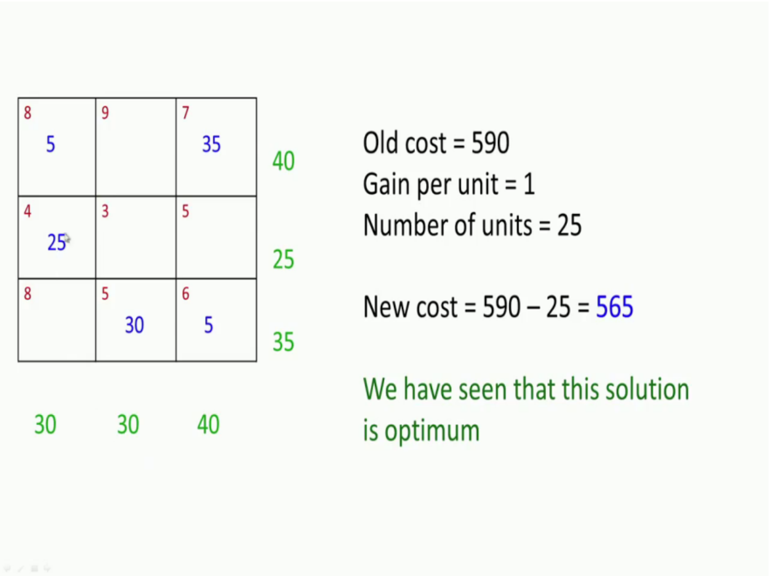
(Refer Slide Time: 13:30)



So, we now move on to see how much we can put in that position, so to find out how much we can put in that position, we now say let us put theta in that position. So, if I put theta in this position to balance it this become 25 minus theta, this becomes 5 plus theta, this becomes 30 minus theta 10 plus theta and 30 minus theta. Now, what is the I know that when I put a plus 1 I get a benefit, so I want to find out how much I can put such that I get as much benefit as I can, so that is given this theta.

So, I keep putting theta equal to 1 theta equal to 2 and so on, so when I put theta here 25 minus theta as I keep increasing this theta 25 minus theta is going to reduce 2 0 and it is going to come 0 first. So, 30 minus theta 30 minus theta 25 minus theta, so when theta is equal to 25, this will become 0. So, this will become 25, this will become 0, this will become 5 plus 25 30, this become 30 minus 25 5, 10 plus 25 35 and 30 minus 25 which is 5. So, theta will be equal to 25 which is the smaller of the three values that theta can be get when we said those with minus theta to 0, so we do that.

(Refer Slide Time: 15:07)



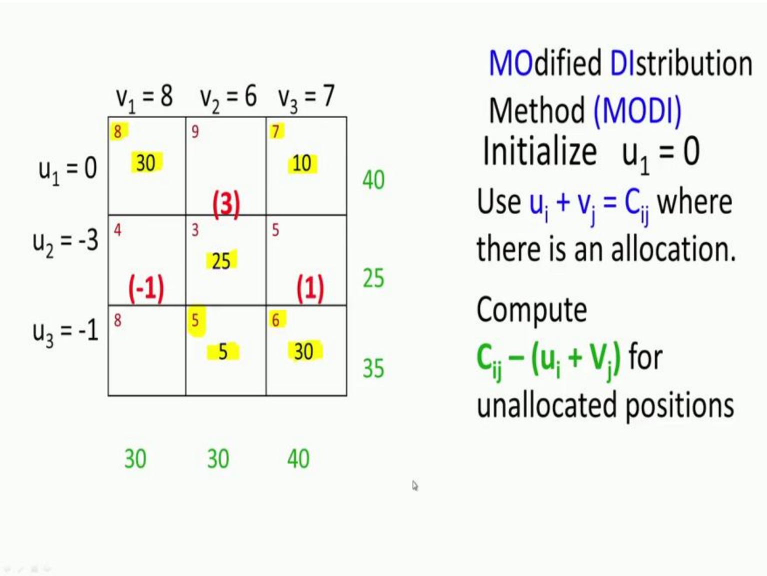
So, now we realize that this has become 25, this has become 30, 5, 35, now we observe that we have this solution. Now, again we also observed that the cost associated with this is 565, now we can we can actually show this 565 from the previous solution of 590 ((Refer Time: 15:32)) which was this, we said the unit gain is 1 or gain is minus 1 minus 1 represents the gain that decrease in cost, the quantity that we were able to put there was 25. So, per unit put the gain is 1 for 25 units the gain is 25, so 590 minus 25 is 565.

Now, to check whether this solution is optimum, we have already seen that this is optimum, but we have to now go back and put a plus 1 here and observed that the cost increases put a plus 1 here observed that the cost increases put a plus 1 here, put a plus 1 here which we have already done, and therefore this is the optimum solution to this problem, now this method is called the stepping stone method.

Now, the stepping stone method requires a starting solution and that starting solution could be from north space corner or min cost or penalty cost or Vogel approximation, given a starting solution the stepping stone method will check whether putting something in an unallocated position can give us an advantage. If there is an advantage, then we will compute the theta the maximum quantity that can be put and implement the stepping stone idea on the new solution.

And this will go on till we observe that for a given solution putting it in the unallocated positions all of them do not give us again, and that is the point the optimum solutions is reached; and for this example, the optimum solution gave us 565.

(Refer Slide Time: 17:23)



We will now look at another method which we will show to get the optimum solutions, this is different from the stepping stone method, this method is called MODI method or MOdified DIstribution Method. Now, this also requires a starting solution, just as stepping stone required the starting solution. So, we look at this solution the same starting solution with which we started one of the stepping stone methods, now this is a starting solution that we have.

Now, with these solutions we are now going to do something else, we have now going to associate a tom with every supply and every demand. So, there are three supply points, so we are going to define a u 1, u 2 and u 3; there are three demand points we will define a v 1, a v 2 and a v 3 and we begin with the initializing u 1 equal to 0. Now, when we initialize u 1 equal to 0 and try to find out use u i plus v j is equal to C i j, where there is an allocation.

So, what does that mean, now there is an allocation here, there is an allocations here u i plus v j, u 1 plus v 1 is equal to C i j which is 8. So, u 1 is 0, so 0 plus v 1 is equal to 8 and that would give us v 1 is equal to 8, now u i plus v j equal to C i j, now here there is another allocation the u is known. So, u i plus v j is equal to C i j u 1 plus v 3 is equal to 7, 7 is your C i j, u 1 plus v 3 is equal to 7, u 1 is 0, therefore v 3 is 7.

Now, that we have found v 3 come back and realize there is an allocation here, there is an allocation here, now for this allocation u i plus v j is equal to C i j. So, u 3 u i u 3 this a third row, so u 3 plus v 3 is equal to C i j is 6, so u 3 plus 7 is equal to 6, therefore u 3 will become minus 1. So, we will now put u 3 will become minus 1, now that we have u 3 and there is an allocation here, so we go back there is an allocation here. So, u i plus v j is equal to C i j which is this 5, so u 3 plus v 2 is equal to 5 minus 1 plus v 2 is equal to 5, therefore v 2 is 6.

Now, having put v 2 there is an allocation here, therefore go back u 2 plus v 2 is equal to 3, u 2 plus 6 is equal to 3. So, u 2 is equal to minus 3. So, we have now calculated all the u’s and the v’s using this and it is possible to calculate all the u’s and the v’s, if we started with our good starting solution. Now, we have to check something whether it is advantages to put something in these unallocated positions, we have use the allocated position to get u’s and the v’s now we want to see the un allocated position.

Now, for the un allocated position compute C i j minus u i plus v j this is an un allocated position, this is an unallocated position. So, C i j is 9 minus u i plus v j 0 plus 6 9 minus 6 is 3 C i j 4 minus u i plus v j minus 3 plus 8 is 5, 4 minus 5 is minus 1 C I j’s is 5 u i plus v j is minus 3 plus 7 which is 4. So, 5 minus 4 is 1 C i j is 8 u i plus v j’s is minus 1 plus 8 7, so 8 minus 7 is 1, now these represents the gains. Now, we observe that there is a negative here, so there is a gain and we can put something in this position. Now, how much we put and how we proceed from here we will now see in the next class.