Selected Topics in Decision Modeling
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Lecture – 38
Particle Swarm Optimization

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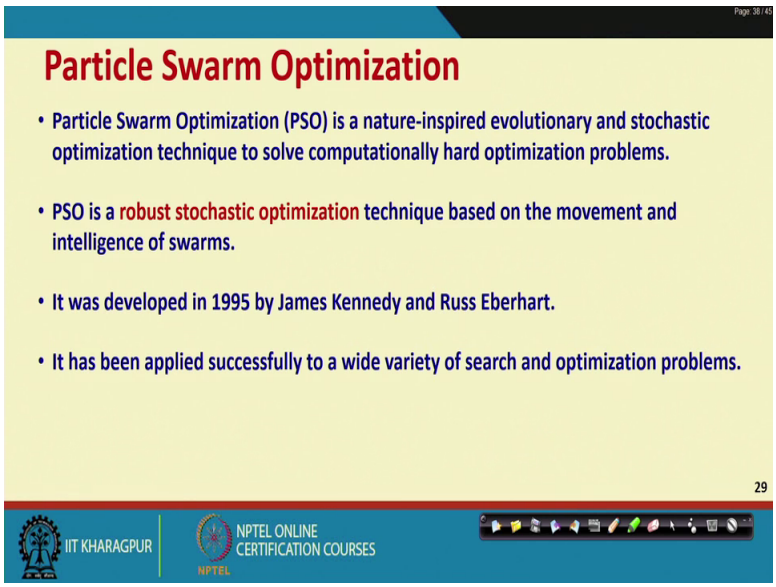

So in our course Selected Topics in Decision Modeling we are in our 38th lecture that is on Particle Swarm Optimization technique. Now, particle swarm optimization is a nature-inspired evolutionary and stochastic optimization technique to solve computationally hard or difficult optimization problems.

PSO as in short we call particle swarm optimization is a robust method for optimization based on movement and intelligence of swarms. Now, what is that robust, robust usually means that means the method works and you know in many different conditions; it really solves problem in all different situations. It was developed by James Kennedy and Russ Eberhart in 1995 and has been successfully applied in a wide range of search and optimization problems.

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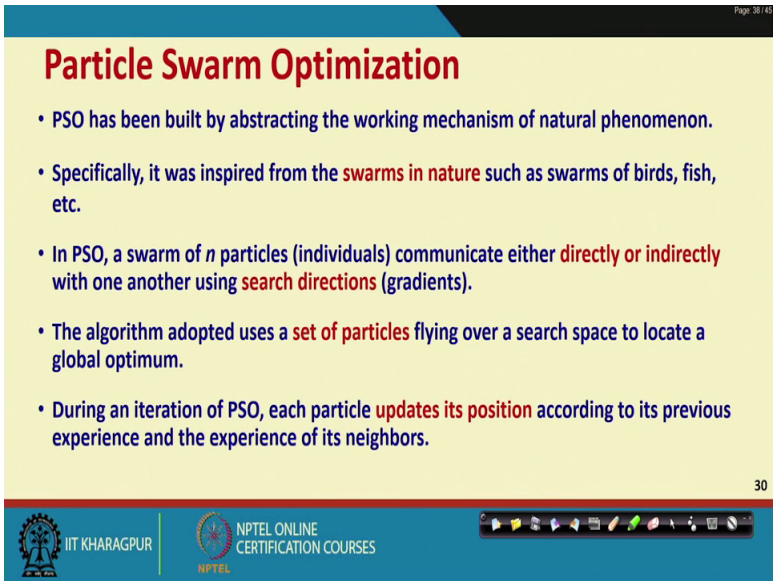

Now, it really abstracts the working mechanism of swarms, like swarms of birds, fish etcetera. What really happens in this method we take a set of particles right. So, supposing we have a solutions space. So, within that solution space, we take several particles and each particle actually represents a particular solution right. And let us say each particle is searching for the optimum solution and you know finally therefore when all the particles finally come to you know certain points around the best possible solution, then slowly we see that the you know the iteration really trying to reach to the best possible solutions.

The question is how it actually happens? How it happens that each particle is continually updating its position according to its previous experience and the experience of its neighbors. That means, a given particle see that is what the birds and fishes actually do when they move in swarm, not only each bird is trying to move based on what that particular bird things is the best direction, but also what the group things as the best direction. So, it takes you know feedback from both the global best and both the local best. So, it is a combination of search for the local best and the search for the global best.

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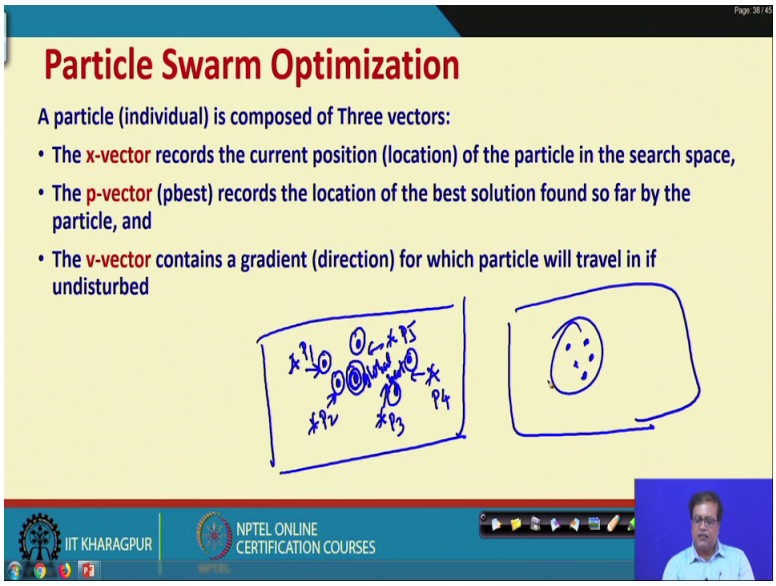

Now, there are three vectors. The first each particle is composed of three vectors, the x-vector, the p-vector and the v-vector. The x-vector records the current position of the particle in the search space. The p-vector records the location of the best solution found so far by the particle and the v-vector contains a gradient for which particle will leave travel if undisturbed; so that means for every particle we have a velocity vector, a position vector right, and you know the p best that means, the best possible solution. So, there is a particle position and the best possible solution that this particular particle is trying to reach you know out of these so that is the also is also recorded.

So, how exactly it happens we shall see through a an example and all these vectors are continually upgraded. Is it all right? So, the positions, the velocity, the best possible solutions - the global best and the local best, they are all continually updated. So, see try to understand suppose this is our solution space and supposing I have my five different particles see P 1, P 2, P 3, P 4 and P 5.

Now, look here. So, each particle is having a velocity and it is trying to move in search of a certain solutions. So, what are those solutions? One is for each particle, there is a local best and there is a global best. So, you see this is the global best. The to the global best every solution is trying to move, but individual particles they are trying to move also to a set of local best. So, you see these are all local best and these one is the global best. So,

each particle is having a velocity from its current position trying to move to a combination of the local best and the global best solutions.

So, what happens after sometime, you may find the particles have narrowed their gaps right, they have narrowed their gaps, and they are very near to the global best solution, is it all right. So, this is the essential idea of the particle swarm optimization we have a number of particles which covers the entire solution space; each solution is trying to move you know in a certain way, so that it tries to reach the global best for the entire group and the local best for its own. So, that finally, it tries to get to the best possible solution as you know quickly as possible right.

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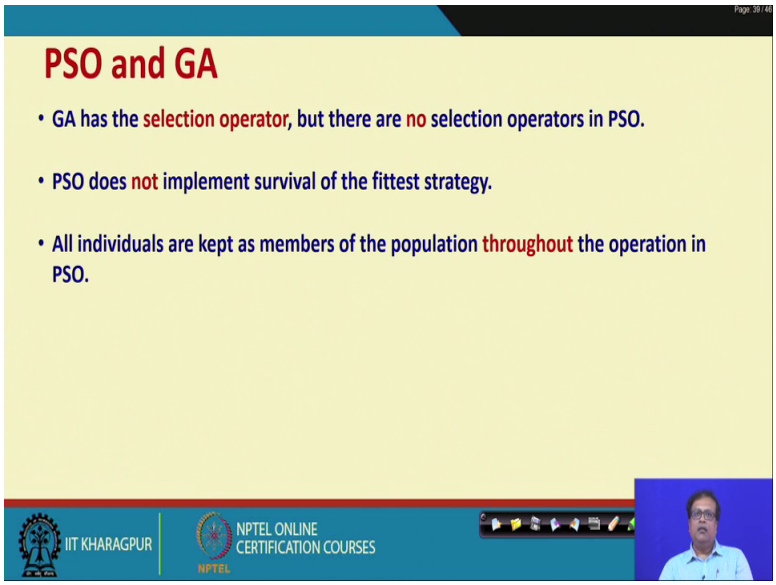

So, PSO and GA, while GA has genetic algorithm has selection operator, PSO does not have. So, there is no selection operator in PSO. PSO does not implement the survival of the fittest right there is no selection crossover etcetera. All individuals are kept as members of the population throughout the operation of the PSO. So, each individual is available, but obviously, those individuals as they move you know they are actually going to a new solution that is what is happening in PSO.

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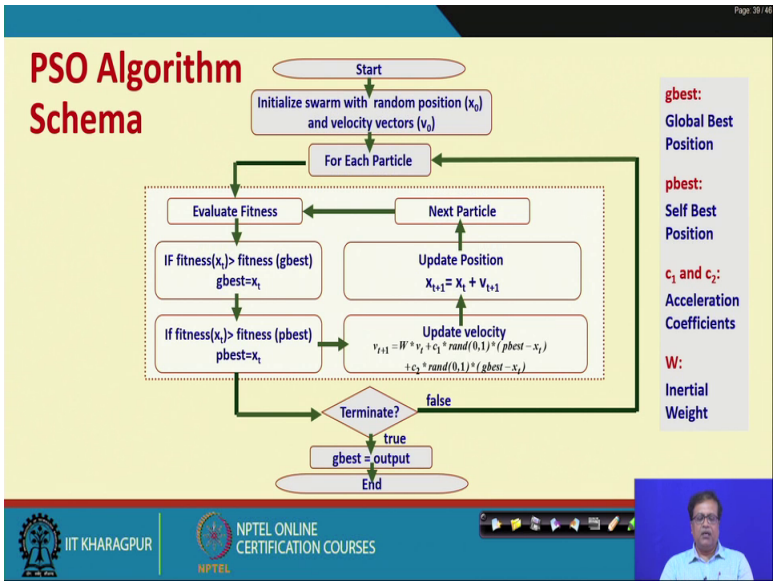

So, this is the framework or the schema for the PSO algorithm that is we start initialized swarm with a random position x 0 and a velocity vector v 0. Then for each particle evaluate fitness, is all right. Then if fitness is better than the gbest then we update the gbest. If the fitness is better than the pbest, then the pbest is updated that means, you keep updating the global best and the particle best. So, pbest and gbest they are updated. And if the terminate it is true, then gbest is output and stop otherwise you know go to the update velocity you know this is the iteration update velocity.

So, you see how the velocity is updated, you know it is updated by c 1 W into v t. So, W is intertial weight, which is with regard to its velocity; c 1 is and c 2 they are acceleration coefficients. So, c 1 into a random number and pbest minus x t, x t is the current position; and c 2 into another random number and gbest minus x t. So, this is the gap from the particle best; this is the gap from the global best multiplied by a random number and a coefficient that is how the velocity is updated.

Now, once the velocity is updated, if you add the current position to that velocity then we get the new position. So, then we have the next particle. So, basically this is done for each of the particles. So, each of the five particles if five particles are taken, this is done, and then after that we see that what is the seen situation and we go for the next iteration right. So, I hope you understood what is really happening that in the solution space, we initially find a certain number of particles. Suppose, you know number of particle should

be quite large for small problem you may take only few particles. For a small problem, let us suppose we have taken five particles then for each particle we keep calculating the velocity as initially we have to do all those initialization. And after that we have to check against the fitness, and g best and p best values, and then update velocity by these kind of formula and keep iterating.

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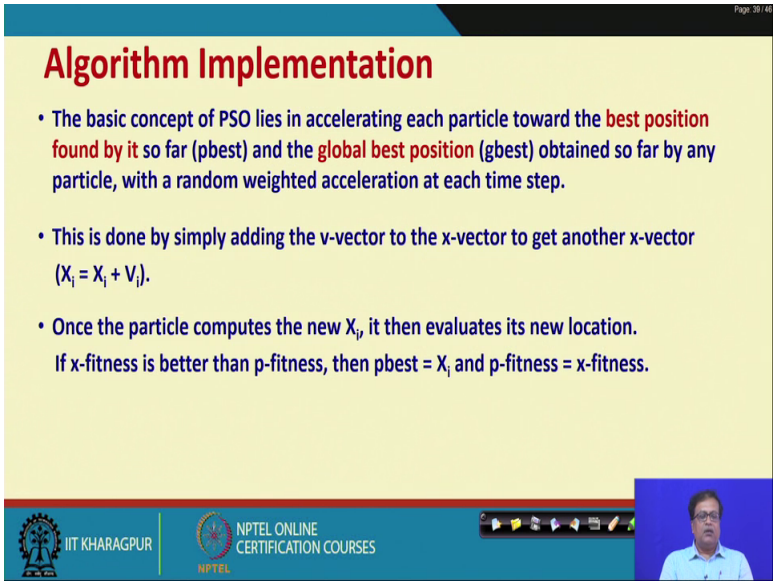

Now, the basic concept of PSO lies in accelerating each particles towards the best position found by it so far that is pbest and the global best position obtained so far by any particle with random weighted acceleration at each state right. And this is done by simply adding the v-vector and x-vector to get another x-vector right. Once the particle computes the new X i it is then evaluates the new location and the p best and the g best and p-fitness and all that.

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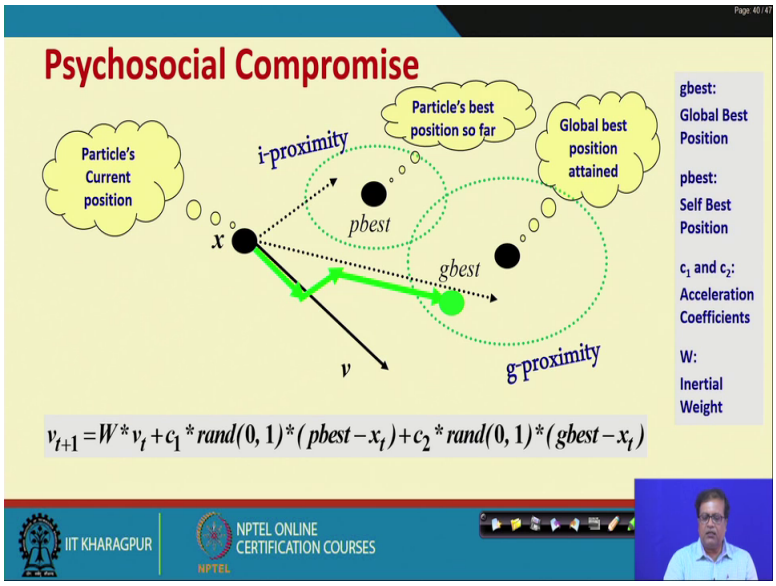

So, I think an example will be very clear for you. So, look here this is an interesting diagram. So, this is the particles current position. So, see look here. So, this is the particles current position. So, one particles current position is this. Now, this is the best possible obtained so far and this is the global best solution obtained so far.

So, the particle has got three components of movement one is towards its velocity direction; another is towards the pbest; another is towards the gbest right. So, the gap of this position from pbest and gbest multiplied by some random numbers and two coefficients, and also an inertial coefficients W for the velocity gives the next velocity thing. Here velocity is more like a position change. So, whatever we have the current position plus velocity will be the new position, is it all right. So, that is what we are calling psychosocial compromise fine so that is why it is.

Now, once we have this then our next calculation, you can see here that is what is being happening. So, particles current position see just see this particles current position, so it moves that i-proximity and g-proximity between pbest and gbest, so particles best position so far and global best position attained. So, the movement of the particles will be something towards v, something toward pbest something towards gbest, is all right. So, it is it is really a balance between diversity and towards a better fitness. Is it all right?

So, when you have number of particles and each particle is moving to its own best and to the global best, it actually ensures that the population diversity that means, the entire

solution space is covered in an adequate manner, is all right, so that is the essential idea of the PSO technique right, so that is where it finally moves.

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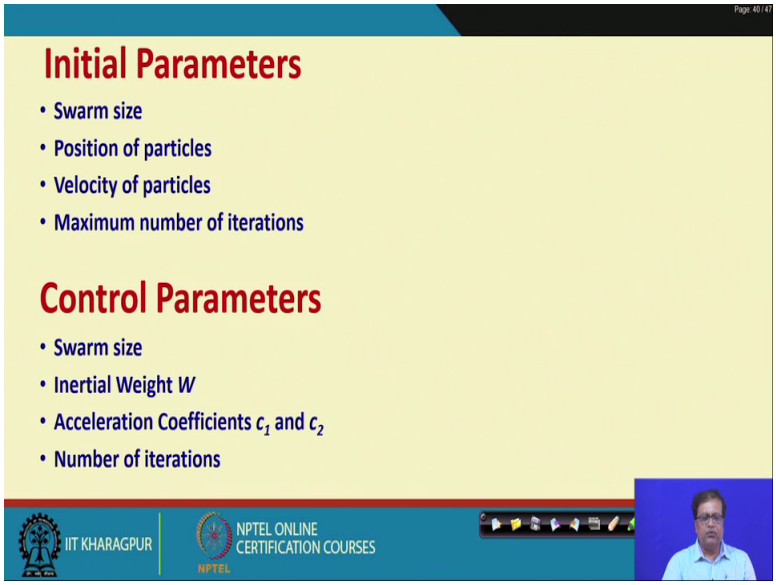

So, some of the initial parameters swarm size, position of particle, velocity of particle, maximum number of iterations and control parameters, swarm size, inertial weight, acceleration coefficients c 1 and c 2, and the number of iterations all right.

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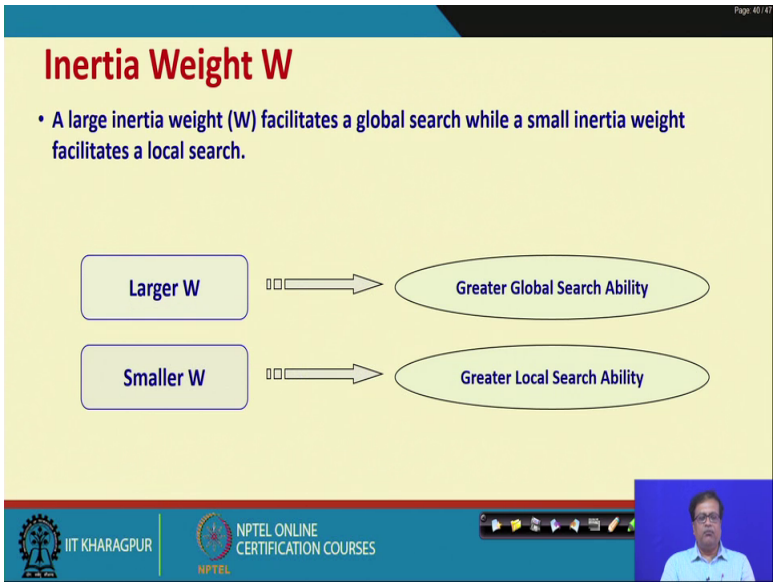

So, this is look here the initial weight, the large inertial weight facilitates a global search; while a small inertial weight facilitates a local search, is it all right. So, should be careful

on that that inertial weight if we take large, then it helps in global search; small it helps a local search.

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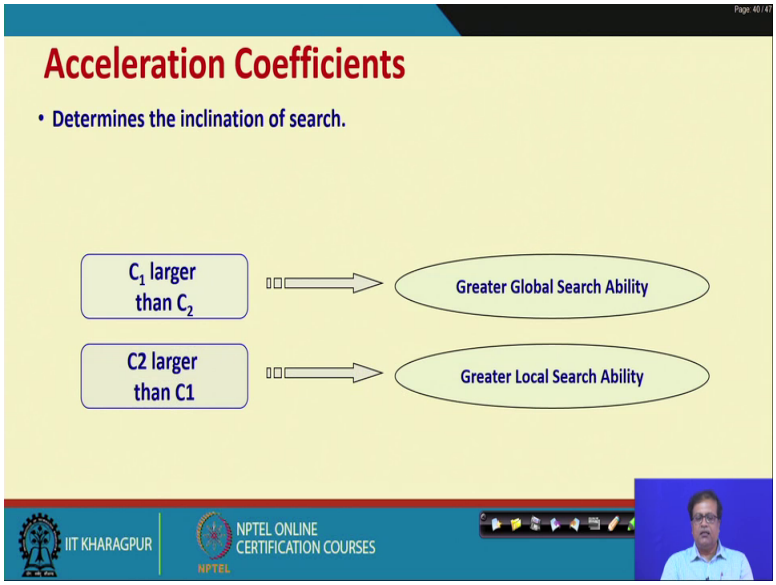

Similarly, the acceleration coefficient C 1 larger than C 2 greater global search ability; C 2 larger than C 1 greater local search ability fine.

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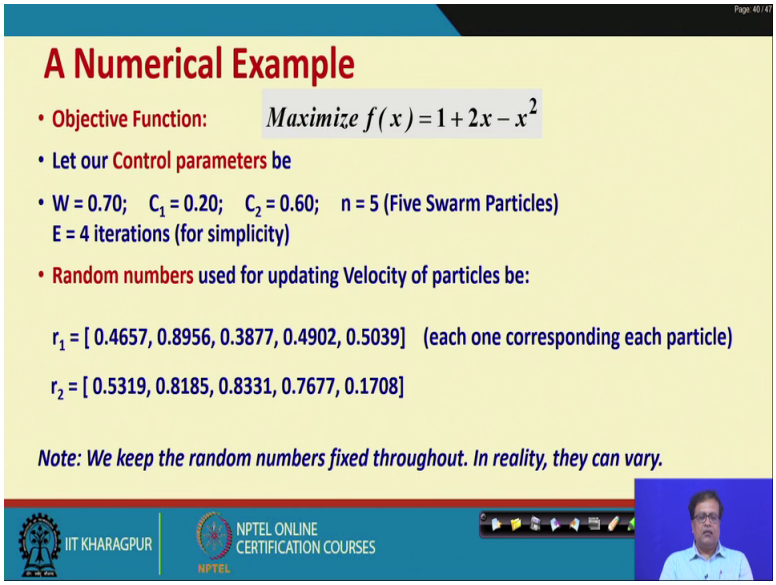

So, let us now look at a numerical example. So, this numerical example is like this. So, first of all objective function is maximized f x equal to 1 plus 2 x minus x square. So, it is a simple problem that we have to maximize these function for the value of x and try to

see how we try to obtain these by the PSO technique. So, we decide that we take the W equal 0.7 that is the weight for the velocity C 1 and C 2, the coefficients for global and local 0.2 and 0.6; and 5 swarm particles we take and for simplicity sake we take only four iterations.

Then we need a certain random numbers for updating velocity. So, you see whenever we talk about random numbers or any number the position we are going to write a matrix of size 5. Why matrix of size 5? The essential is that there are five particles, so each particle is for you know each number is for a particular particle.

Say for example, for the first particle the two random numbers are 0.4657 and 0.5319. So, we decided to keep the random number fixed for the iterations different iterations; in reality we may take different random numbers in different iterations, is all right.

So, maybe you one can take these random numbers for iteration one; in second iteration, it can take another set of random numbers, but we have decided to keep them fixed. So, the problem is clear. We are maximizing f x equal to 1 plus 2 x minus x square we have taken such control parameters, we have taken some random numbers right.

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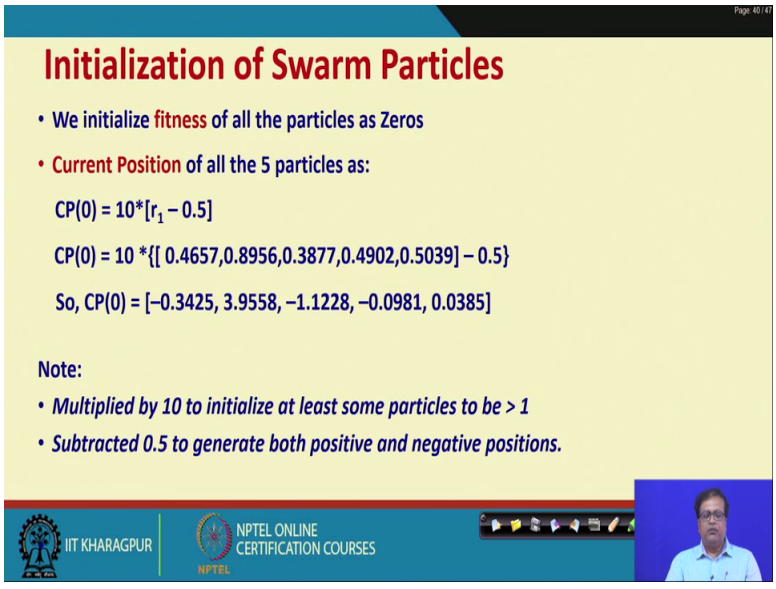

So, now the first thing is we initialize the fitness of all particles to zeros that means initial fitness let us take as zeros. Now, the current positions that random numbers can be used. So, what we do the current position of these at the zeroth iteration can be taken as

10 times r 1 minus 0.5. So, this is our r 1, so minus 0.5 into 10 then this is how the we have found out the current position of the particles, all right.

So, you see multiplied by 10 to initialize at least some particles to be greater than 1; and subtracted 0.5 to generate both positive and negative positions, right. So, we have to ensure that this sort of things are to be done plus another things should be may kept in mind.

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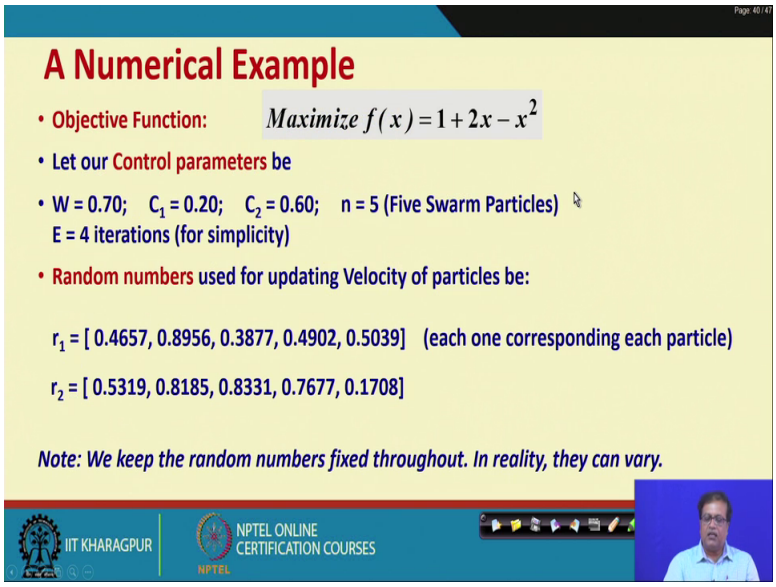

See look at the function. So, you see the position should be such that they cover our solution space, is all right.

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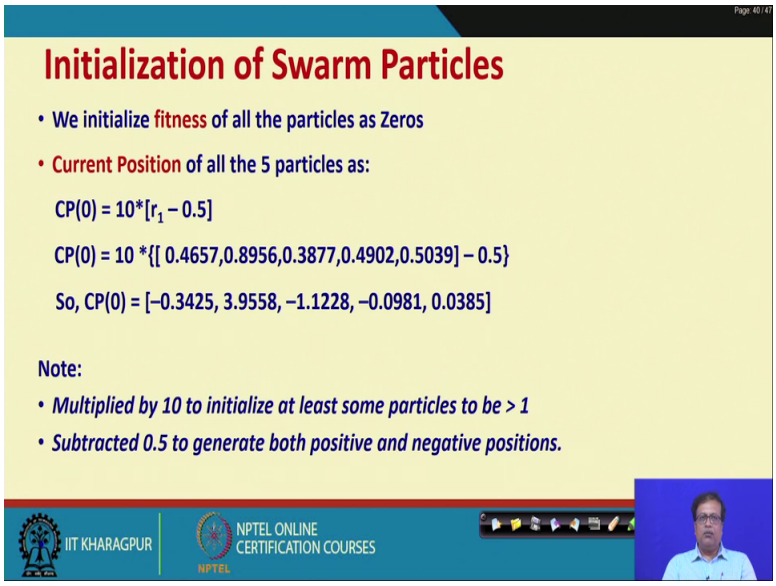

So, the initial positions basically these particle positions actually represent the functional the x value is it not. So, this is a function of x, so these particle positions actually represent the x value. So, the x values should be such so that it covers the available solution space in an appropriate manner. It should not be that you know the solution space is not properly covered. So, it should be ensured that, that is what is done here by multiplied by 10 and then divide minus you know 0.5 to generate both positive and negative positions right.

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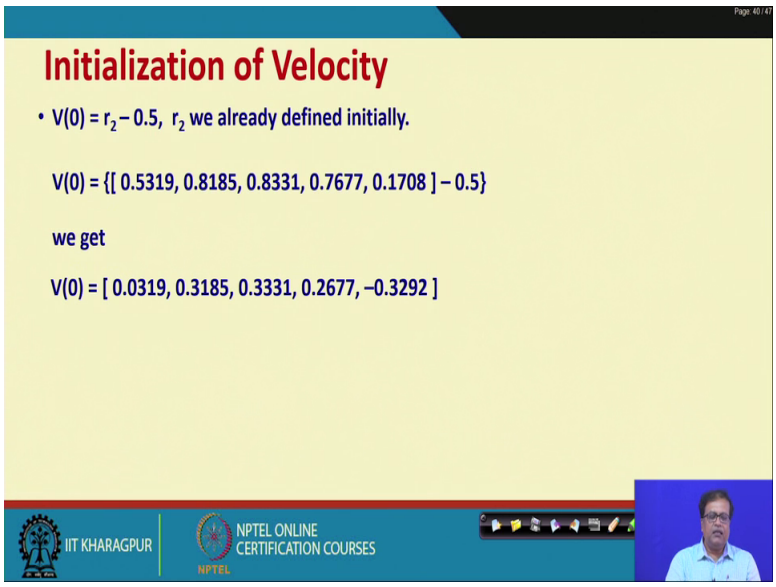

So, similarly the velocity is computed by using r 2; r 2 we already defined initially. So, if I you know in this case minus 0.5 then we obtain the velocities. So, obviously, different problems different kind of solution space and different ways we should be you know redefining our the velocities and position, so that it covers the entire solution space and the velocity should not be too low or should not be too high. So, those kind of fine tuning has to be carried out while initializing the velocity and initializing the position so good.

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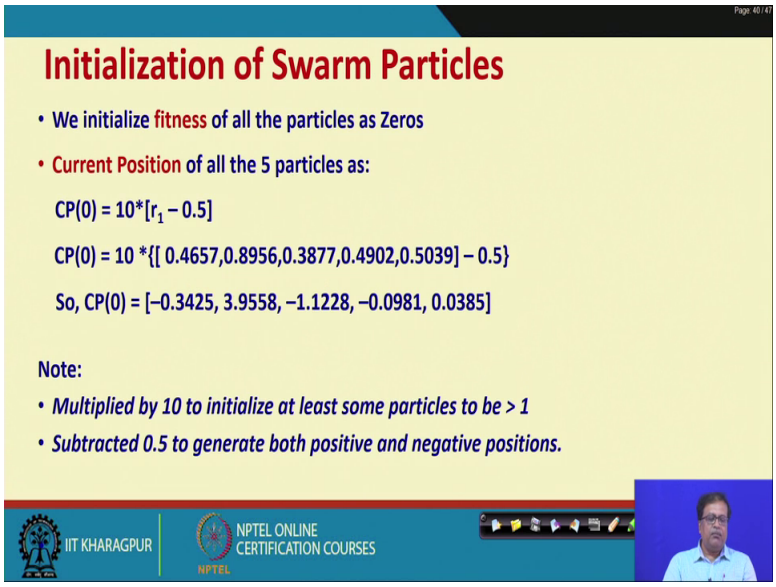

So, what we have done we have obtained the current positions of the particles.

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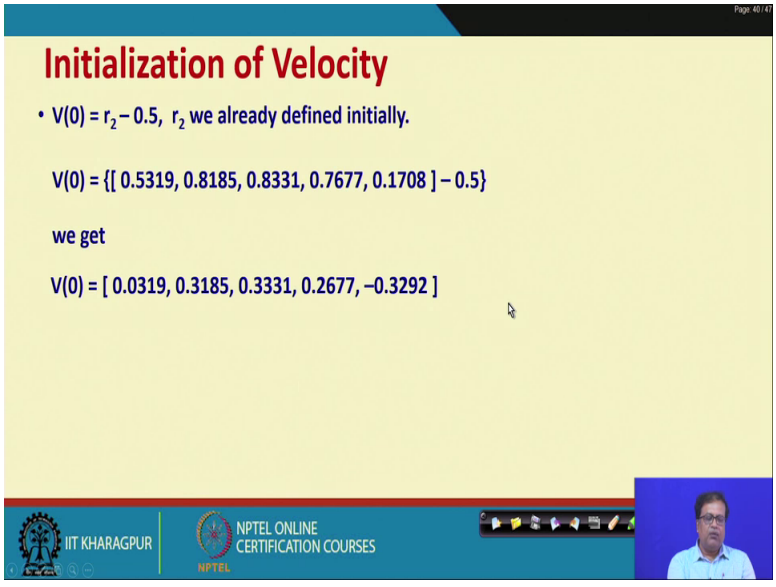

And we have obtained the current velocity of the positions of those particles.

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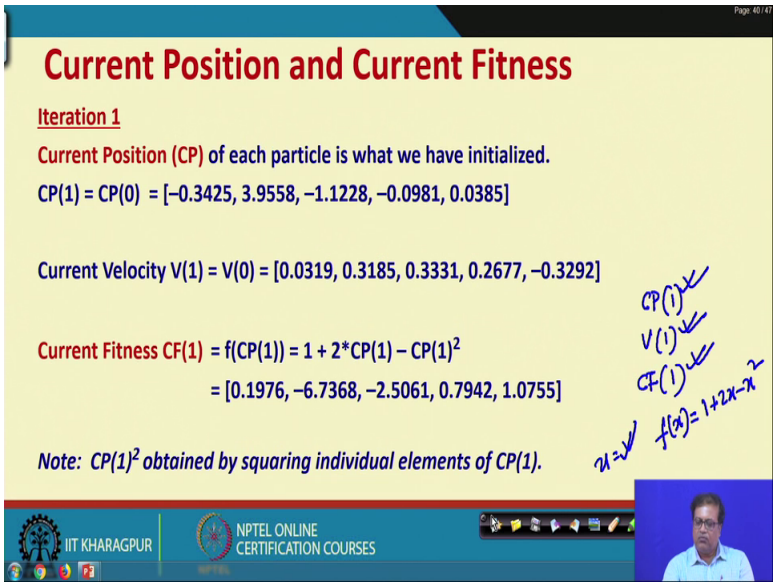

So, now what we see already we got the current positions look here this is our current positions CP 0. So, this is our current positions are already obtained and current velocity is already obtain. So, from the positions, if we do the functional value because you know you remember that our function x f x equal to 1 plus 2 x minus x square, so that was the function. So, if we put the position, and put that we get what is known as the fitness value. So, CP 1 square is obtained by squaring individual elements of CP 1. So, here CP 1 is like x, is it not. The current positions of the particle are like the x values.

So, if we put the function then we get the current fitness. So, what have you done? We have obtain the for the first iteration we have obtained the current positions which is as same as CP 0 we have taken, we have taken the current velocity that is V 1 equal to V 0 so already that is obtained and making use of them we have obtained the current fitness right.

So, what are the things that we have calculated for iteration 1? We have obtained the current positions; we have obtained the current velocity and we have obtained the current fitness right. So these are the three things that we have obtained. Current positions is taken same as iteration 0; current velocity taken same as iteration 0; current fitness we have found by putting in the original function f x.

So, x is known and f x was 1 plus 2 x minus x square. So, instead of x we put the CP positions. So, like CP 1 is 0.0319. So, when you put 0.0319, we got 0.1976. When you put 0.3185, we got this like this we obtain the current fitness values right.

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So, we obtained this current fitness. Now, the question is which one is the best local best positions. Look here the local best position are nothing but its current position because we do not have anything else, so we have to assume whatever is the current position that is also the local best position at the iteration 1. So, our CP 1 was these values. So, same values are going to be our local best positions. And what are our local best fitness? Again it should be current fitness because we do not have any other thing. So, this is going to be our current fitness values.

Now, what are the best possible values? Look here you remember the problem was one of maximization. So, out of these different fitness values the best one is nothing but one point see look at these numbers 1.0755 is the best. So, we call it our global best. So, global best is 1.0755 for the fifth particle, so that is our global best fitness. So, this is the maximum of the local best fitness. And what is the global best position for which position is this particular value that was the 0.0385, because for these x these f x has been obtained so that means global best position will be 0.0385.

So far not nothing very difficult I hope everyone has understood that what exactly we have done. We have obtained the velocity, the current position, the local best positions,

the local best fitness, the global best fitness and global best position that was for the iteration 1. Now, from iteration 1 as we go to iteration 2.

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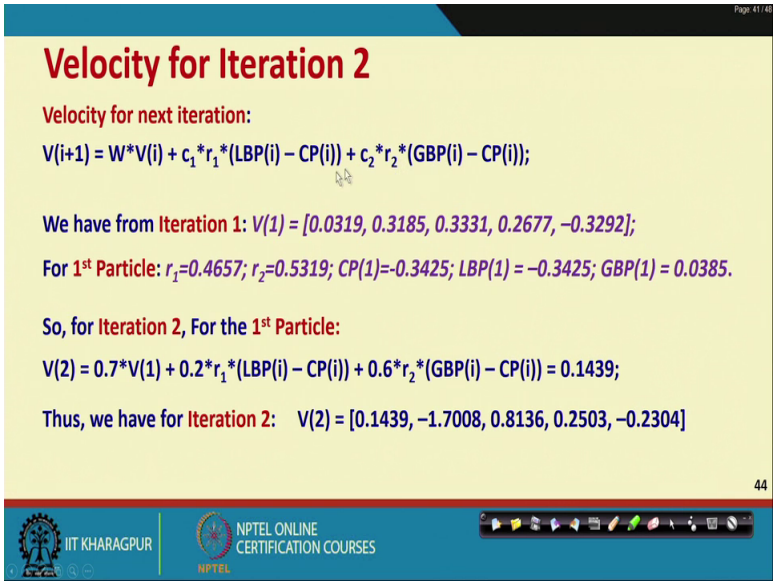

Now, we apply what is known as the velocity that is psychosocial criteria. So, as I have told before, the new velocity will have weight to the previous velocity the current velocity then the c 1 and c 2 two coefficients along with two random numbers r 1 and r 2, the local best position minus current position and global best position minus current position. So, these weight has to be obtained.

Now, we have from iteration 1, the V 1 value. So, this is our V 1 values and we decided to take these random numbers. So, these are our random numbers. So, for the first particle, see there are five particles. So, these are the first particle random numbers. The CP 1 for this you know the position was minus 0.3425; the local best was minus 0.3425; and global best was 0.0385. So, all these things you see they are all there. So, all these values we can make use of.

Then V 2 that for the first particle the V 2 will be then 0.7 into V 1 that is you know the whatever is the 0.0319, 0.2 into r 1 LBP 1 minus CP 1 0.6 into r 2 GBP i minus CP i, so it comes to 0.1439. So, using all of these for iteration 2, velocities for all the five particles can actually be obtained right. So, we have obtained the velocity for the iteration 2.

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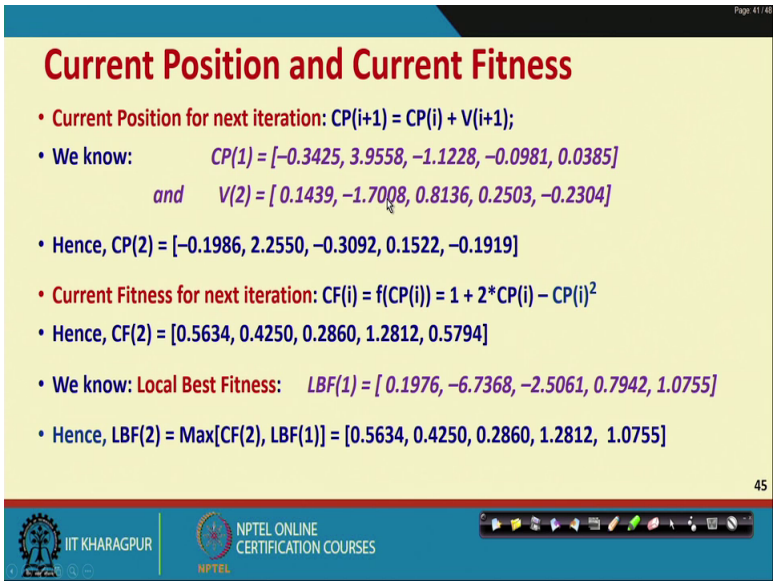

Now, once we have found iteration 2 velocity what will be the current position of the next iteration? It will be the old current position plus velocity 2. See, if you add these values the CP 1 values to the V 2 values we get the next current position. So, this is going to be our next current position which is nothing but a simple addition of this CP 1 and V 2 terms. And what will be the current fitness? The current fitness could be obtained by putting the functions. So, taking as them as x, finding the f x values will be the current fitness values.

So, what will be the local best fitness? See look here LBF 1 was these values. So, what will be LBF 2, the local best fitness of iteration 2? So, you see 0.5634 and 0.1976, this is better. Then 0.4250 point for this is better; 0.2860 that is better; for 1.28 that is better, but out of these 0.5794 and this, this is better, so that will be our LBF 2. So, only if the fifth position is different.

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So, then from there we can find out that we know CP 2 we can find out LBF 2, now from there we see the global best and the global best fitness and global best position right. And also the local best positions can be obtained as well. So, all of these things could be very easily calculated for the different iterations and all these are tabulated here.

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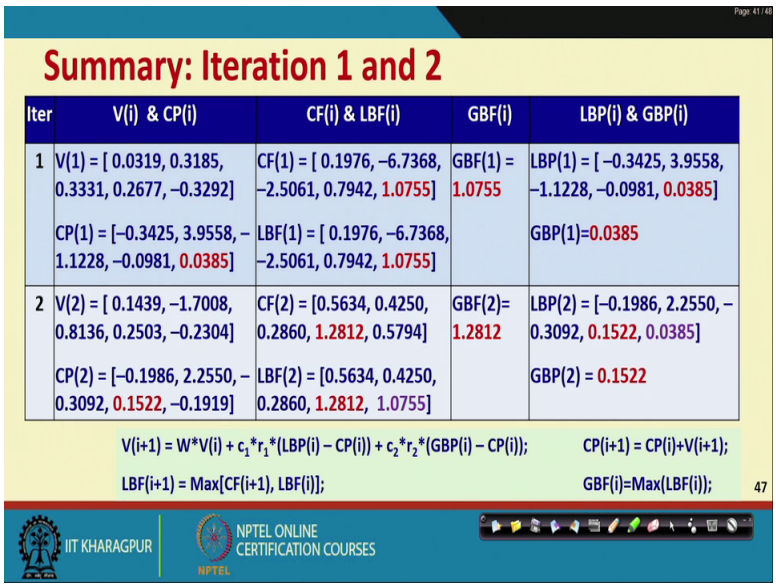

So, look here so that first of all from V 1, so these are the equations that are used that V i plus 1 equal to W V i c 1 r 1 LBP i local best minus CP, then global best minus CP. Then CP equal to CP i plus V i plus 1 local best fitness maximum of CF i plus 1 and LBF i and

global best fitness is the maximum of LBF. So, all these calculations are noted here, is all right.

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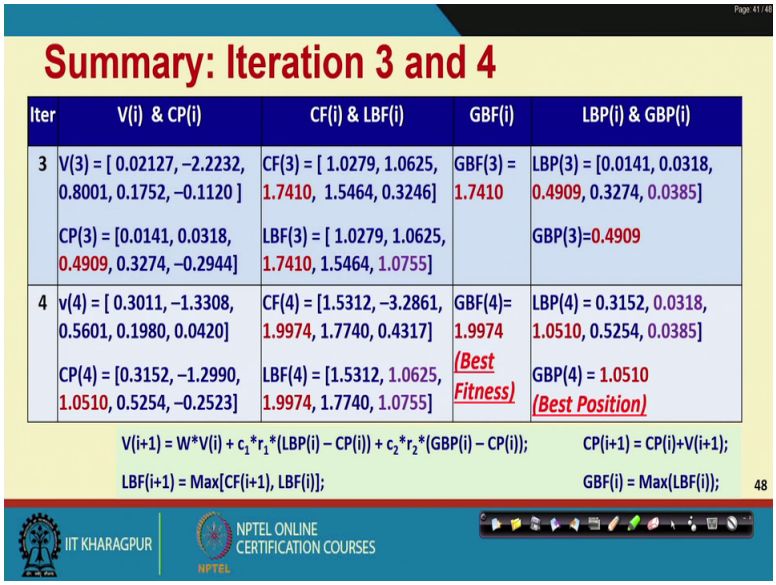

Now, on similarly when you calculate for two more rounds that is the third iteration and the fourth iteration then we can find out after these two iterations, we have found out the global best after four iterations as 1.9974 and the you know for the value the global best position that is 1.0510 right, so that is going to be our best position and best fitness obtained after four iterations right.

So, it is just calculate the velocity, calculate the current position, calculate the current fitness from there find out the local best fitness till that iteration global best fitness, local best position and global best positions do those calculations, is it all right.

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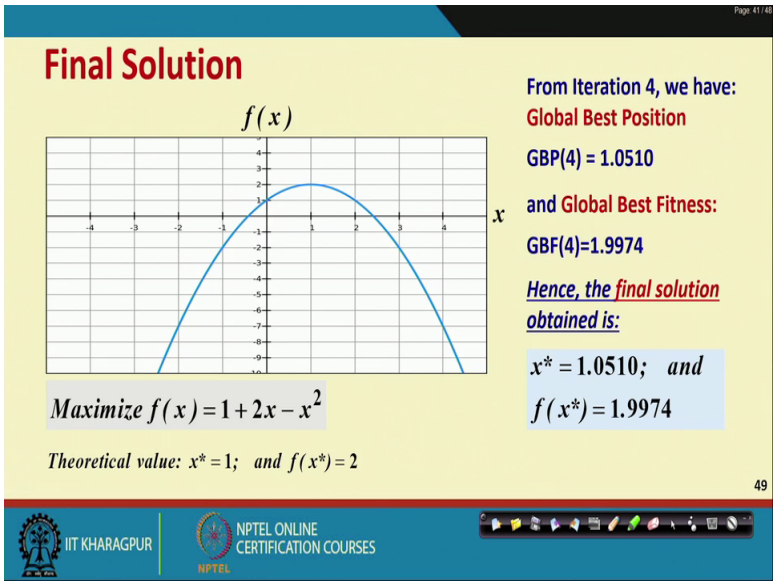

So, this is how the summary that this is the function for which we are trying to find out the final solution Maximize f x 1 plus 2 x minus x square, theoretical value is 1. So, this is 1, x 1, and the value is 2. But what we got through our iteration? We got 1.0510, and f x star is 1.9974.

So, we got a very near solution by only in four iterations maybe if you do more, we may be more near to the theoretical value, is it all right. So, this is how the PSO is applied. I am sure you can do more problems of PSO and you will be able to learn it in a much better manner right.

Thank you very much for patiently hearing.