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Lec 46: Active Hyperbolic Metamaterials for Various Spectral Band



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Hello everyone, welcome to Lecture 46 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on active hyperbolic metamaterials for various spectral bands.

Lecture Outline

- **Active Hyperbolic Metamaterials (HMMs) for Visible to Near-Infrared Spectral Band**
 - Chalcogenide Phase Change Material based Active HMMs
- **Active Hyperbolic Metamaterials (HMMs) for Mid-infrared to THz Spectral Band**
 - Graphene-based Active HMMs
 - Topological Insulator-based Active HMMs
 - Superconductor-based Active HMMs



So, here is the lecture outline. We will look into active hyperbolic metamaterials for the visible and near-infrared spectral bands. We will discuss chalcogenide phase change material-based active metamaterials. And then we will move on to discussing active hyperbolic metamaterials for mid-infrared and terahertz spectral bands.

There we will discuss graphene-based active metamaterials; HMMs are hyperbolic metamaterials, right? Topological insulator-based active HMMs and superconductor-based active HMMs.



Active HMMs for Visible to Near Infrared Spectral Band



So, let us first look into this topic, which is active HMMs for the visible to near-infrared spectral band.

Active HMMs for Visible to Near-Infrared Spectral Band

Tunable Hyperbolic Dispersion in Visible and NIR Wavelengths:

- Approaches Using Active Materials:
 1. Liquid Crystals (LCs):
 - Nanosphere-dispersed LC (NDLC) HMMs proposed for NIR tunability.
 - ✓ External electric fields change the NLC director orientation.
 - ✓ Alters effective permittivity and dispersion relation.
 - ✓ Enables tuning of reflection/refraction from normal to negative under TM-polarized light.
 - LC + silver nanowire HMMs for visible wavelengths:
 - ✓ Tunable dispersion range: 454–475 nm.
 - ✓ Limitation: Modest spectral tunability due to small refractive index contrast ($\Delta n = \Delta n_e - \Delta n_o$).
 2. Phase-Transition Materials (e.g., VO₂):
 - VO₂ transitions from metal to insulator at 68 °C.
 - Its volatile phase behaviour makes it attractive for NIR tunability.
 - VO₂/TiO₂ multilayer stacks:
 - ✓ Use transition of VO₂ to tune dispersion from elliptic to hyperbolic.



Source: Choudhury, Pankaj K., ed. "Metamaterials: technology and applications" CRC Press, 2021

So, the tunability in the visible and near-infrared range is crucial for many advanced optical applications, right? So, what are the approaches to using active materials? You can think of using liquid crystals, so you can use nanosphere-dispersed liquid crystals as that kind of HMMs for NIR tunability. So, in this case, the external electric field can change the orientation of the liquid crystals, and that will change their optical properties.

So, it alters the effective permittivity and the dispersion relation, and it enables tuning of the reflection. or refraction from, you know, normal to negative under TM-polarized light. Now, you can use liquid crystal plus silver nanowire HMMs for visible wavelengths. So, they typically show a tunable dispersion range between 454 to 475 nanometer and the limitation here is that you get this modest spectral tunability mainly coming from the small refractive index contrast you can calculate the change in the refractive index that is $\Delta n = n_e - n_o$, e is the extraordinary axis, o will be the ordinary axis, right. The other approach to making active metamaterials is to use phase change materials such as vanadium oxide or VO₂.

So, VO₂ basically shows the transition from the metal to insulator phase at 68 degrees centigrade. So, its volatile phase behavior can make it attractive for air tunability. So, you can stack VO₂ and TiO₂ multilayers, and by using this metal-insulator phase transition in VO₂, the dispersion can be tuned from elliptic to hyperbolic, right? So, that is how you can obtain active tunability.

Active HMMs for Visible to Near Infrared Spectral Band

Chalcogenide Phase Change Materials(PCMs)-based Active HMMs

- Comparison to LC and VO₂:
 - PCMs retain their structural phase after switching.
 - Energy is only needed during switching, not continuously.
- Example: Ideal PCM – Ge₂Sb₂Te₅ (GST): observe significant optical constant changes:
 - Switches reversibly between amorphous and crystalline states.
 - Can switch in sub-nanosecond scales, billions of cycles.
- Optical Behavior (Fig. 1):
 - Shows high refractive index (n) contrast in the infrared range when phase changes.
 - Low optical loss (k) at wavelengths >1500 nm.
- Key Advantage:
 - Optical constants of PCMs can be thermally, optically, or electrically activated for rapid tuning.

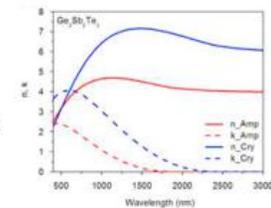


Figure 1: Optical constants of GST in amorphous and crystalline phases

Next, we will look into chalcogenide phase change materials based on active. So, if you compare them with the previous two cases, that is, liquid crystal and VO₂ based, this kind of phase change material basically retains its structural phase after switching.

So, in that case, you know you need energy only during the switching phase, not for retaining that phase. So, you do not need energy continuously; that means this kind of mechanism is much more energy efficient, right? So, an ideal phase change material is

germanium antimony telluride. It is also called GST or GST-225. This can show significant optical contrast change when it is changed from amorphous to crystalline state, okay. And it can switch on a sub-nanosecond scale; that is, you know it can go for billions of cycles, right? So, you can see the optical behavior in figure 1, where the optical constants of this GST are shown.

In the two phases, amorphous and crystalline are plotted. So, here you see $n_{\text{amorphous}}$, $k_{\text{amorphous}}$ and $n_{\text{crystalline}}$ and $k_{\text{crystalline}}$. So, it tells you that the contrast in the Infrared region is very large between the two, okay. So, these solid lines tell you about the n , which is the real part of the refractive index, and the dashed lines basically tell you. About k , that is the absorption coefficient or the imaginary part of the refractive index, right? So, the first observation is that the contrast is large in the near-infrared range.

Okay, the second observation is that the optical loss is very minimal for wavelengths greater than 1500, right? And the key advantage here is that you know the optical constants of this kind of PCMs can be thermally, optically and electrically activated for rapid tuning.

Active HMMs for Visible to Near Infrared Spectral Band

Chalcogenide Phase Change Materials(PCMs)-based Active HMMs

- **Applications:**
 - Enable photonic data storage and reconfigurable photonic devices in the visible and NIR wavelengths.
- **Limitation:**
 - In the visible range, GST is a lossy dielectric with low index contrast.
- Therefore, GST is ideal for tunable devices in the infrared spectrum but less effective in the visible.
- GST-Based Active HMM for NIR Tunability:
 - A multilayer HMM composed of GST/Ag/Ge is proposed for tuning dispersion in the NIR range (Fig. 2).

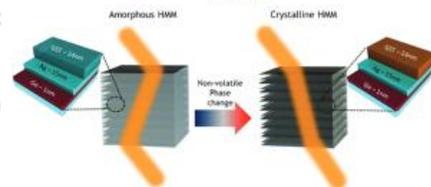
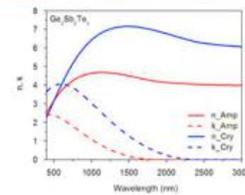


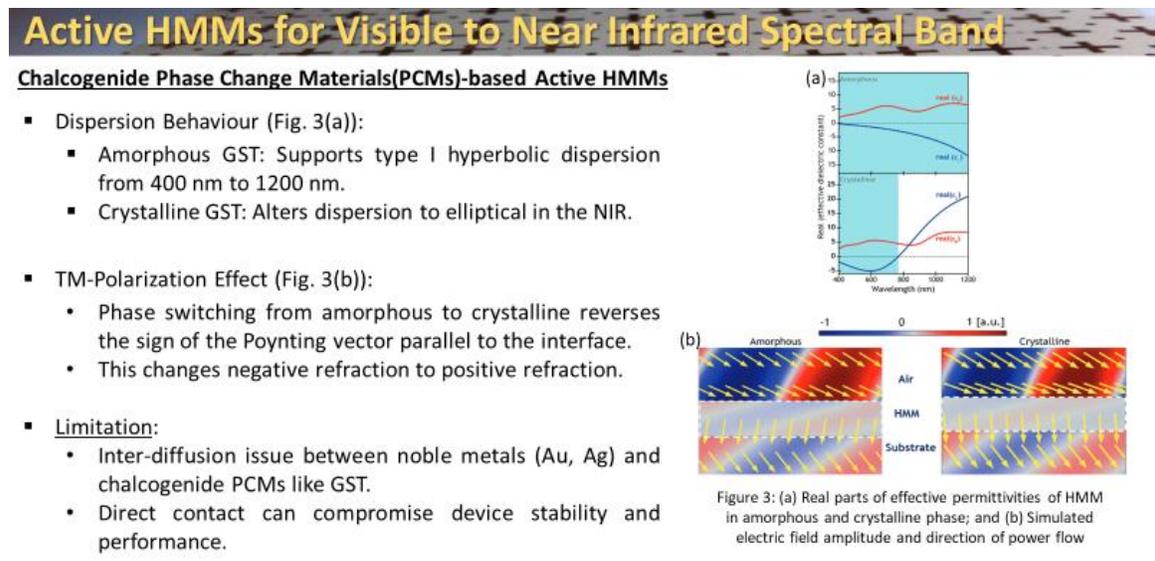
Figure 2: Schematic of chalcogenide-based HMM in the as-deposited configuration with the GST layers in the amorphous phase and after crystallization of GST layers by thermal annealing

Now, what are the applications of this kind of chalcogenide phase-change materials-based active hyperbolic metamaterial? So, the applications involve photonic data storage and reconfigurable photonic devices that work in the visible and near-infrared range. So, the limitation here is that you know, in the visible range, this GST material is a lossy

dielectric. With low index contrast, as you can see here, okay. However, if you look into the NIR region, it gives you pretty good contrast between the two phases, and it can be used for Tunable devices are particularly effective in this range of infrared, but you can also see that they are not very effective in the visible.

Now this particular figure shows you how you can use a GST in an active HMM (hyperbolic metamaterial) for NIR tunability. So, here you consider a multi-layer HMM that is made of GST, silver and germanium, okay. The thicknesses are written. So, this is basically multi-layer stack, okay and what happens in this case is that here different layers of this material are considered, okay. So here you can see in the amorphous phase of GST, okay It basically the overall structure which is a multi-layer of this particular stack is behaving like a hyperbolic metamaterial, okay and that is showing you negative refraction.

However, if you do thermal annealing that will give you nonvolatile phase change of this material. So, your GST will go into the crystalline phase. in that case, it will basically support a regular kind of refraction. So, it is basically tuning from hyperbolic dispersion to an elliptical dispersion in the near infrared regime. So here you can clearly see that the structure is basically comprised of seven periods of this kind of multi-layer unit cell stack, you can say.



So, let us look into the dispersion behaviour which is clearly shown here, okay. So, this is the real part of the effective dielectric constant, and this is the wavelength scale. So,

you can see that this supports it. So, this is the real part of the parallel permittivity; this is the real part of the perpendicular permittivity. So, this is for the amorphous phase, and this is for the crystalline phase, right? So, you can see that the wavelength ranges from 400 to 1200.

So, in the case of amorphous, it supports a hyperbolic dispersion over the entire range of 400 to 1200. However, in the case of crystalline, it alters the dispersion into an elliptical shape. So, when you move into the NIR range, okay, before that it is behaving like hyperbolic dispersion, but then, there is a transition in the crystalline phase, right? So, you can clearly see the TM polarization effects here. So, when the phase switches from amorphous to crystalline, it basically reverses the sign of the Poynting vector, which is parallel to the interface. So, this particular figure shows you both the electric field amplitude and the background red and blue you are seeing, okay? That is the electric field amplitude, and the direction of the power flow is shown through these yellow arrows.

So, you can see here that this is your HMM layer, this is air, and this is substrate. So, in the first case when it is amorphous, it is behaving like that type 1 hyperbolic metamaterial. So, you have the pointing vector, where the component parallel to the interface is basically in the left direction. But when it transitions to the crystalline phase, that component basically changes sign. and it is on the right towards the right direction okay.

So, that is the change that is happening from negative refraction to positive refraction, okay. So, that way you know that many of the GST tunable structures can be used for tunability. However, there is a limitation. So, inter diffusion issues between noble metals like gold, silver, and this kind of chalcogenide phase change material. Like GST, which comes in this kind of structure where the noble metals are basically in direct contact with chalcogenides.

So, they can compromise on the, you know, device stability and performance. So, this inter-diffusion basically refers to the mutual diffusion of atoms across the interface of two different materials when they are in contact. Especially when subjected to elevated temperatures, such as during phase change cycles in this kind of phase change material. So, it usually occurs at high temperatures during the amorphous-crystalline switching of materials like GST. Gold and silver atoms, whichever you are using—in this case, it was silver they can basically diffuse into the GST layer and the tellurium or the other atoms from GST can also diffuse into the metal.

So, this basically forms, you know, inter-metallic compounds like, you know, gold

telluride, and they may disrupt the optical properties and the phase change functionality of this kind of material.

Active HMMs for Visible to Near Infrared Spectral Band

Low-Loss Tunable HMM for Visible Wavelengths – (Sb_2S_3 -TiN Design):

- Problem with existing materials:
 - GST is lossy in the visible range.
 - Noble metals (Au, Ag) cause strong energy dissipation at visible wavelengths.
 - Noble metals also face inter-diffusion issues when paired with chalcogenide PCMs.
- Proposed solution:
 - A low-loss, tunable Sb_2S_3 -TiN HMM for visible wavelengths.
- Material advantages:
 - Sb_2S_3 :
 - ✓ A phase-change material (PCM) with low optical loss.
 - ✓ Has a bandgap of 2 eV, making it transmissive in the visible spectrum.
 - ✓ Shows substantial dielectric function changes when switched from amorphous to crystalline (Fig. 4).
 - TiN:
 - ✓ Selected as a plasmonic material to avoid diffusion issues in noble metals.

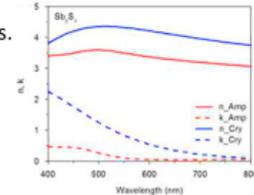


Figure 4: Optical constants of Sb_2S_3 in amorphous and crystalline phases

So, now we will also see a low-loss tunable hyperbolic metamaterial design for visible wavelengths. But this time we will be using another material, which is antimony trisulfide (Sb_2S_3) with titanium nitride. So, why do we have to go for this? Because we have understood that there are some problems with the existing materials. First of all, GST is basically lossy in the visible range, as you can see from this graph; also, if you go back and look.

So, GST is basically lossy; here you have a very high k component in the visible range, okay. Next, we have also seen that you know when you use noble metals like gold or silver, they cause strong energy dissipation in the visible range, okay? and also there are issues with this inter diffusion okay when you put PCM materials Phase change materials like GST are close to noble materials like gold and silver. Now, how do you overcome these problems? First, you have to go for a low-loss material selection, okay? So, you can also look for titanium nitride, which is basically an alternative plasmonic material that will try to, you know, Help counter that effect of the diffusion problem coming from the nopal material. And the overall loss is also less; you can see that if you use antimony trisulphide, the K values are much lower in this case. So, that will give you a low-loss tunable solution if you make an HMM from this kind of material, antimony trisulphide and titanium nitride.

So, these are the material advantages; as I mentioned, antimony trisulphide is a phase change material. But with low optical loss, that is a good thing. It has a band gap of 2 electron volts, making it transmissive in the visible spectrum. So, it does not absorb visible light. So, it shows substantial changes in the dielectric function when you switch from amorphous to crystalline.

And on the other hand, titanium nitride, as I already mentioned, is an alternative plasmonic material. that can avoid the diffusion issues that are coming from the noble metal.

So, the phase change is the phase switching process. You can think of it as you know that your antimony trisulfide is initially in an amorphous phase.

Active HMMs for Visible to Near Infrared Spectral Band

Low-Loss Tunable HMM for Visible Wavelengths – (Sb₂S₃-TiN Design):

- Phase switching process:
 - Starting state: Sb₂S₃ in amorphous phase.
 - To switch to crystalline phase:
 - ✓ Anneal sample at 300 °C for 30 minutes.
 - ✓ Conduct annealing in low-pressure argon since crystallization temp > 285 °C.

- Fabrication details:
 - HMM consists of 10 alternating layers of Sb₂S₃ and TiN on a glass substrate.
 - This design has sharp, non-diffused interfaces between Sb₂S₃ and TiN.
 - Use effective medium theory to compute effective permittivity components.

- Dispersion Tuning in Sb₂S₃-TiN HMM: (Observations from figure (5))
 - Amorphous HMM shows type I hyperbolic dispersion at wavelengths ≥ 580 nm.
 - Crystalline HMM shifts the hyperbolic band slightly blueward to 564 nm.
 - ✓ Caused by a reduction in effective index after crystallization.

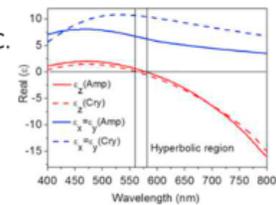


Figure 5: Real parts of uniaxial permittivity components of Sb₂S₃-TiN HMM when Sb₂S₃ is in the amorphous and crystalline phases

When you try to switch to the crystalline phase, you have to first anneal the sample at 300 degrees centigrade for 30 minutes. Conduct annealing in low-pressure argon since the crystallization temperature is typically more than 285 degrees centigrade. So, how do you make this HMM hyperbolic metamaterial? It will consist of 10 alternating layers of antimony trisulphide and titanium nitride on a glass substrate. And this design typically shows sharp non-diffused interfaces between the two materials. And you can use effective medium theory to compute the effective permittivity components, which you can also see over here, right? So, these are the real parts of the permittivity. So, the solid lines tell you about the amorphous phase; the red one is showing this, and this is the two basically.

Showing epsilon z and these two showing you epsilon x and epsilon y, okay. So, the solid lines are amorphous and the dash lines are crystalline. So, if you mark this axis, this is 0. So, below this, you can see that it is showing the hyperbolic region, right? So, the amorphous, this is basically the amorphous. Yeah, there is a small region here also, okay.

If you look into the graph carefully, you can see that the amorphous HMM basically shows. A type 1 hyperbolic dispersion for wavelengths greater than 580 nanometers, right? The crystalline HMM, as shown using the rest lines, basically shifts the hyperbolic band slightly toward the blue word at 564 nanometers, okay. And this is basically caused by a reduction in the effective index due to the crystallization process. So, that gives us a good idea that what kind of material we will be able to use for those spectral bands.



Active HMMs for Mid-infrared to THz Spectral Band

Now, we will move on to the next discussion on active hyperbolic metamaterials working in the mid-infrared and terahertz spectral bands.

Active HMMs for Mid-infrared to THz Spectral Band

Graphene-based Active HMMs

- Graphene is a plasmonic material effective in mid-infrared (MIR) to THz frequencies.
- A multilayer graphene-based HMM: (figure 6)
 - 2D graphene sheets are separated by dielectric spacers.
 - External gate voltage is used to tune carrier density, changing dispersion from elliptic to hyperbolic.
 - This tuning mechanism modulates in-plane conductivity and optical properties.
- Graphene-Based HMM for Negative Group Index Tuning:
 - Proposed structure: Graphene/dielectric stack.
 - Al_2O_3 is used as a dielectric material for Mid-IR wavelengths.
 - Consists of 12 alternating layers of graphene and dielectric with graphene on top (figure 6).

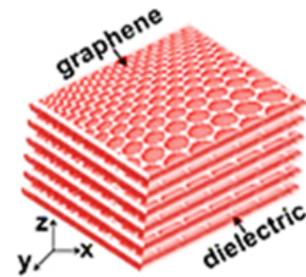


Figure 6: Schematic of graphene-based HMM

So, let us first look into graphene-based active hyperbolic metamaterials. So, you all know that graphene is a plasmonic material effective in the mid-infrared and terahertz frequencies. So here is a multilayer graphene-based HMM structure. So, you have got 2D graphene sheets that are separated by a dielectric spacer. You can apply an external gate voltage to tune the carrier density, which basically changes the dispersion from elliptical to hyperbolic.

And this tuning mechanism basically modulates the in-plane conductivity, and hence there will be a different optical behavior. You can use graphene-based HMMs for negative group index tuning. So, you can see the proposed structure shown here, which is a graphene dielectric multilayer stack. So, for mid-infrared wavelength operation, you can choose aluminum or Al_2O_3 as the dielectric material. Though the structure that is considered here is basically having 12 alternating layers of graphene and dielectric with graphene on the top.

Active HMMs for Mid-infrared to THz Spectral Band

Graphene-based Active HMMs: Graphene-Based HMM for Negative Group Index Tuning

- Modeling approach:
 - ✓ Single-layer graphene (SLG) modeled as a surface conducting sheet.
 - ✓ Conductivity expression:

$$\sigma_g = \frac{ie^2\mu}{\pi\hbar^2(\omega + i/\tau)}$$

where ω : angular frequency; μ : chemical potential (depends on carrier density ($\mu \propto \sqrt{N_c}$)); and τ : electron relaxation time

- Complex dielectric constant of graphene:

$$\epsilon_g = 1 + \frac{i\sigma_g\eta_0}{k_0t_g}$$

where k_0 : vacuum wave vector ($k_0 = \omega/c$) ; t_g : effective graphene thickness and η_0 : free space impedance (377Ω)

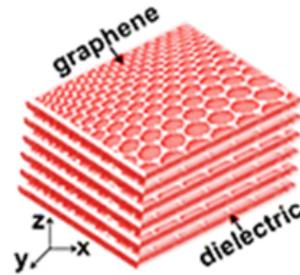


Figure 6: Schematic of graphene-based HMM

So, how do you model this? You can model the single-layer graphene as a surface conducting sheet where the conductivity is given by This expression sigma g (σ_g) is given as $\sigma_g = \frac{ie^2\mu}{\pi\hbar^2(\omega+i/\tau)}$. Omega is nothing but the angular frequency; mu here is something interesting, as it is the chemical potential that depends on. The carrier density is because mu is proportional to the square root of N_c i.e. $\mu \propto \sqrt{N_c}$, and tau tells you about the electron relaxation time. So, once you can estimate this conductivity, you can always get the complex dielectric constant of the graphene, given as epsilon g (ϵ_g) = $1 + \frac{i\sigma_g\eta_0}{k_0t_g}$. So, k_0 is basically the vacuum wave vector that is omega divided by c.

t_g is the effective graphene thickness, and η_0 is the free space impedance, which is 377 ohms.

Active HMMs for Mid-infrared to THz Spectral Band

Graphene-based Active HMMs

Effective Permittivity and Tunable Hyperbolic Dispersion in Graphene HMMs:

- Fig. 7(a) shows EMT-derived real parts of the effective in-plane permittivity (ϵ_{\parallel}) versus frequency.
 - Hyperbolic dispersion occurs when $\text{Re}(\epsilon_{\parallel}) < 0$.
 - In the elliptical region, $\text{Re}(\epsilon_{\parallel}) > 0$, approaching the permittivity of the dielectric at higher frequencies.
- For chemical potential $\mu = 0.2$ eV and thickness of dielectric layer (t_d) = 50 nm:
 - Elliptic-to-hyperbolic transition occurs at 24 THz.
- Fig. 7(b) demonstrates:
 - Tuning of the hyperbolic spectral band with increasing chemical potential.
 - Critical frequency shifts:
 - ✓ At $\mu = 0.2$ eV \rightarrow 24 THz and at $\mu = 0.8$ eV \rightarrow 49 THz
 - This reflects broadband tunability with slight changes in graphene's carrier density.

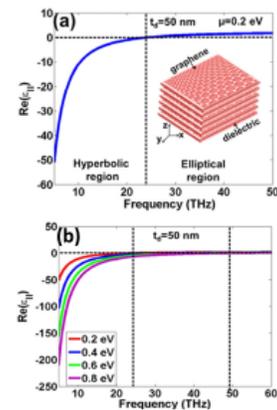


Figure 7: (a) Variation of real parts of ϵ_{\parallel} with frequency; (b) Variation with chemical potential

So, with this, you can use effective medium theory. And you can estimate the real parts of the effective in-plane permittivity, and here is the plot. That is shown versus frequency at different chemical potentials, right? So, what you can observe is that hyperbolic dispersion basically occurs when you have the real part of the parallel permittivity being negative. So, that is here for the case of t_d equals 50, and in this particular case, it is considered μ equals 0.2, okay. So, this frequency So, you can see the transition basically happens from elliptic to hyperbolic at 24 terahertz right. And in this particular figure, you can also see how you can tune this hyperbolic spectral band by changing the chemical potential.

So, when you have the chemical potential μ equal to 0.2 electron volts. that transition happens at 24 terahertz, but when you change the chemical potential to 0.8 electron volts, you can change that transition frequency to 49 terahertz. Now, this μ is the chemical potential, which basically depends on the carrier density, right? So, that is where you can actually change the carrier density. Okay, in graphene, to change its chemical potential, and that will change this transition frequency from elliptical to hyperbolic material. Dispersion curve, and that will dramatically change its optical response, or you can say the, you know, response to that particular wave.

Active HMMs for Mid-infrared to THz Spectral Band

Topological Insulator-based Active HMMs

- Topological Insulators (TIs) have gained interest in THz plasmonics:
 - Prompt exploration for tunable hyperbolic metamaterials (HMMs).
- Common TIs: Bismuth Selenide (Bi_2Se_3) and Bismuth Telluride (Bi_2Te_3)
 - Known for gapless edge/surface states and a bulk insulating properties.
- Bi_2Se_3 (quintuple-layered): analyzed for hyperbolic dispersion in the THz range.
- TI-based HMMs supports:
 - Highly directional, sub-diffractive hyperbolic phonon polaritons.
 - Dispersion tunability via surface state doping.
- Enable realization of high-speed, tunable THz photonic devices.

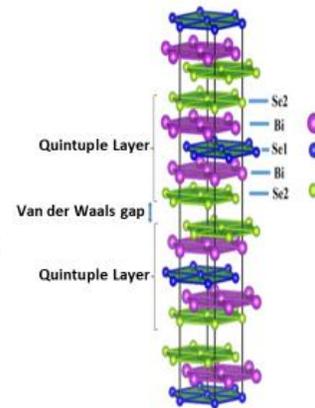


Figure 8: Schematic diagram of crystal structure of Bi_2Se_3

Now we will look into another type, which is topological insulator-based active metamaterials: hyperbolic metamaterials.

So, topological insulators we have briefly discussed earlier. They have gained particularly particular interest in this terahertz plasmonics and they allow you to make these tunable hyperbolic metamaterials. Common types of topological insulators are bismuth selenide, Bi_2Se_3 , and bismuth telluride, which is Bi_2Te_3 . And they are known for gapless edge or surface states and bulk insulating properties. So, let us consider this one bismuth selenide which is basically a quintuple layered that has been analyzed for hyperbolic dispersion in the terahertz range, as we have already discussed.

In the second application of active hyperbolic metamaterials, there is super collimation of terahertz light. Now, topological insulator-based HMMs are particularly interesting because they support highly directional sub diffractive hyperbolic phonon polaritons right. So, they have dispersion tunability that comes from surface state doping, and they enable realization of high-speed tunable terahertz photonic devices. So, if you carefully look into this particular figure, it tells you about the quintuple structure of this bismuth selenide, which is a topological insulator. So, it is basically having those 5 atomic layers which are stacked in a particular sequence you have.

Se Bi Se, Bi Se, that is the sequence, okay. So, these quintuple layers are basically the fundamental building blocks of this crystal structure. and they are weakly bonded together by van der Waals attraction. So, this is basically repeated, right? And this

particular topological insulator is particularly interesting because you can make it dynamically tunable terahertz photonic devices using this kind of material.

Active HMMs for Mid-infrared to THz Spectral Band

Topological Insulator-based Active HMMs

- Proposed structure: Superlattice structure of Bi₂Te₃ (TI) - GeTe (PCM) HMM.
 - Comprises alternating layers of Bi₂Te₃ and GeTe as shown in figure 9.
- Bi₂Te₃ structurally similar to Bi₂Se₃; GeTe is a phase-change material.
- Spectral tunability: Negative group index band can be tuned from THz to MIR frequencies.
- Permittivity modeling (for effective permittivity calculations):
 - Permittivities of Bi₂Te₃ obtained via Drude-Lorentz model:

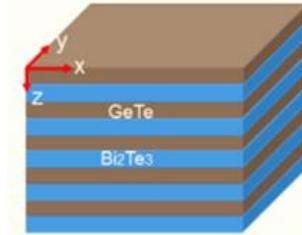


Figure 9: Multilayer structure of Bi₂Te₃ and GeTe HMM

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_D^2}{\omega^2 + i\omega\gamma_D} + \sum_{j=1} \frac{\omega_{p,j}^2}{\omega_{0,j}^2 - \omega^2 - i\omega\gamma_j}$$

ε_{∞} : High-frequency permittivity; ω_D : Drude plasma frequency; γ_D : Damping rate for the Drude (free-carrier) term; $\omega_{p,j}$: Oscillator plasma frequency, $\omega_{0,j}$: Resonance frequency, and γ_j : Damping coefficient for the j^{th} Lorentz oscillator

So, here we will take one example, but we take the example of the other one this time the other topological insulator material that is the bismuth telluride, and we take germanium telluride (GeTe) as a phase change material, and we make their alternating layers form this multilayer structure that will work as hyperbolic metamaterials. So, these are the alternating layers of these two materials. So, why do we use this? Because this particular material, bismuth telluride, is structurally similar to bismuth selenide. And we use germanium telluride as a phase-change material here. And you will see that the negative group index band can be tuned from terahertz to mid-infrared frequencies in this particular case.

So, again, you can use effective medium theory for modeling the permittivities. So, you can first consider the dielectric permittivity of bismuth telluride. It can be obtained using a Drude-Lorentz model. We discussed this earlier. So, you can use this particular model:

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_D^2}{\omega^2 + i\omega\gamma_D} + \sum_{j=1} \frac{\omega_{p,j}^2}{\omega_{0,j}^2 - \omega^2 - i\omega\gamma_j}$$

So, this is basically the group term, and then you have the Lorentz oscillator model. So, you have a summation over j equals 1 to how many Lorentz oscillators you will be using. Then you have $\omega_{p,j}^2$ divided by $\omega_{0,j}^2 - \omega^2 - i\omega\gamma_j$, right? So, here ε_{∞} is basically the high-

frequency permittivity, ϵ_d is the Drude plasma frequency, and γ_D is the damping rate, okay. And these are the plasma frequency, resonance frequency, and the damping coefficient for the j th oscillator.

So, with that, you can model bismuth telluride.

Active HMMs for Mid-infrared to THz Spectral Band

Topological Insulator-based Active HMMs

- GeTe (amorphous): Dielectric permittivity = 15 at THz frequency.
- For a TI fill fraction of 0.35, the Bi_2Te_3 -GeTe HMM structure exhibits broadband type II hyperbolic dispersion from THz to MIR frequencies (figure 10(a)):
 - $\epsilon^x(\omega) < 0$ and $\epsilon^z(\omega) > 0$
- Uniaxial permittivity for crystalline GeTe in Bi_2Te_3 -GeTe HMM is shown in figure 10(b).
 - With crystalline GeTe, the HMM displays both:
 - ✓ Type I hyperbolic bands
 - ✓ Type II hyperbolic bands
- Tunable optical properties achieved by:
 - ✓ Phase change of GeTe (amorphous \leftrightarrow crystalline)
 - ✓ Doping Bi_2Te_3 surface states.

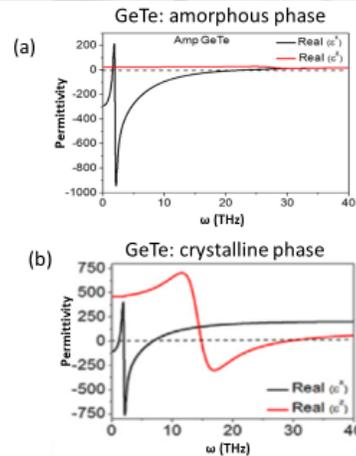


Figure 10: Real parts of the uniaxial permittivity components for Bi_2Te_3 - GeTe HMM structure

For the case of germanium telluride, you can see that the amorphous phase has a dielectric permittivity of 15 at terahertz frequencies. Okay. So, if you consider this bismuth telluride or the topological insulator to have a filling fraction of 35 percent or 0.35, So, this structure can actually give you an HMM kind of structure that will exhibit a broadband type II hyperbolic dispersion. From terahertz to mid-infrared frequencies, which you can also see here.

So, this is considering germanium telluride in the amorphous phase. So, here you obtain ϵ_x to be negative and ϵ_z to be positive, okay. For the germanium telluride in crystalline phase, the permittivities are plotted here. So, here you see that when you have a crystalline phase, the HMM will display both type 1. and type 2 hyperbolic bands, and that is a very unique property, okay.

So, what are the applications? This kind of tunable optical property is basically achieved from the phase change of germanium telluride. So, it is basically changing from amorphous to crystalline, and also from, if you dope this bismuth telluride surface states, okay. You can get tunable optical properties that can be used for different applications.

Active HMMs for Mid-infrared to THz Spectral Band

Superconductor-based Active HMMs

- High- T_c superconductors (HTS) enable hyperbolic dispersion in infrared and THz frequencies.
 - Superconductivity occurs above liquid nitrogen temperature, making them practical.
- Superconductor-based HMMs may help raise the critical temperature, offering a metamaterial-based approach to high- T_c superconductivity.
- Proposed structure: Multilayered HMM with:
 - Alternating layers of YBCO (high- T_c superconductor) and LAO (dielectric). (figure 11)
- Why YBCO & LAO?
 - YBCO: Common high- T_c superconductor.
 - LAO: Dielectric that supports lattice-matched deposition of YBCO films.

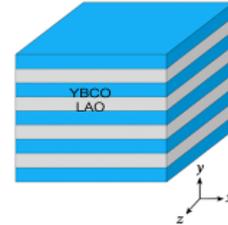


Figure 11: Schematic of YBCO-LAO type II HMM

Next, we will look into superconductor-based active hyperbolic metamaterials. So, in this case high critical temperature superconductors or HTS, they also can enable hyperbolic dispersion in the infrared and terahertz frequencies.

So, superconductivity here occurs above liquid nitrogen temperature, making it practical for regular operations. So, superconductor-based HMMs can also help raise the critical temperature that they offer. Metamaterial-based approach for high critical temperature superconductivity. So, the proposed structure basically comprises multi-layers of YBCO, which is yttrium barium copper oxide, and LAO, which is lanthanum aluminate. Now, why did you choose these two materials? So, YBCO is basically a common high critical temperature superconductor, and LAO (lanthanum aluminate) is a dielectric that basically supports lattice-matched deposition of these YBCO films.

So that becomes a natural choice. Now, what are the material properties?

Active HMMs for Mid-infrared to THz Spectral Band

Superconductor-based Active HMMs

- Material properties:
 - LAO permittivity: ~ 25 , temperature-independent.
 - YBCO permittivity: Varies significantly with temperature and frequency.
- Figure 12 shows the real and imaginary parts of parallel (ϵ_{\parallel}) and perpendicular (ϵ_{\perp}) permittivity components across temperatures.
- Temperature range considered: 20K to 100K (YBCO's superconducting transition temperature: 80 – 93 K).
- At 100 K (above T_c):
 - Both real and imaginary permittivities are positive.
 - Indicates elliptical (dielectric) dispersion.

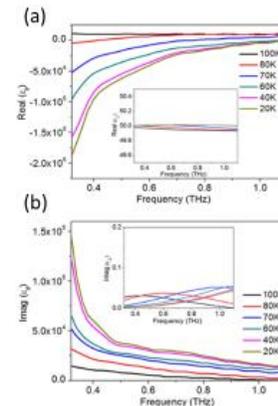


Figure 12: EMT-derived permittivity components: (a) Real parts & (b) Imaginary parts of parallel and perpendicular permittivity for different temperatures.

The material properties indicate that LAO permittivity is around 25. So, here you can see that these are the effective medium theory-derived permittivity components. The (a) tells you about the real parts and (b) tells you about the imaginary part at different temperatures. So, this is for the overall material, but in doing this calculation, you also need to take care of the constituent material permittivity. So, here, LAO permittivity is considered to be around 25, and it is temperature-independent.

So, it is flat, okay, and then if you consider YBCO permittivity, that varies significantly with temperature and frequency, right? So, this particular figure shows the real and imaginary parts of the parallel. So, this is actually epsilon parallel. And perpendiculars are being plotted. So, this is the perpendicular one, this is the parallel one, and this is the imaginary one of the parallel. An imaginary representation of the perpendicular components, okay? So, the temperature range considered here is from 20 K to 100 K.

Because that is particularly interesting, the YBCO superconducting transition temperature is within this range of 80 to 93 K. So, above 100 K that is above your critical temperature you will see that both real and imaginary permittivities are positive that basically indicates an elliptical or dielectric dispersion curve.

Active HMMs for Mid-infrared to THz Spectral Band

Superconductor-based Active HMMs

- Below YBCO's transition temperature:
 - Real part of ϵ_{\parallel} becomes negative, signaling type II hyperbolic dispersion.
- Hyperbolic dispersion bandwidth increases as temperature decreases:
 - Due to increasing frequency-dependent conductivity of YBCO in the superconducting phase.
- Demonstrates a temperature-driven transition from:
 - Elliptical (dielectric) to hyperbolic dispersion.
 - Broadband tunability of hyperbolic response via temperature change.

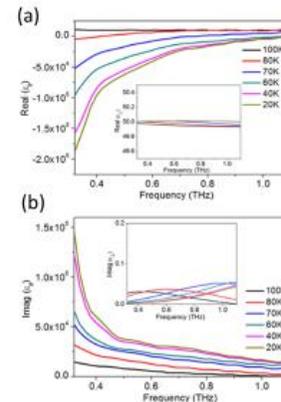


Figure 12: EMT-derived permittivity components: (a) Real parts & (b) Imaginary parts of parallel and perpendicular permittivity for different temperatures.

Now, what happens below this transition temperature? You will see that the real part of the epsilon parallel becoming negative signals a type II kind of hyperbolic dispersion. And it is also clear that the hyperbolic dispersion bandwidth increases as the temperature decreases. And this is mainly due to the increasing frequency-dependent conductivity of this atrium barium copper oxide.

It is due to the increasing frequency-dependent conductivity of YBCO in the superconducting state. And this finally demonstrates that you have a temperature-driven transition from elliptical That is a dielectric to hyperbolic kind of dispersion. You can have broadband tunability, as seen here for the hyperbolic response that is coming from temperature change, right.



Thank You

So, with that, we will conclude our discussion. So, in the next section, we will talk about metamaterials for electromagnetic cloaking. So, if you have any queries regarding this lecture, you can drop an email to this email address mentioning the lecture number and the course name in the subject line. Thank you.