Course Name: Introduction to Microwave and Optical Metamaterials
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Week-2
Lecture-6

Lec 6: Basic Electromagnetic Theory-I

Hello students, welcome to Lecture 6 of the online course on Introduction to Microwave and Optical Metamaterials.

Lecture Outline

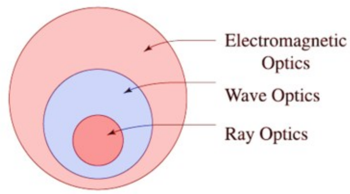
- Electromagnetic Optics – Overview
- Divergence, Curl and Gradient Operations
- Gauss's Theorem and Stokes Theorem
- Constitutive Relations
- Maxwell's Equations - Overview

James Clerk Maxwell (1831–1879) advanced the theory that light is an electromagnetic wave phenomenon. He formulated a set of fundamental equations of enormous importance that bear his name.

Today's lecture will be on basic electromagnetic theory, and here is the lecture outline. So, we will give you a very brief overview of electromagnetic optics, take you through the divergence, curl, and gradient operations, briefly discuss Gauss's theorem and Stokes's theorem, the causative relations, and finally, we will give you an overview of Maxwell's equations.

Electromagnetic Optics — Overview

- **Electromagnetic optics** is a vector theory comprising an electric field and a magnetic field that vary in time and space.
- **Wave optics** is an approximation to electromagnetic optics that relies on the wave function, a scalar function of time and space.
- **Ray optics** is the limit of wave optics when the wavelength is very short.
- In short, **Electromagnetic optics** encompasses wave optics, which in turn reduces to ray optics in the limit of short wavelengths



Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So, electromagnetic optics is basically a vector theory comprising an electric field and a magnetic field that vary both in time and space. So, here is the right way to present it: you have electromagnetic optics, and then you have wave optics, which is an approximation to electromagnetic optics that relies on wave functions. Which is basically a scalar function of time and space.

And then you have ray optics, which is again a limit of wave optics when the wavelength is very short; that means the objects are much, much larger than the wavelength involved. So, in short, you can say that electromagnetic optics encompasses wave optics, which in turn reduces to ray optics in the limit of short wavelengths. Right. So, from this particular figure, we can understand that wave optics has a far greater reach than ray optics.

Remarkably, both approaches provide similar results for many simple optical phenomena involving paraxial waves, such as the focusing of light by a lens and the behavior of light in graded index media and periodic systems. But wave optics can offer something that ray optics cannot, and that is basically the ability to explain phenomena such as interference and diffraction. However, wave optics is unable to quantitatively account for some simple observations in optical experiments, such as the division of light at a beam splitter, where some fraction of light is reflected and transmitted, which depends on the polarization of the incident light. Which means light must be treated in the context of a vector in that case, not as a scalar. And this is where the electromagnetic optics enter the picture, okay.

Electromagnetic Optics — Overview

- In common with radio waves and X-rays, **light** is an electromagnetic phenomenon that is described by a vector wave theory.
- Electromagnetic radiation propagates in the form of two **mutually coupled vector waves**, *an electric-field wave and a magnetic-field wave*.

Radiation Type	Frequency (f)	Wavelength (λ)
Microwave	300 MHz to 300 GHz	1 mm to 1 m
Visible	430 THz to 790 THz	380 nm to 700 nm

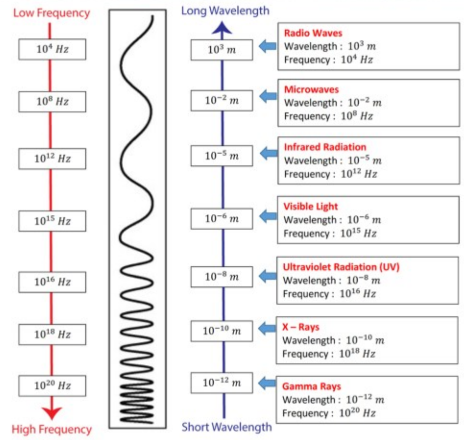


Figure. The electromagnetic spectrum from low frequencies (long wavelengths) to high frequencies (short wavelengths).

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Source: https://igcsezone.com/3-3-electromagnetic-spectrum/

So, in common with radio waves and X-rays, light is an electromagnetic phenomenon that is described by a vector wave theory. So, electromagnetic radiation propagates in the form of two mutually coupled vector waves: an electric field vector or wave and a magnetic field wave. So, in this particular course, we are going to focus on radiation types: microwave and visible. We have already seen that this frequency will be from 300 megahertz to 300 gigahertz, with a wavelength ranging from 1 mm to 1 meter, which is for microwave. For visible light, we will be talking about 430 terahertz to 790 terahertz, which implies a vacuum wavelength of 380 nanometers to 700 nanometers.

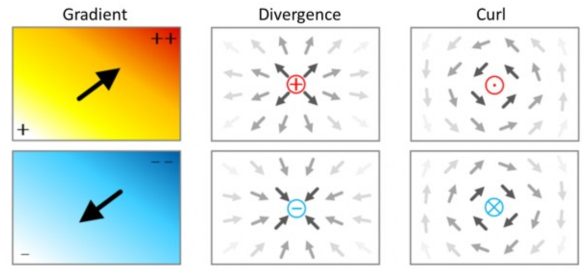
If you quickly have a look at the electromagnetic spectrum from low frequencies to high frequencies, okay, the wavelength basically moves the other way. So, from short wavelength to long wavelength, you can see radio waves, then microwaves, infrared radiation, visible light, and then UV, X-ray, and gamma rays. So, this way the frequency keeps increasing and the wavelength keeps reducing, right? So, you can wonder how vast this range is; the wavelength starts from, you know, 10 to the power of 3 meters, which is like a kilometer, to 10 to the power of minus 12 meters, which is a picometer. So, the variation in wavelength over this range is 10 to the power of 15 meters; it is huge. Now, before proceeding to Maxwell's equations for electromagnetism, which basically guide the relationship between the electric and magnetic fields, and also the materials involved.

Divergence, Curl and Gradient Operations

- The three main operators in **vector calculus** quantify changes in fields:
  - **Gradient** - change in magnitude of scalar field
  - **Divergence** - source of vector field
  - **Curl** - rotation of vector field

- The basic operations allow extracting information about the distribution of electromagnetic fields, energy associated with the field, electromagnetic radiation, and so on.

- The four Maxwell's equations are typically written in the vector calculus notation.



grad (scalar field) = vector | *div (vector field) = scalar* | *curl (vector field) = vector*

We can go through some of the important operators that are used in vector calculus, and these operators are going to be very, very useful for the calculation of electromagnetic phenomena. So, there are three main operators in vector calculus that quantify changes in the field. The first one is called the gradient that changes the magnitude of a scalar field. So, here you can see that this basically shows you the gradient. Here, you have some positive charges, and here is some negative and more positive charges.

So, this is the way it shows the change in magnitude in this direction; here, more negative means less negative. So, this is the gradient, okay. So, if you take the gradient of a scalar field, you are basically going to get a vector. Next is divergence; it tells you about the source of the vector field, right? So, here you can see that the divergence of a vector field is basically a scalar field. So, if you have a positive charge, you have a field diverging outward; if you have a negative charge, you will have a field that is coming towards you, okay, converging fields.

So, curl basically tells you about the rotation of a vector field. So, here you can see that this is one particular way of rotating the electric field vector; this is another rotation, okay. So, these are the two different orientations. So, this tells you about the arrow coming out of the plane, and this tells you about the arrow that goes inside the plane, okay. So, the basic operations allow for extracting information about the distribution of electromagnetic fields.

Divergence, Curl and Gradient Operations

Nabla ∇ operator

- In a 3D space, vectors can be split into orthogonal components, and partial derivatives can be calculated accordingly for each directional component.
- The del operator ∇ is a vector differential operator written as:

$$\vec{\nabla} \equiv \boldsymbol{\nabla} \equiv \nabla \equiv \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Laplacian operator

- Nabla can be used in the "Laplacian" operator, referred to sometimes as "nabla squared" or "del squared", denoting effectively double differentiation:

$$\vec{\nabla}.\vec{\nabla} \equiv \boldsymbol{\nabla}.\boldsymbol{\nabla} \equiv \nabla.\nabla \equiv \vec{\nabla}^2 \equiv \boldsymbol{\nabla}^2 \equiv \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

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Energy associated with the fields, electromagnetic radiation, and so on. So, the four Maxwell equations are typically written in this vector calculus form. So, if we think of the differential form of Maxwell's equations, we will see that the equations are basically written in the form of a vector differential operator, which is also called the nabla operator. So, in a 3D space, vectors can be split into orthogonal components, and the partial derivatives can be calculated accordingly for each directional component. So, the nabla or del operator is a vector differential operator that can be written as this.

So, i cap dot o x plus j cap dot o y plus k cap dot o z, right? Nabla can be used in the Laplacian operator as well, where it is referred to as nabla squared or del squared, which denotes, you know, basically double differentiation. So, del dot del or you can just write it as nabla square or del square is nothing but you know dou square dou x square plus dou square dou y square plus dou square dou z square or dou z square, ok.

$$\vec{\nabla} \equiv \boldsymbol{\nabla} \equiv \nabla \equiv \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla}.\vec{\nabla} \equiv \boldsymbol{\nabla}.\boldsymbol{\nabla} \equiv \nabla.\nabla \equiv \vec{\nabla}^2 \equiv \boldsymbol{\nabla}^2 \equiv \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

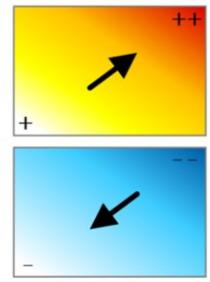
Divergence, Curl and Gradient Operations

Gradient of a Scalar

- A scalar field's gradient is a **vector field** whose magnitude represents the rate of change and which points in the general direction of the scalar field's greatest rate of increase.
- If ∇ is made to operate on a scalar function *F* (such as scalar field), then the following notation for the **gradient** is used, with the **result being a vector**:

In a Cartesian system of coordinates	
(simplified notation)	gradient (F) ≡ grad (F) ≡ $\vec{\nabla}$F ≡ **∇**F ≡ ∇F
(full notation)	$\nabla F \equiv \left(\hat{\imath}\frac{\partial}{\partial x}+\hat{\jmath}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)F \equiv \hat{\imath}\frac{\partial F}{\partial x}+\hat{\jmath}\frac{\partial F}{\partial y}+\hat{k}\frac{\partial F}{\partial z}$

Gradient



grad (scalar field) = vector

Now, coming to the gradient, the gradient of a scalar is a vector, right? So, how does it work? A scalar field gradient becomes a vector field whose magnitude basically tells you about the rate of change, and the direction tells you in which way the change is increasing. So, it tells you that in general, or you can say it points in the general direction of the scalar field's greatest rate of increase, okay. So, if this del is made to operate on a scalar field f, and you consider f as a scalar function here, then the following notation can be used.

So, you can simply write grad f, or you can just write, you know, del f. So, what will del f be? The del f is nothing but del is given by this: i cap dot o x plus j cap dot o y plus k cap dot o z times f, or you can simply write it like this: i cap df/dx or ∂f/∂x plus j cap ∂f/∂y plus k cap ∂f/∂z. So, that way you can find out the gradient.

$$\text{gradient (F)} \equiv \text{grad (F)} \equiv \vec{\nabla}F \equiv \boldsymbol{\nabla}F \equiv \nabla F$$

$$\nabla F \equiv \left(\hat{\imath}\frac{\partial}{\partial x}+\hat{\jmath}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)F \equiv \hat{\imath}\frac{\partial F}{\partial x}+\hat{\jmath}\frac{\partial F}{\partial y}+\hat{k}\frac{\partial F}{\partial z}$$

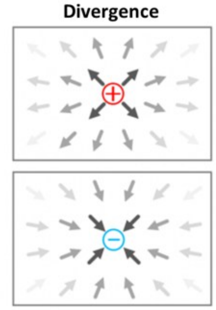
Divergence, Curl and Gradient Operations

Divergence of a Vector:

- Divergence quantifies the magnitude only (no direction) of the amount of a vector field which "flows" out or into a specific region. In other words - the divergence calculates the amount of source (or sink) for a given field.
- If ∇ is made to operate on a vector function ***F*** (such as vector field), then the following notation for the **divergence** is used, with the **result being a scalar** (even though the input is a vector field).

In a Cartesian system of coordinates	
(simplified notation)	$\text{divergence}\,(\vec{F}) \equiv \text{div}\,(\vec{F}) \equiv \vec{\nabla}.\vec{F} \equiv \boldsymbol{\nabla}.\mathbf{F} \equiv \nabla.\mathbf{F}$
(full notation)	$\boldsymbol{\nabla}.\mathbf{F} \equiv \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)$

Divergence



div (vector field) = scalar

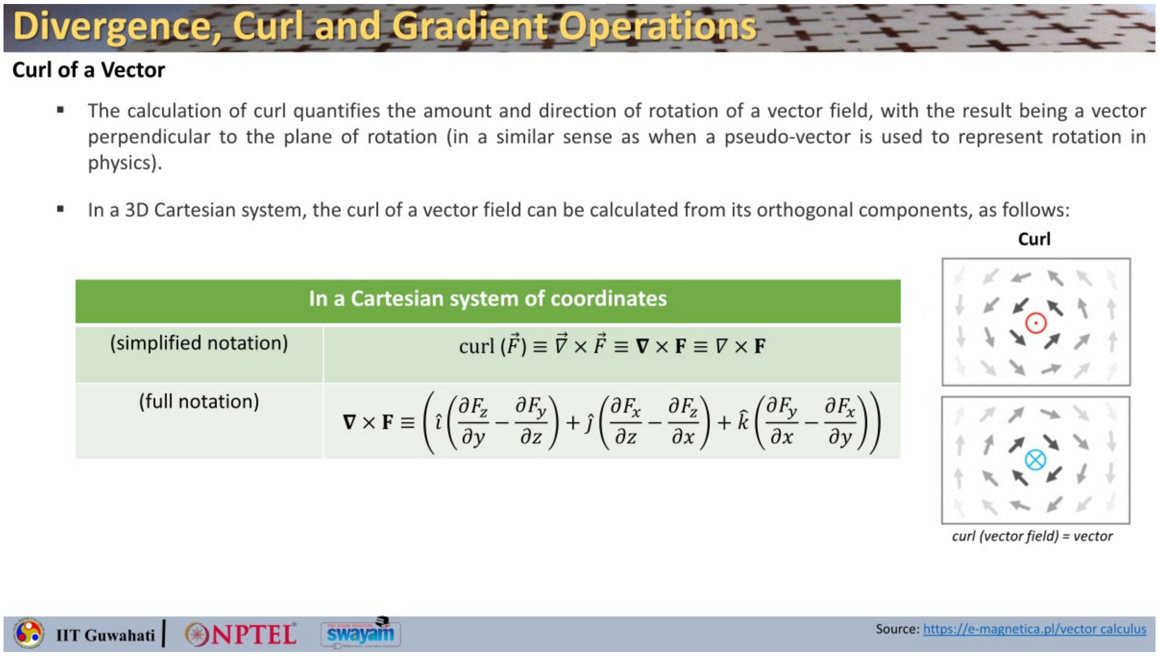
Now, the next important operator is the divergence of a vector. So, the divergence basically quantifies the magnitude only; it does not tell you about the direction of the vector field that flows out of or into a particular region.

So, in other words, you can say that the divergence basically calculates the amount of source or sink for a given field. So, the divergence of a vector field will be a scalar. So, if del is made to operate on a vector function f, and you take f as any vector field, then the following notation can be used, and you can see that the divergence turns out to be a scalar quantity. So, divergence of f can simply be written as ∇ dot f. That is also, you know, divergence.

So, the full notation will be this field has its x, y, and z components; the del operator also has its partial. You know, you can take the partial derivative along each component, and once you do that, you get this: ∂fx/∂x + ∂fy/∂y + ∂fz/∂z. So, this is what your divergence is. So, as I already mentioned, when there is a source, you have the field flowing out; if there is a sink, there will be, you know, a field flowing in.

$$\text{divergence}\,(\vec{F}) \equiv \text{div}\,(\vec{F}) \equiv \vec{\nabla}.\vec{F} \equiv \boldsymbol{\nabla}.\mathbf{F} \equiv \nabla.\mathbf{F}$$

$$\boldsymbol{\nabla}.\mathbf{F} \equiv \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)$$



The next important parameter is curl, the curl of a vector.

So, the calculation of curl basically quantifies the amount and direction of rotation of a vector field, which is the result; here, the result is also a vector that is perpendicular to the field of rotation. So, in a sense, when a pseudo vector is used to represent rotational notation in physics. So, you can see that if you have, you know, a rotation around the right-hand rule, the thumb basically tells you that the vector direction is outward. Right. So, it is basically telling you the direction, and the vector is perpendicular to this plane of rotation.

So, this screen is basically the plane of rotation, and the vector is basically coming out of the plane. On the other rotation, if it is going clockwise in this direction, you will see it is doing like this. So, in that case, the vector from the plane of polarization is basically pointing inward. So, in a 3D Cartesian system, the curl of a vector field can be calculated from its orthogonal components. The curl of f can be written as del cross f; del is a vector and f is a vector.

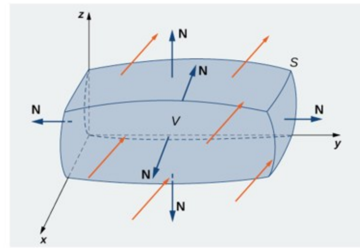
So, you just do the cross product. So, it will be $\hat{i}\ \partial f/\partial y - \partial f/\partial z + \hat{j}\ \partial f/\partial z - \partial f/\partial x + \hat{k}.\ \partial f/\partial y - \partial f/\partial x$. So, that way you can calculate the curl of a vector field, which is also a vector.

$$\mathrm{curl}\,(\vec{F}) \equiv \vec{\nabla} \times \vec{F} \equiv \mathbf{\nabla} \times \mathbf{F} \equiv \nabla \times \mathbf{F}$$

$$\mathbf{\nabla} \times \mathbf{F} \equiv \left(\hat{\imath}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\right)$$

Gauss's Theorem or Divergence Theorem

- The theorem states that the flux of a vector quantity outward through a closed surface **S** is equal to the integral of the **divergence** of the function in the enclosed volume **V**,



$$\iiint_V (\nabla . \mathbf{F}) dV = \oiint_S (\mathbf{F} . \hat{\mathbf{a}}) dS$$

- Therefore, if the given volume does not contain a source (or sink) of the vector field then the net flux through that volume must be zero **(i.e. all flux entering the volume must also leave that volume).**
- It is possible to find such volume that will entrap an electric charge, because **each electric charge represents an electric monopole.**

Leads to Maxwell's First Equation

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So, with a basic understanding of the 3 main operators in vector calculus, we can move on to have a quick recap of the Gauss theorem or the divergence theorem.

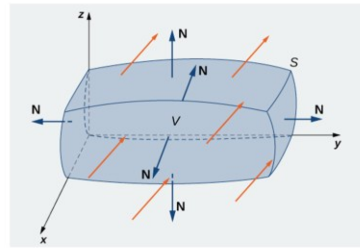
These are important for calculating different fields. So, we are having a very quick overview of it. The theorem states that the flux of a vector quantity outward through a closed surface S is equal to the integral of the divergence of the function in the enclosed volume V. So, whatever the volume this closed surface is enclosing is okay. So, if you take the divergence of this vector field over that volume, that will be the same as, you know, the outward flux that is going out from this closed surface S.

$$\iiint_V (\nabla . \boldsymbol{F}) dV = \oiint_S (\mathbf{F} . \hat{\mathbf{a}}) dS$$

Gauss's Theorem or Divergence Theorem

- The theorem states that the flux of a vector quantity outward through a closed surface **S** is equal to the integral of the **divergence** of the function in the enclosed volume **V**,

$$\iiint_V (\nabla.\mathbf{F})dV = \oiint_S (\mathbf{F}.\hat{\mathbf{a}})dS$$



- But it is not possible to find a volume which entraps a magnetic charge, so the magnetic field is "divergenceless" and thus there are **no magnetic monopoles**.

→ Leads to Maxwell's Second Equation

So, how are V and S basically related? So, this closed surface basically encloses a volume V. So, you can say if the given volume does not contain a source or a sink of the vector field, then the net flux through that volume must be 0. So, if it contains a positive charge, then there will be electric flux lines coming out of it, or even if it contains a negative charge, there will be something. But if there is no net charge, then there will be no net flux through this particular volume. Okay, that means all the flux entering the volume must also leave that volume.

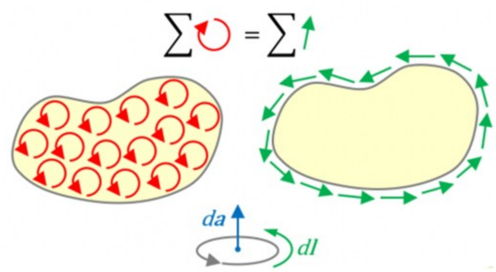
$$\iiint_V (\nabla.\boldsymbol{F})dV = \oiint_S (\mathbf{F}.\hat{\mathbf{a}})dS$$

So, it is possible to find a volume that will entrap an electric charge because each electric charge represents an electric monopole. and that leads to the Maxwell's first equation. And if you try to reapply this to a magnetic field, you will see that it is not possible to find any volume that can entrap a magnetic charge. So, the magnetic field is divergence-less, and hence, you can say that there are no magnetic monopoles. And this basically leads to Maxwell's second equation, right?

Stokes Theorem

- Stokes' theorem states that the surface integral of the curl of the vector field **F** over an open surface **S** is equal to the closed line integral of the vector along the contour enclosing the open surface.

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{a} = \oint_C \boldsymbol{F} . \mathrm{d}\boldsymbol{l}$$



- In other words, the circulation of a vector around a given boundary is equal to net curl over the whole surface of the patch limited by that boundary.

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Source: https://e-magnetica.pl/vector_calculus

Now, let us look into the other theorem, which is also very important: the Stokes theorem.

That states that the surface integral of the curl of a vector field F over any open surface S is ok. So, if you want to take the surface integral of a vector field over any open surface S, it will be equal to the closed line integral of the vector along the contour enclosing the open surface. Right. So, it means if you want to calculate all these small curves over this entire surface, why don't you just calculate the curl or the integral of the vector along the contour that encloses this particular open surface? So, you can write that the curl of f dot dA, when you take the integration over this entire surface, will be the same as the contour line integral or closed line integral of the vector over this particular contour. So, it basically relates the surface integral of the curl of a vector to the line integral of the vector itself, right? That is simply, you can say.

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{a} = \oint_C \boldsymbol{F} . \mathrm{d}\boldsymbol{l}$$

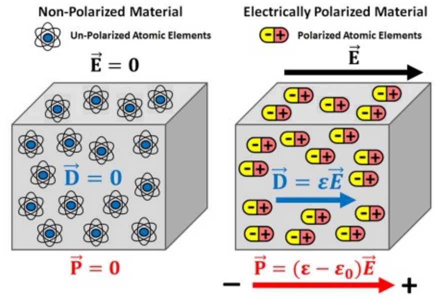
So, in other words, the circulation of a vector around a given boundary is equal to the net curl over the whole surface of the patch limited by that boundary. So in this particular equation, you can see S is basically denoting the analyzed surface, F is basically the analyzed vector field, A is the vector normal to the surface, and C tells you the closed curve, as you can see here, okay. L is basically the tangent to, you know, the curve C; you can make L like this, ok. So, this is an easy way to remember how Stokes' theorem works.

Constitutive Relations

- **Dielectric permittivity** (ε) is defined as the ratio between the electric field (***E***) within a material and the corresponding electric displacement (***D***):

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$$

ε_0 = Dielectric constant of vacuum = 8.85 × 10^{-12} [F/m]



- When exposed to an electric field, bounded electrical charges of opposing sign will try to separate from one another.

Next comes the constitutive relationship.

So, that is important where the light-matter interaction is taken into consideration. So, in order to apply Maxwell's macroscopic equations, it is necessary to specify the relations between the displacement field D and the electric field E, as well as the magnetizing field H with the magnetic field B. So, equivalently, we have to specify the dependence of polarization P, which comes from the bound charges, and also magnetization M, which is basically the bound current, on the applied electric and magnetic fields. So the equations specifying this response of the material are called the constitutive relationship. So, dielectric permittivity is basically diagnosed as a diagnostic physical property that characterizes the degree of electrical polarization a material can experience under the influence of an external electric field.

So, how do you measure it, or how do you define it? Dielectric permittivity can be defined as a ratio between the electric field within a material and the corresponding electric displacement D. So, here you can see the equation D equals epsilon at E. Right. So, this is an unpolarized material. So, the polarization is 0, and the electric field is also 0 when it is not present.

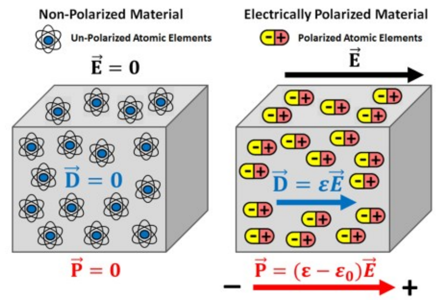
$$\boldsymbol{D} = \boldsymbol{\varepsilon_0 E}$$

So, displacement is also 0, but as soon as. So, what is happening to this material when exposed to an electric field is that the bound electrical charges of opposite sign try to separate from one another, right? For example, the electron cloud of the atoms will try to, you know, move away in the direction of the electric field. Or you can say that the electron clouds of the atoms will shift their position relative to their nuclei. So, in that case, there will be, you know, a scarcity of negative charges, so the positive charges are left behind. So, you can think of those as positive ions, and then you have the electron cloud shifted, so those are the negative charges.

Constitutive Relations

- For example, the electron clouds of atoms will shift in position relative to their nuclei.
- The extent of the separation of the electrical charges within a material is represented by the **electric polarization** (***P***).
- The electric field, electric displacement and electric polarization are related by the following expression:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \boldsymbol{\varepsilon E}$$



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Source: https://em.geosci.xyz/content/physical_properties/dielectric_permittivity/index.html

So, these are basically the bound charges that lead to polarized atomic elements. So the extent of the separation of the electrical charges, or how far they will separate within a material, is basically represented by the electric polarization P. Now the electric field, electric displacement, and electric polarization—these three terms are related by this particular equation, which is called D equals epsilon naught E plus P. So, epsilon naught is vacuum permittivity, okay. So, this is how you write it: epsilon d equals epsilon e.

This epsilon is basically taking care of both the vacuum and the polarization aspects of the material.

$$\boldsymbol{D} = \boldsymbol{\varepsilon_0 E} + \boldsymbol{P} = \boldsymbol{\varepsilon E}$$

Constitutive Relations

- When exposed to an applied magnetic field, the collection of individual magnetic dipole moments within most materials will attempt to reorient themselves along the direction of the field.

- This generates an induced magnetization, which contributes towards the net magnetic flux density inside the material.

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} = \boldsymbol{\mu H}$$

$$\mu_0 = permeability\ of\ free\ space = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

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Similarly, when you expose any material to an applied magnetic field, the collection of the individual magnetic dipole moments within most materials will attempt to reorient themselves along the direction of the field. So, this will basically generate an induced magnetization in the material, which will contribute to the net magnetic flux density inside the material. So, you can say you will have B as the magnetic flux. So, this generates an induced magnetization that contributes to the net magnetic flux density inside the material.

So, B equals μ_0H plus μ_0M. Altogether, that can be written as B equals μH. μ_0 is basically the permeability of free space, which is 4π × 10 to the power of -7 henry per meter. So, you can write B equals mu naught mu h B equals mu h. So, the degree to which the induced magnetization impacts the magnetic flux density basically depends on the material's magnetic permeability. So, magnetic permeability is mu, which is defined as the ratio between magnetic flux density B within the material and the intensity of the applied magnetic field H, provided that both fields are very weak, right? So, B equals mu H.

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} = \boldsymbol{\mu H}$$

Constitutive Relations

- The degree in which the induced magnetization impacts the magnetic flux density depends on the material's magnetic permeability.
- **Magnetic permeability** (μ) defines the ratio between the magnetic flux density ***B*** within a material, and the intensity of an applied magnetic field ***H***; provided the fields are sufficiently weak.
- **Note:** For now, we will focus on material for which

$$\boldsymbol{M} = \boldsymbol{0} \longrightarrow \boldsymbol{B} = \boldsymbol{\mu_0 H}$$

You can understand that mu will be nothing but B divided by H, okay? So, for now we can only focus on materials that are not magnetic. So, the magnetization is 0; in that case, you can simply write B equals mu naught h.

$$\boldsymbol{M} = \boldsymbol{0} \qquad \boldsymbol{B} = \boldsymbol{\mu_0 H}$$

Maxwell's Equations — Overview

- An **electromagnetic field** is described by two related vector fields that are functions of position and time:

  Electric field *E*(*r, t*) and Magnetic field *H*(*r, t*)

- After the myriad of researches carried out for fundamental reasons behind the source of electromagnetic field and relation between electric and magnetic fields by pioneer scientists-

  Ampere, Coulomb, Faraday and Gauss

- The revolution in the Electromagnetic Fields happened when **James Clerk Maxwell** proposed a set of fundamental equations in 1865.

James Clerk Maxwell (1831–1879) advanced the theory that light is an electromagnetic wave phenomenon. He formulated a set of fundamental equations of enormous importance that bear his name.

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Now, with that, we understand the fundamentals that are required for Maxwell's equations, and then let us go and describe these four golden equations that are vital in understanding the behavior of an electromagnetic wave. So we see that the electromagnetic field is basically described by two related vector fields that are functions of both position and time. So, you can write the electric field as E(r,t) and the magnetic field as H(r,t).

So, Maxwell's equation did not just come up like that after the myriad of research you know was carried out for fundamental reasons behind the source of the electromagnetic field and the relationship between the electric and magnetic fields. by pioneering scientists like Ampere, Coulomb, Faraday, and Gauss. Maxwell was the one who combined those four equations to describe the electromagnetic field behaviors. So, he proposed a set of fundamental equations in 1865, okay, and that is him. So, he basically advanced the theory that light is an electromagnetic wave phenomenon, and he formulated that fundamental set of equations, and that is why those four equations bear his name, okay.

Maxwell's Equations — Overview

Maxwell's Equations

$$\nabla . \mathbf{E} = \frac{\rho_v}{\varepsilon} \qquad \text{(Gauss's Law)}$$

$$\nabla . \mathbf{H} = 0 \qquad \text{(Gauss's Law for Magnetism)}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \text{(Ampere's Law)}$$

E = Electric field vector
H = Magnetic field vector
D = Electric flux density
B = Magnetic flux density
ρ = charge density
J = current density

The Maxwell's equations are valid for both static and dynamic electromagnetic fields in a media.

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Though actually those four equations are basically, you know, all done by other scientists, like the first equation, which is basically telling you that the divergence of the electric field equals rho v by epsilon, is Gauss's law or Gauss's law of electricity or electromagnetism. And then del dot h equals 0; that is basically Gauss's law for magnetism, okay? The third and the next two are the curl of E and the curl of H; these are also basically Faraday's law and Ampere's law, but you will see that Maxwell has made a very important contribution here and combined these four equations to describe the electromagnetic phenomena of light, and that is why this set of four equations bears his name, right? So, in general, what you have seen are the 6 scalar functions of position and time. They are required to describe the electromagnetic field in a medium. So, fortunately, these 6 functions are interrelated. So, the electric and the magnetic are both present because E and H each have E x E y E z and H x H y and H z.

$$\nabla . \mathbf{E} = \frac{\rho_v}{\varepsilon} \qquad \text{(Gauss's Law)}$$

$$\nabla . \mathbf{H} = 0 \qquad \text{(Gauss's Law for Magnetism)}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \text{(Ampere's } Law\text{)}$$

So, they are all interconnected, and basically, you know, you have to satisfy the set of coupled differential equations, which are nothing but Maxwell's equations. So, if you look carefully, as I mentioned, the first equation is derived from Gauss's law, which tells you that the electric flux out of any closed surface is basically proportional to the charge density enclosed within the surface. The second equation tells you about Gauss's law for magnetism, which states that the net magnetic flux out of any closed surface is going to be 0 because there are no magnetic monopoles. The third equation is basically Faraday's law, which tells you that a changing magnetic field will produce a circulating current, okay. And the fourth one is called Ampère-Maxwell's law, which tells you that the generation of a magnetic field can be done in two methods: either through electric current or by having a changing electric field.

So, here you can see the representation of these bold characters: E is for the electric field vector, H is the magnetic field vector, and D and B are basically the electric and magnetic flux densities, respectively. Rho is the charge density; j is the current density, right? So, Maxwell's equations are basically valid for both static and dynamic electromagnetic fields in a medium. Now, let us carefully look into each of these equations one last time in both integral and differential form, okay. So, Maxwell's equations can be written in both integral and differential forms, as we will see.

Maxwell's Equations — Overview

- Gauss's law for electric fields: While the *area integral of the electric field* gives a measure of the net charge enclosed, the *divergence of the electric field* gives a measure of the density of sources.

Maxwell's Equations	
Integral Form	**Differential Form**
$Q_e(t) = \oiint_S \vec{D}(t).d\vec{s} = \iiint_V \rho_V(t)dv$	$\nabla.\vec{D}(t) = \rho_V(t)$

So, the Gauss law for electric fields tells you that while the area integral of the electric field gives a measure of the net charge that is enclosed, you can see that by taking the area integral of the electric field, it gives you the net charge, or you can just take the volume integral of the charge density, which is also fine; you are getting the net charge.

$$Q_e(t) = \oiint_S \vec{D}(t).d\vec{s} = \iiint_V \rho_V(t)dv$$

$$\nabla.\vec{D}(t) = \rho_V(t)$$

But when you go for the divergence, it actually gives you the measure of, you know, the density of sources. So, the divergence of D gives you ρV. It is important to understand that both forms of Maxwell's equation can be written in integral form as well as in differential form. The choice between the differential and integral forms depends on the problem statement or the kind of problem you are going to solve and the desired level of detail. So, typically, people use the differential form, which is useful for calculations in localized regions, while the integral form is helpful for relating fields within a region to those on the boundaries.

Okay, so one can say to use the differential form when you need to understand the local behavior of the fields at a specific point. Or when dealing with complex or less symmetric situations, you can use the integral form when you want to relate the fields within a region to those on its boundaries. Especially when dealing with the symmetric distribution of charges and currents.

Maxwell's Equations — Overview

- Gauss's law for Magnetism: The *net flux* will always be zero for dipole sources.

Maxwell's Equations	
Integral Form	**Differential Form**
$\oiint_S \vec{B}(t).d\vec{s} = 0$	$\nabla.\vec{B}(t) = 0$

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The next one, the Gauss law for magnetism, tells you that the net flux is always going to be 0 for dipole sources, okay, and this is the integral form and the differential form for the second law. So, del dot B equals 0, and here the surface integral of B(t) ds is 0, okay.

$$\oiint_S \vec{B}(t).d\vec{s} = 0$$

$$\nabla.\vec{B}(t) = 0$$

So, if there were a magnetic monopole source, you would have obtained a nonzero area integral, but since there are no possible magnetic monopoles, you just get 0 in this case.

Maxwell's Equations — Overview

- Faraday's law: The line integral of the electric field around a closed loop is equal to the negative of the *rate of change of the magnetic flux* through the area enclosed by the loop.

Maxwell's Equations	
Integral Form	**Differential Form**
$V_{\text{emf}}(t) = \oint_L \vec{E}(t).d\vec{l} = -\iint_S \left[\frac{\partial \vec{B}(t)}{\partial t}\right].d\vec{s}$	$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{dt}$

Faraday's law tells you that the line integral of the electric field around a closed loop is equal to the negative rate of change of the magnetic flux through the area enclosed by the loop. And that is in integral form. You can also see from here that the time-varying magnetic flux gives you a rotating electric field. And the fourth law, Ampère-Maxwell's equation, gives the total magnetic force around the circuit in terms of the current through the circuit plus any varying electric field through that particular circuit, which is also called displacement current.

$$V_{\text{emf}}(t) = \oint_L \vec{E}(t).d\vec{l} = -\iint_S \left[\frac{\partial \vec{B}(t)}{\partial t}\right].d\vec{s}$$

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{dt}$$

Maxwell's Equations — Overview

- Ampere-Maxwell equation: This gives the total magnetic force around a circuit in terms of the *current through the circuit*, plus any *varying electric field* through the circuit (that's the "displacement current").

Maxwell's Equations	
Integral Form	**Differential Form**
$I\,(t) = \oint_L \vec{H}(t).\,d\vec{l} = -\iint_S \left[\vec{J}(t) + \frac{\partial \vec{D}(t)}{\partial t}\right].\,d\vec{s}$	$\nabla \times \vec{H}(t) = \vec{J}(t) + \frac{\partial \vec{D}(t)}{dt}$

So, Maxwell's equations in integral form can be written like this: it is nothing but, you know, the line integral of H(t)dl, which is the same as the negative surface integral of the current, okay? So, you are basically taking the current density and, you know, doing a surface integral. So, you are getting the current plus the change in the displacement field. So, that is basically the displacement current. So, in that differential form, it looks a bit tidy.

$$I\,(t) = \oint_L \vec{H}(t).\,d\vec{l} = -\iint_S \left[\vec{J}(t) + \frac{\partial \vec{D}(t)}{\partial t}\right].\,d\vec{s}$$

$$\nabla \times \vec{H}(t) = \vec{J}(t) + \frac{\partial \vec{D}(t)}{dt}$$

So, you can say the curl of H equals J(t) plus dot d dot t. So, this is basically the displacement current component.

Thank you. So, this is all for this lecture. So, we will go into a bit more detail about Maxwell's equations and the wave equation in the next lecture. So, if you have any doubts or queries regarding this lecture, you can drop an email to this particular email address mentioning the lecture number and the course title.