Basic Electrical Circuits
Dr Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture - 59
Mesh Analysis

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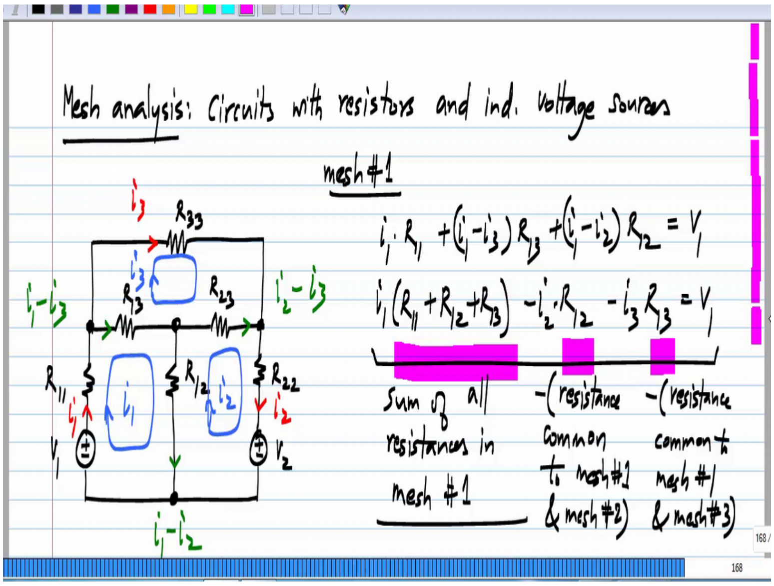

Now let us proceed with a mesh analysis of circuits with resistor and independent voltage sources. I have put down the same circuit that I consider the earlier. And I will identify the three mesh currents, let me call this mesh number 1, so this is current I 1 and this is I 2,and this is I 3. And as we saw the current in this branch is I 1, this is I 2 and this I 3, whereas current here is I 1 minus I 3, current over there is I 2 minus I 3, and current over there is I 1 minus I 2. So, what I will do is right Kirchhoff’s voltage law equations around each mesh. So, let me take mesh number 1. And let say I will go in the same direction as the mesh current that is clockwise direction, and I will write the voltage drops across all the resistances to be equal to the voltage rise across the voltage source in the loop. So, this is the way I choose to write it.

Just like for nodal analysis while writing Kirchhoff’s current law equation; I wrote the sum of all currents going out of a node through conductance equals the sum of independent currents being injected in to the node. Here also I will write some of voltage drops will traversing the mesh to be equal to the voltage rise due to the independent voltage source in the mesh. So, if I do that what will I get the voltage drop across R 1 1 is simply I 1 times R 1 1; and the voltage drop across R 1 3 is its current I 1 minus I 3 times

R 1 3. And the voltage drop across R 1 2 is I one minus I 2 that is the current through R 1 2 times R 1 2 and that will be equal to the rise of voltage V 1. Now this is the Kirchhoff’s voltage law equation around this loop. We have implicitly used Kirchhoff’s current law because this current here that is the mesh current I 1 the current here is I 3. So, we wrote the current in R 1 3 to be I 1 minus I 3 that is basically Kirchhoff’s current law applied at this node. So, we have implicitly use Kirchhoff’s current law also while writing this.

This is exactly the same thing that we did with nodal analysis we were writing Kirchhoff’s current law equations, but we were implicitly using Kirchhoff’s voltage law when we wrote down that the voltage across each brands is different between two node voltages, so that is for mesh number one, and if I group terms containing the same mesh current I will have I 1 times R 1 1 plus R 1 2 plus R 1 3 minus I 2 times R 1 2 minus I 3 times R 1 3 to be equal to V 1. What I have done is have collected the coefficients of the mash currents. Now the pattern must be very obvious to you what is the coefficient of given mesh current I am writing the equation for mesh number one and I 1 is the current for mesh number one.

So, then the coefficient of I 1 will be the sum of all resistances in that mesh. So, I 1 belong to mesh one. So, coefficient of I 1 will be sum of all resistances and mesh number one, when you are writing the equation for mesh number one. And what is the R 1 2, you can see that that is the boundary between mesh 1 and mesh 2 or R 1 2 belongs to both mesh 1 and mesh 1. So, the coefficient of I 2 will be negative of the resistance common to mesh 1 and mesh 2. And finally, R 1 3 what is that that is a resistance common to mesh 1 and mesh 3, so that is the coefficient of I 3, that is negative of resistance common to mesh number 1 and mesh number 3.

So, what is the summary here, if you right the Kirchhoff’s voltage law equation around a particular mesh, it will have terms containing all the mesh currents. Now the coefficient of the mesh current for that mesh would be the sum of all resistances and mesh, and the coefficient of the other mesh current would be the negative of resistance which is common to this mesh and that particular mesh. So, again the pattern is exactly the same as nodal analysis.

So, let us write the equation for all meshes, mesh number one we already done that it is I 1 R 1 1 plus R 1 2 plus R 1 3 minus I 2 minus I 2 R 1 2 minus I 3 R 1 3 equals V 1. Let us do it for mesh 2 then again we have voltage drops right and I will take all the voltage drops as I go in the clock wise direction. So, if I take the voltage drop in that direction that is like this you see that it is I 2 minus I 1 times R 1 2 in this direction it was I 1 minus I 2 times R 1 2 in the opposite direction I 2 minus I 1 times R 1 2. So, will have I 2 minus I 1 times R 1 2 R 2 3 here the current through that is I 2 minus I 3. So, I 2 minus I 3 times R 2 3. Remember I am going around this mesh, and I am considering voltage drops in that direction. The voltage that drops as a drives the loop and clock wise direction so that is I 2 minus I 3 times R 2 3 plus I 2 times R 2 2, because the current total R 2 two is just I 2 and that is equal to the voltage rise, as I go in the clock wise direction.

So, when I go from here to there, the voltage actually falls by V 2 or the voltage rises by minus V 2. So, this is equal to minus V 2. And again I collect all the coefficients, I will have minus I one times R 1 2 plus I 2 times R 1 2 plus R 2 2 plus R 2 3 minus I 3 times R 2 3 to be equal to minus V 2. So, this is again something that follows the same pattern for mesh number 2 the coefficient of I 2 will be sum of all resistances in the mesh coefficient of I one will be the negative of resistance common to mesh one and 2 coefficient of I 3 will be negative of resistance common to meshes 2 one 3 and just for completeness I will prove it fully for mesh number 3 as well and you see that going in the clock wise direction again we have the voltage drops this way is I 3 minus I 2 times R one 3 voltage drop that way is I 3 R 3 3 and the voltage drop this way is I 3 minus I 2 times R 2 3. So, if you are writing the equation for mesh number 3, the current I 3 will have positive coefficient that is something to keep in mind. So, I have I 3 minus I 2 times R 1 3 plus I 3 R 3 3 plus I 3 minus I 2 R 2 3 to be equal to 0, there is no independent voltage source and this mesh again collecting coefficients I have minus I 1 R 1 3 minus I 2 R 2 3 plus R 1 3 plus R 2 3 plus R 3 3 times I 3 to be equal to 0.

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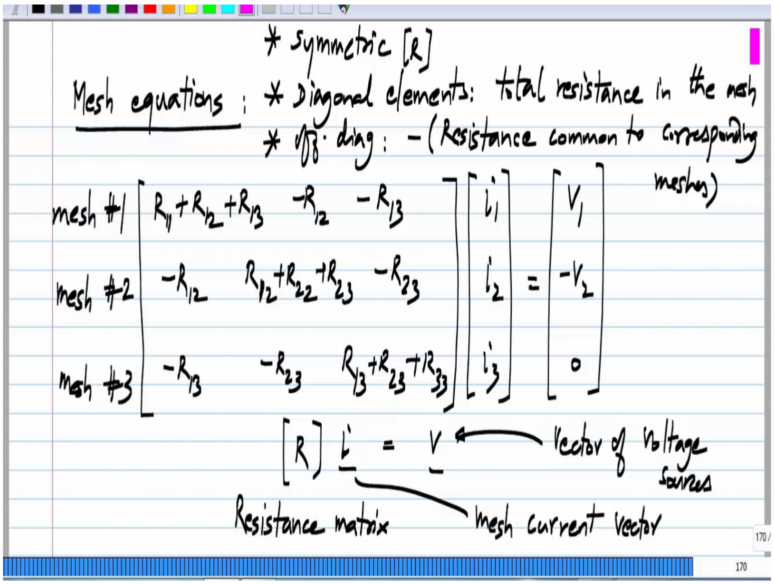

Now, I will put this in a matrix form which is convenient I have this times mesh current variables I 1 I 2 I 3 being equal to the independent sources, the source vector which is V 1 minus V 2 and 0 for our particular circuit. So, this is of the form the resistance matrix times in the vector of unknowns, the vector of mesh currents to be equal to the vector of independent voltage sources. Three rows are the equations for the three meshes and you see exactly similar properties so what we had when we had nodal analysis with resistors and independent current sources. We have a symmetric matrix, symmetric resistance matrix and the diagonal elements correspond to total resistance in the mesh. And off diagonal, it is the negative of resistance common to corresponding meshes.

So, now you can by matrix inversion solves for the unknown mesh currents I, and from those currents, you can get the individual branch currents. For instance we have already identify that the current through R 1 1 is I 1 R 2 2 is a I 2 and R 3 3 is the I 3. Similarly the current through R 1 3 is I 1 minus I 3 R 1 2 is I 1 minus I 2 and R 2 3 is 2 minus I 3. So, from the mesh currents, you identify all the branch currents and from that you can identify all the branch voltages, of course, for voltage sources the branch voltages is independent of the current, but in general you get the branch voltages from the current set you have calculated and the I-V relationships of the element.

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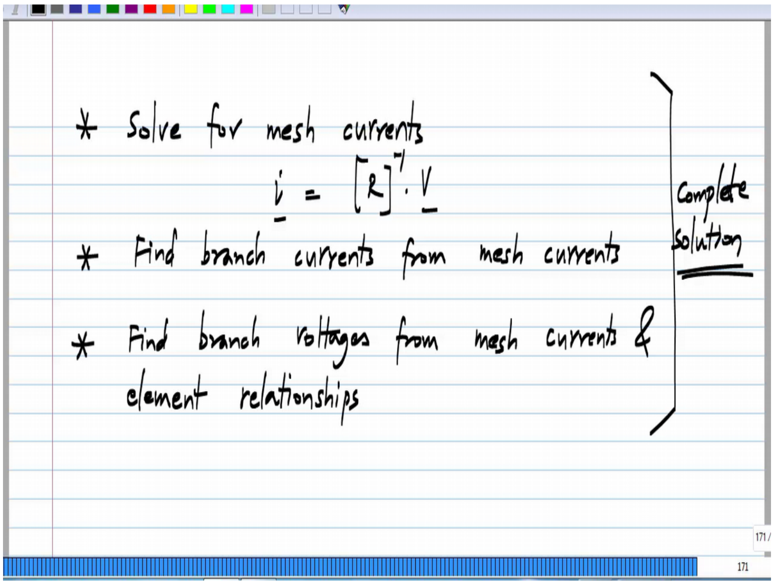

So, first solve for mesh currents the mesh current vector equals the inverse of the resistance matrix times the vector of sources. And you find all the branch currents from the mesh currents and finally, find all the branch voltages from mesh current and element relationships. So, with this, you have the complete solution to the circuit.