Course Name: Introduction to Microwave and Optical Metamaterials
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Week-10
Lecture-48

Lec 48: Examples and Applications of Transformational Electromagnetics

Hello students, welcome to Lecture 48 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on the applications of transformational electromagnetics.

Lecture Outline

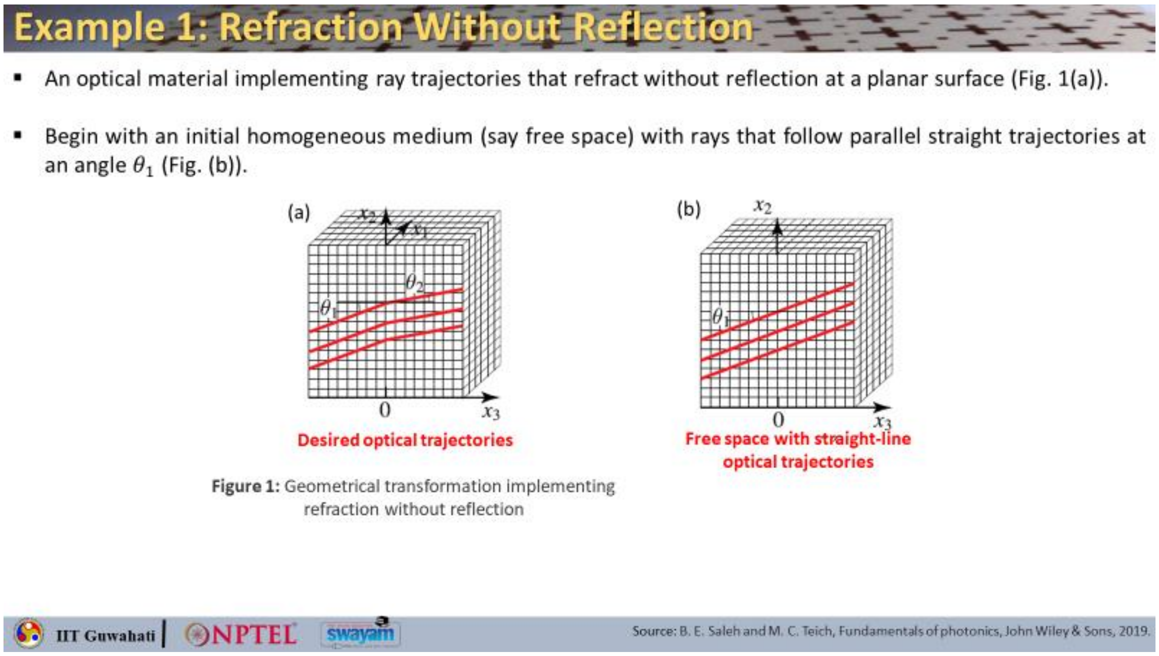
- Examples of Transformational Electromagnetics (TEM)
  - Refraction Without Reflection
  - Refraction at Normal Incidence
  - Cylindrical Focusing
- Applications of Transformational Electromagnetics (TEM)
  - Invisibility Cloaking using Optical Metamaterials
  - Invisibility Cloaks

So, here is the lecture outline. We will first look at a few examples of transformational electromagnetism. We will see how we can have refraction without reflection and refraction at normal incidence. And how we can achieve cylindrical focusing; after that, we will move on to discussing some applications of transformational electromagnetics.

We will discuss how invisibility clocking can be obtained using optical metamaterials and we will discuss Very briefly about invisibility clocks, which will be continued in the next lecture.

Examples of Transformational Electromagnetics

So, let us look at some examples of transformational electromagnetics.



So, the first example we will be handling today is refraction without reflection. So, you know that in a normal scenario, whenever there is refraction, there is some light that is also reflected, right? So, in this particular case, we will see how you can develop material artificially that can provide you with refraction without any reflection.

So, in this case, the desired optical trajectory is something like this. So, this is an optical material that implements these ray trajectories, which show you that There is refraction without any reflection right at a planar surface. So, these are the axes x1, x2, and x3, right? So, this is a geometrical transformation okay that will be implementing your reflection without reflection. So, this is basically a geometric transformation that implements refraction without reflection. So, let us first begin with an initial homogeneous medium, namely free space with rays that basically follow parallel straight trajectories at an angle of theta 1, as shown in this particular figure. So, you see, these are basically parallel straight trajectories.

Refraction Without Reflection

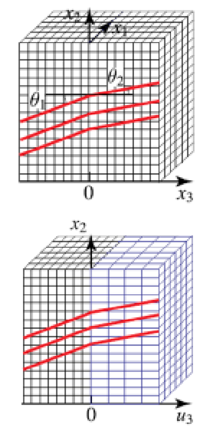
- Apply a geometrical transformation that stretches the coordinate system by a scale factor s along the x_3 direction in the region $x_3 > 0$.
- Stretching the coordinate system by the factor s for $x_3 > 0$ causes the rays to change slope and follow the desired trajectories.
- The desired refraction is achieved by choosing s as the ratio of the initial and desired slopes:

$$s = \tan\theta_1 \,/\, \tan\theta_2$$

- This transformation is implemented by the relations:

$$u_1 = x_1, \qquad u_2 = x_2, \qquad u_3 = s^{-1}x_3$$

- This type of scaling of the Cartesian coordinate system, in which the directions of the axes do not change, converts a cube into a cuboid.



Stretching the coordinate system by the factor s

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Now, after you apply a geometrical transformation in which you basically stretch the coordinate system by a scale factor s along this particular direction, right x3, okay for the positive half of this x_3, that is $x_3 = 0$. So, in that way, when you stretch, what will happen? You know this stretches the coordinate system. By a factor of s, the rays will change its slope.

So, from this straight parallel slope, you will see that when you are stretching only the positive part of x_3, you will see that the rays develop a slope, and this angle can be marked as theta 2. Now, you will see that the desired refraction is essentially achieved by choosing this stretching factor, s. And that is basically a ratio of the initial slope to the desired slope. So, s is basically $s = \tan\theta_1 \,/\, \tan\theta_2$. Now, what is this transformation? The coordinate transformation can be written as you know that there is nothing changing along the x_1 direction.

So, u_1 will be the same as x_1; u are basically the new ones, right? u_1, u_2, and u_3 are basically the new coordinates. Same, you have not changed anything along x_2. So, u_2 will be the same as the old x_2. However, stretching has occurred along the x_3 direction. So, u_3 is nothing but $s^{-1}x_3$, right? So, this type of scaling of the Cartesian coordinate system in which the directions of the axes are do not change; it will basically convert a cube into a cuboid, correct.

So, you will basically see that this particular thing has now changed into a cuboid, right. This happens when you stretch the coordinate system by a factor of s along only one direction.

So, once you understand that these are my new axes and these were the old axes, you can always calculate.

Refraction Without Reflection

- Calculate Jacobian matrix (A):

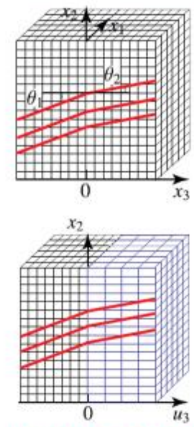
$$A_{ij} = \frac{\partial u_i}{\partial x_j} \qquad i, j = 1,2\,3 \ldots$$

- Based on above equation:

$$A = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial x_2} & \frac{\partial x_1}{\partial x_3} \\ \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial x_2} & \frac{\partial x_2}{\partial x_3} \\ \frac{\partial s^{-1}x_3}{\partial x_1} & \frac{\partial s^{-1}x_3}{\partial x_2} & \frac{\partial s^{-1}x_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s^{-1} \end{bmatrix}$$

- The Jacobian matrix A is diagonal matrix with diagonal elements $(1, 1, s^{-1})$ and its determinant:

$$\det A = s^{-1}$$



Stretching the coordinate system by the factor s

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

The Jacobian matrix A can be given as $A_{ij} = \frac{\partial u_i}{\partial x_j}$, where i and j can both be 1, 2, or 3. and then you can calculate this because you already know the values. So, the Jacobian matrix can now be calculated. So, you see, the stretching only happens in one direction.

So, u_3 will have some kind of role to play here, and finally, you will get the matrix to be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s^{-1} \end{bmatrix}$, right.

So, as you can see, this Jacobian matrix is basically a diagonal matrix where the elements are 1 and 1 and s^{-1} right and you can obtain the determinant of this matrix determinant of A $(\det A) = s^{-1}$, ok. So, with that you can now correlate to the material property which can implement this kind of coordinate transformation right.

Refraction Without Reflection

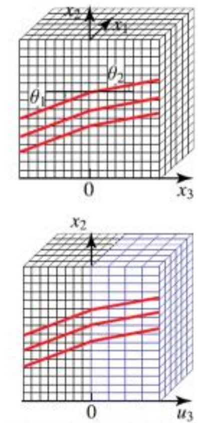
- Using $\varepsilon_0^{-1}\varepsilon' = \mu_0^{-1}\mu' = |\det \mathrm{A}|^{-1}\mathrm{A}^{\mathrm{T}}\mathrm{A}$ & $\det \mathrm{A} = s^{-1}$

$$\Rightarrow \quad \varepsilon_0^{-1}\varepsilon' = \mu_0^{-1}\mu' = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s^{-1} \end{bmatrix}$$

- Since the matrices ε' and μ' are diagonal, the anisotropic material has principal axes pointing along the axes of the coordinate system.

- The principal values are: $\varepsilon_1 = s\varepsilon_0$, $\varepsilon_2 = s\varepsilon_0$, and $\varepsilon_3 = s^{-1}\varepsilon_0$

with $\mu_1 = s\mu_0$, $\mu_2 = s\mu_0$, and $\mu_3 = s^{-1}\mu_0$



Stretching the coordinate system by the factor s

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, you can write $\varepsilon_0^{-1}\varepsilon'$ will be equal to $\mu_0^{-1}\mu'$. So, ε' and μ' will be the new material permittivity and permeability, and that should be equal.

The $|\det \mathrm{A}|^{-1}$ is that, and then you have $\mathrm{A}^{\mathrm{T}}A$. We have already calculated that the determinant of A is nothing but 1 over S, okay.

So, this will simply boil down to this equation. So, you have $\varepsilon_0^{-1}\varepsilon' = \mu_0^{-1}\mu'$, Okay, that will be a 3-by-3 matrix, again a diagonal one. So, you have $\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s^{-1} \end{bmatrix}$.

So, as I mentioned, these matrices epsilon prime and mu prime are basically diagonal. The isotropic material has its principal axis pointing along the axes of the coordinate system. So, that is a good thing, and what are the values of the principal axes? You can see that ε_1 will be simply, $s\varepsilon_0$ and ε_2 will also be $s\varepsilon_0$, but ε_3 will be $s^{-1}\varepsilon_0$. The same thing will also apply to μ because I have the same relationship; So, μ_1 will be $s\mu_0$, μ_2 will also be $s\mu_0$, and μ_3 will be $s^{-1}\mu_0$.

So, the next thing would be that we understand that you know the material properties are obtained.

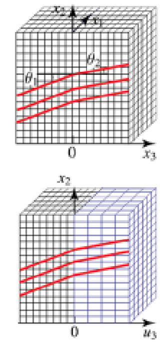
Refraction Without Reflection

- The parameters of the equivalent material may also be obtained

  by matching the phase shift encountered when a plane wave crosses the stretched free-space segment with that encountered when the wave is transmitted through an unstretched segment filled with the new material.

- To determine the parameters, we consider three waves in turn, each with the electric field along one of the coordinates:
  - Wave 1 is a plane wave traveling along the x_3 direction with electric and magnetic fields in the x_1 and x_2 directions, respectively.
    - ✓ The appropriate permittivity and permeability are thus ε_1 and μ_2 so that: $k = \omega\sqrt{\varepsilon_1 \mu_2} = \omega\sqrt{s\varepsilon_0 s\mu_0} = sk_0$, corresponding to a refractive index $n_1 = s$

- Therefore; the phase shift accumulated over the distance (d) is $sk_0 d$ and impedance: $\eta_1 = \sqrt{\mu_2/\varepsilon_1} = \eta_0$



Stretching the coordinate system by the factor s

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Now, some other parameters of this material should also be obtained.

So that you know, there is no reflection. So, that can be obtained by matching the phase shift encountered when a plane wave crosses this stress-free space segment is encountered when the wave is transmitted. Through an unstressed segment filled with the new material, right? So, to determine the parameters, you basically have to consider three waves in turn, each with an electric field along one of the three coordinates. So, we are now trying to do the matching of the phase shift.

So, let us first consider wave 1, which is basically a plane wave traveling along the x_3 direction. With its electric and magnetic fields in the x_1 and x_2 directions, respectively, So, in that case, you will see that the permittivity and permeability are basically epsilon 1 and mu 1. Because you know it is along x_1 and x_2, ε_1 and μ_2 . So, from that, you can obtain what your k is, which is the wave number that is $k = \omega\sqrt{\varepsilon_1 \mu_2}$, okay. So, here you have ε_1 and μ_2.

So, that goes in here, okay. Now you know that ε_1 is nothing but $s\varepsilon_0$, and μ_2 is nothing but $s\mu_0$. So, s will come out, and this whole thing is giving you sk_0 , which is the vacuum wave number, okay. OK. So, this tells you that it is giving you a corresponding refractive index of n_1, which is basically s. So, this is what is important: you need to see that you know along the plane.

Where the wave is incident, you actually see the same refractive index. So, in this case, you are seeing that you know the wave is experiencing n_1 equal to s. So, you can say the phase shift that will be accumulated over a distance d will be simply given as sk_0d.

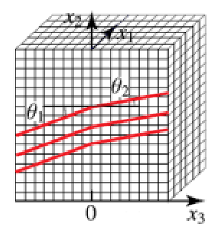
and that is the phase shift, and the impedance will be η_1 , which can be expressed as $\eta_1 = \sqrt{\mu_2/\varepsilon_1}$. Now this is the same as η_0 because they are both, you know, $s\mu_0$ divided by $s\varepsilon_0$.

So, s cancels out. So, this is simply $\sqrt{\mu_0/\varepsilon_0}$, which is nothing but η_0 . So, there is no change in the impedance and that will ensure that there is no reflection.

Refraction Without Reflection

- Wave 2 is also taken to travel along the x_3 direction but the electric and magnetic fields are now in the x_2 and $-x_1$ directions, respectively.
  - ✓ This wave also travels with a refractive index $n_2 = s$ and has an impedance $\eta_2 = \eta_0$.
- Wave 3 travels along the x_2 direction with electric and magnetic fields in the x_3 and x_1 directions, respectively.
  - ✓ The appropriate permittivity and permeability are ε_3 and μ so that:

    $k = \omega\sqrt{\varepsilon_3\mu_1} = \omega\sqrt{s^{-1}\varepsilon^0 s\mu_0} = k_0$, corresponding to a refractive index $n_3 = 1$
  - ✓ The phase shift is k_0d since there is no stretching in the x_2 direction and
  - ✓ The impedance $\eta_3 = \sqrt{\mu_1/\varepsilon_3} = s\eta_0$.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Next, we consider wave 2, which is taken along the x-ray direction again, but this time the electric field and the magnetic fields are in the x_2 and $-x_1$ directions, respectively. So, in this case, if you do a similar kind of analysis, you will see that the wave is basically traveling. With a refractive index n_2 that is equal to s, it also has an impedance η_2 that is the same as η_0 , Okay, and then we consider the third wave, which is wave 3, that will be traveling along the x_2 direction.

And in this case, the electric and magnetic fields will be in the x_3 and x_1 directions, respectively, right.

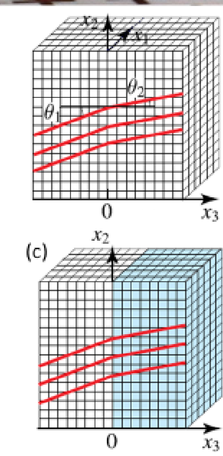
So, in this case, you can approximate the permittivity and permeability to be ε_3 and μ_1. So that will give you, you know, the wave number $(k) = \omega\sqrt{\varepsilon_3\mu_1}$. So, omega remains like that: $\sqrt{\varepsilon_3}$ can now be written as $s^{-1}\varepsilon_0$. So, this has to be a there is a typo it is ε_0 and μ_1 is

$s\mu_0$.

So, s basically cancels out, and you get k_0, Okay. So, that is basically giving you that you know the refractive index n_3 is equal to 1. So, in this case the phase shift will be simply k_0d since there is no stretching in the x_2 direction okay. that is the direction of wave propagation in this case, and the impedance η_3 will be simply $\sqrt{\mu_1/\varepsilon_3}$, right? And that is $s\eta_0$, okay.

Refraction Without Reflection

- Using these results, we conclude that the final design is a piecewise homogeneous medium with free space in the left half plane and an anisotropic uniaxial material in the right half plane (Fig. (c)).
- The anisotropic material is birefringent with $n_1 = s$, $n_2 = s$, and $n_3 = 1$, but it introduces no reflection at the boundary with free space since the impedances are the same as that of free space: $\eta_1 = \eta_2 = \eta_0$.
- Two factors distinguish refraction at the boundary of the synthesized anisotropic medium from conventional refraction at the boundary of a homogeneous and isotropic medium:
    1. Refraction is not accompanied by reflection; and
    2. The relationship between the angle of refraction and the angle of incidence, $s\tan\theta_2 = \tan\theta_1$, differs from Snell's law.



Equivalent anisotropic, homogeneous material that causes the rays to change slope in an identical way

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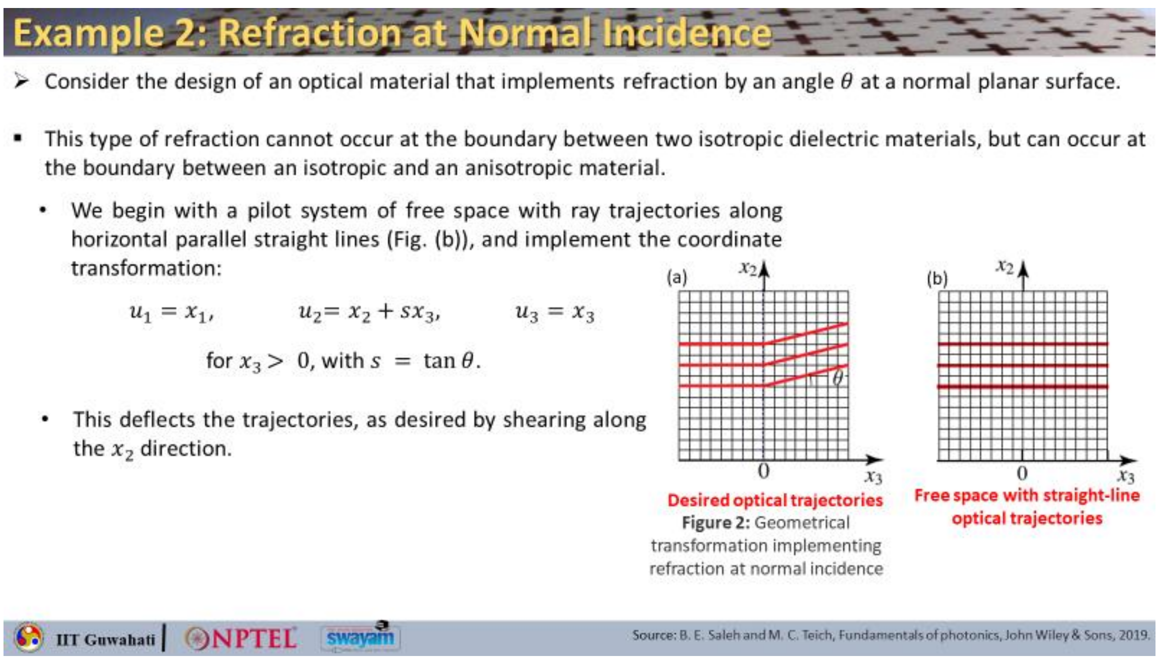
So, there is a change in the impedance. So, what we understood is that using these results, you can conclude that the final design is basically, it is a piecewise homogeneous medium where you have free space in the left half-plane. And then you have anisotropic uniaxial material in the right half-plane, which is shown here, right? So, what are the properties of this material? We have seen that n_1, which is the refractive index along the x_1 direction, is s; n_2 is also s. That is the refractive index, and n_3, which is along this one, is basically 1.

So, in this case, you can understand that this also has a refractive index of 1. In this direction, the refractive index is again 1. So, when you look at the reflection coming from the boundary with the free space, that is basically your x_1, x_2 plane. So, in the x_1, x_2 plane, you can see that the impedance present is basically the same as that in free space. Because that is why we computed η_1 and η_2 separately.

And both η_1 and η_2 came out to be the same as η_0 . So, there is no impedance mismatch.

So, there is no reflection right. So, these two factors distinguish refraction at the boundary of this synthesized anisotropic medium. From a conventional reflection that takes place at the boundary of a homogeneous and isotropic medium, right? So, here is what we have understood: refraction is not accompanied by reflection, okay.

So, this is something completely new, and also the relationship between the angle of refraction and the angle of incidence is not governed by Snell's law here; it is rather given by this formula, which is $s\tan\theta_2 = \tan\theta_1$, right? So, this is how you can actually achieve refraction without reflection, and that is from an anisotropic homogeneous material. That will help you achieve it. So, now let us look at the second example, which is basically refraction at normal incidence, okay.

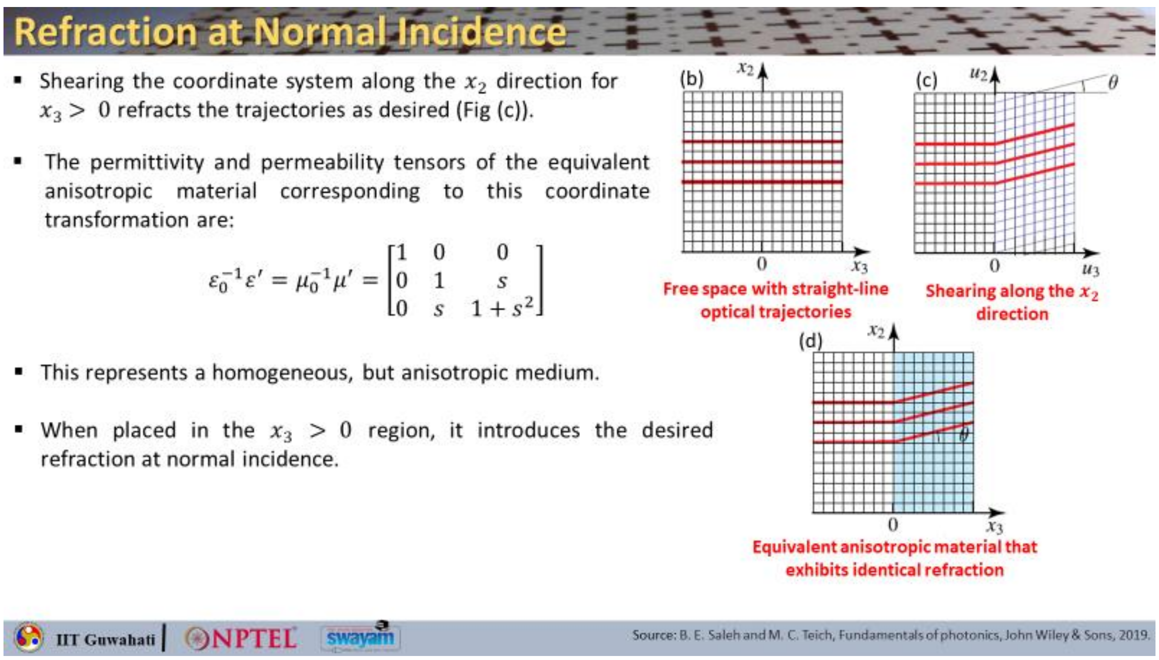


So, let us consider the design of an optical material that implements refraction at an angle theta at a normal planar surface, right? So, this is how the desired optical trajectory looks. So, you have normal incidence, and then you get refraction.

So this type of refraction cannot occur at the boundary between two isotropic dielectric materials. But this can occur at the boundary between an isotropic and anisotropic material, right? So, to begin with, you take a pilot system of free space with a rate trajectory that is shown as horizontal, parallel straight lines. And then you can basically implement the coordinate transformation that can give you this kind of refraction or bending of the rays. So, if you do that, u1 will be equal to x_1, u2 will be equal to $x_2 + sx_3$, okay. And then u3 remains the same as x_3 , okay. And this kind of stretching is happening because, you know, when x_3 is greater than 0, it is in the positive half-plane.

And s will now have the tangent dependency on theta correctly. So, this will basically deflect the trajectory, and this is done as per the requirements and as desired by the shearing.

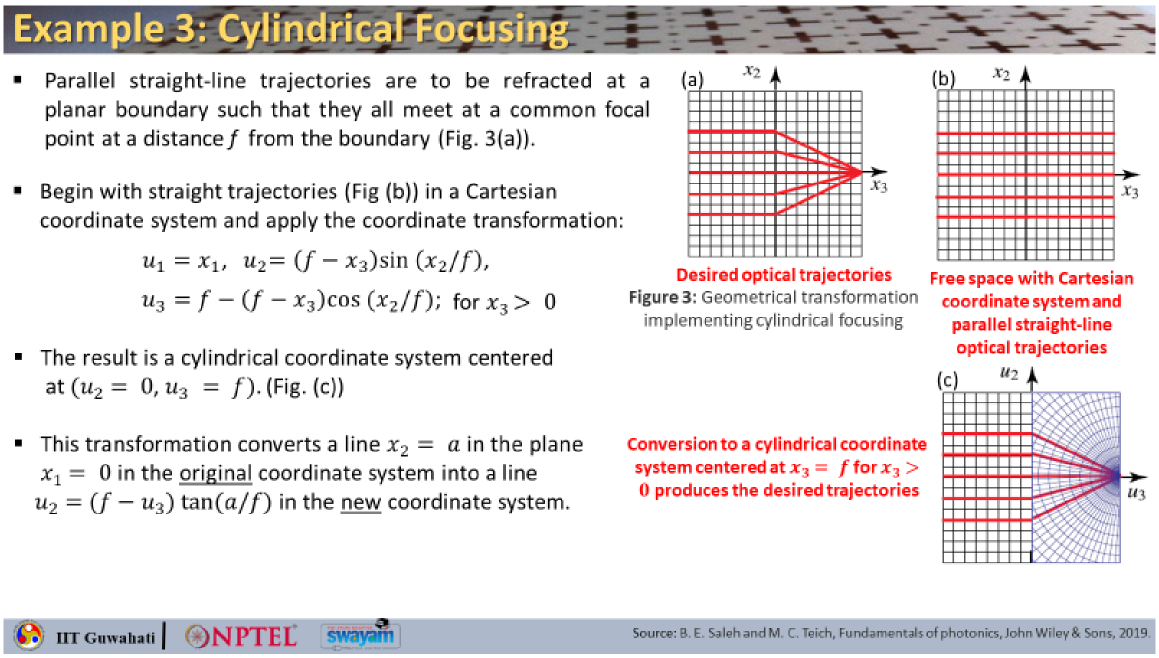
So, you can actually do this by shearing along the x2 direction.



So, you can see the coordinate system is basically sheared along the x2 direction like this by an angle of theta. That basically gives you the bending of the rays, and this is done only for the positive half-plane of x_3 , that is, where x_3 is greater than 0.

Now, if you try to put this in terms of the permittivity and permeability tensor, as we have done earlier. So, if you do that computation, you will see that you will get $\varepsilon_0^{-1}\varepsilon' = \mu_0^{-1}\mu'$ given as: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & s \\ 0 & s & 1+s^2 \end{bmatrix}$, okay. So, this basically represents a homogeneous yet anisotropic medium. So, if you place this kind of medium in the positive x_3 plane, it will basically give you the desired bending, right. So, this is where we started: we started with free space and a straight-line optical trajectory.

And then we introduced shearing along the x_2 direction, allowing the waves to bend, and then we mapped this property into the new material. And we have gone back to the old coordinate system, which is x_1, x_2, and x_3, and that is how we do all this all the time, right.



A new example of how we can achieve cylindrical focusing is needed. So, here we also start with parallel straight-line trajectories, as you can see in the left half-plane x3, on the negative side of x3. So, these are to be refracted at a planar boundary in such a way that they meet. At a common focal point at a distance f from the boundary.

So, normally this is not possible from a flat boundary between two isotropic mediums, right? So, you can again think of that as starting with rays that have a straight trajectory and Then you have to introduce a Cartesian coordinate system and then some coordinate transformation where your u_1, which is x_1; that is the direction in and out of this screen that remains unchanged, but in this plane, okay. In the x_2-x_3 plane, you will have this coordinate transformation happening. So, you can write $u_2 = (f - x_3)\sin(x_2/f)$, f is basically the focal length, ok and $u_3 = f - (f - x_3)\cos(x_2/f)$, and that is for the positive x_3, okay.

So, once you implement this, you will see that you are basically getting a kind of cylindrical coordinate system centered at $u_2 = 0$, Okay, and $u_3 = f$. So, you will get something like this, you know. So, you are basically doing a conversion to a cylindrical coordinate system that is centered here. that is how your rays will be now you know converging to this particular point.

So, this transformation basically converts a line that is, say, x_2 equals a in a plane where x_1 equals 0. There is this particular plane in the original coordinate system into a line that is, say, $u_2 = (f - u_3)\ tan(a/f)$ in the new coordinate system. So, that is how it achieves this kind of cylindrical focusing.

Similarly, you can see what is happening to any line over here.

Cylindrical Focusing

- It also converts a line $x_3 = b$ in the plane $x_1 = 0$ into a circle $u_2^2 + (f - u_3)^2 = (f - b)^2$ of radius $(f - b)$ centered at the point $(u_2, u_3) = (0, f)$.

- Using: $\varepsilon' = |\mathrm{detA}|^{-1} \mathrm{A}^{\mathrm{T}} \varepsilon \mathrm{A}$ & $A_{ij} = \frac{\partial u_i}{\partial x_j}$ $i, j = 1,2\ 3 \ldots$

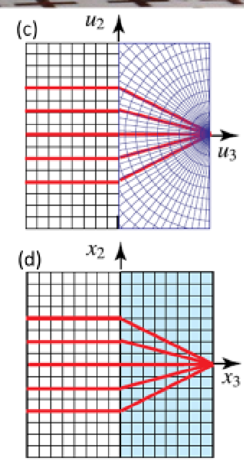
  $\mu' = |\mathrm{detA}|^{-1} \mathrm{A}^{\mathrm{T}} \mu \mathrm{A}$

  The transformation yields the diagonal matrix: $\varepsilon_0^{-1}\varepsilon' = \mu_0^{-1}\mu' = \begin{bmatrix} s & 0 & 0 \\ 0 & s^{-1} & 0 \\ 0 & 0 & s \end{bmatrix}$

  and $s = \frac{f}{|x_3 - f|}$

- Therefore, permittivity and permeability tensors of the equivalent medium have principal axes along the (x_1, x_2, x_3) axes.

- The principal values of permittivity and permeability tensors are dependent on the position x_3, *i.e.*, the equivalent material is graded along the x_3 direction with larger anisotropy near the focal line.(Fig. (d))



Equivalent anisotropic material with identical trajectories

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- So, if you have a line x_3 equals b in this plane, okay. That is basically being converted into a circle that has the equation: $u_2^2 + (f - u_3)^2 = (f - b)^2$.

  So, it is basically being converted into a circle of radius $(f - b)$ that is centered at this point where u_2 and u_3 are essentially 0 and f. So, once you understand this kind of conversion, you can always calculate the new material property ε', μ' from this formula is acceptable where the Jacobian matrix can be calculated. So, once you do this, you can always practice and find out that you know ε_0' and $\varepsilon_0^{-1}\varepsilon'$, which is also equal to $\mu_0^{-1}\mu'$, is basically a diagonal matrix that has elements s, 0, 0, 0, s^{-1} , 0, 0, 0, and s.

  And what is this? It is basically, $s = \frac{f}{|x_3 - f|}$. So, that way you can find out the permittivity and permeability tensors of the equivalent media that have principal axes along x_1, x_2, and x_3. So, the principal values of the permittivity and permeability tensors depend on the position x_3. That is, you know the equivalent material is basically graded along the x_3 direction. Some larger anisotropy is okay near the focal line, as you can also see it here.

So, this is how the equivalent anisotropic material achieves cylindrical focusing. but back in the original coordinate system can be designed. It is a little mathematically intensive; I am just telling you about this kind of transformational optics or, like coordinate transformation, you can achieve this kind of extraordinary manipulation of the behavior or trajectories of electromagnetic waves.

So, with that, we move on to the next one. The topic is the application of transformational electromagnetism.

Application of Transformational Electromagnetics

Invisibility Cloaking

- Artificially structured metamaterials have enabled unprecedented flexibility in manipulating electromagnetic waves and producing new functionalities, including the cloak of invisibility based on coordinate transformation.
- The first experimental demonstration of such a cloak at microwave frequencies.
- However, that design cannot be implemented for an optical cloak, which is certainly of particular interest because optical frequencies are where the word 'invisibility' is conventionally defined.

"An invisibility cloak"

Microwave Cloaking

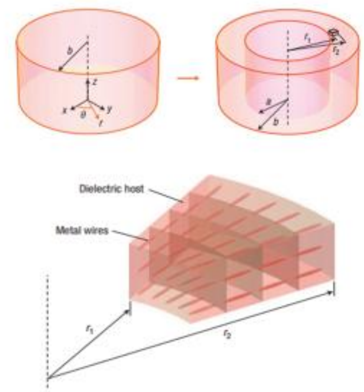
So, the first application is invisibility cloaking. So, this kind of artificially structured metamaterials has enabled this unprecedented. Flexibility in manipulating electromagnetic waves and producing new functionalities is something like this. You know this kind of cloak of invisibility based on coordinate transformations.

So, the first experimental demonstration of such a cloak was conducted at microwave frequencies, where normally you have a solid object. And if you have microwave sources, the rays are reflected and it is basically detected, but if you have an invisibility cloak. around this object it is possible that the rays can bend around the object and pass now disturbed as if there is no material in between. And this ring is basically this kind of a structure that is made of splitting resonators that can allow you band this path. However, the design cannot be implemented for the optical cloak, which is particularly interesting.

Because you know that in optical frequencies, only the word "invisibility" is conventionally defined, right? So, this particular image I will just want to tell you about this that this particular image shows you that How a person holding a cloak can simply become invisible means that the cloak is allowing it. The light rays bend around the person, and you can see the background. So, this is a kind of, you know, schematic prediction of how an invisibility cloak should look. So, you can obtain invisibility cloaking by using optical metamaterials.

Invisibility Cloaking using Optical Metamaterials

- **Transformation-Based Cloaking (Optical Cloak Design)**
  - Utilizes coordinate transformation to guide electromagnetic waves around an object.
  - The inner region $r < b$ is mathematically compressed into a shell region $a < r < b$.
  - This compression results in anisotropic, spatially varying permittivity and permeability values—forming the basis of the cloak.
- **Design Implementation Using Metal Wires (shown in fig.)**
  - The cloak structure consists of metal wires embedded in a dielectric host, forming an artificial anisotropic medium.
  - Wires are aligned perpendicular to the cylindrical surface.
  - Wire arrangement can be random; perfect periodicity is not required, making fabrication more flexible.
- **Design for Larger Cloaks**
  - For large-scale cloaks, the metal wires can be subdivided into shorter segments.
  - These segments must remain subwavelength in size to ensure effective medium behavior.



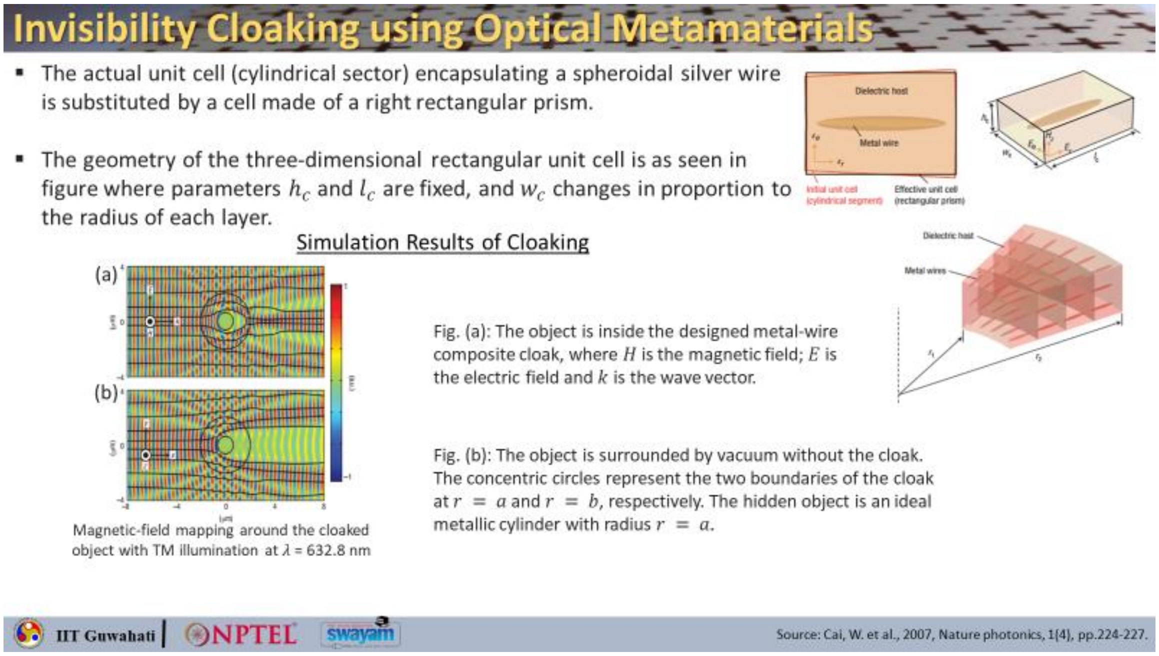
Source: Cai, W. et al., 2007, Nature photonics, 1(4), pp.224-227.

So, the first thing you have to do is to know that you use transformation-based cloaking.

So, it basically uses a coordinate transformation to guide electromagnetic waves around an object. So, the linear region that you see where r < b needs to be mathematically compressed into a shell region. where r is between a and b. In that case, whatever you are keeping inside this region will be cloaked. Because you are basically mapping this entire space into only this shell region.

So, this compression results in anisotropic, specially varying permittivity and permeability values, and it will form the cloak. So, you can implement this using metal wires, okay. So, that allows you to have the physical realization okay of this kind of cloak. So, these are basically metal wires embedded in a dielectric host, which is forming an artificial anisotropic medium. So, the wires are basically aligned perpendicularly to the cylindrical surface.

And the arrangement can be random. So perfect periodicity is not required. So, that gives your fabrication little bit flexibility. So, for large-scale cloaks, these metal wires can be subdivided into shorter segments and One requirement is that the segment must remain sub-wavelength in order to ensure that you get effective medium behavior right.



So, you can also do this by taking a small metallic wire embedded in a dielectric host. Usually, it is a cylindrical segment, but you can kind of, you know, approximate using a rectangular prism, Okay, like this, and then you periodically repeat it, and these are some results from. Electromagnetic simulation using ComSol Multiphysics software, where you see what happens when you place an object inside a cloak. The light rays bend around the cloaked object, and after that, you will see the wave front remains somewhat similar. So, the object remains undetected; this is the magnetic field map at 632.8 nanometers.

So, in optical frequency, this is the same object without the cloak being installed. So, you can see that the waves are basically being disturbed here. So, you can identify this object if the cloak is not present. So, the invisibility cloak has always been a dream for a lot of people to achieve.

This idea of invisibility has long fascinated the human imagination. It seemed, although confined only to science fiction and movies, that you know the recent advancements in optical technology. has brought us much closer to our very own, you know, real-life invisibility cloak, so here is a picture of Demonstration of optical camouflage technology at Tokyo University. Okay, so as you can see, although it is not perfect, it is, you know, kind of. I'm still telling you that you can move towards that technology with further improvement, right.

Invisibility Cloaks

A demonstration of optical camouflage technology at Tokyo University, conducted by Faculty of Engineering professor Susumu Tachi, in Tokyo. Credit: Tokyo University, 2003.

Although not nearly as impressive or easy to use as the cloaks shown in *Star Trek* or *Harry Potter*, these yet early devices are redefining the meaning of the word stealth.

How does an invisibility cloak work?

"Visibility depends on the action of the visible bodies on light. Either a body absorbs light, or it reflects or refracts it, or does all these things. If it neither reflects nor refracts nor absorbs light, it cannot of itself be visible." – H.G. Wells, *The Invisible Man*

It is not as catchy as the cloak shows shown in Star Trek or the Harry Potter movie, but you know. They are still telling you that this is the way forward, so how does an invisibility cloak work? Now, the visibility basically depends on the action of visible bodies on light.

Invisibility Cloaks

- Normally, when light interacts with an object, it is either absorbed or reflected, making the object visible.
- At its core, invisibility technology relies on manipulating light waves, which are responsible for our visual perception.
- Researchers have developed innovative approaches that bend and redirect light, effectively concealing objects from view.
- However, achieving a full-blown invisibility cloak across the entire visual spectrum is a huge challenge.
- Previously, scientists have made fighter jets invisible to radar and made thermal invisibility jackets that hide soldiers from enemy thermal cameras.
- But to conceal something from the naked eye as if it never was there requires some serious engineering.

Figure: An illustration of real life invisible cloaks.

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So, either you know a body basically absorbs light, or it reflects it, or it refracts it, or it does all of these things.

Invisibility Cloaks

- True transparency would require light to pass through an object undisturbed, as if it were not there.
- To achieve this, a cloaking device would need to redirect light from all directions around the object, so that it appears invisible from any angle.
- Metamaterials, engineered materials with unique properties not found in nature, play a pivotal role in creating invisibility cloaks.
- By designing these materials with carefully arranged nanostructures, scientists can control the behavior of light waves.
- This field, known as transformation optics, allows for the manipulation of light around an object, making it appear invisible.

Figure: An illustration of real life invisible cloaks.

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Source: https://www.zmescience.com/feature-post/technology-articles/inventions-1/real-life-invisibility-cloaks/

But if it neither reflects nor absorbs, that means it is not doing anything to the light, and that is where it becomes invisible, right.

Invisibility Cloaks

- Several cloaking mechanisms have been proposed and developed to achieve invisibility.
- One approach involves bending light around an object, creating a "cloak" that renders it optically transparent.
- Another method utilizes the redirection of light, making an object appear as if it were not there.
- These techniques require precise control over the speed, direction, and intensity of light waves to achieve the desired effect.

Figure: An illustration of real life invisible cloaks.

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So, we will continue this discussion in the next lecture.

So, we will stop here for this right now. Thank you, and if you have any queries regarding this lecture, please drop an email to this email address. Thank you.