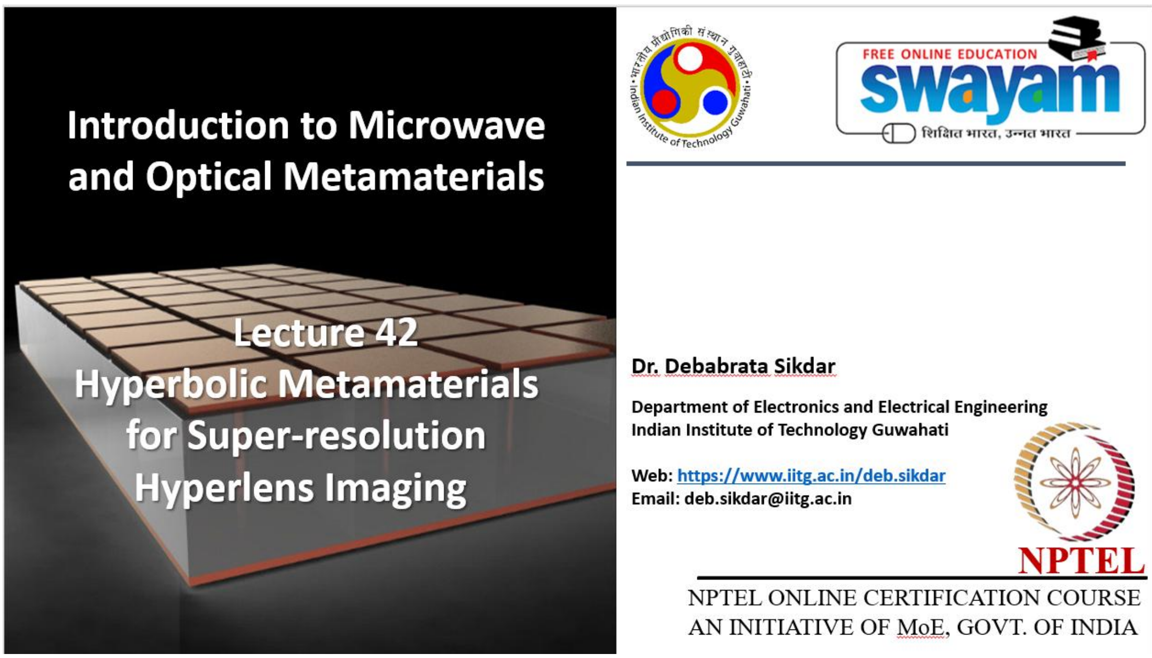
Course Name: Introduction to Microwave and Optical Metamaterials
Professor Name: Dr. Debabrata Sikdar
Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-9
Lecture-42

Lec 42: Hyperbolic metamaterials for nanoscale optical cavities



Hello students, welcome to lecture 42 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on hyperbolic metamaterials for super-resolution hyperlens imaging.

Lecture Outline

- History & Introduction to Super-resolution Hyperlens Imaging
- Physics of the Hyperlens
- Design of Spherical Hyperlens

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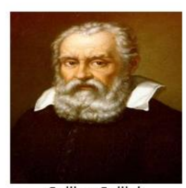
So, we will briefly look into the history and then introduce you to super-resolution hyperlens imaging. We will discuss the physics of the hyperlens and the design of the spherical hyperlens.

History of Super-resolution Hyperlens Imaging

- **1590:** Optical microscopy, invented by Zaccharias Janssen and his son Hans using a tube with lenses.
  - **1609:** Galileo Galilei enhanced the design with a compound microscope (convex + concave lenses).
  - **1632–1723:** Anton van Leeuwenhoek developed lens-making techniques and first observed bacteria, protozoa, and blood cells.
- Conventional optical lenses face a diffraction limit:
  - Restricting resolution to about half the wavelength of light.
  - This limit is due to subwavelength details being carried by evanescent waves, which exist only in the near field.
- Immersion microscopes (introduced by Abbé) improve resolution slightly:
  - Resolution enhanced to $\lambda_0/2n$, where n is the medium's refractive index.
  - Limitation: Limited range of transparent materials with high n.



Galileo Galilei

Anton van Leeuwenhoek

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Source: Smolyaninov, Igor I. Hyperbolic metamaterials. Morgan & Claypool Publishers, 2018.

So, the history of super-resolution hyperlens imaging dates back to, you know, the first imaging dates back to 1590.

So, that is when optical microscopy invented by Janssen ok and his son Hans using a tube with lenses. Then in 1609 Galileo enhanced this design. With a compound microscope that has convex and concave lenses and then in this period, Anton could develop lens-making techniques.

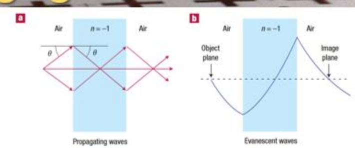
Then he was the first one to observe bacteria, protozoa, and blood cells. Now, conventional optical lenses typically face the diffraction limit. That means the resolution of that imaging is basically restricted to about half the wavelength. of the light being used. So, this limit is basically due to sub wavelength details are being carried by evanescent waves, which could exist only in the near field, okay.

So, in order to capture those, immersion microscopes were introduced by Abbe that could improve. the resolution slightly and the resolution was enhanced to lambda naught by 2 n ok. Here, n is the medium's refractive index. But you know you cannot make n as high as possible. Because there is a limited range of transfer and materials, okay.

So, you know overcoming this diffraction limit has remained a major focus in the research. where we need to find ways to not lose the sub-wavelength details of a particular object. So, in that query, you know people have continuously developed, and once Professor Sir John Penry Predicted the superlens using a negative refractive index medium, okay.

History of Super-resolution Hyperlens Imaging

- Superlens based on Pendry's proposal, allows near-field sub-diffraction imaging but still limited in range.
- For years, nanometer-scale resolution is obtained using near-field scanning optical microscopy (NSOM) :
  - NSOM detects evanescent waves close to the sample surface.
  - NSOM produces images point-by-point via scanning.
  - Limitation: Indirect and slow scanning.
- Breakthrough: Introduction of surface plasmon-assisted microscopy.
- SPP-based imaging principle: Exploits SPP dispersion at the metal-dielectric interface.
  - Utilizes short-wavelength 2D optics of SPPs to form localized magnified images.



A slab of negative-refractive-index medium acts as a perfect lens.

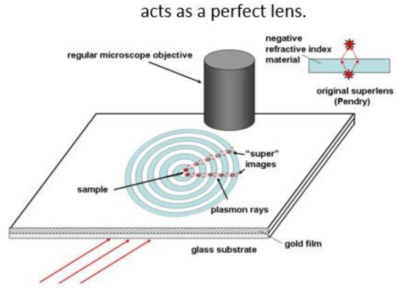


Fig.1: Plasmon microscope operating in "hyperlens mode".

Source: Smolyaninov, Igor I. Hyperbolic metamaterials. Morgan & Claypool Publishers, 2018.

That could give an idea that How a super lens can give you, you know, the perfect lens can give you a perfect image. So, you see air, and then you have a slab of negative refractive index material.

So, it can perfectly focus the propagating waves. Also, the sub-wavelength feature comes from the evanescent waves. Okay, which typically decay in a normal medium, they get amplified in this negative refractive index medium. And then, after this medium decay back in the image plane, it basically gives you the same amplitude as it was in the object plane. So, you are actually also able to capture the details from the subweb length features, right? So, this was the idea of a perfect lens, and for years, people have used nanoscale resolution to obtain.

You know about this kind of small detail using near-field scanning optical microscopy, which is also known as NSOM. So, NSOM can detect the evanescent waves that are close to the sample surface. So, NSOM can produce images point by point by scanning. So, obviously, it is indirect, and it is a slow scanning method, right? So, typically what happens is that NSOM will basically like the versatility and the convenience compared to the far-field optical systems. Also, the tip-induced perturbation could limit the use of real-time imaging.

Then next breakthrough claims when people introduce surface plasmon assistant microscopy. So, here people have used surface plasmon polaritons, which are surface electron waves that can propagate at Sub-wavelength scale is okay, and using the SPP or the surface plasmon polariton dispersion at the metal-dielectric. Interface people try to, you know, capture the details of the sub-wavelength feature. So, here is a picture that shows a plasmon microscope that is operating in the hyperlens mode. So, it's basically a gold film on a glass substrate, and this is at the bottom from the bottom.

You can do the laser excitation, so you can see. The plasmons are generated by the sample that is kept at the center of a hyperlens. So, it's a kind of cylindrical hyperlens you can think of. So, you have alternating layers of metal, dielectric, metal, dielectric, okay. So, the lateral distance between the plasmonic rays you can see will grow with the radius.

And after that, when it comes out, it basically grows. So, here it is, above the diffraction limit. So, it can be captured by the regular microscope objective, right? So, that is how you can see a sub-wavelength feature from this kind of imaging, right? So, what is it doing? It basically utilizes the short wavelength 2D optics of the surface plasma polyatoms to form localized magnified images.

History of Super-resolution Hyperlens Imaging

- 2D "mirror" structure:
  - Implemented using parabolic droplets on the surface.
  - Reflects/scatters SPPs from surface objects to form a magnified 2D image.
- Image detection done via far-field microscopes, using light scattered from:
  - Surface roughness, or
  - Lithographically created structures.
- Initial hyperlens demonstrations were limited to 1-D, UV-wavelength focused.
- Later improvements include impedance-matched, flat, oblate, aperiodic, and acoustic hyperlenses.
- Despite progress, practical limitations remain:
  - No 2D magnification in the visible spectrum has been realized.

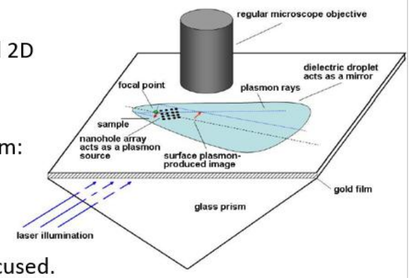


Fig.2: Plasmon microscope operating in "geometrical optics" mode.

Source: Smolyaninov, Igor I. Hyperbolic metamaterials. Morgan & Claypool Publishers, 2018.

So, a 2D mirror structure can also be formed this way. So, this is a plasmon microscope that operates in geometrical optics mode.

So, it is implemented using parabolic droplets on the surface, and it is able to reflect or scatter. The SPPs from surface objects form a magnified 2D image. So, you can see here that there is a nano hole array. Okay, this is the nano hole array which is illuminated by the laser excitation; so, this is the whole setup: it will have a glass prism at the bottom. So, you can send light through the glass prism that will excite the surface plus bond at the metal-dielectric interface.

So, this is the gold film, and this is the dielectric or the glass prism. So, this light basically acts as a source of the surface plasmon which is emitted in all directions. Upon interaction with the sample that is positioned near the focal point of the parabola. You see here a parabolically shaped dielectric droplet. So, this is the dielectric droplet, and from the reflection that is coming from the edges, okay.

The plasmas basically form a magnified planar image of the samples. So, you can see here that the image is formed, right? And that image is basically seen by... This regular microscope objective.

So, the droplet edge, okay, here you can see it can act as. An efficient plasmon mirror because of the total internal reflection is correct. So, the image detection here is also done via a far-field microscope. But using the light that is scattered from the surface roughness or lithographically created structures. So, initially, these hyperlens demonstrations were basically limited to 1D and focused mainly on the UV wavelength range.

As also discussed in the previous lecture, later people made improvements, including, you know, matching. The impedance, making them flat, oblate, or periodic, or even going for an acoustic kind of superlenses, is right. So, despite all this progress, the physical limitations remain, and typically no 2D magnification in the visible spectrum is realized.

History of Super-resolution Hyperlens Imaging

- To overcome these limitations, a novel approach called optical hyperlens for super-resolution imaging allows:
  - Overcoming current diffraction limit far beyond using hyperbolic metamaterials.
  - Sphere-shaped design and new material combinations for:
    - ✓ 2D magnification
    - ✓ Visible wavelength operation.
  - Easy integration with commercial confocal microscopes for real-world applications.

- Optical hyperlens marks a major breakthrough:
  - Uses curved, alternating dielectric-metal multilayers to achieve strong optical anisotropy.
  - Converts high spatial frequencies (sub-diffraction details) into propagating waves.
  - The cylindrical geometry compresses lateral wavevectors adiabatically as waves propagate radially outward.
  - Enables magnification of sub-diffraction features into far-field viewable range.

IIT Guwahati | NPTEL swayam Source: Smolyaninov, Igor I. Hyperbolic metamaterials. Morgan & Claypool Publishers, 2018.

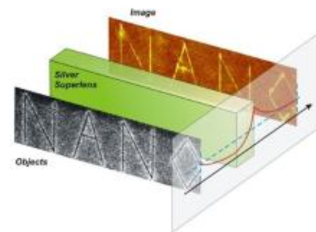
Now, to overcome this limitation okay a novel approach called optical hyperlens, has been used for super-resolution imaging that would allow overcoming the current diffraction limit far beyond Using hyperbolic metamaterials, you will see spherical-shaped designs and new material combinations that can be used. For 2D magnification and also in the ridge operation in the visible range, they can be easily integrated.

with the commercial confocal microscopes for real world applications. So, optical hyperlens can mark major peak too. By using curved alternating metal dielectric multilayers to achieve strong optical anisotropy. And the whole idea here is that you need to convert high spatial frequencies, which means the sub-wavelength. Features are propagated into waves so that they can be captured by the regular microscope.

So, the cylindrical geometry compresses the lateral wave vectors adiabatically as the waves propagate radially outward. And we will see that this enables the magnification of the sub-diffraction feature into the far field with a viewable range. So, normally optical microscopes have been used significantly.

Introduction of Super-resolution Hyperlens Imaging

- Conventional optical lens systems are limited by the diffraction limit: Restricts resolution.
- Pendry's "perfect lens" concept inspired the development of superlenses with sub-diffraction resolution.
- Optical superlens use silver slabs to enhance evanescent waves but are limited to near-field imaging.
- Techniques like near-field scanning microscopy and fluorescence-based imaging bypass the diffraction limit but sacrifice imaging speed.
- Lens-based projection imaging is preferred for high-speed applications.
- Artificial metamaterials offer new possibilities to overcome this limitation.

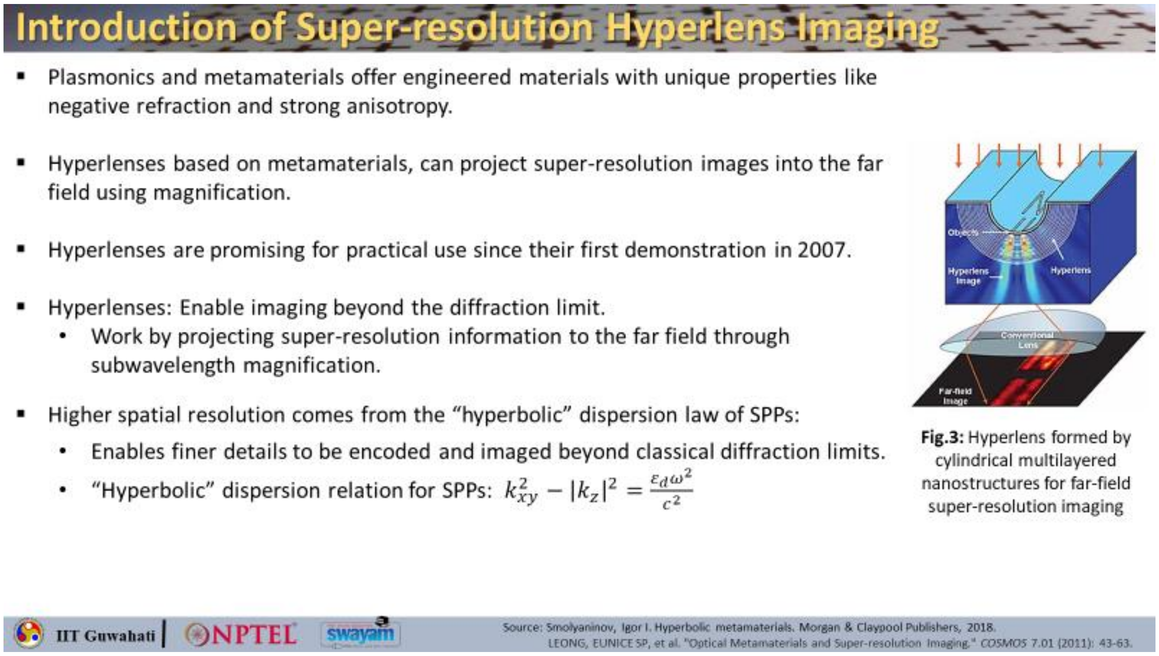


Source: Smolyaninov, Igor I. Hyperbolic metamaterials. Morgan & Claypool Publishers, 2018.

So, you will see that optical microscopy is typically used in many fields such as microelectronics and biology. And medicine, right, and conventional optical lens systems, as we know, are also limited by diffraction limit that basically restricts the resolution, and once Penry could predict the perfect lens, there is a concept inspired by the development of superlenses using negative refractive index material. It was possible to perform sub-diffraction imaging. So, the first optical superlens was realized using silver slabs that could enhance the evanescent waves. OK, but are limited to near-field kinds of imaging. So, in this particular work, they have imaged objects that are slits as narrow as 40 nanometers.

This silver super lens, which is also just 35 nanometers thick, is remarkable. So, in contrast, you can see the current optical microscopes, which can only resolve objects down to around 400 nanometers. Here it was possible to, you know, detect objects that are just 40 nanometers or so, okay. So, that was the kind of, you know, first true super-resolution imaging at optical frequencies using a silver superlens. So, instead of the negative refractive index material, they used a silver superlens, which is called a negative permittivity, right? So, this actually allowed for the enhancement of the decaying evanescent field.

that evanescent field was having this information which was captured, right. So, techniques like near-field scanning microscopy and fluorescence-based imaging could bypass. The diffraction limit that we have already discussed, but it sacrifices the imaging speed. So, you need lens-based projection imaging, okay, to ensure that it is used for high-speed applications. So, that is where artificial metamaterials can be used to now overcome this particular limitation.



So, plasmonics and metamaterials could offer engineered materials that have unique properties like Negative refraction and strong anisotropy; if you make hyperlenses based on metamaterials, they can project super-resolution images into the far field using magnification. So hyperlenses have been promising for practical use since their first demonstration in 2007. So, they basically enable imaging beyond the diffraction limit; hyperlenses are basically made of hyperbolic metamaterials, right? So, they work by projecting the super-resolution information into the far field through subwavelength magnification, and that is shown here. So, you have got an object which is subwavelength and then you have hyperlens which is made of hyperbolic metamaterials that will magnify the feature to a scale which can be picked up by conventional lenses and then you can do far field imaging So, higher spatial resolution basically comes from this hyperbolic dispersion law of the SVPs that enables finite details to be encoded and imaged beyond the classical diffraction limit. So, if you remember, the hyperbolic dispersion relation for SPP will look like this: $k_{xy}^2 - |k_z|^2 = \frac{\varepsilon_d \omega^2}{c^2}$.

Physics of the Hyperlens

- Light emission/scattering consists of:
  - Propagating waves (low wave-vectors):
    - ✓ Carry large-feature information.
    - ✓ Reach the far field.
  - Evanescent waves (high wave-vectors):
    - ✓ Carry fine, subwavelength detail.
    - ✓ Are non-propagating in normal materials and remain confined to the near field.

- Diffraction-Limited Imaging:
  - Arises because evanescent waves can't reach the far field.
  - As a result, subwavelength details do not contribute to the final image.

- Hyperlens Realization – Key Requirements: A material that supports high wave-vector propagation.
  - A magnification mechanism to convert high wave-vector waves into low wave-vector waves, enabling far-field transmission of super-resolution information.

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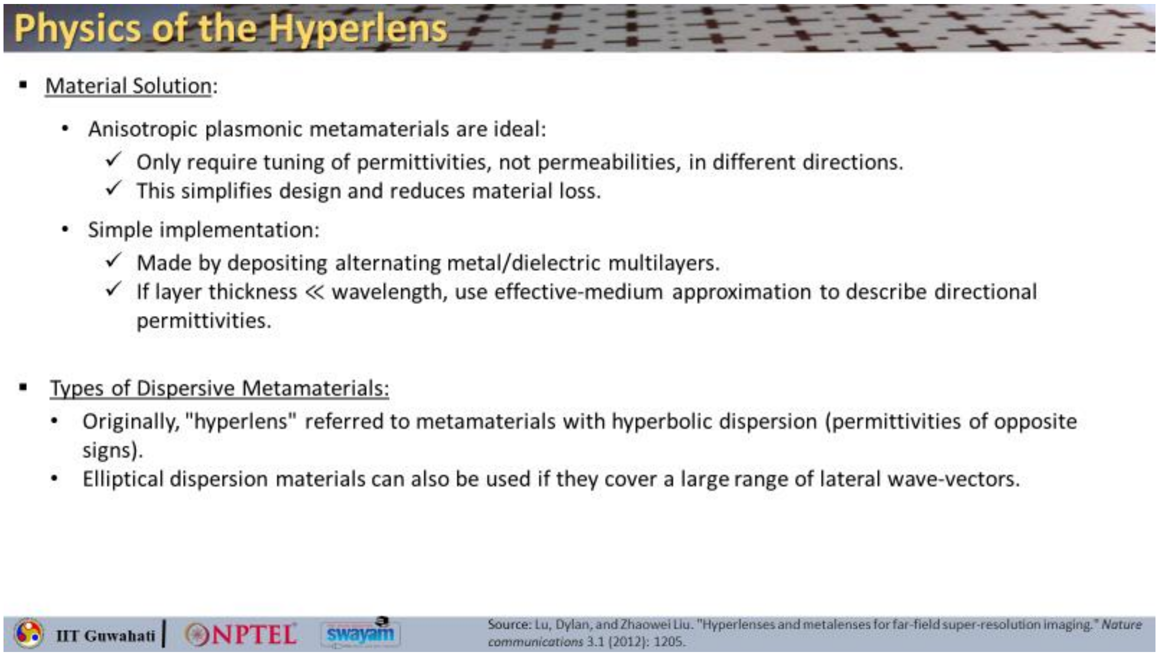
Source: Lu, Dylan, and Zhaowei Liu. "Hyperlenses and metalenses for far-field super-resolution imaging." *Nature communications* 3.1 (2012): 1205.

Now what is the physics behind the hyper lens? The first thing is that light emission and scattering basically consist of two waves. One is the propagating wave that has the low wave vectors. Typically, they carry large feature information and can reach the far field. And the other part is evanescent waves which basically correspond to high or large wave factors, they carry fine sub wavelength details Typically, they are not propagating in normal materials, and they remain confined within the near field.

Okay. So, when you look into diffraction-limited imaging, that means this limitation is basically coming from the fact that Evanescent waves are not able to reach the far field before they are captured. So, you are losing information. So, it means that surveillance details are not being contributed to the final images. So that is why you need hyperlens and the key requirement here will be a kind of material that supports high wave propagating wave vector propagation. So that you know, evanescent waves coming from sub-wavelength features can be propagated correctly.

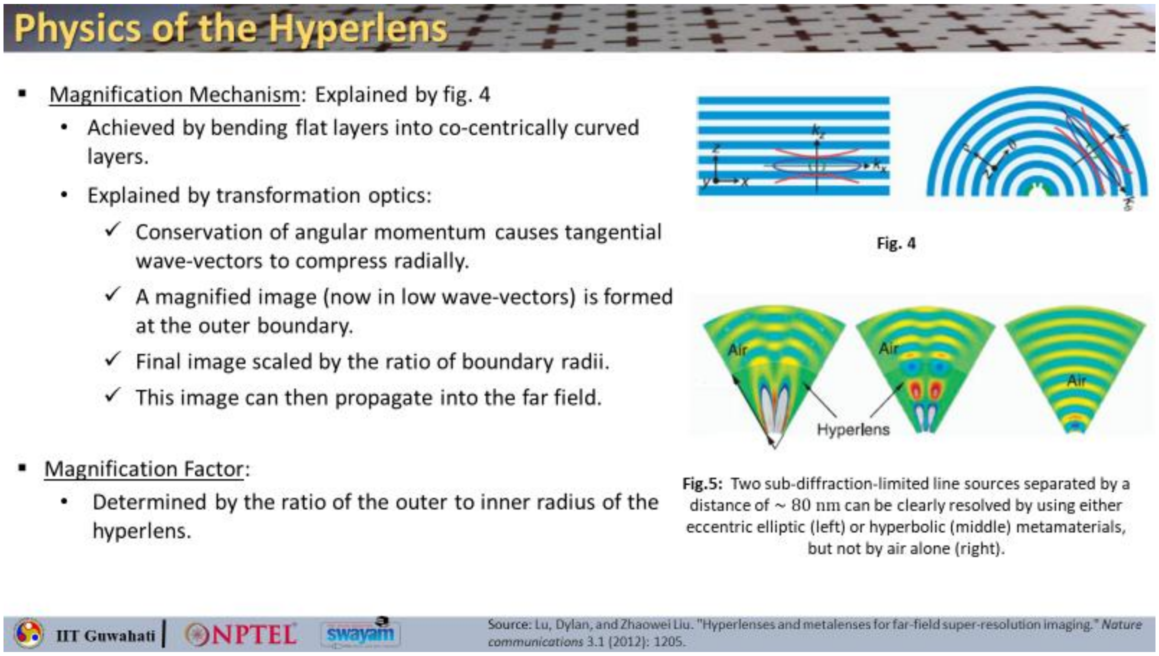
So, a magnification mechanism is needed to convert high wave-vector waves. which are the evanescent waves to low wave vector waves that are propagating waves. So that you can enable far-field transmission of this kind of, you know, super-resolution information.



So, what is the material solution for this? The first thing is you have to understand that anisotropic plasmonic metamaterials are ideal in this kind of scenario because they only require tuning of permittivities not permeabilities ok in different directions and that simplifies the design and also reduces the material loss. Now the simple implementation could be that they are made of alternating metal-dielectric multilayers.

like a kind of multilayer stack in the form of a 1D periodic array that we have seen. And you have to ensure that the layer thickness is much, much sub-wavelength. So, that effective medium theory approximation can be used to describe the directional permittivities epsilon parallel and epsilon perpendicular. Now, there are different types of dispersive metamaterials. So, if you see hyperlens, they basically refer to metamaterials that have hyperbolic dispersion.

That means the permittivities are of opposite signs. So, epsilon parallel and epsilon perpendicular are of opposite signs. And if you use elliptical dispersion materials, they can also be used if they cover a large range of lateral wave vectors. In that case, they will also be able to pick up that large or high wave vector information from the subwavelength features. and pass it on to low wave vector waves that are propagating waves.

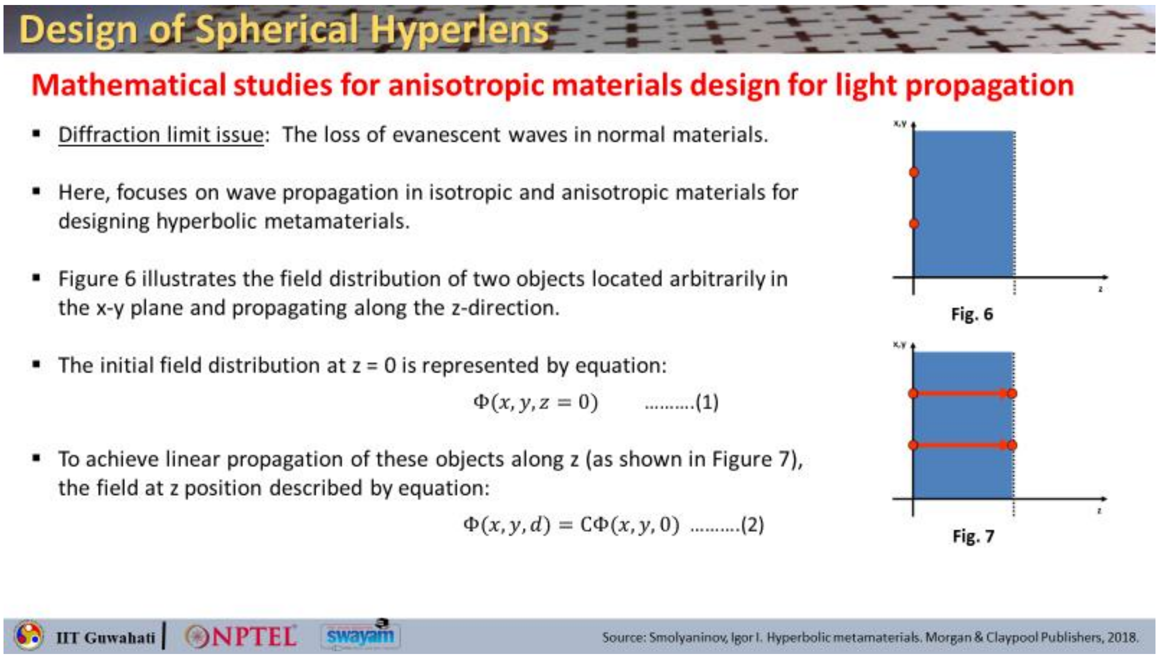


So that you can pick up that information. So, the magnification mechanism is explained here, okay. So, this is a material which has an elliptical kind of dispersion curve. So, you can see bend the flat layers into a concentric curve layers you can get this kind of a structure. So, this can be explained by transformation optics. So, here is what happens: the conservation of angular momentum causes the tangential wave vectors to compress radially.

So, these are the two cases. So, now we will see that a magnified image, which is basically a low-vector image, will be formed. in the outer boundary of each case and the final image is scaled by the ratio of the boundary radiia. So, what is happening? So, this is the portion where the hyperlens is placed; you can see these are the hyperlens. So, here are the sub-wavelength features and the two small features, as you can see. Now, what is the objective? You want to image it here in the far field.

So, this part is normally if you consider this one. So, there is no other material; you just have two sub-wavelength features, and you will see that, you know. Only here are you capturing the low wave vectors. So, they are not able to resolve the two objects correctly. Now you can do that okay here assume that these two objects are separated by a distance of say 80 nanometers. So, you can either use an eccentric elliptic kind of metamaterial, which is this one, okay.

So, in that case also you can convert this high wave vector information to a lower wave vector information like this is this high wave vector information ok. To a lower wave vector information, and then you can come up to this boundary, which can further propagate. So, here you can resolve the two objects. You can also use a hyperlens where you can clearly see that the two objects are resolvable, but not only in air. So, what is the magnification factor? The magnification factor is basically the ratio of the outer to the inner radii, right? So, that basically tells you the magnification factor of this hyperlens.



So, what is the method for designing this kind of spherical hyperlens? So, let us look into the mathematical studies for this kind of anisotropic material design that can support light propagation from diffraction limited objects. So, the diffraction limit issue is mainly due to the loss of the evanescent waves in normal materials. So, our focus here will be to see the wave propagation in isotropic and in anisotropic material that allows us to design this hyperbolic metamaterial. So, first focus on figure 6 that illustrates the field distribution of the two objects which are located arbitrarily in the xy plane and propagating along the z direction. So, the initial field distribution at z equals 0 that is in this plane can be written as φ (x, y, z) = 0.

Now to achieve linear propagation of these objects along z okay, so the field at any z position okay, so z equals d can be written as $\Phi(x, y, d) = C\Phi(x, y, 0)$, right.

Design of Spherical Hyperlens

- To overcome the diffraction limit, the field distribution must be linearly transferred:
  - From the object plane to the image plane.
  - Should occur without disturbance or change at any single point (illustrated in Figure 8).
- Objective: To find or design materials that enable such linear propagation along the z-direction (or any desired direction).
- The propagation of electromagnetic waves is governed by an equation:

$$\Phi[x,y,d] = \int_{x'y'} \Phi[x',y',0]\left(\int_{k_x k_y} e^{ik_x(x-x')}e^{ik_y(y-y')}e^{ik_z[k_x,k_y]d}dk_x dk_y\right)dx'dy' \quad \text{..........(3)}$$

(for z-directional propagation).

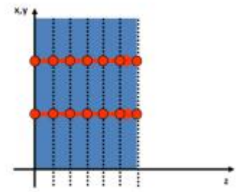


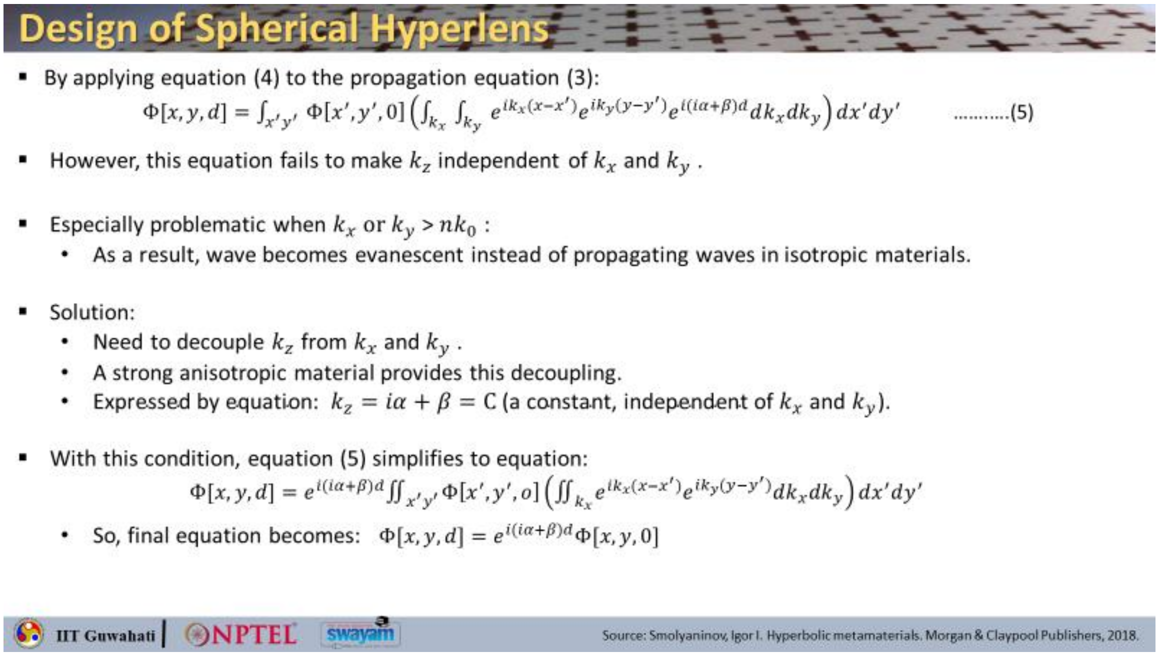
Fig. 8

- Isotropic materials have a dependency between three wavevectors k_x, k_y and k_z, expressed by equation: $k_z[k_x,k_y] = \sqrt{n^2k_0^2 - k_x^2 - k_y^2}$ ....(4)

IIT Guwahati | NPTEL swayam Source: Smolyaninov, Igor I. Hyperbolic metamaterials. Morgan & Claypool Publishers, 2018.

So, to overcome the diffraction limit, the field distribution must be linearly transformed. So, that means it should occur from the object to the image plane without disturbance or any change at any single point, right? So, the objective here is to find or design materials that enable such linear propagation through the z direction or say in any other direction. And in this case, you can write that the propagation of the electromagnetic wave will be defined or governed by this particular equation. So, $\Phi[x,y,d] = \int_{x'y'} \Phi[x',y',0]\left(\int_{k_x k_y} e^{ik_x(x-x')}e^{ik_y(y-y')}e^{ik_z[k_x,k_y]d}dk_x dk_y\right)dx'dy'$.

So, this is what the z-directional propagation. So, you will see that isotropic materials have a dependency between the 3 wave vectors kx, ky and kz which can be expressed by this particular equation where you can write $k_z[k_x,k_y] = \sqrt{n^2k_0^2 - k_x^2 - k_y^2}$.



Now, if you apply this equation to the propagation equation, it simplifies to this particular form, okay. So, you can write $\Phi[x,y,d] = \int_{x'y'} \Phi[x',y',0]\left(\int_{k_x}\int_{k_y} e^{ik_x(x-x')}e^{ik_y(y-y')}e^{i(i\alpha+\beta)d}dk_x dk_y\right)dx'dy'$ at 0 that is at z equals 0. So, this particular term here $e^{i(i\alpha+\beta)d}$, this term is constant, okay. So, the final equation satisfies the condition for linear propagation of objects along the z direction, as you can see here. So, this equation fails to make k z independent of k x and k y, right? So, it is not like in the previous case, okay. So, here k z is still dependent on k x and k y, and especially it is problematic when you see k x or k y being greater than n k 0.

So, as a result what you will see the wave becomes evanescent instead of propagating wave you will get evanescent wave in the isotropic material. So, what is the solution? You need to decouple this kz from kx and ky that means you have to bring some strong anisotropy to do this decoupling. So, that is where you can express you know this as a constant. So, if you consider this value as a constant k, z, this is basically k z, okay. If you write $i\alpha + \beta = C$ that is a constant that means it is independent of k x and k z.

In that case you can write this equation as this ok. So, this equation can be written as φ (x, y, d). So, the propagation along z is not going to alter your wave, okay. So, $e^{i(i\alpha+\beta)d}\iint_{x'y'}\Phi[x',y',o]\left(\iint_{k_x} e^{ik_x(x-x')}e^{ik_y(y-y')}dk_x dk_y\right)dx'dy'$ at z equals 0 and then you have integration over kx and ky.

So, this is a constant okay. So, it turns out okay. So, the final equation can be simply written as $\Phi[x,y,d] = e^{i(i\alpha+\beta)d}\Phi[x,y,0]$. So, here you can see that this particular term is constant. So, you can see that you are actually getting a linear propagation of objects along the z direction which was mentioned in the earlier equation 2.

Design of Spherical Hyperlens

Design of spherical geometry for far-field hyperlens imaging

- Suppose, two nanoscale objects (e.g., 50 nm diameter holes) are placed 50 nm apart, beyond the diffraction limit.
  - Medium: Isotropic (such as air) and
  - Wavelength of incident light: 410 nm.
- When light shines from below (as shown in figure 9).
- Light passes through the holes but cannot propagate into the air.
- Reason: The 50 nm separation is much smaller than the diffraction limit at 410 nm.
- As a result:
  - The light becomes an evanescent wave that cannot propagate along the z-direction.

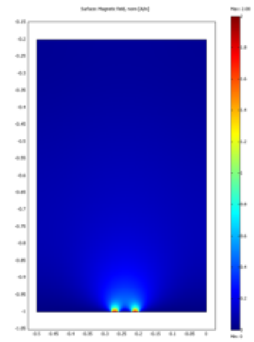


Fig. 9: Simulation for light propagation of two very small nanoscale objects in normal isotropic materials (blue color), and their distance is within diffraction limit.

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Source: Z. Liu *et al.*, "Far-field optical hyperlens magnifying sub-diffraction-limited objects", *Science* **315**, 1686 (2007)
LEONG, EUNICE SP, et al. "Optical Metamaterials and Super-resolution Imaging" *COSMOS* 7.01 (2011): 43-63.

So, suppose that we consider two nanoscale objects; for example, 50-nanometer diameter holes that are also placed 50 nanometers apart. So, these are beyond the diffraction limit, right? So, here let us see why we need the spherical geometry of hyperlens and the problem in a you know isotropic medium with this kind of consideration.

So, first thing we are considering the medium to be isotropic such as air, wavelength of the incident light is considered to be 410 nanometers. So, when light shines from the below as you can see here, these are the two spherical holes or nanoscale objects ok, circular holes you can say. 50 nanometer in diameter and they are 50 nanometers apart. So, when light passes through the holes you can see the field is getting diminished in the very close proximity that means they are not able to propagate in air.

Okay. The reason is that this is sub-wavelength information, and a 50-nanometer separation is much smaller than the diffraction limit at 410. So, typically, the diffraction limit is half the wavelength. So, below this means below 200 nanometers or 205 nanometers will not be able to resolve the information, and that is what we are seeing here. okay and this is only because that the sub wavelength information is basically carried by high wave vector components and these components cannot propagate in air due to their limited dispersion relation okay because you know your kx, ky and kz are interdependent okay because your kx, ky and kz are interdependent in this kind of medium. So, they will not be able to support this kind of high-wave vector components.

So, what is the result here? You can see the light becomes an evanescent wave that cannot propagate along the z direction.

Design of Spherical Hyperlens

- If a hyperbolic metamaterial is filled with air:
  - Light can propagate through the metamaterial in the z-direction due to the hyperbolic dispersion.
  - This is because the material supports high in-plane wavevectors (k_x, k_y).
- Important observation from Figure 10(b):
  - After light escapes the hyperbolic metamaterial, it decays exponentially.
- This decay limits the detection of sub-diffraction images at the image plane.
- Challenge: Practical, far-field imaging is necessary for convenient super-resolution applications.

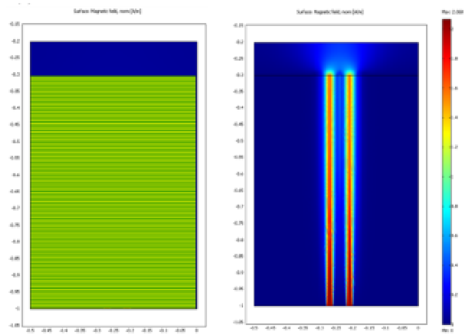

Fig. 10: (a) Schematic of a multilayer stacked hyperbolic metamaterial for light propagation of two objects beyond diffraction limit (yellow/green color).
(b) Simulation for light propagation of two very small nanoscale objects in a multilayer stacked hyperbolic metamaterial.

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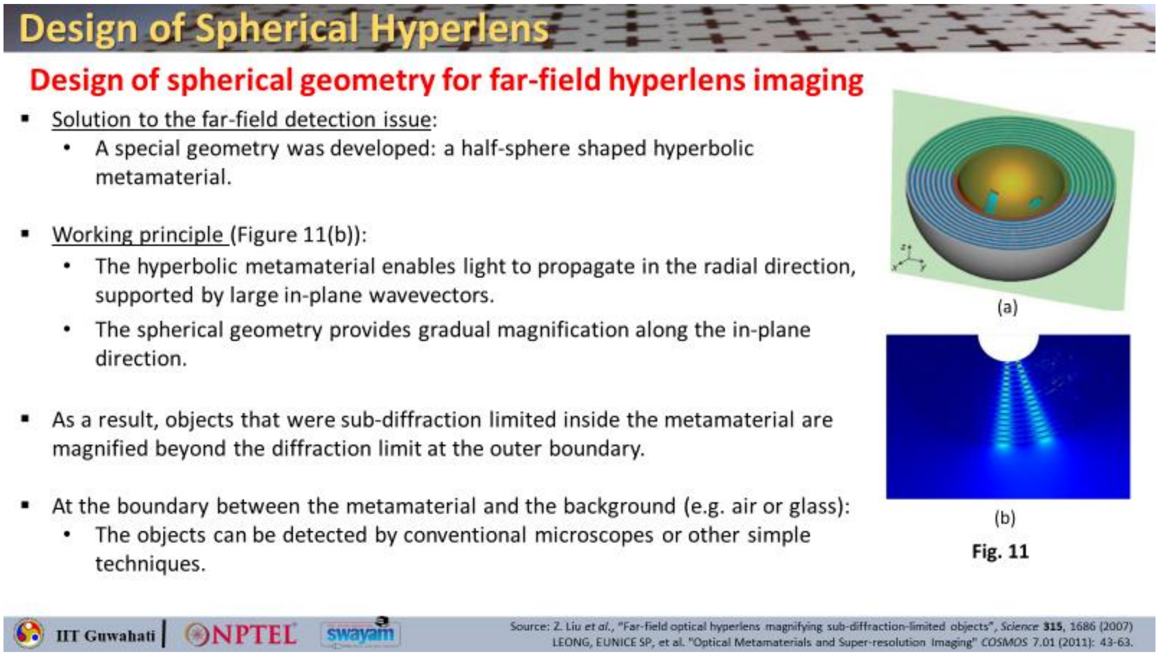
Source: Z. Liu *et al.*, "Far-field optical hyperlens magnifying sub-diffraction-limited objects", *Science* **315**, 1686 (2007)
LEONG, EUNICE SP, et al. "Optical Metamaterials and Super-resolution Imaging" *COSMOS* 7.01 (2011): 43-63.

So now if you fill a hyperbolic metamaterial with the air okay, you can see that light can propagate through the metamaterial in the z direction due to the hyperbolic dispersion because that is where your kz can be independent of kx and ky and it can support very high wave vector okay. So, this is where you can see the hyperbolic metamaterial present and light propagating along this direction. This is air again, so this is where it basically stops propagating, right? So, these are important observation that as soon as the light exit from the hyperbolic metamaterial, it again decays exponentially and you cannot see this further. So, this decay basically limits the detection of sub-diffraction images at the image plane.

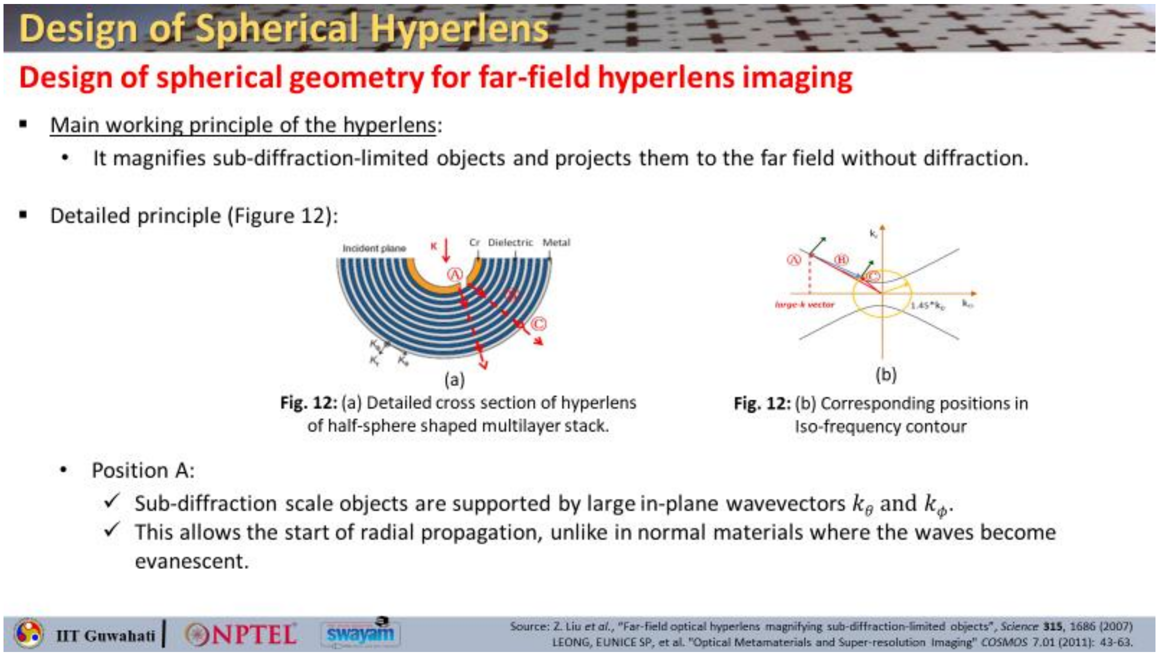
So, here they are able to propagate, but then as soon as they exit you are not able to capture it because the sub-wavelic information is basically you know getting exponential decayed. So, what do you want? You want to also specially separate out this information, so that you can do far-field imaging. So, the current challenge that remains is that you need practical far-field imaging to be done for this kind of super resolution application.

That is where you can take the help of this spherical geometry.



So, you consider a half sphere shaped hyperbolic metamaterial. So, you are having these alternating layers of metal dielectric, but increasing radius. So, this hyperbolic metamaterial will basically enable light propagation along the radial direction. Okay, which are supported by a large in-plane wave vector. So, these are sub, like below the diffraction limit, but as they propagate, they basically get spread out. So, you can actually now get this information above the diffraction limit which can now couple to the far field and you can get that imaging done.

So, this is where the spherical geometry can provide this gradual magnification along the in-plane direction, right. So, here you see the schematic of a spherical superlens which comprises of 9 pair of silver and titanium oxide layers and in this particular figure you see sub diffraction limited objects which are propagating in the radial direction are with gradual magnification along the plane direction in plane direction that is phi and theta. So, as a result the objects which were actually below diffraction limit They were now magnified beyond the diffraction limit at the outer boundary of this material and at the boundary between the metamaterial and the background of the glass, they can be now detected by conventional microscope because they are above the diffraction limit and that is how you can do the imaging.



So, the main working principle of this kind of hyperlens is that it magnifies sub wavelength or you can say sub diffraction limited objects and projects them to the far field without diffraction. Here you can see what is happening; this is the detailed cross-section of the hyperlens of a high-sphere-shaped multilayer stack.

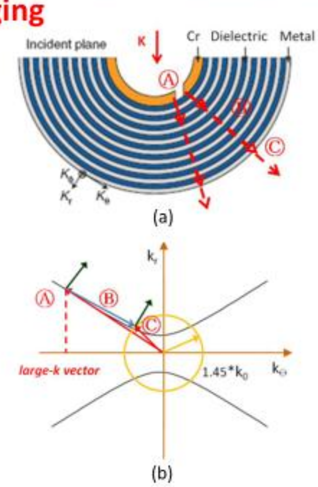
So, you have a chromium material that has this small piece of information, okay. So, this is the information that is diffraction-limited; there is a slit, then you have this metal-dielectric layer alternating layers, okay. Here you can see carefully so that it marks your k theta; this is k r, and k phi is going into the screen. Ok. So, this particular figure tells you that you have got a sub wave length image that is curved on this chromium layer and then you have this metal dielectric oxide multilayer hyperlens ok. The transverse magnetic component of the unpolarized light relative to the plane is basically leveled as k.

Okay, that you see here. So, what happens in position A is that it is basically a sub-diffraction scale object. which are supported by large in-plane wave vector k theta and k phi okay. So, you have this one large k vector. So, this is the corresponding position in the isofrequency contour, okay. So, this is the hyperbolic metamaterials contours and then you are basically you are allowing to start this radial propagation, ok because this hyperbolic metamaterial and they support this large wave vector.

Design of Spherical Hyperlens

Design of spherical geometry for far-field hyperlens imaging

- Position B:
  - ✓ k_θ and k_ϕ are gradually compressed along the hyperbolic curve as the radial position increases.
  - ✓ This is because optical momentum ($k_{in} \times r$) is conserved at every point.
  - ✓ As the radius r increases, k_{in} (in-plane momentum) decreases.
- Position C:
  - ✓ The distance between the two objects becomes large enough so that it is no longer diffraction limited.
  - ✓ The light is now supported by the dispersion relation of normal material (e.g. glass substrate).
  - ✓ The light can propagate into free space after exiting the hyperbolic metamaterial.
  - ✓ The magnified image can be easily detected by a conventional microscope.



Source: Z. Liu *et al.*, "Far-field optical hyperlens magnifying sub-diffraction-limited objects", *Science* **315**, 1686 (2007)
LEONG, EUNICE SP, et al. "Optical Metamaterials and Super-resolution Imaging" *COSMOS* 7.01 (2011): 43-63.

So, evanescent waves can propagate. So, they are propagating, and they are coming to position B, which is here, okay. So, at this position, what is happening? That your k theta and k phi are gradually compressed along this hyperbolic curve as your radial position is constantly increasing, right? So, this is because the optical momentum, which is k crossed with r, is basically conserved at every point. Now, with the radius being increased, you will see that the in-plane momentum, which is k in, is basically decreasing. So, that's why it is going down the slope and at point C the distance between the two points become large enough or you can say the two objects become large enough so that they are no longer diffraction limited. So, in this case, the light will now be supported by the diffraction relation of normal material, which is the glass substrate.

So, you can see if you take this as the isofrequency point of glass. So, here you can have either this phase matching or momentum matching. So, light can now propagate into free space after exiting from this particular point at C. So, this is where you see the light can escape into free space after exiting the hyperbolic metamaterial. So, this is how it works. So, you were initially in position A Then, you are moving through this point that is your B and then when you are coming out of this metamaterial okay and you can match the momentum of free space waves, you can couple to the free space wave and can you can be detected directly.

So, in this case, the sub-wavelength information can be captured as a magnified image that can be detected by a conventional microscope.

So, with that we conclude this lecture. if you have got any query regarding this lecture, you can drop an email to this particular email address mentioning the course title and the lecture number in the subject line.