Course Name: Introduction to Microwave and Optical Metamaterials
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Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-9
Lecture-43

Lec 43: Hyperbolic metamaterials for nanoscale optical cavities

Introduction to Microwave and Optical Metamaterials

Lecture 43
Hyperbolic Metamaterials for Nanoscale Optical Cavities

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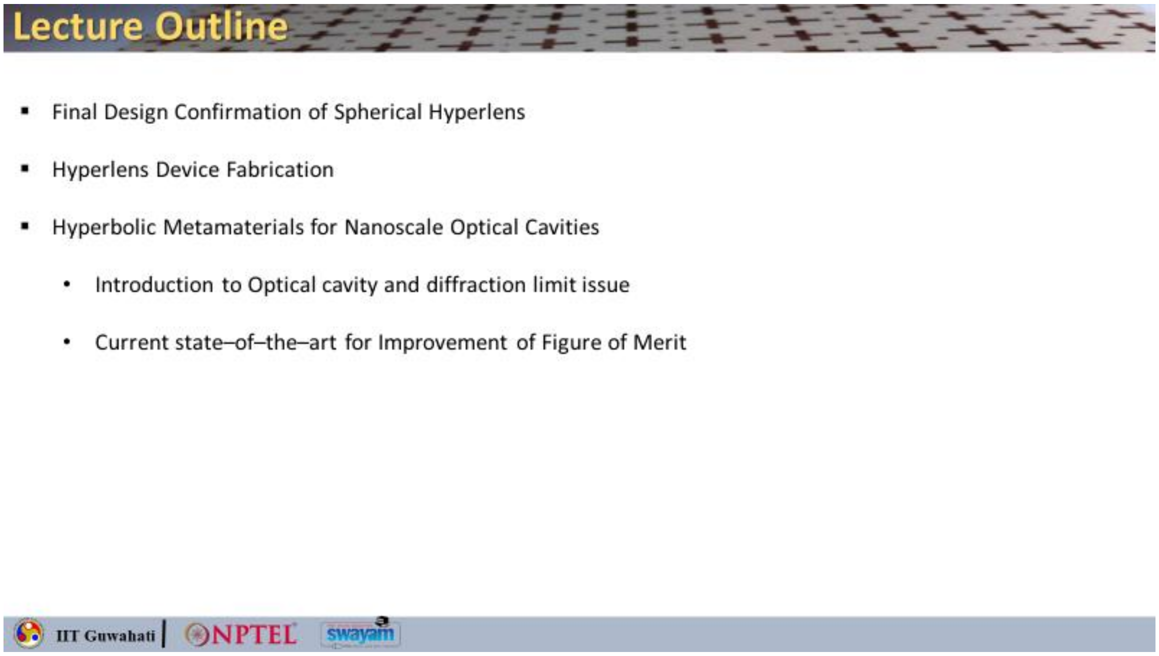
Hello, students. Welcome to Lecture 43 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on hyperbolic metamaterials for nanoscale optical cavities.

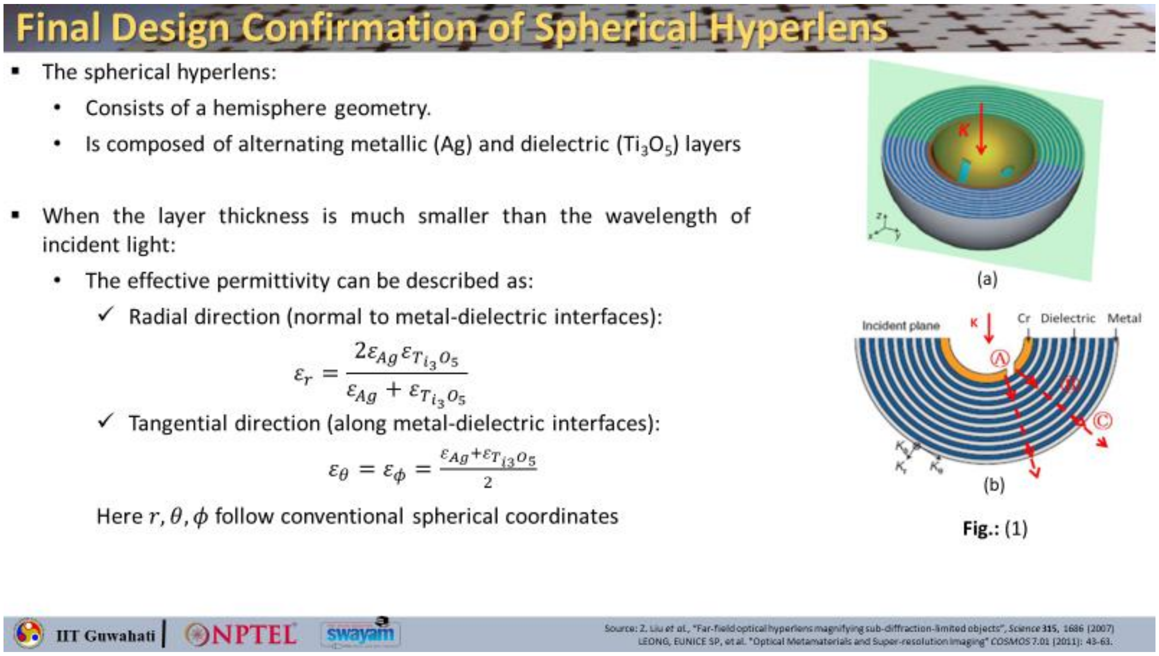


Lecture Outline

- Final Design Confirmation of Spherical Hyperlens
- Hyperlens Device Fabrication
- Hyperbolic Metamaterials for Nanoscale Optical Cavities
  - Introduction to Optical cavity and diffraction limit issue
  - Current state–of–the–art for Improvement of Figure of Merit

So, here is the lecture outline: we will have the final design confirmation of the spherical hyperlens that we discussed in the previous lecture. Then we will discuss the fabrication methods for hyperlens devices. We will see how hyperbolic metamaterials can be used in nanoscale optical cavities.

So, in doing that, we will first introduce the optical cavity and the issue of the diffraction limit. We will discuss the current state of the art in improving the figure of merit of these cavities.



Final Design Confirmation of Spherical Hyperlens

- The spherical hyperlens:
  - Consists of a hemisphere geometry.
  - Is composed of alternating metallic (Ag) and dielectric (Ti_3O_5) layers
- When the layer thickness is much smaller than the wavelength of incident light:
  - The effective permittivity can be described as:
    - ✓ Radial direction (normal to metal-dielectric interfaces):

$$\varepsilon_r = \frac{2\varepsilon_{Ag}\varepsilon_{Ti_3O_5}}{\varepsilon_{Ag} + \varepsilon_{Ti_3O_5}}$$

    - ✓ Tangential direction (along metal-dielectric interfaces):

$$\varepsilon_\theta = \varepsilon_\phi = \frac{\varepsilon_{Ag}+\varepsilon_{Ti_3O_5}}{2}$$

Here r, θ, ϕ follow conventional spherical coordinates

Fig.: (1)

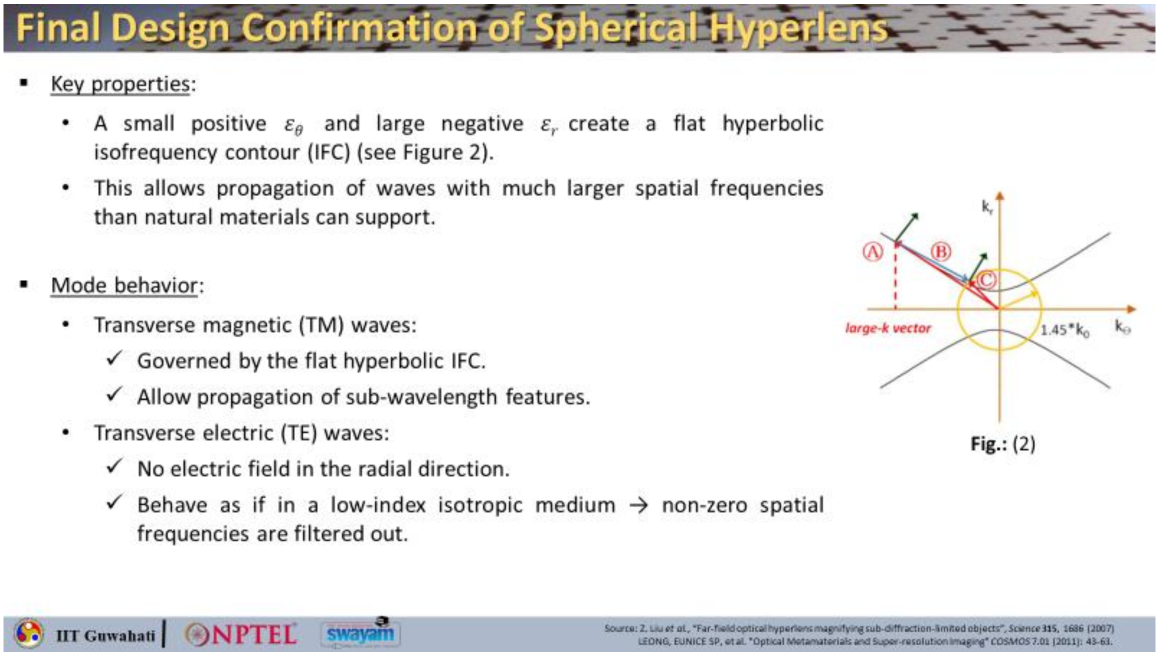
Source: Z. Liu *et al.*, "Far-field optical hyperlens magnifying sub-diffraction-limited objects", *Science* **315**, 1686 (2007)
LEONG, EUNICE SP, et al. "Optical Metamaterials and Super-resolution Imaging" *COSMOS* 7.01 (2011): 43-63.

So, the spherical hyperlens we saw earlier was also in the previous lecture. So, this basically consists of hemispherical geometries.

And just to recap, this is composed in this particular application of alternating silver and Ti3O5, which are titanium oxide layers. Now, this is the schematic that shows the final design of this hemispherical hyperlens, which has. Sub-diffraction features are on the top layer, and then TM light is excited from the top layer. Now, the things to remember here are that you need to ensure the layer thickness you are using is appropriate. The hyperlens should be much smaller than the wavelength of the incident light.

In doing so, you will be able to use effective permittivity okay. So, you can describe the effective permittivity along the radial direction that is normal to this metal-dielectric interface. So, these are like metals and dielectrics. The light-colored one is the metal, and the dark-colored one shows the dielectric. This is the wave factor; okay, that's the sub-wavelength feature.

So, this is the interface. So, this is the radial direction. So, you can see k r, which is marked here. This is basically k theta, and the k phi basically goes inside, right? So, these are typically the spherical coordinate conventions. So, what you see in the radial direction is that the permittivity epsilon r can be written as $\varepsilon_r = \frac{2\varepsilon_{Ag}\varepsilon_{T_{i_3}O_5}}{\varepsilon_{Ag}+\varepsilon_{T_{i_3}O_5}}$. In the tangential direction, which is along theta and also in the phi direction, you will see that the permittivity is basically the average of the two because it is along the metal-dielectric interface. So, you can write it as $\varepsilon_\theta = \varepsilon_\phi = \frac{\varepsilon_{Ag}+\varepsilon_{T_{i3}O_5}}{2}$, okay. So, this is done in the spherical coordinate system.



Final Design Confirmation of Spherical Hyperlens

- Key properties:
  - A small positive ε_θ and large negative ε_r create a flat hyperbolic isofrequency contour (IFC) (see Figure 2).
  - This allows propagation of waves with much larger spatial frequencies than natural materials can support.
- Mode behavior:
  - Transverse magnetic (TM) waves:
    - ✓ Governed by the flat hyperbolic IFC.
    - ✓ Allow propagation of sub-wavelength features.
  - Transverse electric (TE) waves:
    - ✓ No electric field in the radial direction.
    - ✓ Behave as if in a low-index isotropic medium → non-zero spatial frequencies are filtered out.

Fig.: (2)

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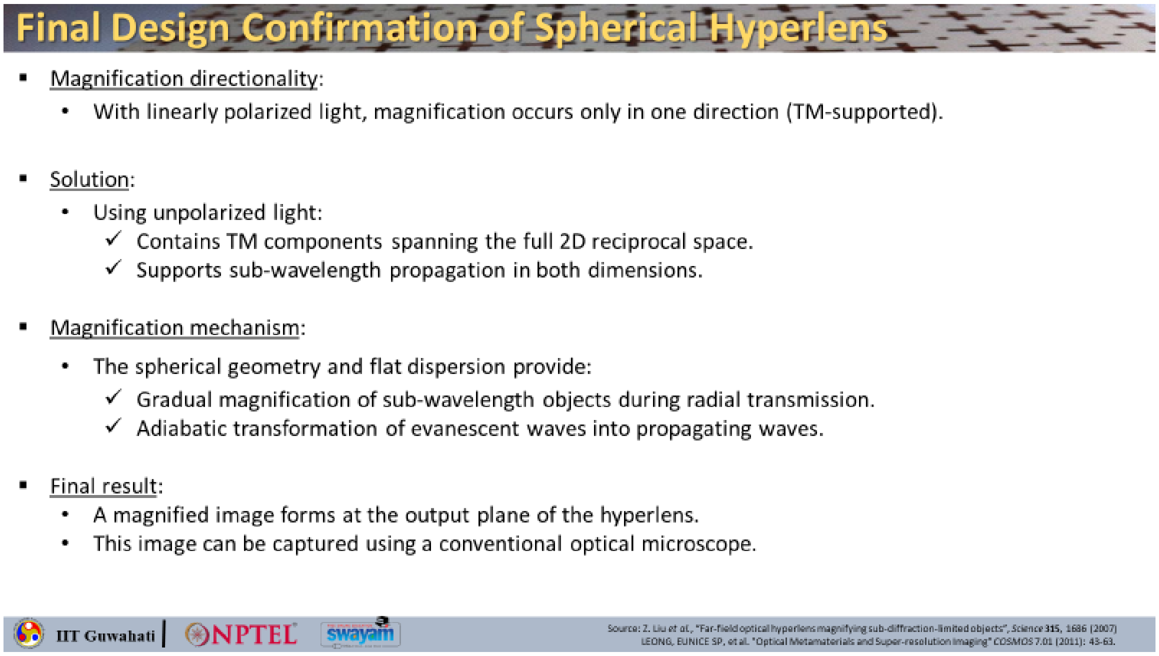
Source: Z. Liu *et al.*, "Far-field optical hyperlens magnifying sub-diffraction-limited objects", *Science* **315**, 1686 (2007)
LEONG, EUNICE SP, et al. "Optical Metamaterials and Super-resolution Imaging" *COSMOS* 7.01 (2011): 43-63.

Now, the key properties of this particular design of a spherical hyperlens are that you need a small positive epsilon and theta and you require a large negative epsilon r so that you get this kind of flat hyperbolic isofrequency contour. Right. Now, it is important because you want this kind of hyperbolic dispersion curve that basically supports. Much larger spatial frequencies, or you can say large k vectors, than the other natural materials can normally support are present, Okay. So, if you talk in terms of the mode behavior, you will see that the transverse magnetic (TM) waves are basically governed by these flat hyperbolic isofrequency contours, they allow the propagation of sub-wavelength features. So, this is the trick. So, you allow the propagation of the sub-wavelength features by supporting the high wave vector, and then you gradually magnify them. In the radial direction, and then at a certain point when the feature is no longer diffraction limited.

You can simply couple it to the normal wave vectors that allow for far-field propagation. So, that is how you are actually capturing the evanescent waves and allowing them to propagate. And then convert it into, you know, the free-space propagating wave. In the case of transverse selective electromagnetic (EM) waves or TE waves, you will see that there is no electric field in the radial direction. So, it behaves as if it is in a low-index isotropic medium, and that allows you to know it is non-zero spatial frequencies that are filtered out mean that propagation for the TE mode is not allowed. That's why we have shown that initial slide only for TM waves.



Final Design Confirmation of Spherical Hyperlens

- Magnification directionality:
  - With linearly polarized light, magnification occurs only in one direction (TM-supported).

- Solution:
  - Using unpolarized light:
    - ✓ Contains TM components spanning the full 2D reciprocal space.
    - ✓ Supports sub-wavelength propagation in both dimensions.

- Magnification mechanism:
  - The spherical geometry and flat dispersion provide:
    - ✓ Gradual magnification of sub-wavelength objects during radial transmission.
    - ✓ Adiabatic transformation of evanescent waves into propagating waves.

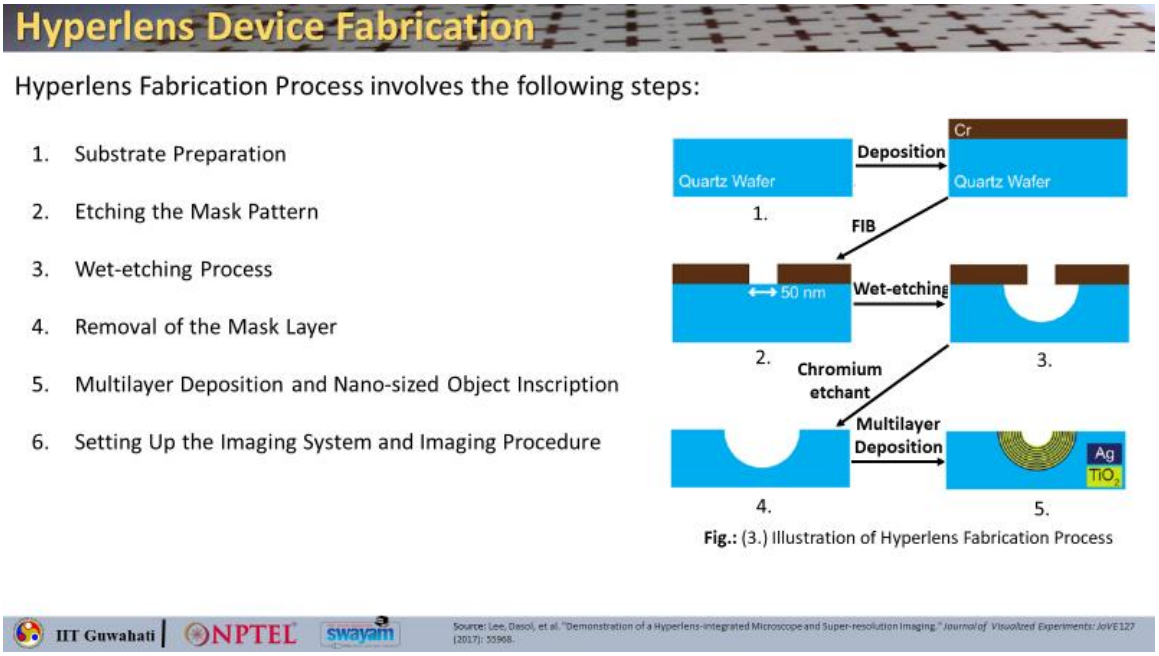
- Final result:
  - A magnified image forms at the output plane of the hyperlens.
  - This image can be captured using a conventional optical microscope.

Source: Z. Liu *et al.*, "Far-field optical hyperlens magnifying sub-diffraction-limited objects", *Science* **315**, 1686 (2007)
LEONG, EUNICE SP, et al. "Optical Metamaterials and Super-resolution Imaging" *COSMOS* 7.01 (2011): 43-63.

Now, the magnification directionality depends on the fact that, in this case, magnification of linearly polarized light will occur. Only in one direction, okay? That is for the TM-supported modes or in the TM direction. So, when you are using unpolarized light that contains TM components spanning the entire 2D reciprocal space, and thus, it will support subwavelength propagation in both dimensions.

So, what is the magnification mechanism, as we have discussed, that you are using with spherical geometry and a flat dispersion profile in that isofrequency contour? That will allow for gradual magnification of the sub-wavelength objects in the radial direction because the radius continues to increase. So, you will see the magnification in that direction; additionally, there will be an adiabatic transformation of evanescent waves into propagating waves. So, that is where you know, finally, that you will be able to use your regular optics. A microscope to pick up those particular waves and see those diffraction-limited objects is needed. So, what is the final result in this case? You are getting a magnified image.

That is coming out of the plane of the hyperlens, and this image can be captured using a conventional optical microscope. Because you have already coupled the evanescent wave information and converted it into propagating wave information.



Hyperlens Device Fabrication

Hyperlens Fabrication Process involves the following steps:

1. Substrate Preparation
2. Etching the Mask Pattern
3. Wet-etching Process
4. Removal of the Mask Layer
5. Multilayer Deposition and Nano-sized Object Inscription
6. Setting Up the Imaging System and Imaging Procedure

Fig.: (3.) Illustration of Hyperlens Fabrication Process

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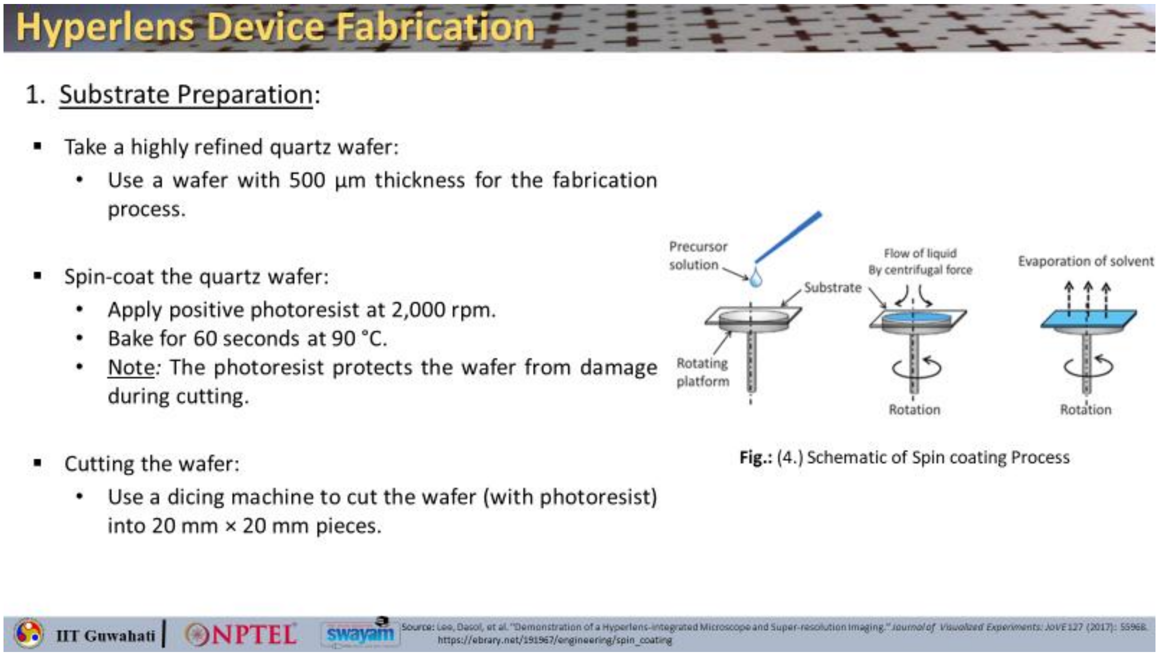
Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

So, now let's look at the fabrication of hyperlens devices. The hyperlens fabrication process typically involves the following steps: substrate preparation, first, and then you do the deposition to make an etching on the mask pattern; after that, you do the third step, which is basically. The wet etching process creates this particular shape, and then you remove it. So, that is done by using a chromium agent, and then you perform multilayer deposition.

Here you can consider silver and TiO2 as the two alternating layers, and then finally, you create a small dimension. Or you can have a small intrusion as the object; then you can set up the imaging system here, and you can do the imaging process.



Hyperlens Device Fabrication

1. Substrate Preparation:

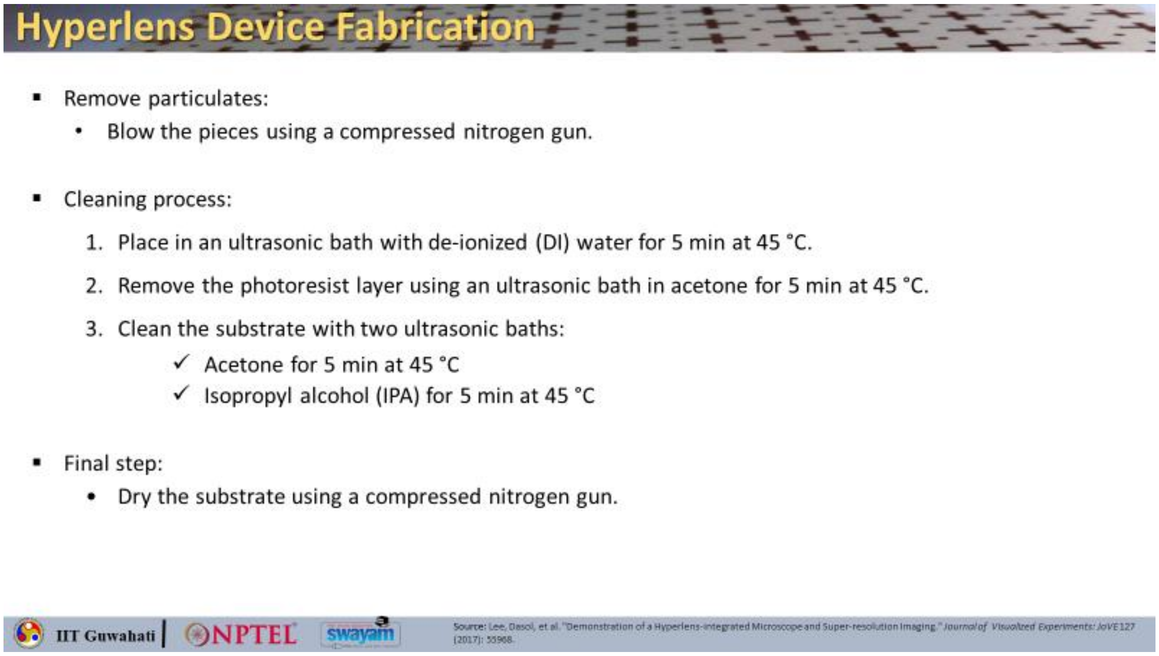
- Take a highly refined quartz wafer:
  - Use a wafer with 500 µm thickness for the fabrication process.
- Spin-coat the quartz wafer:
  - Apply positive photoresist at 2,000 rpm.
  - Bake for 60 seconds at 90 °C.
  - Note: The photoresist protects the wafer from damage during cutting.
- Cutting the wafer:
  - Use a dicing machine to cut the wafer (with photoresist) into 20 mm × 20 mm pieces.

Fig.: (4.) Schematic of Spin coating Process

Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.
https://ebrary.net/191967/engineering/spin_coating

So, if you go into the details of each of these processes, the first thing that starts is substrate preparation, as you have already seen. So, first, you need to take a highly refined quartz wafer. So, you use a wafer with a thickness of 500 microns for this fabrication process, and then you spin-coat the wafer.

So, this is basically a rotating platform that is placed on top of the substrate. So, you put a precursor solution, then apply a photoresist at 2000 rpm, and then bake it for 60 seconds at 90 degrees centigrade. That way, you know you will be able to make a layer of photoresist that can protect the wafer from damage during cutting. And then finally you do the cutting. So, you use a dicing machine to cut the wafer along with the photoresist into 20 mm by 20 mm pieces.



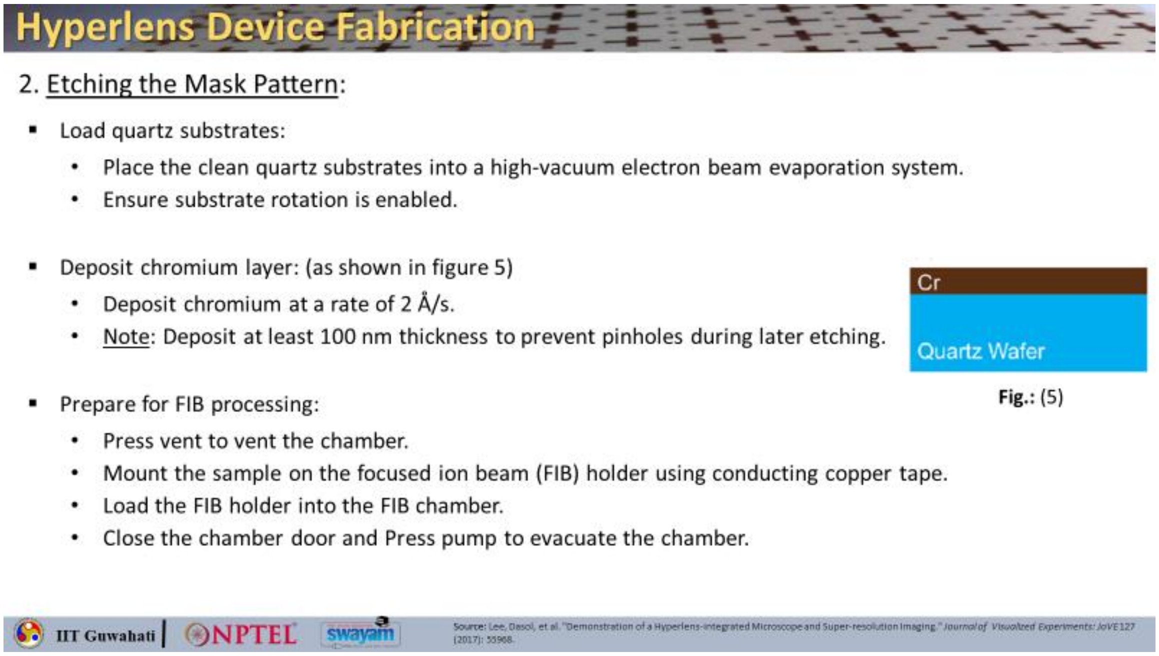
Hyperlens Device Fabrication

- Remove particulates:
  - Blow the pieces using a compressed nitrogen gun.
- Cleaning process:
  1. Place in an ultrasonic bath with de-ionized (DI) water for 5 min at 45 °C.
  2. Remove the photoresist layer using an ultrasonic bath in acetone for 5 min at 45 °C.
  3. Clean the substrate with two ultrasonic baths:
     - ✓ Acetone for 5 min at 45 °C
     - ✓ Isopropyl alcohol (IPA) for 5 min at 45 °C
- Final step:
  - Dry the substrate using a compressed nitrogen gun.

Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

So, once it's done, you remove the particulates. So, you blow the pieces using a nitrogen gun and compressed nitrogen gas, and the cleaning process involves that placing those samples into an ultrasonic bath with deionized water for around 5 minutes at 45 degrees Celsius is okay. And that removes the photoresist layer using, you know, the ultrasonic bath, but this time with acetone. For 5 minutes at the same temperature of 45 degrees Celsius.

So, finally, you clean the substrate with two ultrasonic baths. One is acetone for five minutes at 45 degree and then you take isopropyl alcohol for five minutes at 45 degree centigrade. So, the final step is to dry the substrate using a compressed nitrogen nozzle. So, that is how your substrate is prepared.



Hyperlens Device Fabrication

2. Etching the Mask Pattern:

- Load quartz substrates:
  - Place the clean quartz substrates into a high-vacuum electron beam evaporation system.
  - Ensure substrate rotation is enabled.
- Deposit chromium layer: (as shown in figure 5)
  - Deposit chromium at a rate of 2 Å/s.
  - Note: Deposit at least 100 nm thickness to prevent pinholes during later etching.
- Prepare for FIB processing:
  - Press vent to vent the chamber.
  - Mount the sample on the focused ion beam (FIB) holder using conducting copper tape.
  - Load the FIB holder into the FIB chamber.
  - Close the chamber door and Press pump to evacuate the chamber.

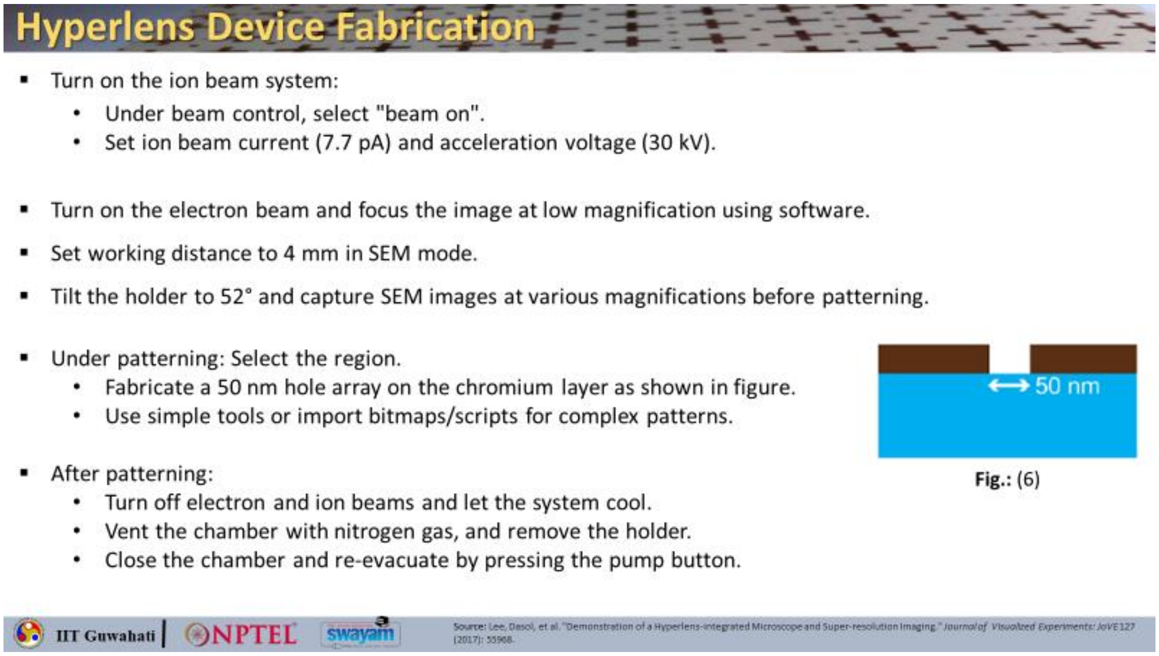
Fig.: (5)

Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

The next step would be to etch the mask pattern, right? So, for that, you should load the quartz substrates.

So, you have to place the clean quartz substrate in a high-vacuum electron beam deposition system, okay? And you have to ensure that column rotation is enabled. So, you have to deposit chromium, okay? As you can see in this figure. So, the deposition rate is typically 2 angstroms per second, and you need to deposit at least 100 nanometers in thickness. So, you can prevent pinholes during later etching steps. And after that, you prepare for the milling process, which is a focused ion beam milling process.

So, for that, you need to vent the chamber. Okay, you mount the sample on this focused ion beam holder using some conductive copper tape. Then you load the FIB holder into the FIB chamber. And you close the chamber door and use the pump to evacuate the chamber again.



Hyperlens Device Fabrication

- Turn on the ion beam system:
  - Under beam control, select "beam on".
  - Set ion beam current (7.7 pA) and acceleration voltage (30 kV).
- Turn on the electron beam and focus the image at low magnification using software.
- Set working distance to 4 mm in SEM mode.
- Tilt the holder to 52° and capture SEM images at various magnifications before patterning.
- Under patterning: Select the region.
  - Fabricate a 50 nm hole array on the chromium layer as shown in figure.
  - Use simple tools or import bitmaps/scripts for complex patterns.
- After patterning:
  - Turn off electron and ion beams and let the system cool.
  - Vent the chamber with nitrogen gas, and remove the holder.
  - Close the chamber and re-evacuate by pressing the pump button.

Fig.: (6)

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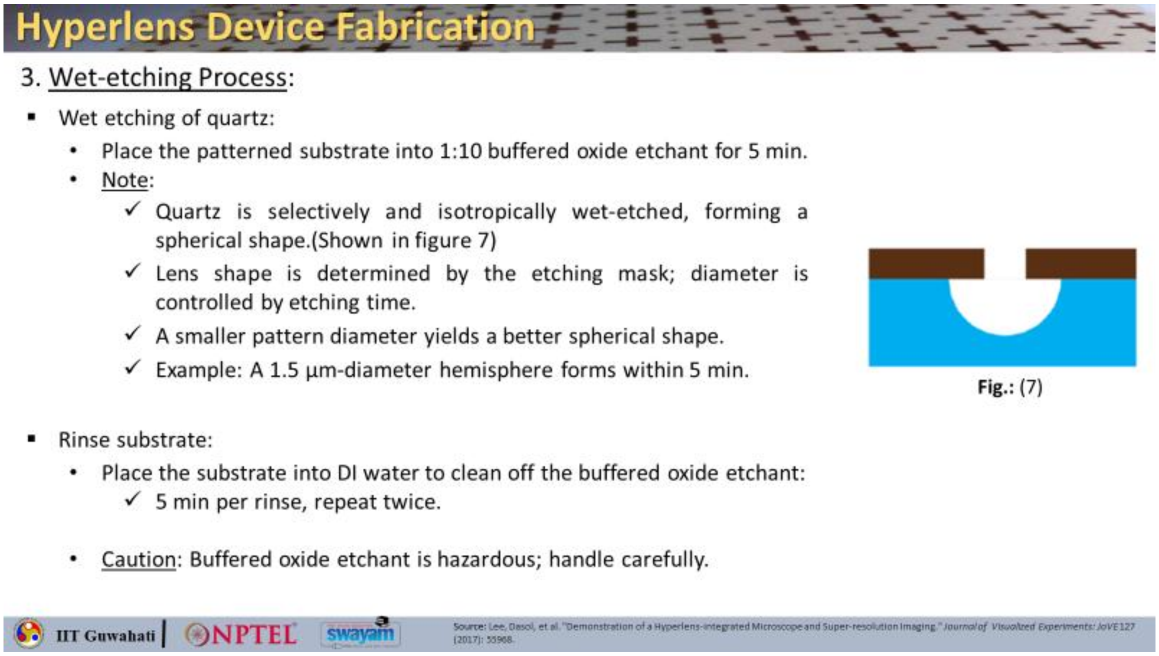
Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

After that, you turn on the ion beam.

So, under the beam control, you select "Beam On." So, once you set the ion beam current to around 7.7 picoamperes and the acceleration voltage to around 30 kV, Okay. So, you have to set the working distance to 4 mm in this SEM mode and tilt the holder. 52 degrees, and capture the SEM image at various magnifications before doing the patterning.

So, under the patterning what you have to do you got to select the region first Okay, for that you can now consider fabricating a 50-nanometer whole array in chromium. Okay, as you can see here, you can use simple tools or import bitmaps or scripts for any kind of complex structure. So, here it is, whole and simple. So, if it is a complex structure, you can create that pattern using scripts or by importing the bitmaps. So, after the patterning is done, you have to turn off the electron and ion beams and let the system cool down.

You vent the chamber with nitrogen gas, remove the holder, and finally close the chamber before re-evacuating it by pressing the pump. Right button. So, with that, your sample is ready for etching.



Hyperlens Device Fabrication

3. Wet-etching Process:

- Wet etching of quartz:
  - Place the patterned substrate into 1:10 buffered oxide etchant for 5 min.
  - Note:
    - ✓ Quartz is selectively and isotropically wet-etched, forming a spherical shape.(Shown in figure 7)
    - ✓ Lens shape is determined by the etching mask; diameter is controlled by etching time.
    - ✓ A smaller pattern diameter yields a better spherical shape.
    - ✓ Example: A 1.5 µm-diameter hemisphere forms within 5 min.

Fig.: (7)

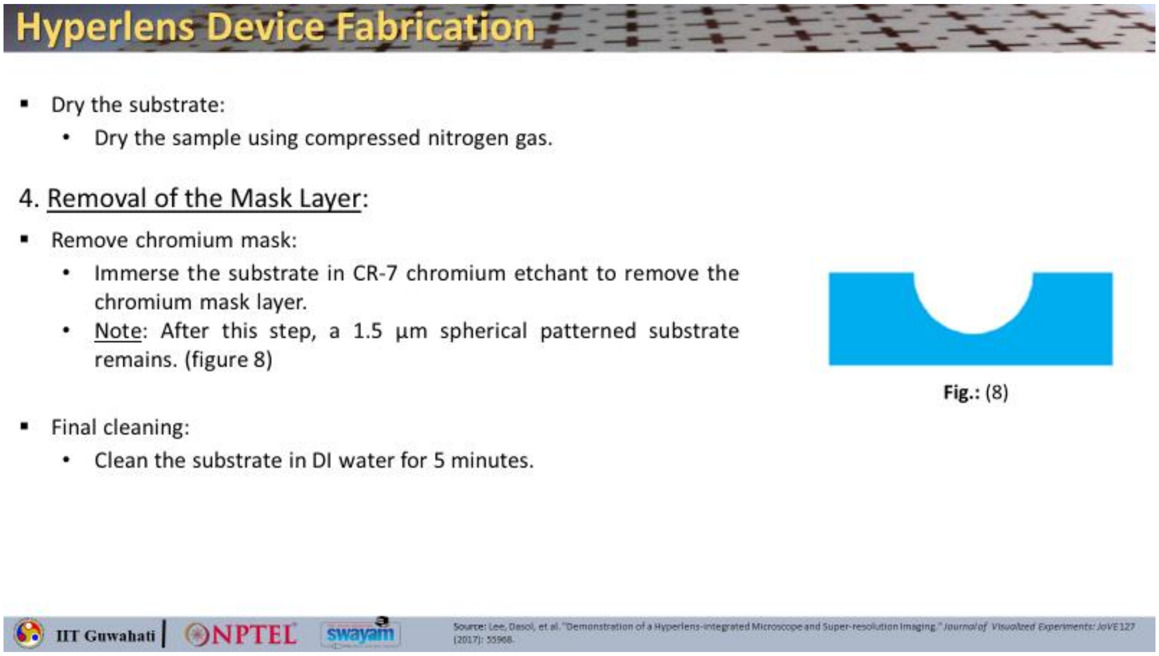
- Rinse substrate:
  - Place the substrate into DI water to clean off the buffered oxide etchant:
    - ✓ 5 min per rinse, repeat twice.
  - Caution: Buffered oxide etchant is hazardous; handle carefully.

Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

So, the next step will be the wet etching process. So, you are doing the wet etching of the quartz, right. So, you have to place the patterned substrate into a 1:10 buffered oxide etchant for about 5 minutes.

So, you will see that quartz is selectively and isotropically wet etched. So, that will form a spherical shape, and the length is determined by this mask, while the diameter essentially controls the etching time. So, a smaller pattern diameter basically yields a better spherical shape. So, you will see, for example, that a 1.5-micron-diameter hemisphere forms within 5 minutes.

So, after doing that, you have to rinse the substrate. So, you place the substrate in deionized water to clean off the buffered oxide etchant. So, you have to do it for 5 minutes per rinse, and you repeat this step two times, okay? So, one thing to remember here is that these buffered oxide agents are hazardous. So, you have to handle the sample carefully.



Hyperlens Device Fabrication

- Dry the substrate:
  - Dry the sample using compressed nitrogen gas.

4. Removal of the Mask Layer:

- Remove chromium mask:
  - Immerse the substrate in CR-7 chromium etchant to remove the chromium mask layer.
  - Note: After this step, a 1.5 µm spherical patterned substrate remains. (figure 8)

Fig.: (8)

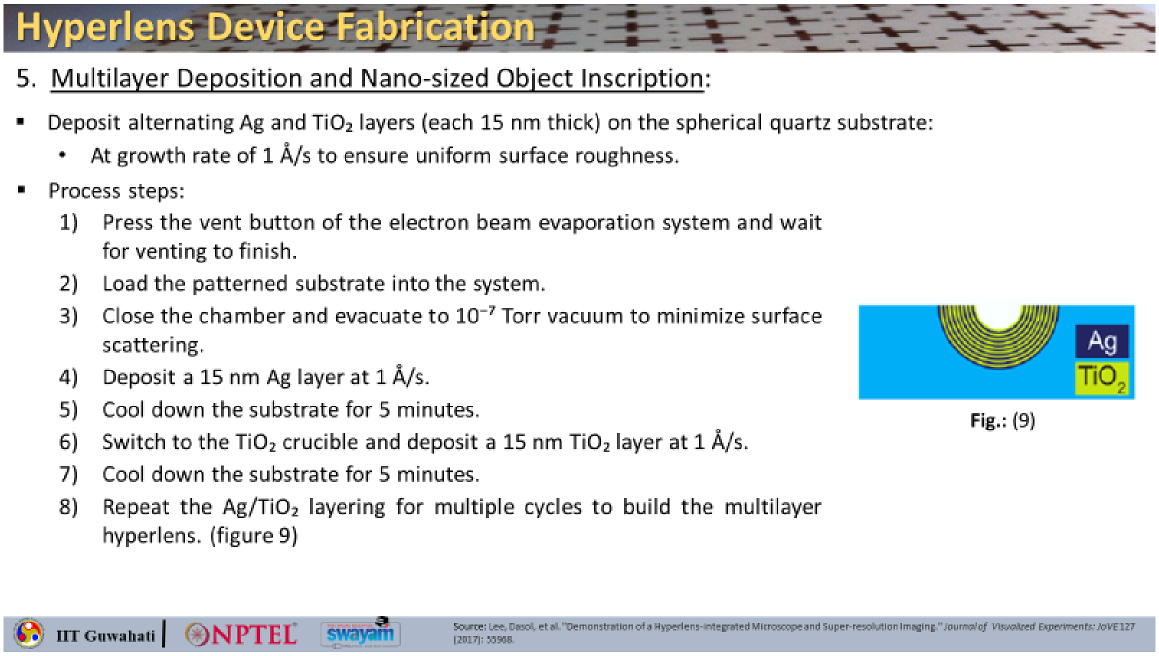
- Final cleaning:
  - Clean the substrate in DI water for 5 minutes.

Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

The next step will be to dry the substrate.

So, you can dry the substrate using compressed nitrogen gas, and finally You remove the mask layer because your formation of the hemispherical well is complete. So, you remove the chromium mask. So, for that, you immerse it in CR-7 chromium etchant, which will remove your chromium mask layer, okay.

So you got rid of the mask. Okay. So, after this step, you will see a 1.5-micron spherical patterned substrate that remains with you. And the final cleaning will be done again with deionized water for 5 minutes.



Hyperlens Device Fabrication

5. Multilayer Deposition and Nano-sized Object Inscription:

- Deposit alternating Ag and TiO_2 layers (each 15 nm thick) on the spherical quartz substrate:
  - At growth rate of 1 Å/s to ensure uniform surface roughness.
- Process steps:
  1) Press the vent button of the electron beam evaporation system and wait for venting to finish.
  2) Load the patterned substrate into the system.
  3) Close the chamber and evacuate to 10^{-7} Torr vacuum to minimize surface scattering.
  4) Deposit a 15 nm Ag layer at 1 Å/s.
  5) Cool down the substrate for 5 minutes.
  6) Switch to the TiO_2 crucible and deposit a 15 nm TiO_2 layer at 1 Å/s.
  7) Cool down the substrate for 5 minutes.
  8) Repeat the Ag/TiO_2 layering for multiple cycles to build the multilayer hyperlens. (figure 9)

Fig.: (9)

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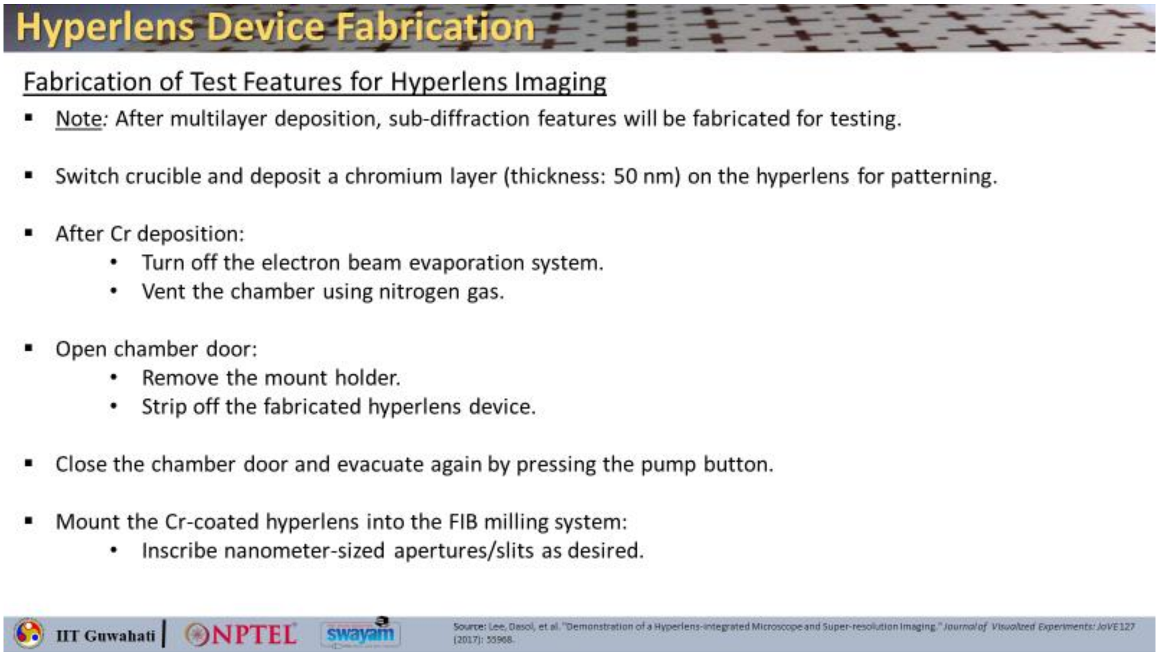
Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

and after that, you have your substrate ready for multilayer deposition, correct? So, in this process, you will deposit alternating layers of silver and TiO_2. So, in this case, we are considering each to be the same thickness: 15 nanometers, Okay, and on the spherical quartz substrate, this is also the reason mentioned in the initial slide. If you see that the volume fraction was not mentioned and was taken as half, then you can see in this particular case, okay. Here, the thicknesses of the metal and dielectric layers are considered to be the same. So, what is the growth rate? The growth rate is typically 1 angstrom per second, and the slow growth rate ensures uniform surface roughness. So, the process step involves that you first have to press the vent button of the electron beam evaporation system.

And you have to wait for the venting to finish. Then you load the patterned substrate into the system, close the chamber, and evacuate it to 10 to the power of minus 7 torr vacuum. And that will minimize surface scattering, and then you will deposit a 15-nanometer silver layer. At the deposition rate of 1 Armstrong per second. Let the substrate cool down for 5 minutes, and then switch to the $TiO2$ crucible, and with that, deposit a 15 nanometer $TiO2$ layer at 1 Amstrong per second again. So, with time, you can basically calibrate the thickness that is being deposited.

Let the substrate cool down for another five minutes, and in this way, you can repeat the silver $TiO2$ layering for multiple cycles. And that will allow you to build this multi-layer hyperlens.



Hyperlens Device Fabrication

Fabrication of Test Features for Hyperlens Imaging

- Note*:* After multilayer deposition, sub-diffraction features will be fabricated for testing.
- Switch crucible and deposit a chromium layer (thickness: 50 nm) on the hyperlens for patterning.
- After Cr deposition:
  - Turn off the electron beam evaporation system.
  - Vent the chamber using nitrogen gas.
- Open chamber door:
  - Remove the mount holder.
  - Strip off the fabricated hyperlens device.
- Close the chamber door and evacuate again by pressing the pump button.
- Mount the Cr-coated hyperlens into the FIB milling system:
  - Inscribe nanometer-sized apertures/slits as desired.

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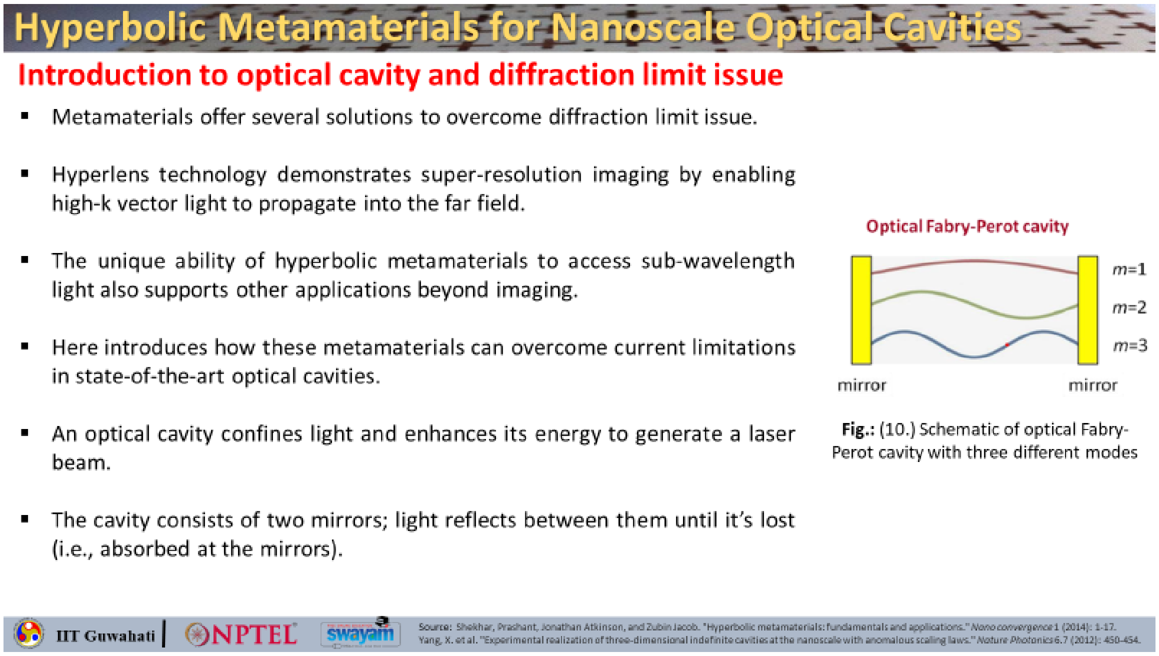
Source: Lee, Dasol, et al. "Demonstration of a Hyperlens-integrated Microscope and Super-resolution Imaging." *Journal of Visualized Experiments: JoVE* 127 (2017): 55968.

Now, the final step will be the fabrication of the test feature for this hyperlens imaging. So, after the multilayer deposition is done, subwavelength features can be fabricated for testing. So, you switch the crucible and deposit a chromium layer of 50-nanometer thickness on the hyperlens for patterning.

So, after the chromium deposition, you turn off the electron beam evaporation system and vent the chamber with nitrogen gas. And then you open the chamber door, remove the mount holder, and strip off the fabricated hyperlens device. So, after that, you close the chamber again and evacuate. Now, close the chamber door; you will evacuate again by pressing the pump button. So, finally, you mount the chromium-coated hyperlens in the FIB milling system.

So, that is exactly where you inscribe those nanometer-sized apertures and slits. as your test features that you will be used for hyperlens imaging right. So, with that, we understood how a hyperlens is fabricated.



Hyperbolic Metamaterials for Nanoscale Optical Cavities

Introduction to optical cavity and diffraction limit issue

- Metamaterials offer several solutions to overcome diffraction limit issue.
- Hyperlens technology demonstrates super-resolution imaging by enabling high-k vector light to propagate into the far field.
- The unique ability of hyperbolic metamaterials to access sub-wavelength light also supports other applications beyond imaging.
- Here introduces how these metamaterials can overcome current limitations in state-of-the-art optical cavities.
- An optical cavity confines light and enhances its energy to generate a laser beam.
- The cavity consists of two mirrors; light reflects between them until it's lost (i.e., absorbed at the mirrors).

Optical Fabry-Perot cavity

Fig.: (10.) Schematic of optical Fabry-Perot cavity with three different modes

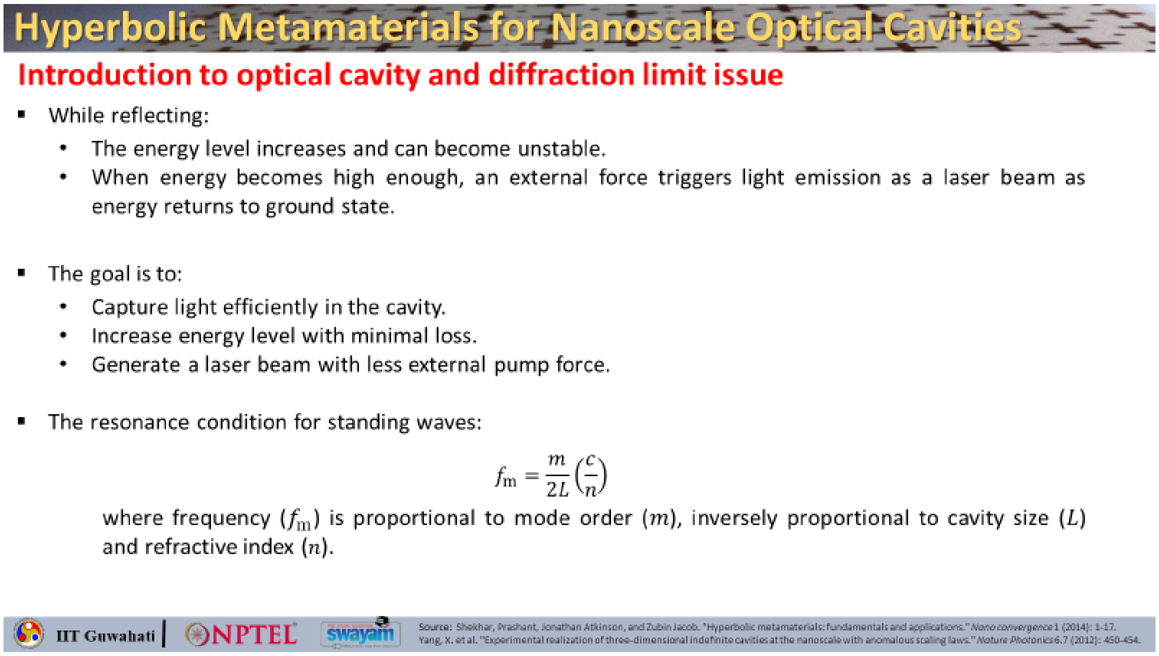
Source: Shekhar, Prashant, Jonathan Atkinson, and Zubin Jacob. "Hyperbolic metamaterials: fundamentals and applications." *Nano convergence* 1 (2014): 1-17.
Yang, X. et al. "Experimental realization of three-dimensional indefinite cavities at the nanoscale with anomalous scaling laws." *Nature Photonics* 6.7 (2012): 450-454.

Now, let us look into some other applications of hyperlenses, or you could say hyperbolic metamaterials in general, for nanoscale optical cavities. So, these are basically optical cavities, and we will see how the diffraction limit issue is troubling them in providing you with a higher figure of merit.

So, normally you will see that metamaterials can offer several solutions to overcome the issue of the diffraction limit. So, the first thing is that you know hyperlens technology can demonstrate super-resolution and that we understand it well by now that it enables high k vector to propagate you know light and then you know take it into far field. So, this unique ability of hyperbolic metamaterials allows one to access subwavelength light. which are typically the, you know, evanescent waves, okay, and also support other applications beyond imaging. So, this is what we will be seeing today: that, other than imaging, we can also use this for other applications.

So, right now we will discuss that how this kind of metamaterials can be used to overcome the current limitations in the state-of-the-art optical cavities. So, an optical cavity basically tells you that it is a cavity. So, there are two mirrors at each end that form a typical optical Fabry-Pérot cavity, right? So, what it does, you know, is it basically confines light between two mirrors, and you know, upon multiple rounds of reflection. It enhances the energy to finally generate a laser beam, doesn't it? So, a cavity is formed between two mirrors. And the light gets reflected between them until it is completely absorbed by the medium or by the mirrors.

So, this shows you different modes. So, this is m equals 1, m equals 2, and m equals 3: three different modes are shown here.



Hyperbolic Metamaterials for Nanoscale Optical Cavities

Introduction to optical cavity and diffraction limit issue

- While reflecting:
  - The energy level increases and can become unstable.
  - When energy becomes high enough, an external force triggers light emission as a laser beam as energy returns to ground state.
- The goal is to:
  - Capture light efficiently in the cavity.
  - Increase energy level with minimal loss.
  - Generate a laser beam with less external pump force.
- The resonance condition for standing waves:

$$f_{\mathrm{m}} = \frac{m}{2L}\left(\frac{c}{n}\right)$$

where frequency (f_{m}) is proportional to mode order (m), inversely proportional to cavity size (L) and refractive index (n).

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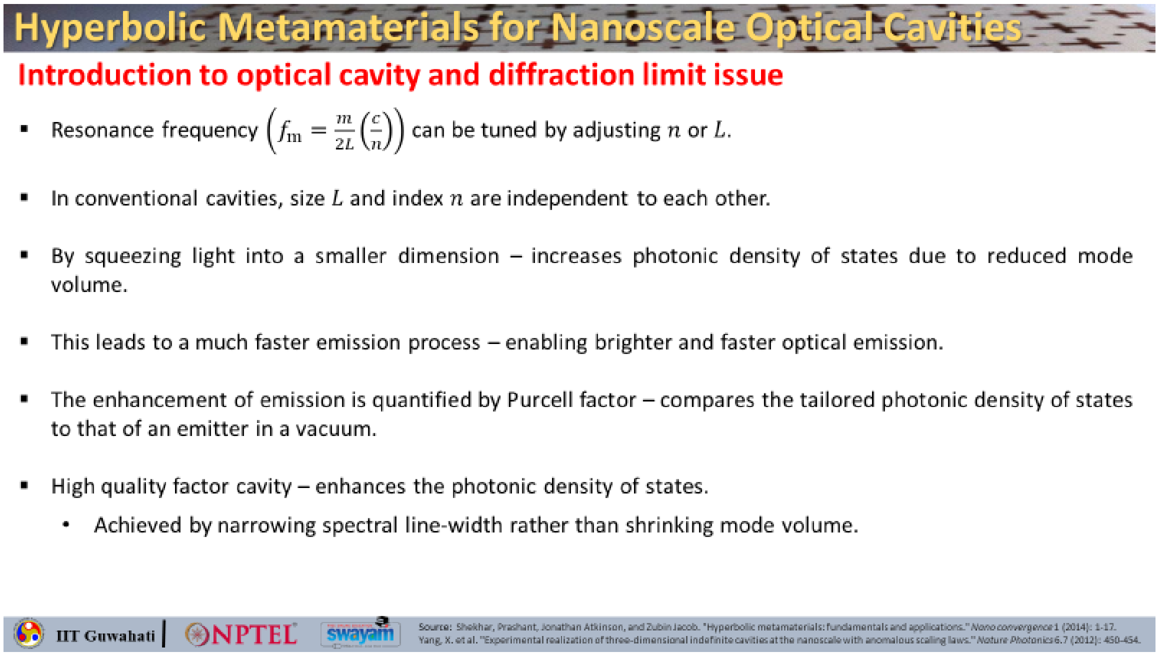
Source: Shekhar, Prashant, Jonathan Atkinson, and Zubin Jacob. "Hyperbolic metamaterials: fundamentals and applications." *Nano convergence* 1 (2014): 1-17.
Yang, X. et al. "Experimental realization of three-dimensional indefinite cavities at the nanoscale with anomalous scaling laws." *Nature Photonics* 6.7 (2012): 450-454.

So, while reflecting on what happens, the energy level basically increases. And it can become unstable. So, when the energy becomes high enough, an external force can trigger light emission as a laser beam.

And that will allow the energy to return to the ground state. So, here the goal is to capture the light efficiently in the cavity and increase the energy levels with minimal loss. And then finally, generate a laser beam with less external pumping force, okay?

So, the resonance condition for the standing waves that we just saw can be written as $f_{\mathrm{m}} = \frac{m}{2L}\left(\frac{c}{n}\right)$; what m is, is basically the mode order. So, we have seen m equal to 1, 2, and 3. So, fm will be the frequency corresponding to each mode, and you can see that it is inversely proportional to them.

The length of the cavity is okay, and the refractive index of the medium is correct. So, the resonance frequency—if you want to tune it, you can simply adjust the length of the cavity. Or is it the refractive index of the cavity medium, correct? Also, if you see that as m increases, the mode number increases, you will get higher frequencies, right? So, that will basically correspond to, on the other hand, you could say that higher modes cause higher frequencies. A shorter wavelength will correspond to a larger refractive index.



Hyperbolic Metamaterials for Nanoscale Optical Cavities

Introduction to optical cavity and diffraction limit issue

- Resonance frequency $\left(f_{\mathrm{m}} = \frac{m}{2L}\left(\frac{c}{n}\right)\right)$ can be tuned by adjusting n or L.
- In conventional cavities, size L and index n are independent to each other.
- By squeezing light into a smaller dimension – increases photonic density of states due to reduced mode volume.
- This leads to a much faster emission process – enabling brighter and faster optical emission.
- The enhancement of emission is quantified by Purcell factor – compares the tailored photonic density of states to that of an emitter in a vacuum.
- High quality factor cavity – enhances the photonic density of states.
  - Achieved by narrowing spectral line-width rather than shrinking mode volume.

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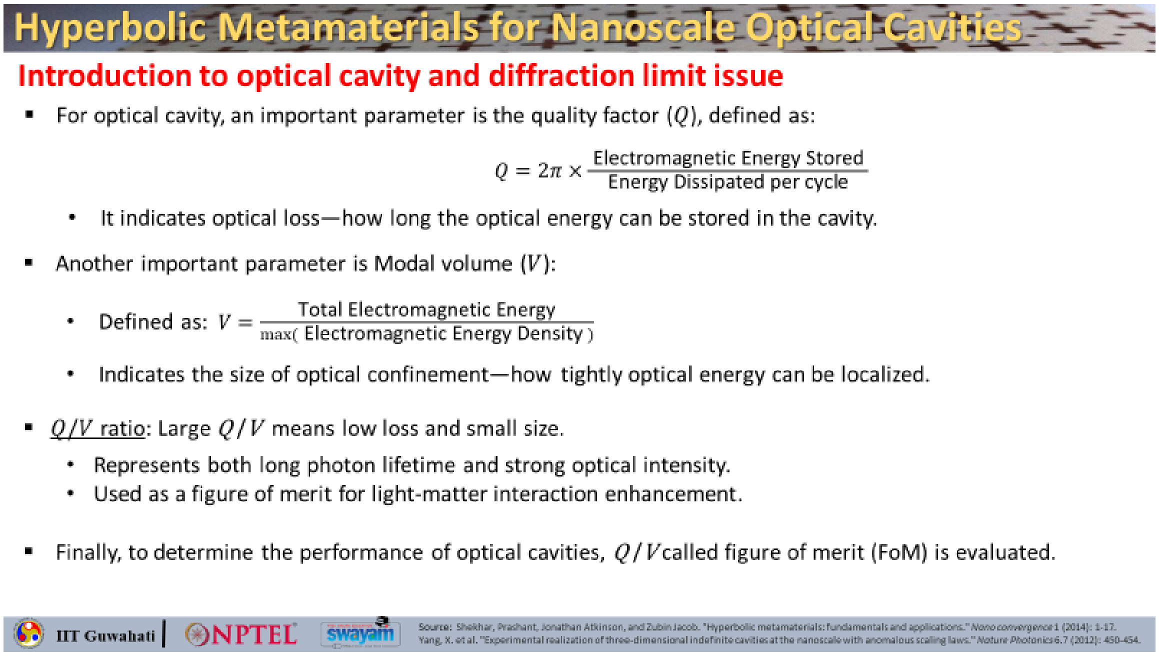
So, in conventional cavities, size L and index n are basically independent of each other.

So, by squeezing light into a smaller dimension, it essentially increases the photonic density of states due to the reduced mode volume. So, what happens in this case it will lead to much faster emission process that will enable a brighter and faster optical emission right. So, this particular independence is something that will be different in the case of metamaterial-based cavities which we will be discussing later ok. So, we will first see that you know this kind of enhancement of emission can be quantified using the Purcell factor. That compares the tailored photonic density of states of an emitter in a vacuum.

So, you know the high-quality factor cavity basically allows you to enhance the photonic density of states. That is achieved by narrowing the spectral linewidth rather than shrinking the mode volume. So, researchers typically introduce a high-quality factor cavity to enhance the photonic density of states. As I mentioned, it is typically done by narrowing the spectral width or spectral line width rather than, you know, reducing the volume of the modes.



Hyperbolic Metamaterials for Nanoscale Optical Cavities

Introduction to optical cavity and diffraction limit issue

- For optical cavity, an important parameter is the quality factor (Q), defined as:

$$Q = 2\pi \times \frac{\text{Electromagnetic Energy Stored}}{\text{Energy Dissipated per cycle}}$$

  - It indicates optical loss—how long the optical energy can be stored in the cavity.
- Another important parameter is Modal volume (V):
  - Defined as: $V = \frac{\text{Total Electromagnetic Energy}}{\max(\text{ Electromagnetic Energy Density })}$
  - Indicates the size of optical confinement—how tightly optical energy can be localized.
- Q/V ratio: Large Q/V means low loss and small size.
  - Represents both long photon lifetime and strong optical intensity.
  - Used as a figure of merit for light-matter interaction enhancement.
- Finally, to determine the performance of optical cavities, Q/V called figure of merit (FoM) is evaluated.

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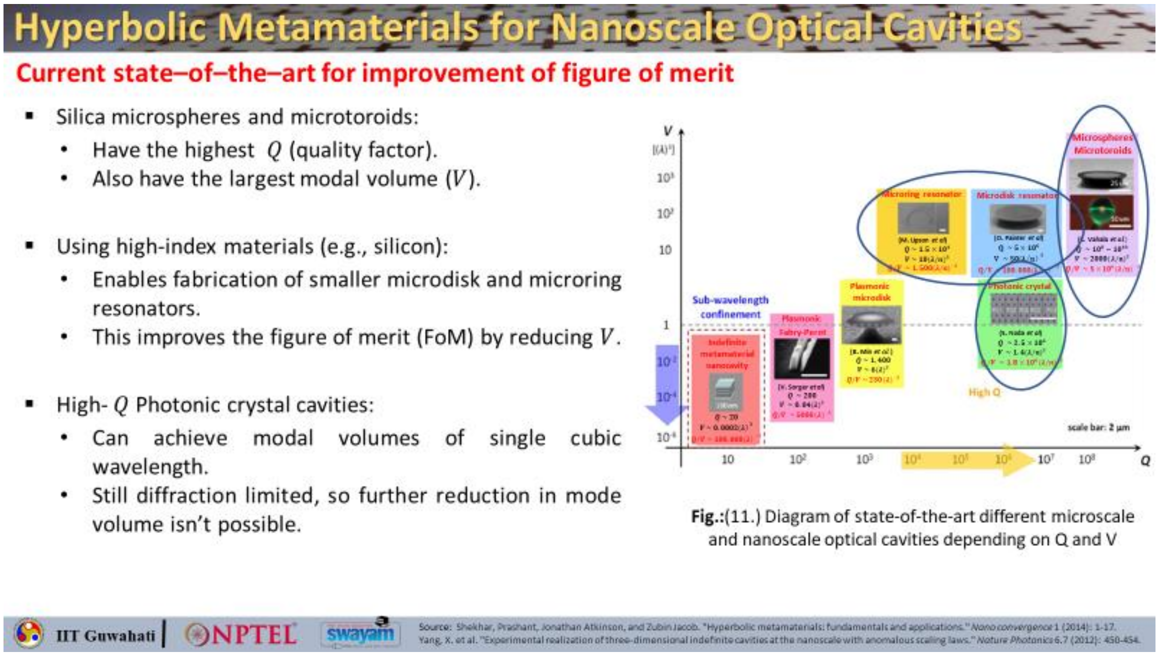
For an optical cavity, an important parameter is the quality factor, which is basically defined as Q equals 2 pi times the electromagnetic energy stored divided by the energy dissipated per cycle, right? So, it basically indicates the optical loss, which means it tells you how long the optical energy can be stored in the resonant cavity.

Another important parameter is the modal volume, which is defined as V equals total electromagnetic energy divided the maximum electromagnetic energy density. So, that tells you about the modal volume, and this gives you an indication of the size of the optical confinement. That means how tightly the optical energy can be localized. So, the Q by V ratio; when you have a large Q by V, that means you have low loss and a small size.

So that is a good thing because it also represents a long photon lifetime and strong optical intensity. Therefore, you know you can use Q by V as the figure of merit for the enhancement of light-matter interaction, right? So, this is a better figure of merit that could quantify the enhancement in light-matter interaction, right? So, we will basically now compare a couple of different cavities and see their performance in terms of Q by V. So, this is our figure of merit, okay for the discussion.



Hyperbolic Metamaterials for Nanoscale Optical Cavities

Current state–of–the–art for improvement of figure of merit

- Silica microspheres and microtoroids:
  - Have the highest Q (quality factor).
  - Also have the largest modal volume (V).
- Using high-index materials (e.g., silicon):
  - Enables fabrication of smaller microdisk and microring resonators.
  - This improves the figure of merit (FoM) by reducing V.
- High- Q Photonic crystal cavities:
  - Can achieve modal volumes of single cubic wavelength.
  - Still diffraction limited, so further reduction in mode volume isn't possible.

Fig.:(11.) Diagram of state-of-the-art different microscale and nanoscale optical cavities depending on Q and V

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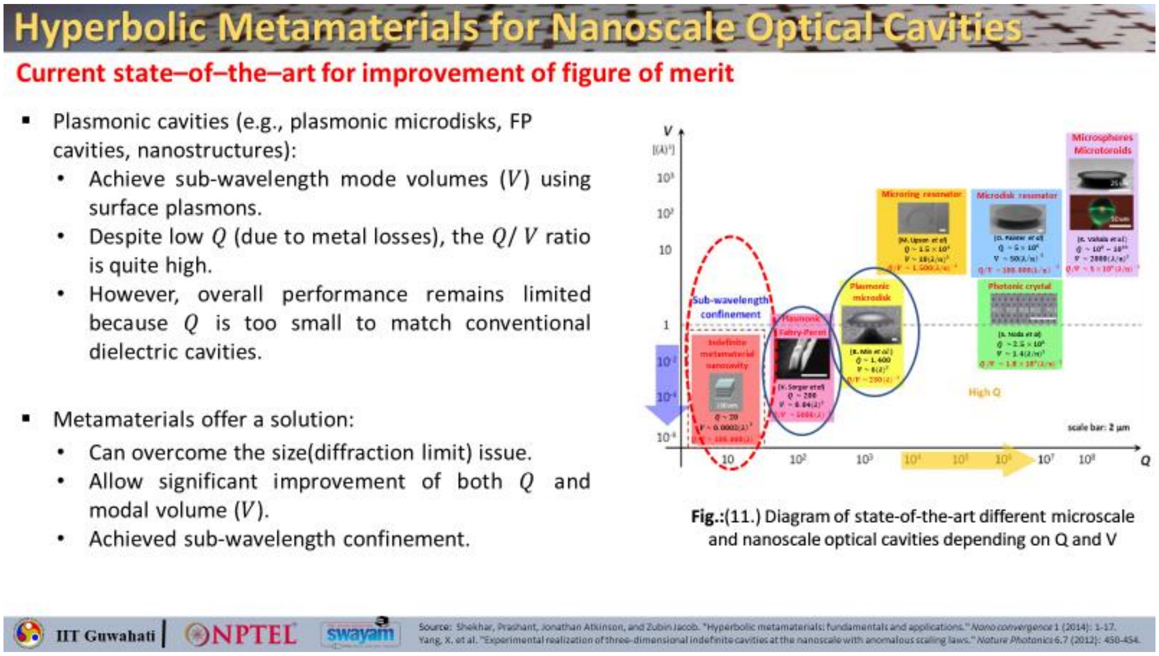
So, we can start with silica microspheres and microtoroids.

So, you can see they have very high Q, okay. So, they basically have the highest Q, on the order of 10 to the power of 8. Okay. But the volume is also very large, and they also have the largest, you know, you can say the modal volume. That is given in terms of 2000 lambda by n cubed, right? So, that tells you that you know the q by v ratio is not that large, okay? So, if you use high-index material like silicon, okay.

In that case, you can bring down the modal volume, okay. So, that allows you to fabricate smaller micro disks and micro ring resonators. So, you can see here the quality factor and the modal volume, which also reduces. So, overall, you can get some improvement in the Q by V ratio. You can also look into the photonic crystal cavity.

So, there you can achieve very little modal volume. So, the volume is on the order of almost a wavelength, 1.4 lambda by n whole cube, and this is the quality factor. So, the Q-to-V ratio is pretty good here. So, however, it is still diffracting limited because you cannot further reduce the mode volume below and you cannot go sub-wavelength.



Hyperbolic Metamaterials for Nanoscale Optical Cavities

Current state–of–the–art for improvement of figure of merit

- Plasmonic cavities (e.g., plasmonic microdisks, FP cavities, nanostructures):
  - Achieve sub-wavelength mode volumes (V) using surface plasmons.
  - Despite low Q (due to metal losses), the Q/V ratio is quite high.
  - However, overall performance remains limited because Q is too small to match conventional dielectric cavities.
- Metamaterials offer a solution:
  - Can overcome the size(diffraction limit) issue.
  - Allow significant improvement of both Q and modal volume (V).
  - Achieved sub-wavelength confinement.

Fig.:(11.) Diagram of state-of-the-art different microscale and nanoscale optical cavities depending on Q and V

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Then you can move toward plasmonic cavities where you can think of plasmonic microdisks.

For Fabry-Perot cavity kind of nanostructures, you can achieve sub-wavelength mode volume. So, you can see V is very, very small—okay, 0.04 lambda cubed. So, that is, like, you know, you can achieve sub-wavelength modal volume. So, although your Q quality factor is not good in this case because of the loss inherent to plasmonic metal, it is acceptable.

Here you can still appreciate that the Q by V ratio is pretty high, okay. However, you know that the overall performance in this case is also limited because Q itself is too small to match any kind of conventional dielectric cavities. So finally, you can see that metamaterials can provide a solution here. You can think of hyperbolic metamaterials as providing you with a cavity that can overcome the size issue, which is the diffraction limit issue. So, you can go very, very sub-wavelength or deep sub-wavelength.

So, the volume can be 0.0002. Lambda cube and the quality factor are around 20. So, you can have you know significantly high Q by V ratio okay using this kind of metamaterial concept okay. So, that also allows sub-wavelength confinement.

Thank You

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Send your queries at deb.sikdar@iitg.ac.in

So, with that, we will conclude this lecture. We will continue the discussion of these hyperbolic metamaterials for optical cavities in the next lecture. If you have any queries regarding this lecture, you can drop an email to this particular email address. Thank you.