Course Name: Introduction to Microwave and Optical Metamaterials
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Week-10
Lecture-50

Lec 50: Carpet Cloaking

Hello everyone, welcome to Lecture 50 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on carpet cloaking.

- Carpet Cloaking:
    - Introduction
    - Theory
    - Applications

So, here is the lecture outline: we will first introduce the concept of Carpet Cloaking, and we will discuss the theory and the applications of Carpet Cloaking.

Carpet Cloaking

So, what is carpet cloaking? So, we have always seen in this case, like you know, that when you do invisibility cloaking, you can actually bend light rays around the object that you are going to hide, and to achieve that, you can use transformation optics or transformation electromagnetics that basically does you know maps the coordinate transformation into the material property, Okay. So, what you have seen in the previous lectures is that to achieve some sort of cloaking properties, you are required to have.

You know the permittivity and permeability both need to be obtained using metamaterials. And at times, the values that are required for that kind of cloaking material are really, really difficult to achieve. So, that is where the carpet cloaking concept will also be very useful and handy because it will be more practical.

So, when you talk about carpet, here we mean that it's a reflective surface.

Carpet Cloaking: Introduction

- **The Carpet:** A reflective surface, often a bump, is created on a reflecting plane. This bump is the "carpet" under which the object is hidden.

- Carpet cloaking is a practical application of transformation optics:
  - A field that uses coordinate transformations to design materials with unusual electromagnetic properties.

- Carpet cloaking (also known as a ground-plane cloak) is a technique that renders an object invisible by making it appear as if it were part of a flat surface, effectively hiding it under a "carpet".

- This is achieved by manipulating how light or other waves interact with the object, causing them to reflect as if from a flat surface, even when an object is present.

- A carpet cloak aims to conceal an object by making it appear as a flat surface:
  - This is achieved by altering the path of light or radio waves around the object.
  - So that they reflect as if from a flat surface.
  - The object is effectively "hidden" beneath a cloaked surface.

Often, a bump is created on a reflective plane. And this bump is basically the carpet under which the object you want to hide can be placed. So, carpet cloaking is basically a practical application of transformation optics, as I mentioned. Transformation optics, what it does is basically use coordinate transformations that can be mapped into material properties. which are some unusually electromagnetic properties like mu and epsilon, the new values that are required.

The carpet cloaking, which is also known as the ground plane cloak, is basically a technique that renders an object invisible. By making it appear as if it were part of the flat surface. So, effectively, it is like hiding it under a carpet. So, the concept basically looks very similar to if you have a kid or if you have seen any kid hiding his chocolate; you will just put it under the carpet. So, a similar kind of concept has been used here, and we will see how it can actually help you achieve the cloaking.

So, this is basically achieved by manipulating how light or other waves interact with

objects. causing them to reflect as if it were coming from a flat surface even when an object is present. So, the scattering signature of the object is somewhat hidden. So, you cannot identify whether the object is there or not. So, a carpet cloaking basically aims to conceal an object by making it appear as a flat surface.

And this is achieved by altering the paths of light or radio waves around the object. So, they reflect as if the reflection is coming from a flat surface. And where the object is placed, it is effectively hidden beneath this cloaked surface.

Carpet Cloaking: Introduction

- Applications of Carpet Cloaking:

  1. Concealing Objects:
  - The most obvious application is to hide objects from detection.

  2. Stealth Technology:
  - In military applications, this could involve concealing vehicles or other objects from radar or other sensors.

  3. Antenna Protection:
  - Carpet cloaks can be used to protect antennas, allowing them to transmit and receive signals while remaining undetectable.

  4. Novel Lenses:
  - Carpet cloaks can also act as lenses, focusing or directing beams of radiation.

Let us look into the applications of carpet cloaking. As you can understand, the first application would be concealing objects.

So, this is the most obvious application of cloaking that you are using to hide an object from being detected. They can be used in stealth technology, typically in military applications. This could also involve, you know, concealing vehicles or other objects from radar or any other sensor. And then you can also use carpet cloaking for antenna protection. So, you can use carpet cloaking to protect antennas, allowing them to transmit or receive signals while remaining undetectable, okay.

And finally, you can also use them for making novel lenses, okay, because this kind of carpet cloaks also. act as lenses that allow focusing or directing beams of radiation.

Carpet Cloaking: Introduction

- Transformation optics can be used to design a cloak of invisibility.

- The only way to achieve actual transparency would be if the electromagnetic waves coming from behind the object could somehow still arrive with the same trajectory, in front of the object, as though the electromagnetic waves were transmitted directly through the object.

- In essence the cloak makes its contents appear to be very small and hence invisible.

- An illustration of reshaping an object via anisotropic metamaterial (AMM) coating as shown in figure.

"An illustration of real life invisible cloaks"
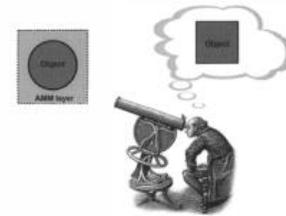
"Reshaping of an object via AMM layer"

So, transformation optics can be used to design this cloak of invisibility. So, the only way to achieve actual transparency would be if the electromagnetic wave were coming from behind. The object could somehow still arrive with the same trajectory in front of it.

As though the electromagnetic waves were transmitted directly through the object. So, here is an illustration of a real-life invisibility cloak. We have seen this picture before. So, you can see a lady holding cloaking material in her hand. So, that allows the light rays to come around her so that the view behind her is not blocked.

So, in a sense, the cloak can also make its contents appear very small. And hence that can also render it invisible to radars, and all is okay. So, there is another way of doing it; it is called reshaping an object, and that can be achieved via anisotropic metamaterial coatings. So, how it works is that you have an object that is circular, but you put an anisotropic metamaterial of square shape around it. So, when it is detected by the radar or perceived by the radar, it will be detected as a square object.

So, you are basically hiding that particular object.

Carpet Cloaking: Introduction

- There are three distinct topological possibilities for cloaking: By squishing an object either

- Crushed to a point
  - ✓ In the process of crushing the object becomes infinitely conducting
  - ✓ Not a problem as the objects have negligible size
- To a line
  - ✓ Can be rendered invisible but requires extreme and singular values as well as being anisotropic.

- To a sheet
  - ✓ Sheet is highly visible
  - ✓ Can only be made invisible if it sits on another conducting sheet so they cannot be distinguished
  - ✓ Although more limited, invisibility can be rendered without extreme values and with isotropic materials

Source: J. Li and J. B. Pendry, Physical review letters, 101(20), 203901, 2008.

So, there are basically three distinct topological possibilities of cloaking: you can either squish an object to a point or to a line. So, if you do this in this process of crushing the object. It becomes infinitely conducting, but it is not a problem here because the objects will have a very negligible size and they can be rendered invisible. So, that is a good thing they can become invisible because of their negligible size, but as you can understand This kind of thing will require extreme and singular values, as well as being anisotropic, okay.

The other method could be to squish an object into a sheet. So, a sheet is highly visible. So, it can only be made invisible if it sits on another conducting sheet so that they cannot be distinguished. Right. So, although you can say that the application here could be more limited, you know.

The invisibility here can be rendered without extreme values and with isotropic materials. So, it is more feasible to make this kind of cloak.

So, let us see the theory of carpet cloaking.

Carpet Cloaking: Theory

➤ A cloak to mimic a flat ground plane.

▪ Carpet cloak does not require singular values for the material parameters; the range for the permittivity and the permeability is much smaller than in the case of a complete cloak.

▪ The regions in cyan are transformed into each other.

▪ Shaded regions represent the ground planes.

▪ The observer perceives the physical system as the virtual one with a flat ground plane.

(a) physical system — object to conceal — y, x
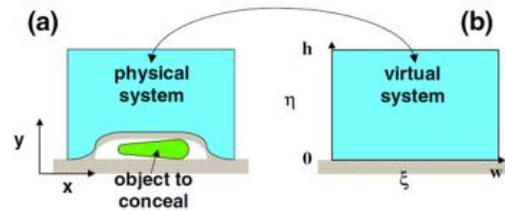
(b) virtual system — h, η, 0, ξ, w

Figure: Carpet Cloaking: the physical and the virtual systems.

So, here is a cloak to mimic a flat ground plane, okay. So, basically, here you have the physical system.

So, you have this object to hide, and then you have this carpet, okay? And then this has to be mapped into a virtual system that looks like a flat ground plane, right? So, this is your virtual system, and this is your physical system, and these are the coordinates. Now, the carpet cloak does not require singular values for the material parameters. The range of the permittivity and the permeability is much smaller than in the case of a complete cloak. So, you can see the regions in cyan are basically transformed into each other, right? The shaded region here basically represents the ground planes. and the observer perceives the physical system as the virtual system that has a flat ground plane, but actually there is an object hidden.

Carpet Cloaking: Theory

- Consider the system as the 2D wave problem with E polarization.

- Fields are invariant in z direction.

- Here; a ground plane means a highly reflective metal surface: regarded as a perfect conductor.

- Suppose an object lies on it, a cloak is covered on the object so that the system is perceived as a flat ground plane again.

- The object is concealed between the cloak and the original ground plane as shown in Fig. (a).

- We assume the cloak (shown in Fig. (a) in cyan color) is a rectangle of size $w \times h$ except that the bottom (inner) boundary is curved upwards to leave enough space for the object.
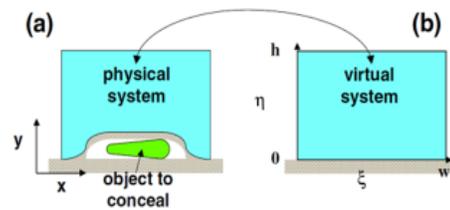

Figure: Carpet Cloaking: the physical and the virtual systems.

Now let us consider the system as the 2D wave problem with electric polarization. So, we can consider the fields to be invariant in the z-direction. So, we are only mentioning the x and y directions here. So, here you can see that a ground plane basically means a highly reflective metal surface that can be regarded as a perfect conductor. Now you assume that an object lies on that particular plane and a cloak is covering the object.

So that you know, the system is perceived as a flat ground plane again, okay? So, here is the original physical system where you can see the object. That has been concealed between the cloak and the original ground plane. Whereas, you know the cyan color here is basically showing you a cloak which is typically rectangular of width w and height h except for the bottom where you need to have that curved shape to leave enough space for the object's height.

- The whole configuration is called the physical system with coordinate (x, y) or (x_1, x_2) in indexed notation.

- The virtual system, shown in Fig. (b), is the configuration observer perceives.

- Its coordinate is labeled by (ξ, η) or (ξ^1, ξ^2).

- In general, we consider a coordinate transform which maps a rectangle $(0 \leq \xi \leq w, 0 \leq \eta \leq h)$ in the virtual system to an arbitrary region (the cloak) in the physical system.

(a) (b)

Figure: Carpet Cloaking: the physical and the virtual systems.

- The **Jacobian matrix** as defined earlier: $A_{ij} = \dfrac{\partial x_i}{\partial \xi_j}$

  where ξ_1, ξ_2 are the basis vectors of the virtual coordinates appearing in the physical system.
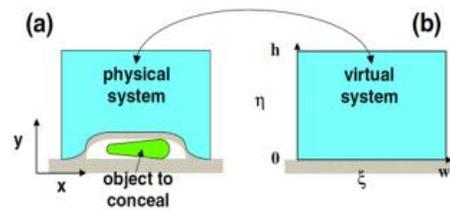
- The **Covariant matrix** as defined then: $g = A^T A$

Now this whole configuration is called the physical system with coordinates x and y. Or you can say x_1 and x_2 in indexed notation, and then this has to be mapped to the virtual system that is shown here, okay? Now the coordinates here can be labeled as (ξ, η) or you could have named them (ξ^1, ξ^2).

So, this basically shows the width and the height, and here it is a perfect rectangular cloak. So, in general, you can see that we are basically considering a coordinate transformation that maps a triangle. which has a width of w and a height of h in the virtual system to an arbitrary region that is the cloak in the physical system. Right. So, how do you do this coordinate transformation? You have to bring back the Jacobian matrix that was discussed earlier.

So, you can write $A_{ij} = \dfrac{\partial x_i}{\partial \xi_j}$. So, you are basically going from here to here, okay. So, here ξ^1 and ξ^2 are the basis vectors of the virtual coordinates that appear in the physical system. So, from that, you can calculate the covariance matrix, which is defined as g, and it is calculated as $A^T A$, okay. Now, for cloaking, the observer basically perceives the physical system as an isotropic, homogeneous medium.

- For cloaking, the observer perceives the physical system as an isotropic homogeneous medium of permittivity ε_{ref} and $\mu_{\text{ref}} = 1$ (**virtual system**).

- The **corresponding physical medium** induced by the coordinate transformation is given by:

$$\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det g}} \quad \text{and} \quad [\mu^{ij}] = \frac{AA^{\text{T}}}{\sqrt{\det g}}$$

- We write μ_{T} and μ_{L} be the principal values of the permeability tensor in the physical medium.

- The corresponding refractive indices be:

$$n_{\text{T}} = \sqrt{\mu_{\text{L}}\varepsilon} \quad \text{and} \quad n_{\text{L}} = \sqrt{\mu_{\text{T}}\varepsilon}$$

for the two (local) plane waves traveling along two principal axes.
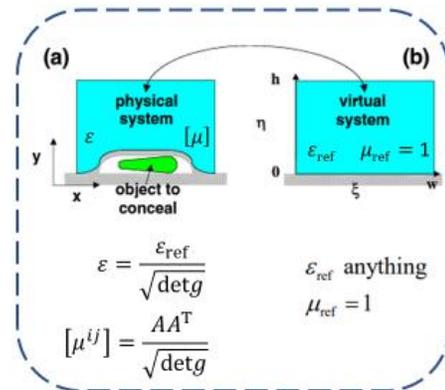
So, you take $\mu_{\text{ref}} = 1$, okay, and epsilon reference can be anything. So, the corresponding physical system that will be induced by the coordinate transformation will have these values. which is, you know, epsilon given as: $\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det g}}$, Which is the covariance matrix, and then you have the magnetic permeability, which is mu, okay. So, that is $[\mu^{ij}] = \frac{AA^{\text{T}}}{\sqrt{\det g}}$.

So, what are these? You can actually see that you have the tangential and longitudinal components of the magnetic permeability.

So it is basically represented as the tensor, whereas this is an isotropic value. So you can write mu T and mu L as the principal values of the permeability tensor in the physical system, right? So, you can obtain the corresponding refractive indices as n_{T}, which will be $\sqrt{\mu_{\text{L}}\varepsilon}$ and n_{L} will be $\sqrt{\mu_{\text{T}}\varepsilon}$, right? So, this will be the refractive index seen by the two plane waves that are traveling along the two principal axes.

- To indicate the extent of anisotropy in the physical medium, the anisotropy factor (a function of position) is defined by:
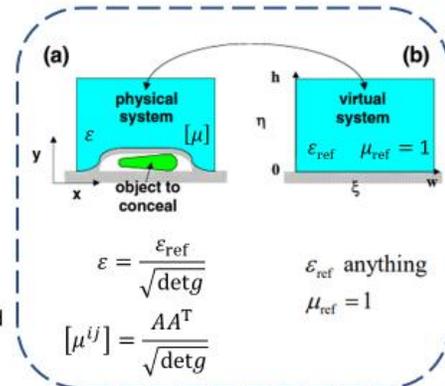
$$\alpha = \max\left(\frac{n_T}{n_L}, \frac{n_L}{n_T}\right)$$

- Using: $A_{ij} = \dfrac{\partial x_i}{\partial \xi_j}$ & $g = A^T A$ & $\alpha = \max\left(\dfrac{n_T}{n_L}, \dfrac{n_L}{n_T}\right)$

it can be proved that: $\alpha + \dfrac{1}{\alpha} = \left(\dfrac{\text{Tr}(g)}{\sqrt{\det g}}\right)$ with $\mu_L \mu_T = 1$

- On the other hand, an averaged refractive index n is defined relative to the reference medium by:

$$n = \sqrt{\frac{n_L n_T}{\varepsilon_{ref}}} \qquad \text{so that:} \quad n^2 = \frac{\varepsilon}{\varepsilon_{ref}} = \sqrt{\frac{1}{\det g}}$$

(a) physical system, ε, $[\mu]$, object to conceal

(b) virtual system, ε_{ref}, $\mu_{ref} = 1$

$\varepsilon = \dfrac{\varepsilon_{ref}}{\sqrt{\det g}}$

$[\mu^{ij}] = \dfrac{AA^T}{\sqrt{\det g}}$

ε_{ref} anything

$\mu_{ref} = 1$

Now, to indicate the extent of anisotropy in the physical medium, the anisotropy factor is okay. Which is also a function of position can be defined by alpha (α), which is basically $\alpha = \max\left(\frac{n_T}{n_L}, \frac{n_L}{n_T}\right)$.

So, if you use the Jacobian matrix A_{ij} which is $\frac{\partial x_i}{\partial \xi_j}$ and g is basically $A^T A$

and then you have obtained this alpha, which is the anisotropy factor. From this you can prove that $\alpha + \frac{1}{\alpha}$ can be written as: $\left(\frac{\text{Tr}(g)}{\sqrt{\det g}}\right)$. Now, the trace is basically, you know, for a square matrix, the sum of the diagonal elements. And if g is a n cross n matrix then it will be like trace of g can be calculated as summation over i ranging from 1 to n g i i ok. So, you are again taking the diagonal elements and adding them on, okay? And this is the assumption that $\mu_L \mu_T = 1$.

So, on the other hand, the averaged refractive index n can also be defined relative to the reference medium. So, you can get $n = \sqrt{\frac{n_L n_T}{\varepsilon_{ref}}}$. So, you can obtain that $n^2 = \frac{\varepsilon}{\varepsilon_{ref}}$, which is nothing but $\sqrt{\frac{1}{\det g}}$.

- Instead of using ε and μ^{ij} to describe the physical medium, we now use α and n which have geometrical meanings in terms of the metric.

- If there is a very fine rectangular grid in the virtual domain with tiny cells of size $\delta \times \delta$, every such tiny square is transformed to a parallelogram in the physical domain with two sides $\xi_1\delta$ and $\xi_2\delta$.

- A smaller anisotropy means a smaller value of $\frac{\text{Tr}(g)}{\sqrt{\det g}}$ while a smaller area of the transformed cell ($\sqrt{\det g}\,\delta^2$) means a larger refractive index n. Since: $n^2 = \frac{\varepsilon}{\varepsilon_{\text{ref}}} = \sqrt{\frac{1}{\det g}}$

- In cloaking, compression of space in the physical domain essentially makes the cloak anisotropic.

- However, the approach here is to minimize the induced anisotropy by choosing a suitable coordinate transform.

- If the anisotropy is small enough, we can simply drop it (by assigning $\alpha= 1$) and only keep the refractive index n.

- In other words, the physical medium becomes just a dielectric profile described with **unit magnetic permeability**.

So now, instead of using ε and μ^{ij}, which are the permittivity and permeability tensors, to describe the physical system now allows you to use α and n, which have geometrical meanings in terms of the metric. If there is a very fine rectangular grid in the virtual domain with tiny cells of size, say, delta by delta ($\delta \times \delta$).

Such a tiny square is basically transformed into a parallelogram in the physical domain because you have to leave that space for the object to hide, ok. And that will have two sides given by $\xi_1\delta$ and $\xi_2\delta$. A smaller isotropy will mean a smaller value of the trace of g divided by the square root of the determinant of g, i.e. ; $\frac{\text{Tr}(g)}{\sqrt{\det g}}$. While a smaller area of the transformed cell is $\sqrt{\det g}\,\delta^2$ will mean a larger refractive index n because n^2 can be written as: $n^2 = \frac{\varepsilon}{\varepsilon_{\text{ref}}}$. And that is nothing but $\sqrt{\frac{1}{\det g}}$.

So, in cloaking, what we will see is that the compression of space in the physical domain is essentially making the cloak anisotropic. However, the approach here is to minimize the induced anisotropy by choosing a suitable coordinate transformation. If the anisotropy is small enough, we can simply drop it by assigning $\alpha= 1$ and only keep the refractive index n. So, in other words, the physical medium becomes just a dielectric profile described by unit magnetic permeability. So, here you can see the transform grid in the physical system with the inner clock boundary, okay.
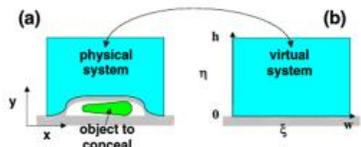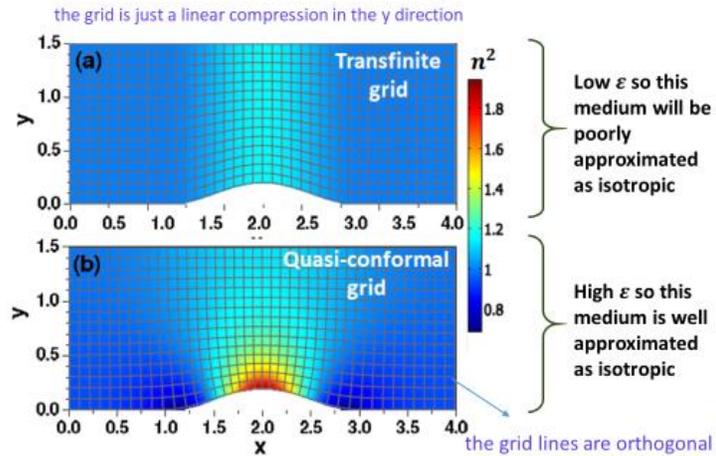
Figure: Carpet Cloaking: the physical and the virtual systems.

- An optimal map is generated by minimizing the Modified-Liao functional:

$$\Phi = \frac{1}{hw} \int_0^w d\xi \int_0^h d\eta \frac{\text{Tr}(g)^2}{\det g}$$

So, this is given by this transfinite grid where the transfinite interpolation is used for a rectangular 40 by 15 grids. So, in this case, you know you can see that the grid is just a linear compression in the y direction, right? So, this color map shows you the value of permittivity, and you can understand that low permittivity is required everywhere, okay. So, this medium will be poorly approximated as isotropic because the variation is very small.

Now, an optimal map can be generated by minimizing the modified-Liao function. So, you can get a quasi-conformal grid for this particular case, okay.

So, this is the generated grid using a quasi-conformal map, and this kind of mapping can be used. Here, the catch is that the grid lines are basically orthogonal. So, the aspect ratio of each cell in this particular case, or you can say the anisotropic factor alpha, basically becomes a constant of 1.042. Okay, while you can see that n squared in this case is basically ranging from 0.68, which is typically the dark blue value, to almost 1.96, okay. So, here n or epsilon remains finite without approaching either 0 or infinity. So, the singular values are not considered. So, it is basically a result of crushing the object to a plane instead of crushing it to a line or a point.
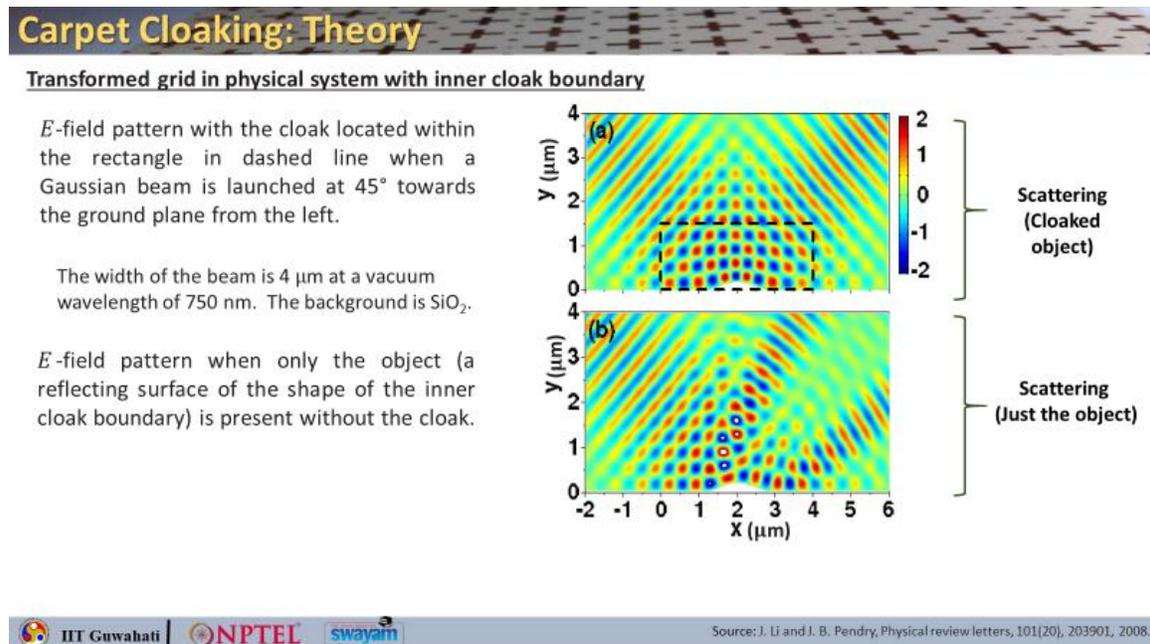
So, you are getting values that are easily achievable, right? So, no singular points basically occur in this kind of coordinate transformation, right? But what you can see is that there are some high permittivities. So, in this case, the medium is well approximated

as isotropic. That is a typo here; it should be anisotropic, yeah. So, now let us see how it works. So, to test the effectiveness of the design cloak, suppose that you have made a cloak that is 4 microns by 1.

5 microns, okay. And the cloak is basically defined relative to silica, which has a epsilon reference of 2.25. So, what we will do is employ the quasi-conformal grid that you have seen earlier. But ignore the anisotropy in the cloak by keeping only the epsilon profile with unit permeability.

So, you are keeping mu equal to 1. So, in this case, the permeability of the clock will vary from, say, 1.5 to 4.4, typically. And this range can be obtained easily by effectively etching or drilling subweb-length holes of different sizes. Along the z direction in a high dielectric material such as silicon.

So, it becomes you know more achievable. Now, outside the cloak, you can again see that you know the silica is working as the background material. Moreover, the inner surface of the cloak is coated with a highly reflective metal. So, to the observer, it is perceived as if this is the actual ground plane. This is relevant to the situation of routing light at our own will in optical integrated circuits. So, this is a clear picture that shows that what happens with a carpet cloaking and without a carpet cloaking, ok.

Carpet Cloaking: Theory

Transformed grid in physical system with inner cloak boundary

E-field pattern with the cloak located within the rectangle in dashed line when a Gaussian beam is launched at 45° towards the ground plane from the left.

The width of the beam is 4 µm at a vacuum wavelength of 750 nm. The background is SiO_2.

E-field pattern when only the object (a reflecting surface of the shape of the inner cloak boundary) is present without the cloak.
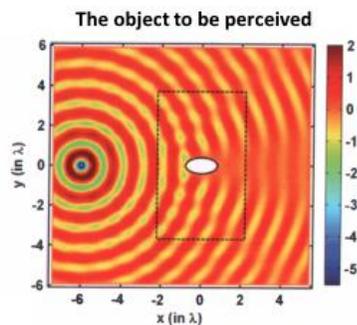
Scattering (Cloaked object)

Scattering (Just the object)

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So, these are nothing but the electric field patterns with the cloak located within the rectangle, as you can see here. This is the dashed line is the boundary ok and this is where

the bump is where the object is hidden And then, on top of that, you have put on the cloak. So, this is basically the physical cloak that you have seen earlier, right? So, we are basically considering a Gaussian beam that is launched at a 45-degree angle towards the ground plane from the left, okay. And the width of the beam is around 4 microns at a vacuum wavelength of 750 nanometers, and we considered silica as the background. So, if you compare this with this, the electric field pattern when only one object or You can say that a reflecting surface in the shape of the inner cloak boundary is present without the cloak.

So, you can see that this is how the scattering will tell you that there is an object here. But as soon as you put on the cloak, it is reflected only in one direction, as if it were reflected from a flat ground plane. So, this is how the cloaking works. The other applications of carpet cloaking, as we briefly mentioned, would be to reshape the scattering of the objects. So, that way, it will also be able to disguise one object in another form, right.

So, the electric field contours in finite element simulations for reshaping a rectangular object into an elliptical object are shown here.
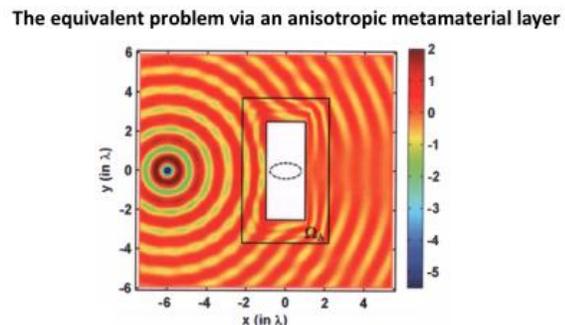

So, this is the object to be perceived and the equivalent problem via anisotropic metamaterials. The layer can be seen here. So, this particular diagram shows you what the scattering profile from an elliptical object is. Now, here is what is happening: you have basically put that elliptical object into this kind of cloaked medium, okay. So, you

are basically getting the scattering from a rectangular object that is embedded in an anisotropic medium. Designed by this transformation optics, this overall thing basically scatters just like an elliptical object.

So, what is happening is that you are basically able to hide your object, which is rectangular in shape. because it will give you its identification as an elliptical object. So, that is called reshaping the scattering of objects. You can also see another example of the electric field contours in a finite element simulation. Shows the reshaping of a square PEC object into a dielectric object.
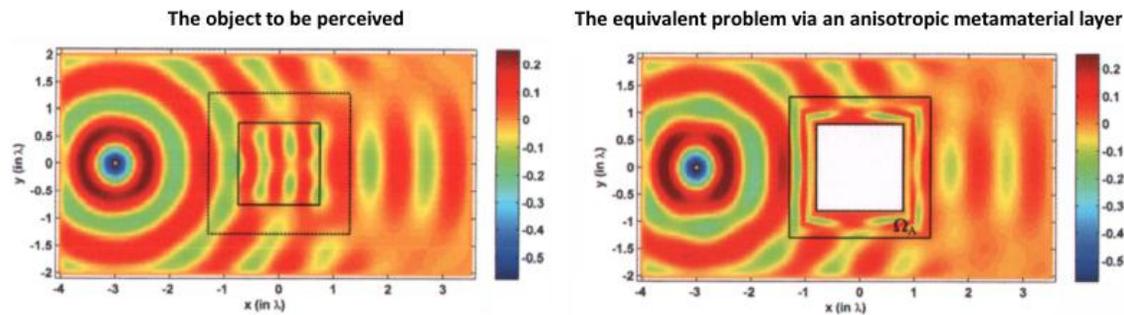
So, this is the object to be perceived, and this is the equivalent problem via an anisotropic metamaterial layer. So you can see that you know. You are reshaping a square PEC into a dielectric object. So, you are basically giving it the scattering properties of a different material. So, you can put any dielectric object, and you can see that this kind of scattering signature will be obtained.

When you put a cloak around it, right? So, this is how carpet cloaking can help you first hide an object on a ground plane, or you can also reshape it. the scattering from any object as if the scattering is coming from different shaped objects. So, these are the two vital applications of carpet cloaking.

So, with that, we will conclude here. We will look into the transformational electromagnetic technology for ender applications and stealth technology in the next lecture.

If you have any queries regarding this lecture, you can drop an email to this email address mentioning the course title and the lecture number in the subject line. Thank you.