Course Name: Introduction to Microwave and Optical Metamaterials
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Week-9
Lecture-41

Lec 41: Fundamentals of Hyperbolic Metamaterials

Hello students, welcome to Lecture 41 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on the Fundamentals of Hyperbolic Metamaterials.

Lecture Outline

- Introduction of Hyperbolic Metamaterials
- Physics of Multilayered Hyperbolic Metamaterials
- Basic properties of Hyperbolic Metamaterials
- Geometries of Hyperbolic Metamaterials
  - ✓ Metal wire arrayed structure
  - ✓ Multilayer metal-dielectric stacked structure
- Applications of Hyperbolic Metamaterials

So, in this lecture, we will introduce you to hyperbolic metamaterials and discuss the physics. Behind multilayered hyperbolic metamaterials are some of their basic properties. We will consider these two geometries of hyperbolic metamaterial. The first one will be a metal wire arrayed structure, and the second one will be a multilayer metal dielectric stack structure.

After that, we will discuss some applications of hyperbolic metamaterials.

Introduction of Hyperbolic Metamaterials

- Hyperbolic metamaterials (HMMs): Highly anisotropic, uniaxial, sub-wavelength artificial nanostructures materials having hyperbolic dispersion.
  - Act like a metal in one direction and a dielectric in the orthogonal direction.
- **Hyperbolic dispersion:** Principal components of permittivity and permeability tensors have different signs.
- **Origin and Purpose:** Originally developed to overcome the diffraction limit of optical imaging.
- **Key feature**: Possess a broadband singularity in the density of photonic states.
- **Material Types**:
  - Initially believed to be achievable only through artificial structuring.
  - Later found in some natural materials within specific frequency ranges.
  - Even physical vacuum may exhibit hyperbolic properties under strong magnetic fields.

Source: Smolyaninov, Igor I. *Hyperbolic metamaterials*. Morgan & Claypool Publishers, 2018.

So, hyperbolic metamaterials these are highly anisotropic, uniaxial, sub-wavelength artificial nanostructures material that has got hyperbolic dispersion. So, if you look into their dispersion diagram, it looks like a hyperbola, right? So, what is so unique about this metamaterial? They basically act like metal in one direction and like a dielectric in the two orthogonal directions. So, normally you will see that all-natural materials, or even the other metamaterials, that We have designed so far; they have the same sign for, you know, electric permittivity and permeability tensor, okay.

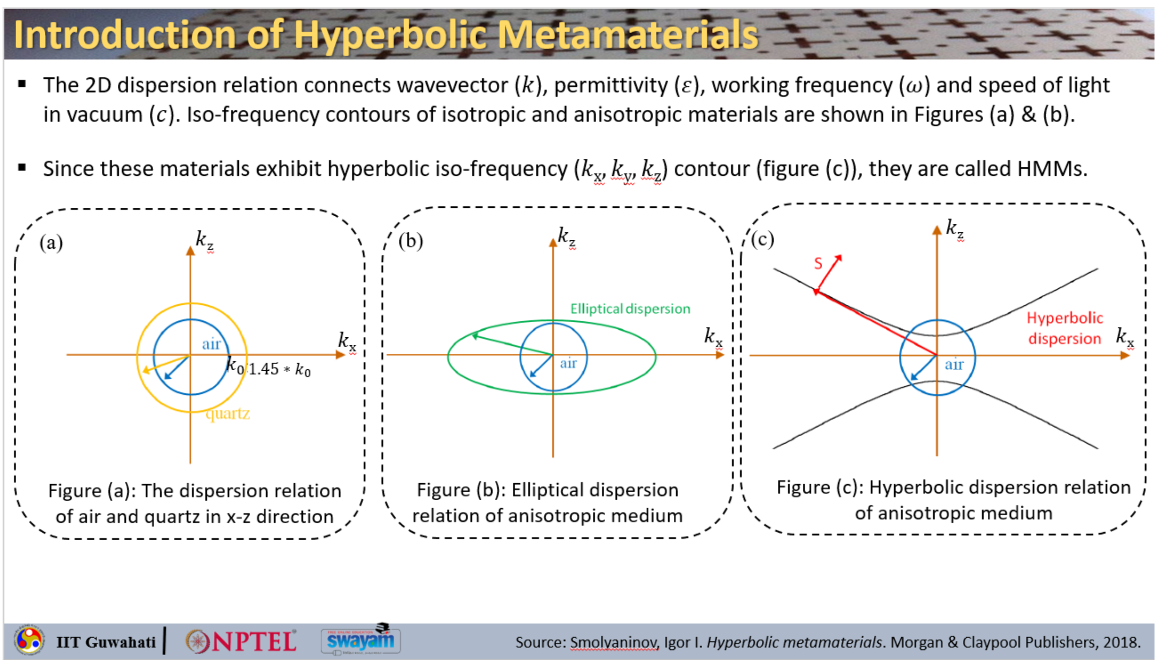
But in hyperbolic dispersion, you will see that the principal components of the permittivity and permeability tensors have different signs. So, the origin and the purpose of this is that it was initially developed to overcome the diffraction limit of optical imaging, and we will discuss how it works in this particular lecture. So, these hyperbolic metamaterials basically possess a broadband singularity in the density of photonic states. Now, when I talk about broadband singularity in the density of photonic states, it basically does Refers to a unique property of some metamaterials where the density of states for light waves. The photon is significantly enhanced over a wide range of frequencies.

This singularity leads to unusual behavior in how light is going to interact with this kind of material that basically influences phenomena like spontaneous emission and light propagation. So, if you consider this photonic density of states that basically describes How many different light wave states exist at a given energy or frequency? And when we talk about singularity in the context of the photonic density of states, a singularity Basically means a large or infinite value at a specific frequency or over a range of frequencies, right? So, here we are talking about broadband singularity. So, broadband means that the singularity basically occurs over a wide range of frequencies, not just a single point. Now, what are these materials with hyperbolic dispersion? So, these materials are basically, as I mentioned, artificial nanostructures or engineered structures. Nanostructures need not be what we continuously keep telling because in the context of Optical metamaterial aspects, if they are in the microwave range, have dimensions that are typically in the millimeter range.

So, these metamaterials are basically artificially engineered structures that can exhibit unique electromagnetic properties. which are not found in nature, we all know this right. So, one such property is this Hyperbolic dispersion where the dielectric permittivity has different signs along different directions. That means, say your epsilon parallel is positive and epsilon perpendicular is negative, right? So, this kind of different sign basically leads to broadband singularity in the photonic density of states. Now, what is the consequence of this? This thing is basically interesting because it can enhance spontaneous emission.

So, light sources kept near this kind of material may emit light much faster than it is. It can shorten the spontaneous emission lifetime, and it can also modify light propagation. So, light can actually propagate in a very unusual way. Including support for very large wave factors due to the large number of available states. So, what are the applications of this kind of hyperbolic metamaterial? These properties are primarily explored for applications in the areas of single-photon sources, quantum communication, and sensing.

So, in essence, you will see that metamaterials with hyperbolic dispersion basically offer a way. To control and manipulate light-matter interaction in a way that is not possible with conventional materials, right? So, it gives very unique features, okay. So, what are the typical types of materials that can show this kind of feature? The first thing is that it was initially believed that it is only achievable through artificial structures. Later it was seen that some natural materials within a specific frequency range can also show this kind of property. Even a physical vacuum may exhibit hyperbolic properties under strong magnetic fields.



So, here you can see the 2D dispersion relation that connects momentum, okay. So, this is basically the dispersion relation of air and quartz, okay. So, this is the k-space diagram. So, these particular rings are basically telling you about the permittivity. The refractive index is okay.

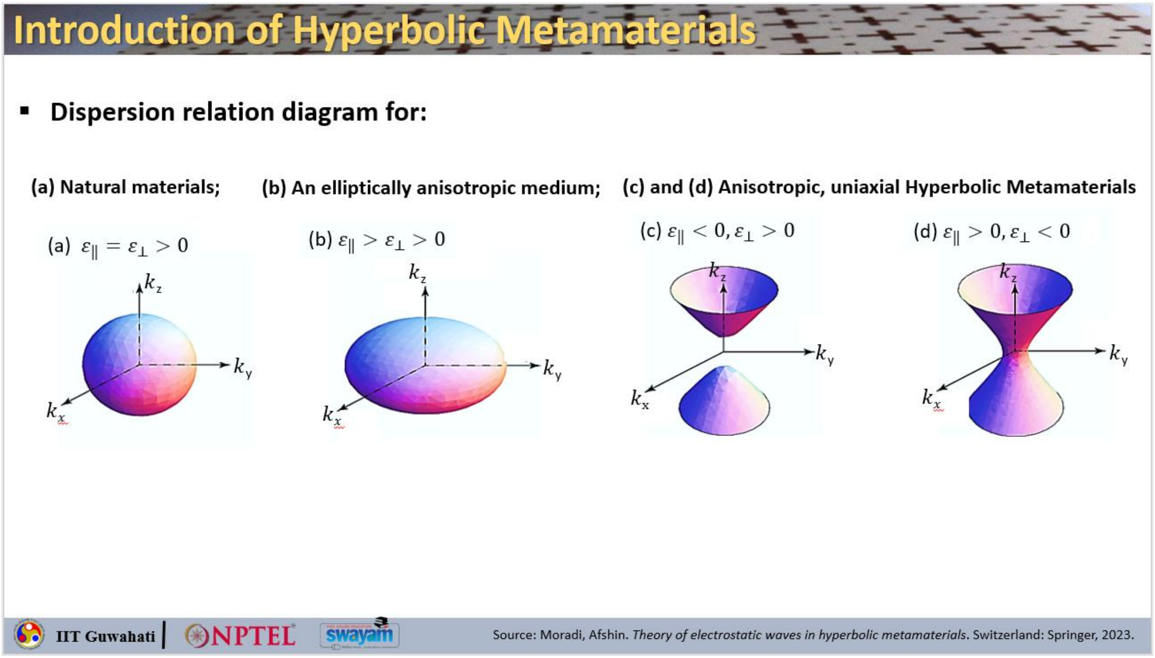
So, this value is basically because this is in k space. So, you are writing everything in terms of the wave vector k. This is in air. You have k naught, and in quartz, you will have 1.45 times k0. Okay, this is the wave factor. So, this is the typical dispersion relation of air and quartz in this particular case, right? Now there are some materials that also show elliptical dispersion relations, which can be found in anisotropic mediums. So, here you can see that the circular one is shown for air. This elliptical one is for some anisotropic material, right? Finally, in the third figure, you can see something. That is called hyperbolic dispersion, where the dispersion diagram looks like a hyperbola.

So, in the k-space, which is basically marking your kx and kz, you can see. these are the you know dispersion curves. Now, these contours are also special contours okay. So, here is what happens: these are called isofrequency contours, okay? Or you can also call them equal frequency contours, which are basically a graphical representation in the. Momentum space, or k-space, shows all the possible wave vectors corresponding to a particular frequency or energy; it is basically a critical tool that allows you to understand.

The wave behavior, particularly in periodic media like photonic crystals and solids. So, the shape of the isofrequency contour basically reveals whether the material is isotropic or anisotropic. It basically tells you about the wave propagation direction and the potential of some. Exotic phenomena like negative refraction and others, right? So, you can see that these isofrequency contours are basically plotted in momentum space or k space. So, each point is basically giving you a value of k that is a wave vector, okay? In a particular contour, all the wave vectors have the same frequency; that's why they are called isofrequency.

So, typically they are used in photonic crystals where this kind of isofrequency is known. Contours help us analyze how light is going to propagate through this structure. So, if you see a circular contour that basically tells you about isotropic propagation, okay, and If you see something like a square or a star shape, it is kind of an isofrequency contour that will Basically, it gives you anisotropic behavior and some new properties, like self-collimation or the super prism effect, okay. You can also see negative refraction from the isofrequency contour, okay? So, you can use that to see whether the light wave is bending in the other direction or not. So, those things you can see in more detail for the isofrequency contour in the case of photonic crystals, right? So, what is more interesting about why we are showing this isofrequency contour in the case of a hyperbola is here.

A hyperbolic metamaterial is that here you see they can extend up to very large values of the wave vector is okay, and the pointing vector is basically normal to the wave vector, right? And this is basically for reference; it shows you what the wave vector is in an air medium, right? Now, here we are plotting it for a natural material that is isotropic. So, in this case, we consider epsilon parallel and perpendicular to be equal, and both are positive.

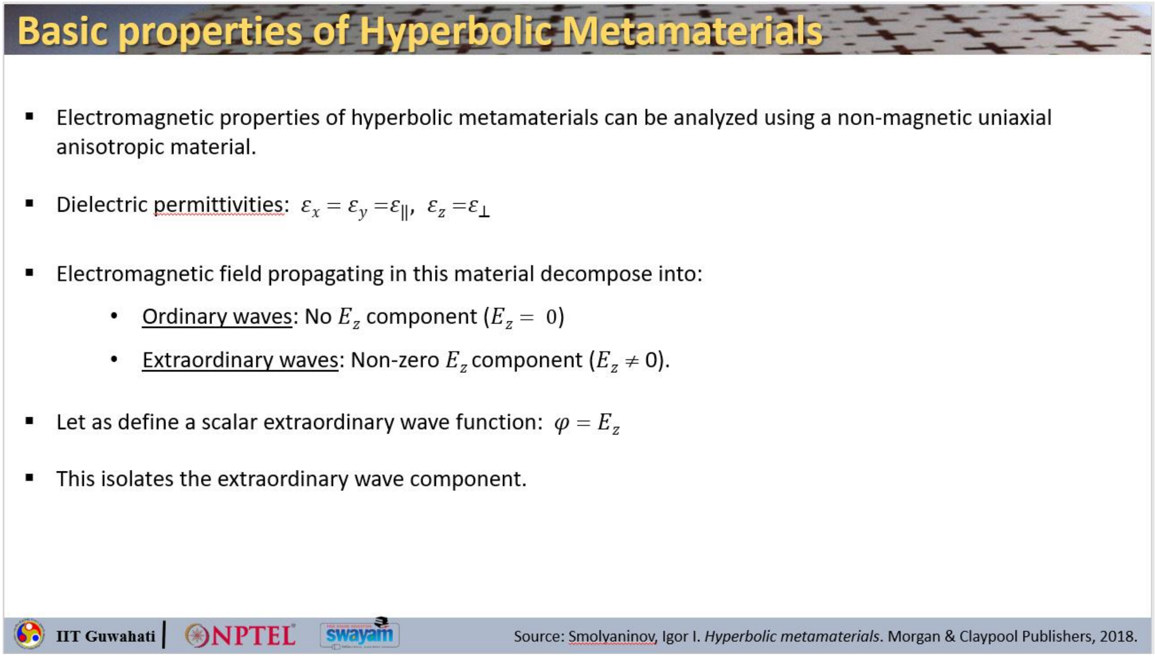


So, in that case, the k-space diagram looks like a spherical shape in 3D. You can see here in the previous case that we are basically taking the cross section of the 2D in the XZ plane, and that is why we were seeing that circle.

So, it is basically the XZ plane. In 3D, it looks like this. Now, if you have a medium that is anisotropic, that means you know the values are both positive. But yet they are not equal. So, in that case, you will get an ellipsoid kind of isofrequency.

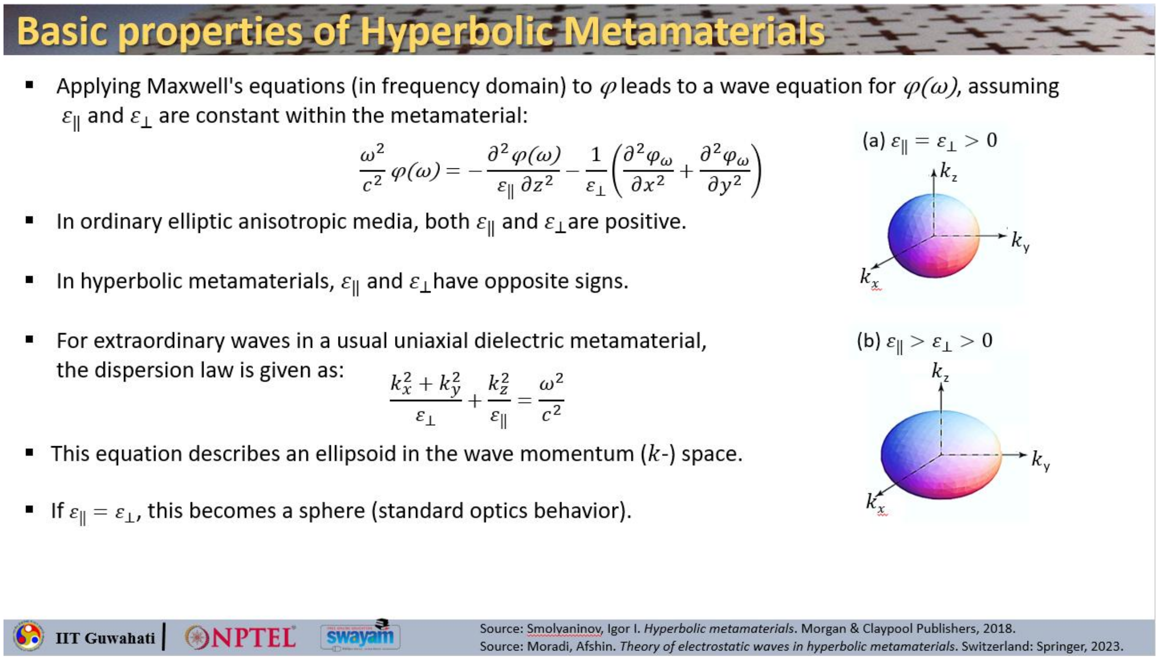
Or you can say this is the dispersion relation that gives you this kind of ellipsoid, okay? So, you can see in the k in the xz that you can get this kind of ellipsoid isofrequency contour. So, this basically tells you about the anisotropy in the medium. Now, these two figures, C and D, are basically telling you about the hyperbolic dispersion relation. Because here you see the parallel and perpendicular components of the permittivity. They are unequal, and they are also of opposite signs.

So, if perpendicular epsilon perpendicular is positive. You get this kind of dispersion diagram, and if the parallel part is positive, the orthogonal part is negative; you get this as your dispersion diagram right in k space.



So, you can understand the electromagnetic properties of this kind of hyperbolic material. can be analyzed using non-magnetic uniaxial anisotropic material, and until now we have seen that the dielectric permittivities typically along the x and y directions are considered to be epsilon parallel and Along the z-direction, we consider epsilon to be perpendicular, right? So, this is what you can also apply here. In the case of, say, some multilayer hyperbolic metamaterial, you can use this kind of convention, right? So, what will happen in this kind of material when it interacts with an electromagnetic field? You will see that you know the electromagnetic field can be decomposed into ordinary waves and extraordinary waves.

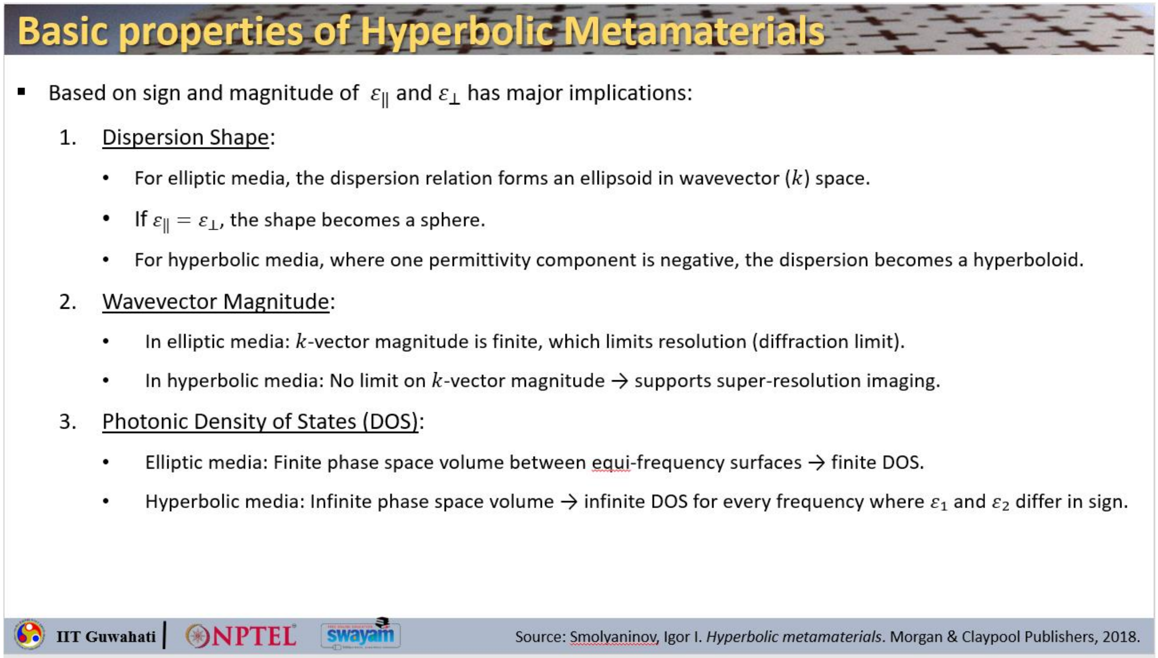
So, you will see that in one case you will find no Ez component, and in the other case you will find a nonzero Ez component, right? So, let us define a scalar extraordinary wave function, which is $\varphi = E_z$, and this basically isolates the extraordinary wave component.



Now, after applying Maxwell's equation in the frequency domain, you will see that this phi basically leads to a wave function for phi omega, assuming that you know the parallel and perpendicular components of the permittivity. Or you can say that parallel and perpendicular permittivity are basically constant within the material. So, you can basically think of this as the isotropic case, and this is the anisotropic case; you can consider this one, okay. So, you can write $\frac{\omega^2}{c^2}\varphi(\omega) = -\frac{\partial^2\varphi(\omega)}{\varepsilon_{\parallel}\,\partial z^2} - \frac{1}{\varepsilon_{\perp}}\left(\frac{\partial^2\varphi_{(\omega)}}{\partial x^2} + \frac{\partial^2\varphi_{(\omega)}}{\partial y^2}\right)$.

Now, in the case of an ordinary elliptical anisotropic medium, you will see that this is the case where both epsilon parallel and epsilon perpendicular is basically positive. But in the case of hyperbolic metamaterials, these two will have opposite signs. So, for an extraordinary wave in an unusual uniaxial dielectric material, the dispersion law can be simply written as $\frac{k_x^2+k_y^2}{\varepsilon_{\perp}} + \frac{k_z^2}{\varepsilon_{\parallel}} = \frac{\omega^2}{c^2}$. So, this basically defines the ellipsoid in momentum space or k space.

Now, if you put in this equation, epsilon perpendicular equals epsilon parallel, you will basically get this as a sphere. and that is the case in an isotropic medium.



Now, based on the sign and amplitude of this epsilon perpendicular and parallel, sorry, parallel and perpendicular, you can have some major differences. The first thing is the dispersion shape. For elliptic media, the dispersion relation basically forms an ellipsoid in the k space.

Now, if you consider the two components epsilon parallel and perpendicular to be equal, this ellipsoid will become a sphere, which means it becomes an isotropic medium. Now, in the case of hyperbolic media, one component is negative. So, in that case, the dispersion Becomes hyperbolic, okay. So, what happens to the web vector? In elliptical media, you will see that. The k vector magnitude is basically finite, which limits the resolution and brings about the diffraction limit.

But in the case of hyperbolic media, because these are hyperbolas, there are No limit on the k vector magnitude means k can be very, very large. Lambda can be very, very small; that means it can support super-resolution imaging. Now, in the aspect of the photonic density of states, we have already briefly discussed. In the case of elliptical media, it has a finite phase space volume between equi-frequency surfaces. So, you have a finite density of states, but in the case of hyperbolic media, you have infinite space.

Volume because of the hyperbola. So, you have an infinite density of states for each frequency. where you know the two permittivities basically have different signs.

Basic properties of Hyperbolic Metamaterials

4. Robustness to Disorder:
   - Even with disorder that modifies ε-values, the hyperbolic shape persists, preserving infinite DOS.
   - This is a core reason for the strong performance of hyperbolic metamaterials.

5. Validity of Effective Medium Approximation:
   - EMA breaks down when wavelength of the propagating mode $\approx$ unit cell size (a).
   - Introduces a natural wavevector cut-off: $k_{max} = 1/a$

- Unit cell size (a) in metamaterials varies by design method (usually a $\sim$ in order of nm).
- The hyperbolic enhancement factor in optical density of states (DOS) scales as: $\rho(\omega) = \rho_0(\omega)\left(\frac{k_{\max}}{\omega/c}\right)^3$

  where: $\rho_0(\omega) \sim \omega^2$ is the DOS in free space and $k_{\max} = 1/a$ is the wavevector cut-off due to finite unit cell size.
- Even with this cut-off, the hyperbolic singularity yields density of states enhancement factor of 10^3 to 10^5.

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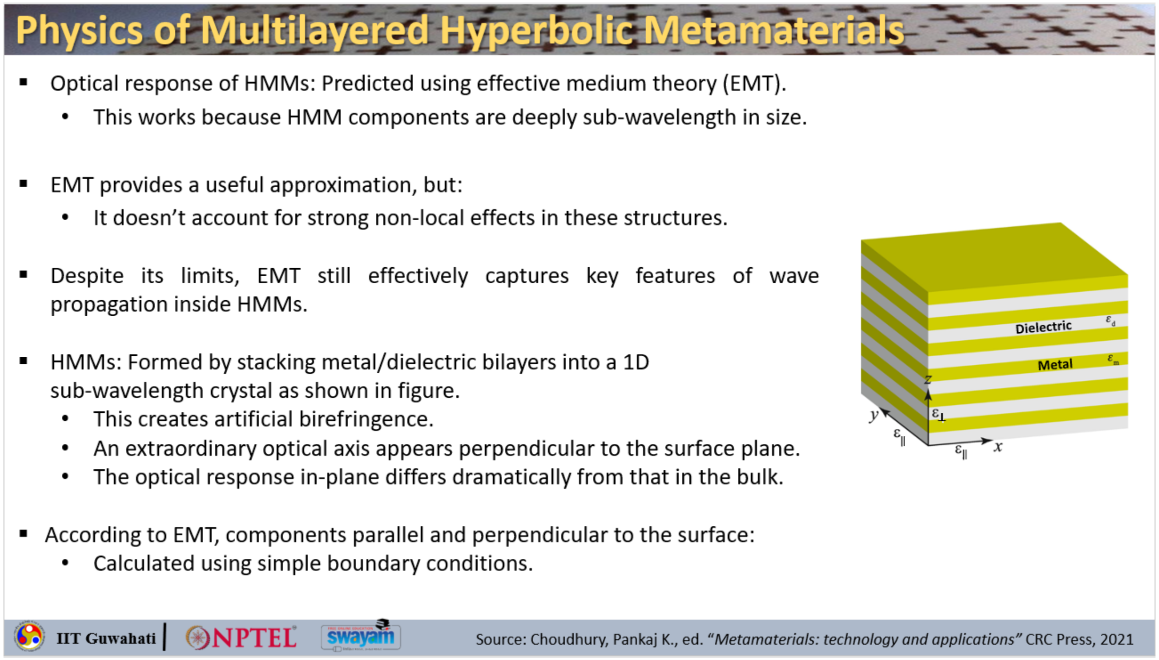
Now, how are they robust to disorder? So, even with disorder that modifies the permittivity values, you will see that the hyperbolic shape. still persists, which means it can still preserve the infinite density of states, and this is one of the main reasons why the performance of these hyperbolic metamaterials is very strong and robust. You can also use effective medium theory to describe this hyperbolic metamaterial.

However, there is a range of validity; thus, we will see that the effective medium approximation basically breaks down. When the wavelength of the propagating mode becomes comparable to the size of the unit cell, okay. So, you can actually introduce a natural wave vector cutoff; you can consider k max to be 1 by a. So, the unit cell size a in metamaterials typically varies with the design; normally, we will see that a is on the order of nanometers. So, the hyperbolic enhancement factor in the optical density of states will scale as $\rho(\omega) = \rho_0(\omega)\left(\frac{k_{\max}}{\omega/c}\right)^3$.

So, you will see that this term rho naught omega is basically proportional to or is almost equal to omega squared, which is the density of states in free space, and then you have k max, which is basically 1 by a, that is the wave vector cutoff due to the finite unit cell size. From that, you can obtain what the typical optical density of states is. So, what do you see is that even with this particular cutoff in place, the hyperbolic singularity yields a density of states enhancement factor of around because there is a cubic factor over here. So, it is 10 to the power of 3 to 10 to the power of 5. So, it is pretty large, right?

So, with that, let us now look into the physics of multilayered hyperbolic metamaterials.



So, the first thing is that the optical response of a hyperbolic metamaterial can be predicted using effective medium theory. And this works because the hyperbolic metamaterial components are typically deeply subwavelength in size. So, effective medium theory can provide you with very useful information. However, it does not account for the strong non-local effects that take place within the structures. However, within these kinds of limitations, you will also see that effective medium theory can Capture the key features of wave propagation inside hyperbolic metamaterials.

So, here is a picture of a typical hyperbolic metamaterial that is formed by the stacking of Metal dielectric layers, okay in a 1D subwavelength crystal. So, the yellow one shows metal, the lighter color shows dielectric, and then you are basically repeating. So, you have ε_d and ε_m as the permittivities. So, this is the x, y, and z directions.

So, x and y are the parallel directions. So, you have epsilon parallel, and z is the perpendicular direction considered as epsilon perpendicular. So, obviously, this crystal brings artificial birefringence, okay, and you can see the extraordinary. The optical axis is basically perpendicular to the surface plane; it is along the z-axis. The optical response in the plane will basically differ from that in the bulk, according to the effective medium theory. Okay, you can calculate the components epsilon parallel and epsilon perpendicular using simple boundary conditions, right? So, let's see how it's done.

Physics of Multilayered Hyperbolic Metamaterials

- Consider an electric field polarized along the HMM surface plane:
  - The field is continuous at metal/dielectric interfaces:

$$E_{\mathrm{m}} = E_{\mathrm{d}} = E_{\mathrm{eff}} \qquad \text{..........(1)}$$

- The effective electric flux is sum of flux densities in metal and dielectric:

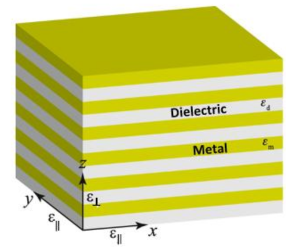
$$D_{\mathrm{eff},\|} = \tilde{\varepsilon}_{\|} E_{\mathrm{eff}} = f_{\mathrm{m}} D_{\mathrm{m}} + f_{\mathrm{d}} D_{\mathrm{d}} \qquad \text{..........(2)}$$

where f is the fill-fraction of individual layers.

- Also; $f_j = t_j / t_{\mathrm{tot}}$,

  where t_j: thickness of the j^{th} material, t_{tot} : total thickness of the unit cell.

- Substituting Eq. (1) into Eq. (2) to obtain $\tilde{\varepsilon}_{\|}$: $\tilde{\varepsilon}_{\|} = \frac{\tilde{\varepsilon}_{\mathrm{d}} t_{\mathrm{d}} + \tilde{\varepsilon}_{\mathrm{m}} t_{\mathrm{m}}}{t_{\mathrm{d}} + t_{\mathrm{m}}}$



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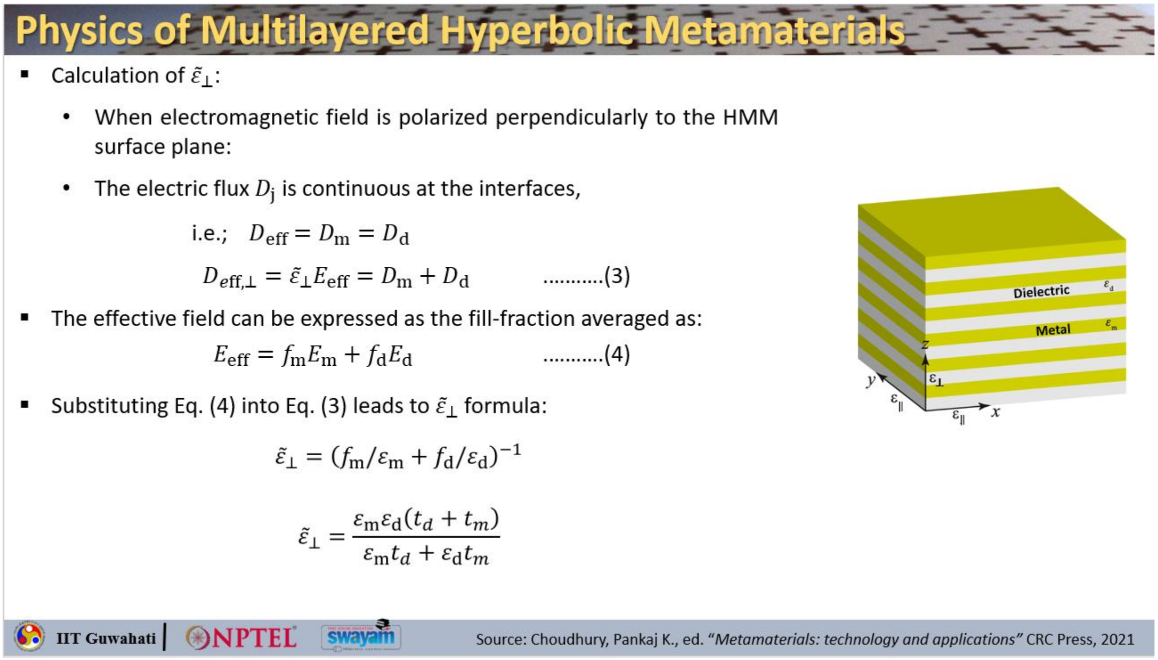
Source: Choudhury, Pankaj K., ed. *"Metamaterials: technology and applications"* CRC Press, 2021

So, let us first consider an electric field which is polarized along the surface plane, okay. So, the wave is falling from the top, and the electric field is polarized along the surface plane. So, in this case, the field is continuous at the metal-dielectric interface. So, you can write $E_{\mathrm{m}} = E_{\mathrm{d}} = E_{\mathrm{eff}}$. So, m and d are basically metal and dielectric, and effective stands for, you know, F stands for effective here.

The field is continuous. So, what can you do? The effective you can see what the effective electric flux will be that is basically. The sum of the flux densities in the metal and the dielectric, so you can write. The effective parallel will be epsilon effective right, and that can be written as $f_{\mathrm{m}} D_{\mathrm{m}} + f_{\mathrm{d}} D_{\mathrm{d}}$. So, fm and fd are basically the volume fractions of the metal and the dielectric. So, obviously, if you calculate one, the other one will be, so fd will be 1 minus fm, like very simple.

But typically, it is given like this: that fi or $f_j = t_j / t_{\mathrm{tot}}$. So, Tj is basically the thickness of the jth material, and Ttot is the total thickness of the unit cell. So, here the unit cell is basically one metal layer plus one dielectric layer thickness; that is the unit cell. So, now you substitute this equation into equation 2, and you can see how this looks. So, you are basically having epsilon parallel ($\tilde{\varepsilon}_{\|}$), which is given as $\tilde{\varepsilon}_{\|} = \frac{\tilde{\varepsilon}_{\mathrm{d}} t_{\mathrm{d}} + \tilde{\varepsilon}_{\mathrm{m}} t_{\mathrm{m}}}{t_{\mathrm{d}} + t_{\mathrm{m}}}$, right. So, that is how you can obtain the parallel permittivity.

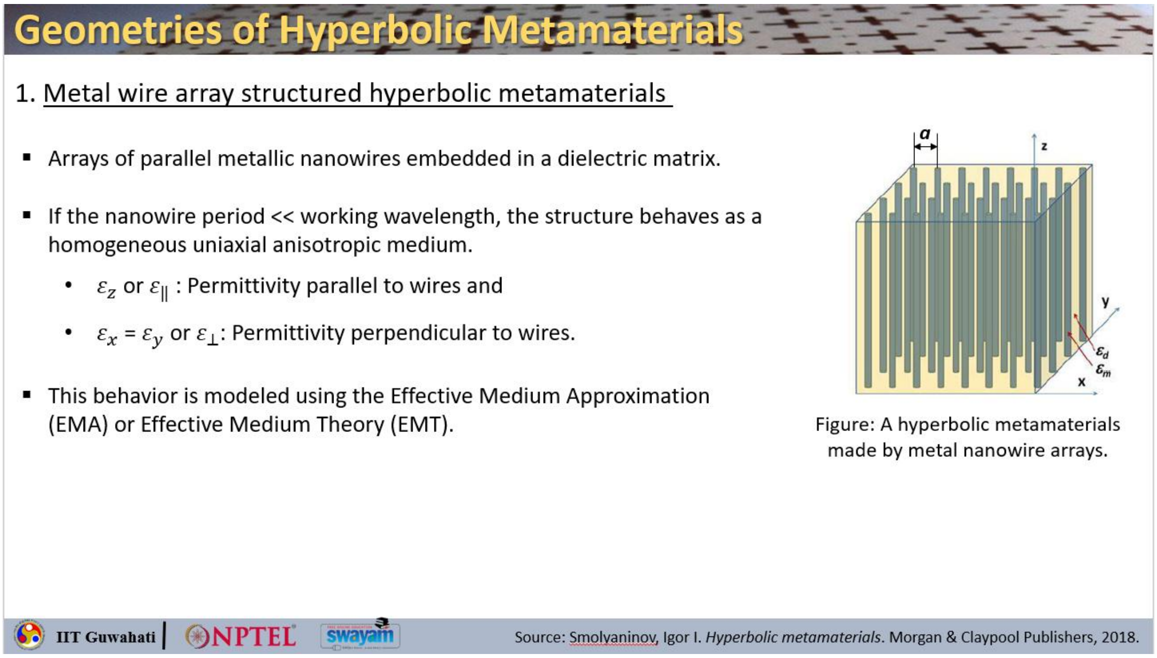


Now, let us look into how we can calculate this epsilon perpendicular.

So, in this case, we have to consider the electromagnetic field that is polarized perpendicularly to the HMM surface plane that is the electric field is along this direction, to be honest, okay. And in this case, the electric flux Dj is considered to be continuous across the interface. In the previous case, the field was continuous; here, the electric flux is continuous.

So, you have D effective, which can be written as Dm equals Dd. And from that, you can write that D effective perpendicular $\left(D_{eff,\perp}\right)$ will be equal to $\tilde{\varepsilon}_\perp E_\mathrm{eff} = D_\mathrm{m} + D_\mathrm{d}$, right? So, the effective field in this case can be expressed as a volume fraction of the averaged value. The E_eff will be $f_\mathrm{m} E_\mathrm{m} + f_\mathrm{d} E_\mathrm{d}$. So, once you substitute this into this equation, you can obtain the formula for epsilon perpendicular.

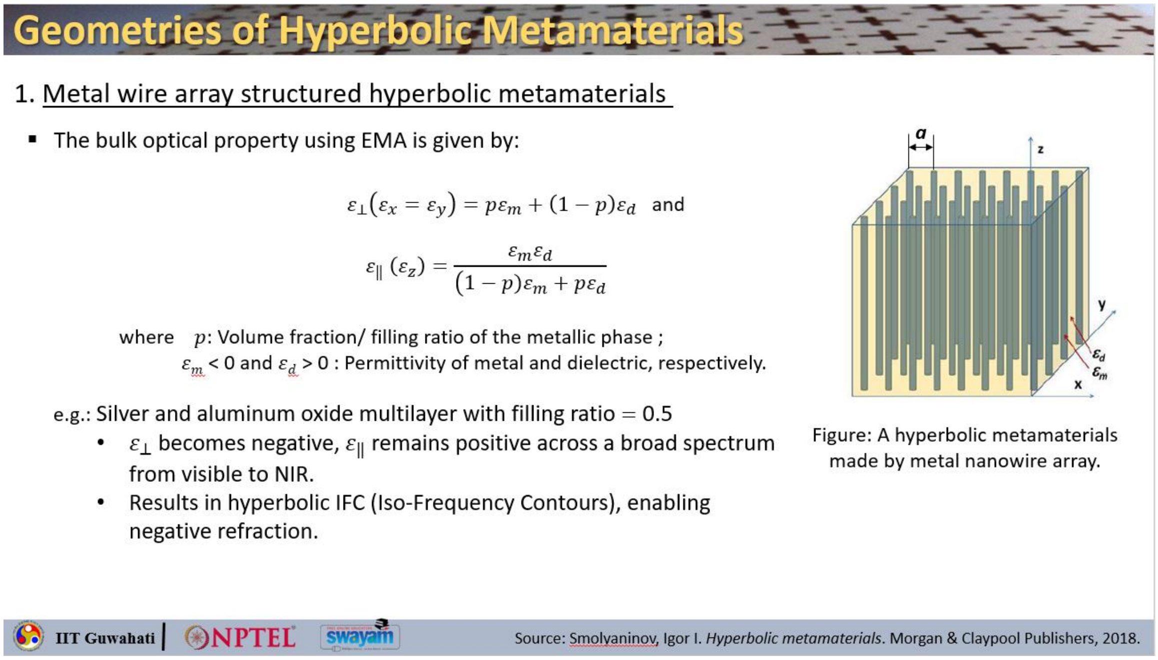
And that turns out to be $\tilde{\varepsilon}_\perp$ should be equal to $(f_\mathrm{m}/\varepsilon_\mathrm{m} + f_\mathrm{d}/\varepsilon_\mathrm{d})^{-1}$. So, once you do this calculation, you will get $\tilde{\varepsilon}_\perp = \frac{\varepsilon_\mathrm{m}\varepsilon_\mathrm{d}(t_d+t_m)}{\varepsilon_\mathrm{m} t_d+\varepsilon_\mathrm{d} t_m}$. So, that is how you can obtain the perpendicular polarizability. So, now we know how to calculate epsilon perpendicular and epsilon parallel for this kind of material.



With that, let us now look into two geometries of hyperbolic metamaterials and estimate these parameters, okay? Let us consider the first one, which is metal, where array-structured hyperbolic metamaterials are used. So, you can see that it is basically hyperbolic metamaterials made of metallic nanowire arrays. So, these are parallel metallic wires embedded in a dielectric matrix. You can see the parameters shown here; the periodicity is okay.

This is the z-axis; this is the x and y. So, metal has εm permittivity, and dielectric has εd, right? So, we consider that the period that is a is much smaller than the working wavelength. So, in that case, you can consider this structure a homogeneous uniaxial isotropic medium. So, you can consider epsilon z or epsilon parallel as the permittivity parallel to the wires. And then you can consider epsilon x and epsilon y, which will be equal to epsilon perpendicular.

because that is perpendicular to the wire's axis. So, this behavior you can model safely using the effective medium approximation because the period is much smaller than the wavelength, right? So, how do you do that?



You can use the formula like this. So, $\varepsilon_{\perp}$, which is $\varepsilon_x = \varepsilon_y$ will be $p\varepsilon_m + (1-p)\varepsilon_d$ and So, p is basically the volume fraction of the metal wire in this material and epsilon parallel. That will be equal to ε_z, which will simply be $\varepsilon_z = \frac{\varepsilon_m \varepsilon_d}{(1-p)\varepsilon_m + p\varepsilon_d}$.

So, this way you can very easily find out the perpendicular and parallel permittivities, right? So, you will see that epsilon m is here because it is metallic. So, epsilon m is basically negative, and epsilon d is positive, right? So, you can consider silver and aluminum oxide for this kind of system. So, you can consider p to be 0.5. So, in that case, you will see that epsilon perpendicular becomes negative and epsilon parallel.

which is along the wire that remains positive across a broad spectrum ranging from visible to NIR. So, that is basically giving you a hyperbolic isofrequency contour, and that can also enable negative refraction. So, negative refraction is referred to as such because at any point, if you take the normal at that location. Hyperbola that will basically give you the direction of the energy flow or the pointing vector. So, you can do that and see that it basically goes in the opposite direction as from the conventional material.

Geometries of Hyperbolic Metamaterials

1. Metal wire array structured hyperbolic metamaterials

- Nanowire array method supports negative refraction due to hyperbolic dispersion with deeply curved IFCs (Iso-Frequency Contours).
- This method enables Poynting vector to deviate negatively—valuable for applications.
- Limitation of Nanowire array method : Challenging fabrication restricts practical use.
- Alternative method: Metal-dielectric multilayer stacking offers a more practical solution.

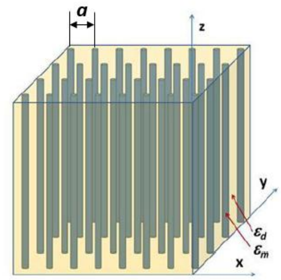


Figure: A hyperbolic metamaterials made by metal nanowire array.

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So, you will get basically negative refraction. So, this kind of nanowire method that supports negative refraction because of the hyperbolic dispersion coming from its deeply curved isofrequency contours. And that basically enables the pointing vector to deviate negatively, which means on the opposite side. And that is applicable to many applications. However, if you see the practicality of this kind.

The fabrication of a structure remains very, very challenging.

Geometries of Hyperbolic Metamaterials

2. Multilayer metal-dielectric stacked structure

- Follows the same principle and effective medium theory as nanowire arrays method.
- Material selection can be tuned to desired working wavelengths.
- Fabrication advantage: Multilayer method is easier to implement if layers are smooth and thin.
- Key requirement: Layers must be much thinner than the working wavelength for effective medium theory to hold.
- Overly thin layers may lose bulk properties, making fabrication precision essential.

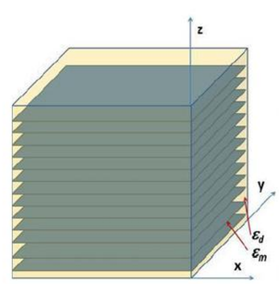

Figure: A hyperbolic metamaterials made by metal-dielectric multilayer.

IIT Guwahati | NPTEL swayam Source: Smolyaninov, Igor I. *Hyperbolic metamaterials*. Morgan & Claypool Publishers, 2018.

And that is why people have opted for a much simpler design, which is a metal dielectric multilayer stack that we have just seen in the earlier figure. That offers a much more practical solution. So, here is a picture of how this multilayer stack looks. So, it works on the same principle, and you can use effective medium theory as long as this.

Periodicity is much smaller than the wavelength, and the material selection can be tuned to the desired specifications. You know the operating wavelength. So, you can actually use silver plus aluminum oxide, which is Typically suitable for ultraviolet, visible, and infrared. You will see different types of other materials like zinc oxide and indium tin oxide, people are also using it. Here, the fabrication is easier. Because multilayer fabrication will be easier to implement if the layers are smooth and thin. One thing to remember is that the layers must be very thin compared to the wavelength. So that you know, you can use the effective medium theory to understand its behavior.

So, what do we require? We require you to know thin layers. But then if you make them too thin, they may, you know, lose their bulk properties, and that is why. You know that maintaining precision in fabrication is very important.

Applications of Hyperbolic Metamaterials

- Key Phenomena and Applications of hyperbolic Metamaterials :
  - Enable super-resolution imaging.
  - Exhibit enhanced quantum-electrodynamic effects.
  - Potential for new stealth technologies.
  - Demonstrate thermal hyperconductivity.
  - Contribute to research on high-temperature superconductivity (high T_c).
  - Offer insights into gravitational theory analogues.

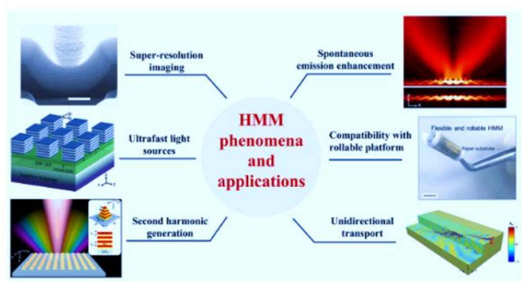

Figure: Phenomena and applications of hyperbolic Metamaterials

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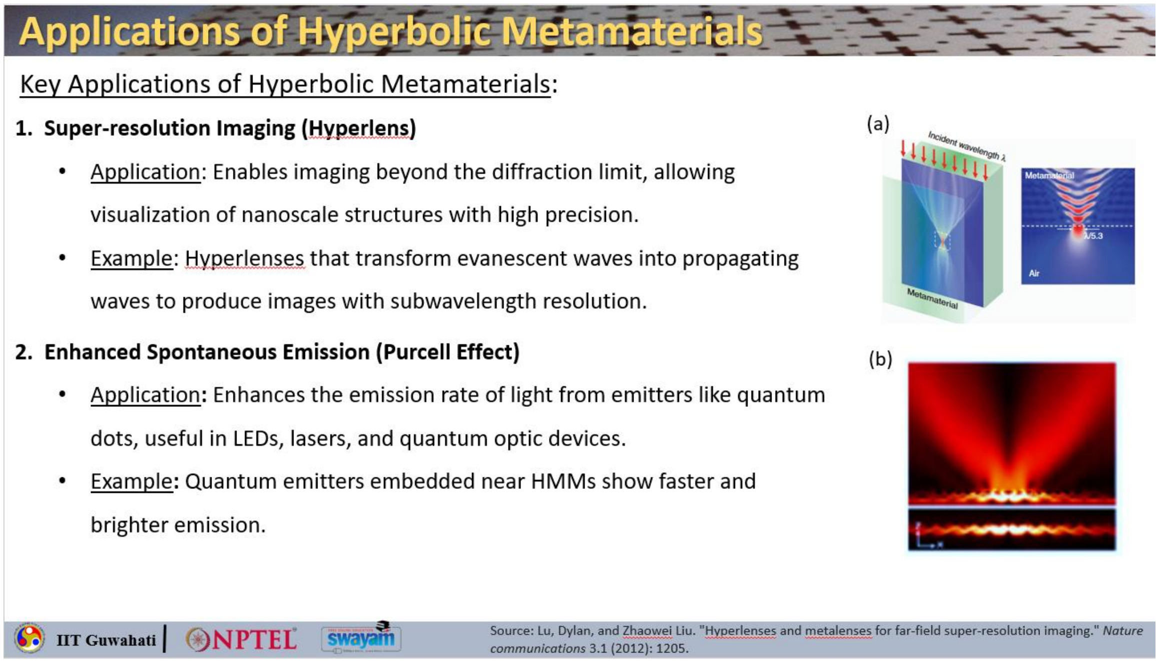
Source: Smolyaninov, Igor I. *Hyperbolic metamaterials*. Morgan & Claypool Publishers, 2018.
Li, Zhitong, and Qing Gu. "Topological hyperbolic metamaterials." *Nanophotonics* 13.6 (2024): 825-839.

So, here are some key applications of hyperbolic metamaterials. The first one is that it allows breaking.

the diffraction limit of light and allows your super resolution imaging. you can use them for enhanced quantum electrodynamic effect, you can use them for stealth application, they can be used for thermal hyperconductivity. It can also contribute to research on high. Temperature superconductivity is okay. Other applications could be, like, you know, spontaneous emission enhancement.

Ultrafast light sources, you can see hyperbolic metamaterials over here. You can also use them for unidirectional transport or second-harmonic generation.

So, here are some key applications for which hyperbolic metamaterials are popular.



The first one is super-resolution imaging or hyperlens. So, the application enables imaging beyond the diffraction limit, allowing visualization of nanoscale features with high precision. So, you can see that you know hyperlens can basically transform the Evanescent waves are converted into propagating waves to produce images with sub-wavelength resolution. Here you can see that, and that is because the hyperbolic metamaterials can support large wave vector modes, okay. You will also see that they support enhanced spontaneous emission, which is the Purcell effect. So, they can increase the local photonic density of states, and that will allow you to enhance. The emission rate of light from emitters such as quantum dots is useful in LEDs and lasers.

Other quantum dot devices, quantum optic devices, right? So that way you know quantum emitters. The hyperbolic metamaterials that are embedded show faster and brighter emission, okay.

Applications of Hyperbolic Metamaterials

3. Ultrafast Light Sources

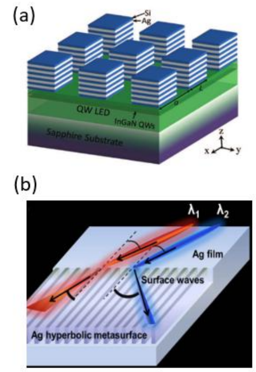
- Enables the creation of high-speed light-emitting devices, essential for fast communication and computing systems.

4. Negative Refraction and Flat Lenses

- Application: Used in flat lenses that focus light without curvature, improving compact imaging systems.
- Example: Veselago's lens concept implemented with HMMs.

5. Thermal Emission and Radiative Heat Transfer

- Application: Enhanced thermal management in electronics, thermophotovoltaics, and near-field heat transfer.
- Example: Subwavelength heat channels using hyperbolic phonon polaritons.



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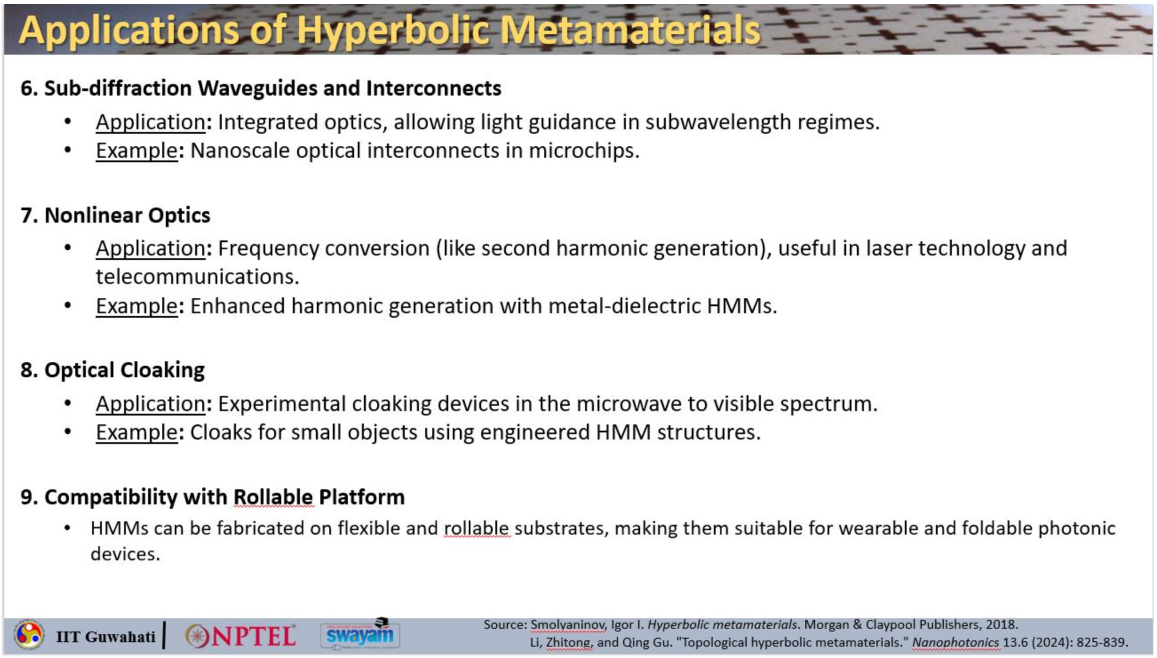
Source: Takayama et al. "Optics with hyperbolic materials." *Journal of the Optical Society of America B* 36.8 (2019)
Source: https://www.zhaowei.us/publications.html

You can also develop ultra-fast light sources, which enable the creation of high-speed light Emitting devices are essential for fast communication and computing, so you can see the figure here.

Where on top of the quantum well LED, you have this kind of structure made of silicon. silver ok. So, these are those hyperbolic metamaterials being created. You can also think of negative refraction and flat lenses; these are used in flat lenses to focus light. Without curvature improving compact imaging applications, one example could be, you know, Conceptual realization of Veselago's lens with HMM. So, here in the figure, you can see that Lambda 1 wavelength just passes through, but lambda 2 wavelength is basically reflected.

So, that is like, you know, you can see it bends light in an unusual way, showing the negative refraction. You can also see applications where HMMs can tailor the spectral and spatial characteristics of thermal radiation. So, they can be used for thermal management in electronics, thermophotovoltaics, and also near-field heat transfer. So, these are some of the important applications where people use hyperbolic metamaterials.



The last one is that you do not know the couple of other applications you can mention. Sub-wavelength sub-diffraction waveguides and interconnects have typical applications in integrated optics. Where it allows light guidance into the sub-wavelength regime again because of the fact that it can support high-k modes. So, you can also use them for nanoscale optical interconnects in microchips. Other applications include non-linear optics, where you can do frequency conversion, like second harmonic generation. Which are very useful in laser technology and telecommunications and can happen in metal dielectric hyperbolic metamaterials.

You can also use this kind of metamaterials to bend and guide light that can render objects. Invisible or make it less visible; that means you can use cloaking devices in the microwave to the visible spectrum. So, you can make cloaks for small objects using this kind of engineered hyperbolic metamaterial. And there is compatibility for this kind of material to be fabricated on flexible and rollable substrates. That makes them suitable for wearable and foldable photonic devices.

So, with that, we come to an end to this lecture. We will continue the discussion of hyperbolic metamaterials in the next lecture. If you have any queries, you can drop an email to this particular email address. Thank you!