Course Name: Introduction to Microwave and Optical Metamaterials
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Week-8
Lecture-37

Lec 37: Negative Index Metamaterials-I

Lecture Outline

- Reversed Phenomena in Negative-Index Media
  - Determining the Sign of the Refractive Index
  - Left-Handed Nature and Anti-Parallel Vectors
  - Modified Snell's Law
  - Examples of Other Reversed Phenomena

- Negative Refraction in Microwave Frequencies
  - Challenges with Naturally Occurring Negative-Index Materials
  - Overcoming Natural Limitations with Metamaterials
  - Early Experimental Demonstrations at Microwave Frequencies

Hello, everyone. Welcome to lecture 37 of the online course on the introduction to microwave and optical metamaterials. Today's lecture will be about negative index metamaterials. Here's the lecture outline. We'll first look into a couple of reversed phenomena. in negative index media.

We will discuss

Determining the Sign of the Refractive Index

The choice of sign for the refractive index n in a medium where both permittivity (ε) and permeability (μ) are negative is resolved by the causality condition.

- **The Ambiguity:** The refractive index is defined as $n = \pm\sqrt{\varepsilon\mu}$. When $\varepsilon = -1$ and $\mu = -1$, it is not immediately clear whether n should be +1 or -1 .

- **The Causality Condition:**
  - For any realistic, passive material, there must be some energy absorption, however small.
  - This means both ε and μ have a small positive imaginary part.
  - We can write them as $\varepsilon = -1 + i\delta_1$ and $\mu = -1 + i\delta_2$, where $\delta_1, \delta_2 > 0$.

- **Mathematical Resolution:** The refractive index becomes:

$$n = \pm\sqrt{(-1 + i\delta_1)(-1 + i\delta_2)} = \pm\sqrt{(1 - \delta_1\delta_2) - i(\delta_1 + \delta_2)}$$

Determining the Sign of the Refractive Index

This can be approximated as:

$$n \approx \pm \left[1 - \frac{i(\delta_1 + \delta_2)}{2}\right]$$

- **Conclusion:**
  - Causality requires that the imaginary part of n must be positive to represent wave attenuation. To satisfy this, the minus sign must be chosen, resulting in $n \approx -1 + i\left(\frac{\delta_1+\delta_2}{2}\right)$.
  - Therefore, when the real parts of ε and μ are negative, the real part of n must also be negative.

determining the sign of the refractive index, left-handed nature, and antiparallel vectors. We will discuss Motivate-Snell's law and take up examples of other reversed phenomena. And then we will look into negative refraction at microwave frequencies. What are the challenges posed by the naturally occurring negative index material and how can you overcome the natural limitations with metamaterials? And at last, we will see some early experimental demonstrations at microwave frequencies.

So, first of all, what would be the sign of the refractive index? If you have a medium where both permittivity ε and permeability μ are negative. So, the choice of the sign can be resolved by the causality conditions. So, how is the refractive index defined? Refractive index n is defined as + or - the square root of ε μ. Now, when ε = - 1 and μ = - 1, it is not immediately clear whether n should be + 1 or - 1. And that is where the ambiguity lies.

Now, there you can bring in the causality condition. It says that for any realistic passive material, there must be some energy absorption, however small it may be. That means that for both ε and μ, there should be a small positive imaginary part. In that case, we should write ε as - 1 + i δ 1; δ 1 is very small, but it is positive. Similarly, μ should also be written as - 1 + i δ 2, where δ 2 is small but positive.

Now, mathematically when you resolve it, you can see that the refractive index will now take this particular form, where you have n which is given as + - square root of - 1 + i δ 1 * - 1 + i δ 2. So, that can be expanded and you can separate the real and the imaginary part and can write as + - square root of 1 - δ 1 δ 2 - i δ 1 + δ 2. Now, this can be approximated as n = + or - 1 - i δ 1 + δ 2 / 2. Now, what is the conclusion here you can see that causality will require that the imaginary part of the n must be positive and that will represent wave attenuation in that medium. So, in order to satisfy that you have to choose this outside - sign, so that you get a n = - 1 + i is small you know positive imaginary part and that is how you actually get this - 1 sign when both ε and μ are negative right.

So, that is the final thing: when the real parts of ε and μ are negative, the real part of n must also be negative. Now, let us look into the left-handed nature and the antiparallel vectors. So, in negative index materials or NIMs, the

Left-Handed Nature and Anti-Parallel Vectors

In Negative-Index Materials (NIMs), the relationship between the electric field ($\boldsymbol{E}$), magnetic field ($\boldsymbol{H}$), and the wave-vector ($\boldsymbol{k}$) is fundamentally different from that in conventional materials.

- **Phase Advancement:**
  - A negative refractive index implies that the phase velocity of a wave is directed against the flow of energy.
  - For a wave described by $e^{i(nkz-\omega t)}$, a negative n means the phase becomes advanced in the direction of propagation, rather than retarded.

- **Vector Relationship:** Maxwell's equations define the orientation of the fields:

$$\begin{cases} \mathbf{k} \times \mathbf{H} = -\omega\varepsilon_0\varepsilon\mathbf{E} \\ \mathbf{k} \times \mathbf{E} = \omega\mu_0\mu\mathbf{H} \end{cases}$$

relationship between the electric field E, magnetic field H and the wave factor K is fundamentally different from the conventional

Left-Handed Nature and Anti-Parallel Vectors

- **Right-Handed vs. Left-Handed:**
  - In a common material $(\varepsilon > 0, \mu > 0)$, the vectors $\mathbf{E}, \mathbf{H}$, and $\mathbf{k}$ form a right-handed system.
  - In a NIM $(\varepsilon < 0, \mu < 0)$, these vectors form a left-handed system.
    - This is why NIMs are often called left-handed materials.
  - Energy Flow: The most crucial consequence is that the wave-vector $\mathbf{k}$ is anti-parallel to the Poynting vector ($\mathbf{S} = \mathbf{E} \times \mathbf{H}^*$), which represents the direction of energy propagation.

materials. First, you can talk in terms of phase advancement. You will see that in a negative index medium, you will have the phase velocity of a wave that is directed against the flow of energy.

That means k and s will basically be in opposite directions. So, for a wave which is described by e to the power i nkz - ω t, if you have negative n that is basically telling you that the phase becomes advanced in the direction of propagation rather than getting retarded. So, that is. You know the feature of phase advancement in negative index materials. Next is the vector relationships.

Now Maxwell's equations define the orientations of the fields in this way. So, you have seen this earlier as well k cross h will be - ω ε naught ε e and k cross e will be equal to ω μ naught μ into h ok. Now, that is where the comparison between the right hand and the left-handed media will come. We discussed this earlier in the previous lecture. So, in the common material where you have both permittivity and permeability positive, you will see that the vectors E, H and K form a right handed system where the x's are mutually orthogonal right.

In the case of a negative index material where both ε and μ are negative, these vectors form a left-handed system. And that is why negative index materials are also called left-handed materials. Another important aspect is the flow of energy. The most crucial consequence is that the wave vector k is going to be antiparallel to the pointing vector s which is calculated as E cross H conjugate, ok. That basically represents the direction of energy propagation.

So, your wave vector is going to be the opposite of that. So, that also gives you some interesting effects like modified Snell's law. So, this is one of the most

Modified Snell's Law

The most intuitive result of a negative refractive index is how it changes the law of refraction.

A schematic of this scenario with negative refraction is depicted in Fig. 1.

- **Negative Refraction Angle:**
  - According to Snell's Law, $n_1 \sin\theta_i = n_2 \sin\theta_t$.
  - When light passes from a positive-index medium ($n_1 > 0$) to a NIM ($n_2 < 0$), the angle of refraction θ_t must be negative.
- **Bending on the Same Side:**
  - This means the refracted beam bends to the same side of the normal as the incident beam.

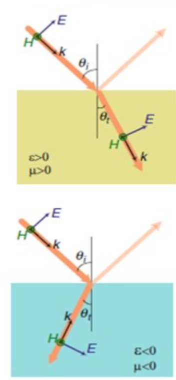

Figure 1: The refraction of a light beam when passed through the boundary of air and a PIM (*top*) or a NIM (*bottom*)

intuitive results of negative refractive index: how it is going to change the law of

Modified Snell's Law

- **Physical Justification:**
  - This behavior is required by the law of momentum conservation, which dictates that the tangential component of the wave-vector $\boldsymbol{k}$ must be continuous across the interface.
  - Since the energy ray (Poynting vector) in the NIM is directed opposite to its wave-vector $\boldsymbol{k}$, this unusual bending is the only possible outcome.

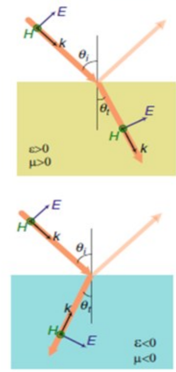

Figure 1: The refraction of a light beam when passed through the boundary of air and a PIM (*left*) or a NIM (*right*)

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refraction. So, in a normal situation, when you have both materials or conventional materials, the permittivity and permeability are positive, okay? You have this sort of refraction, okay? So, you have the incident angles θ i and θ t on the opposite side of the normal, right? But in the case of you know negative refraction angle you will see that you know if N_2 is negative. In that case, you will see that θ t is basically - θ t, okay.

That means to satisfy this equation, because this is negative, the angle also has to be negative. That means the angle of the reflected light, instead of going in this direction or like this, will basically be on the same side of the normal. So, this happens when your light is going to pass from a positive index medium that is say n_1 greater than 0 to a negative index medium where you have n_2 which is negative. So, in that case, the angle of refraction θ t must also be negative. So, that is making the beam or the refracted beam to band on the same side of the normal as of the incident okay so what is the physical justification of this kind of behavior So, this behavior is basically required by the law of momentum conservation which basically dictates that you know the tangential component of the wave factor must be a continuous across the interface.

So, if you do that here between the interface of a normal and a negative index material you will see that you know the k has to be in this particular direction to satisfy this condition. Now, since your energy ray, or the pointing vector in the negative index material, is basically pointing opposite to the wave factor. So, the energy flow is happening this way. So, your k-vector has to be in the opposite direction. So, this is completely unusual, and it is not seen in any regular material.

So, this is the way energy flows. So, in this material in the regular material you see the energy flow and the k is in the same direction, but inside the negative index material the energy flow is downward, but the k is upward. Now, let us look at examples of other reversed

Examples of Other Reversed Phenomena

The unique phase reversal in NIMs leads to the reversal of several other fundamental optical effects.

- **Doppler Effect:**
  - Definition: The Doppler effect is the change in a wave's frequency as perceived by a detector due to relative motion between the wave source and the detector.
  - Governing Formula: The perceived frequency (ω) is described by the equation $\omega = \omega_0 + \mathbf{k} \cdot \mathbf{v}$, where ω_0 is the true frequency, $\mathbf{k}$ is the wave-vector, and $\mathbf{v}$ is the source's velocity relative to the detector.

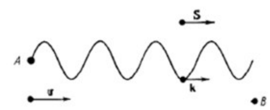

Figure 2: Doppler effect in a right-handed substance

The letter A represents the source of the radiation, the letter B the receiver.

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Examples of Other Reversed Phenomena

- **Normal Doppler Effect:**
  - In conventional materials, the wave-vector $\mathbf{k}$ points from the source to the detector.
  - When the source approaches, $\mathbf{k}$ and $\mathbf{v}$ are in the same direction, making the dot product positive and the perceived frequency higher.
  - When the source moves away, the dot product is negative, and the perceived frequency is lower.

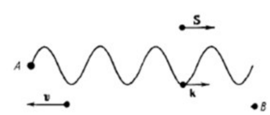

Figure 2: Doppler effect in a right-handed substance

The letter A represents the source of the radiation, the letter B the receiver.

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phenomena. So, this kind of you know unique phase reversal that you see in negative index materials can also lead to the reversal of several other

Examples of Other Reversed Phenomena

The unique phase reversal in NIMs leads to the reversal of several other fundamental optical effects.

- **Reversed Doppler Effect in NIMs:**
  - In negative-index materials, the wave-vector $\mathbf{k}$ is unique because it points from the detector to the source.
  - As a result, when the source approaches, $\mathbf{k}$ and $\mathbf{v}$ are in opposite directions (antiparallel).
  - This makes the dot product negative, causing the perceived frequency to be lower-the direct opposite of the normal effect.

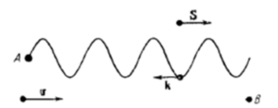

Figure 2: Doppler effect in a left-handed substance.

The letter A represents the source of the radiation, the letter B the receiver.

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fundamental optical The first one is Doppler effect. So, let us first see what the Doppler effect normally is.

Doppler effect is basically the change in a wave's frequency as perceived by the detector due to the relative motion between the wave source and the detector. So, here you can see this is a Doppler effect in the right-handed substance. So, A represents the source of the radiation, and B is your receiver. So, this is the flow of energy and this is also the wave vector and say the source is basically moving with the velocity v towards the detector. In that case the perceived frequency ω will be you know described by this equation where ω = ω naught + k dot v.

So, ω naught is basically the true frequency of the source, k is the wave factor that is along you know this direction and v is basically the source velocity with respect to the detector. Now, what happens in the normal Doppler effect? In conventional materials where you have both ε and μ as positive. your wave factor k points from you know the source to the detector and as the source approaches okay that means you are basically seeing that you know the k and the v are basically in the same direction. So, that will make the dot product of k and v positive, and that will increase the perceived frequency. On the other case, when you have the source moving away from the detector, that means the V is in the opposite direction, that particular dot product k dot V will become negative and that is why you will see the perceived frequency to be lower than the original or true frequency.

Now, what happens when you see the case of a left-handed medium? So, in the left-handed medium, A is again the source and B is the detector. So, the energy flow is happening from the source to the detector. So, this is the direction of S, which is the energy flow, but the wave is flowing in the other direction. So, K is in the backward direction. So, that gives rise to reverse Doppler effect in negative index media.

Because in this kind of negative index material, the k vector is unique, and it basically points from the detector to the source. So, when the source is basically approaching the detector that means you will have V going in this direction which is opposite to the direction of K that means they are antiparallel. So, in that case you will get a negative dot product and that will make your passive frequency lower than that as compared to the normal Doppler effect. So, here it basically works the other way when the source is coming towards the detector; okay, you will see that the frequency is basically reducing. Another interesting phenomenon could be the

Examples of Other Reversed Phenomena

- **Reversed Čerenkov Radiation:**
  - What is Cerenkov radiation?
    - This is radiation emitted when a charged particle travels faster than the phase velocity of light in a medium.
  - Slower than Light:
    - When the electron's speed (βc) is less than the speed of light in the medium (c/n), the electromagnetic waves it generates expand away from it in concentric spheres.
    - The particle never catches up to the waves it creates, so no shockwave is formed.

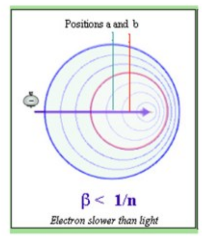

Figure 3: Schematic representation electron's speed (βc) is less than the speed of light

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reversed Charon Cobb

Examples of Other Reversed Phenomena

- **Reversed Čerenkov Radiation:**
  - Faster than Light:
    - When the electron travels faster than light in the medium, it continuously outpaces the spherical wavefronts it generates.
    - These individual wavefronts overlap and interfere constructively, forming a single, coherent wavefront in the shape of a cone.
    - This cone of light is the Čerenkov radiation.

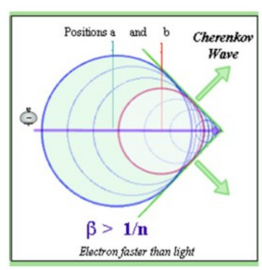

Figure 3: Schematic representation of Cerenkov radiation

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Examples of Other Reversed Phenomena

- **Reversed Čerenkov Radiation:**
  - The Cone Angle (Right Panel):
    - The angle of the cone (θ_c) is determined by the ratio of the two speeds.
    - In the time it takes the light to travel a distance of c/n, the electron travels a distance of βc.
    - The geometry of these overlapping waves forms a right-angled triangle, which gives the precise relationship: $\cos\theta_c = 1/(\beta n)$.
  - In a NIM, since n is negative, $\cos\varphi$ is also negative.
  - So, the angle φ must be obtuse ($> 90°$), and the Čerenkov radiation is emitted backward relative to the particle's direction of motion.

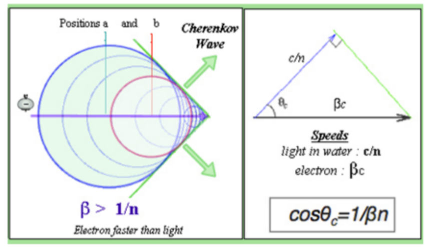

Figure 3: Schematic representation of Cerenkov radiation

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radiation.

So, to understand this, we need to first understand what Charon Cobb radiation is. So, this is radiation emitted when a charged particle travels faster than the phase velocity of light in a particular medium. So here let us consider a situation when you have an electron speed that is β c, which is less than the speed of light, okay. So when you consider the velocity slower than light, what happens? that is the case we will see that this is the electron, these are two positions A and B ok and this β is basically less than 1 by n. So, you have the electrons lower than light ok.

So, when that is the case ok, you will have the electromagnetic wave basically generating from this electron expands away from it in concentric spheres just like this ok. So, in this case what is happening the particle never catches up to the waves it creates and that is why no shock wave is basically formed. So, once again this is the situation when the electron speed β c is less than the speed of light in the medium that is c/n ok. So, the electromagnetic waves it generates basically expand away from it in this kind of concentric spheres. Now, what is the other case when the particle travels at a speed? Faster than light.

So, here we are considering the electron traveling faster than light in that medium. So, what is happening? It can continuously outpace the spherical waveform it is generating. So, in that case this individual wave fronts can overlap and interfere constructively forming a large or you can say forming a single coherent waveform which is in the form of a cone. So, this cone of light that you see is basically Cherenkov radiation. So, on the right-hand side, you can also see that you can determine the cone angle.

So, the angle of this cone is basically determined by the ratio of the two speeds. So, in the time it takes for light to travel a distance of c/n. The electron basically travels a distance of β c, okay. So, that is it. How this particular distance is, this is the electron distance; this is the distance of light in that medium.

So, you can basically see that the geometry of this overlapping waves basically form a right angled triangle, so like this, ok. So, you can actually get this particular angle, which is called θ c. Ok. And you can see that the relationship will be nothing but cos θ c will be equal to 1 by β n, this c and this c will cancel out ok. So, that is how you can obtain this particular cone, okay.

Now, what do you see in a negative index material? Because your n is negative, your cos φ will also be negative. That means you will have reversed a Cherenkov radiation right. So, the angle here you have to understand that because cos φ is becoming negative that means the angle φ must be obtuse that means greater than 90 degree and in that case what will happen the challenge of radiation is basically emitted backward relative to the particles direction of motion. So, this is completely unusual and the opposite of what happens in a conventional material. So, now let us discuss the

Challenges with Naturally Occurring Negative-Index Materials

- **The Core Problem:** No known natural material has a readily observed negative index of refraction.
- **Conditions for Negative Index:** A material would have a negative refractive index if both its electric permittivity (ε) and magnetic permeability (μ) were simultaneously negative at the same frequency.
- **Frequency Mismatch in Nature:**
  - Materials with negative ε (like noble metals) typically exhibit this property at high optical frequencies (terahertz and above).
  - Materials with negative μ (like some anti-ferromagnets) show this property, but their natural magnetic resonance fades away at the high frequencies where negative ε occurs.
  - Conclusion: This mismatch in frequency response prevents natural materials from having both $\varepsilon < 0$ and $\mu < 0$ at the same time.

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challenges that you see with naturally occurring negative-index materials.

The first problem is that no known natural material has a readily observed negative index of refraction. So, you do not find such material in nature where, for a particular frequency, it gives you simultaneously negative permittivity and negative permeability. So, what are the conditions for a negative index that we all know? that a material should have a negative refractive index if both its electric permittivity ε and magnetic permeability μ are simultaneously negative at the same frequency. Now, this is a problem because in nature you will see there is a frequency mismatch. By that, I mean that some materials exhibit negative permittivity, like noble metals, at high optical

Overcoming Natural Limitations with Metamaterials

- **The Critical Step:** To create a Negative-Index Material (NIM), the main challenge was to overcome the low-frequency limit of magnetic response found in conventional materials.
- **The Metamaterial Solution:** Researchers used artificially engineered structures, or "meta-atoms," to create strong magnetic coupling to electromagnetic fields at high frequencies.
- **Successful Demonstrations:** This approach has enabled the creation of NIMs across a wide portion of the electromagnetic spectrum, from microwave frequencies up to visible light.

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Early Experimental Demonstrations at Microwave Frequencies

- **Initial Focus:** The first experimental demonstrations of negative refraction (before 2005) were focused on microwave frequencies.
- **Key Building Blocks:**
  - Early NIMs were constructed by combining two main components:
  - A periodic array of thin wires to produce a negative effective permittivity ($\varepsilon < 0$).
  - A periodic array of Split-Ring Resonators (SRRs) to create a negative effective permeability ($\mu < 0$).

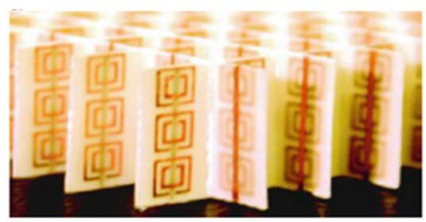

Figure 4: The NIM at microwave frequency built by the University of California - San Diego (UCSD) group.

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frequencies, such as terahertz and above.

However, they do not have, you know, negative permeability at that range. Similarly, there are materials with negative permeability, such as antiferromagnets; they have this kind of negative μ. natural magnetic resonance will fade away at high frequencies where typically the negative permittivity occurs or takes place. So, it is very difficult to have you know the frequency matching where you will have both negative permittivity and permeability at the same time in a particular material. So, what is the solution? The solution here is to overcome this kind of natural limitation with better materials.

So, the critical step here is to create a negative index material ok by overcoming this low frequency limit of the magnetic response that you find in conventional materials. So, you basically have to see magnetism at high frequency; only then will you be able to get this negative permeability at high frequencies. So, that is where metamaterial solutions are important. So, researchers basically used artificially engineered structures or their unit cells are called meta atoms, meta means artificial we have already discussed this earlier. So, they can create a strong magnetic coupling to electromagnetic fields at high frequencies.

So, this approach has basically enabled the creation of negative index materials across a wide portion of the electromagnetic spectrum. And when I say a wide portion, it basically ranges from macro frequencies up to visible light. Now, we will look into some early

Early Experimental Demonstrations at Microwave Frequencies

- **Pioneering Work:**
  - The famous two-dimensional NIM from the University of California - San Diego (UCSD), as shown in Fig. 4a, integrated these wires and SRRs into a periodic array.
  - This structure successfully verified a negative index of refraction by demonstrating refraction according to a modified Snell's law in Fig. 4b.

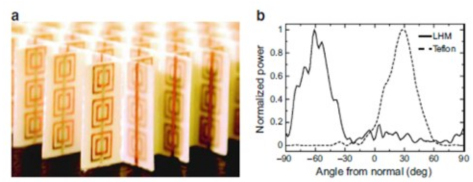

Figure 4: (a) The NIM at microwave frequency built by the University of California - San Diego (UCSD) group. (b) The experimental result verified the modified Snell's law.

- **Confirmation and Legacy:**
  - Later, a three-dimensional NIM cube using a similar design confirmed these results, removing any doubt about the feasibility of fabricating NIMs.
  - This wire-and-SRR combination became the standard prototype for most early research in the field.

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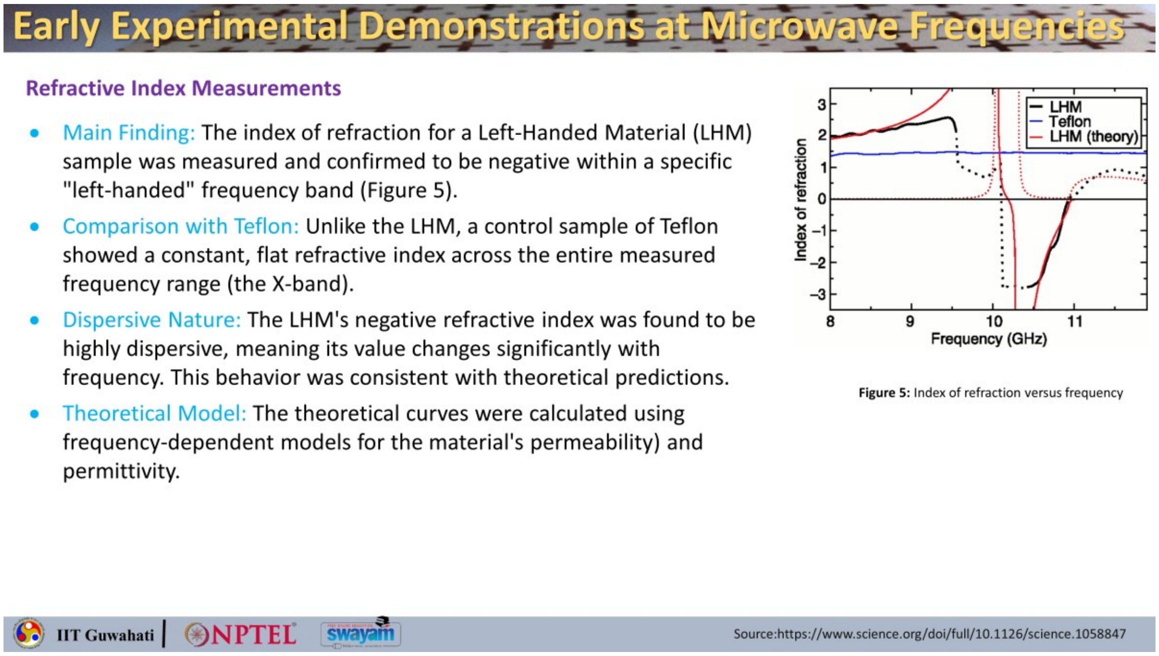

experimental demonstrations of negative index materials at microwave frequencies. So, the first experimental demonstration of this kind of negative refraction that was obtained before 2005 was mainly focused on microwave frequencies, right? So, what are the key building blocks? So, these early negative index materials were basically constructed by combining two main components. First, you would need to have a periodic array of thin wires that can produce a negative effective permittivity.

So, you are continuously getting ε negative. And then you need a periodic array of split-ring resonators that can create negative effective permeability. So, you are getting me also negative. So, here you are seeing the photograph of the first left-handed metamaterial sample that was built by the University of California, San Diego. So, this sample as you can see consists of square copper split ring resonators and also there is a copper wire strip both overlapped on a fiberglass circuit board material. So the rings, as you can see here, clearly the rings and the wires are on the opposite side of the boards.

So you are basically having a array of wires which were giving you negative permittivity and then you also have a array of splitting resonators that were giving you. effective negative permeability and these two things where you know the boards were cut and assembled into an interlocking lattice and that was giving negative permittivity. So, this is a famous work, and this is the first demonstration, as I mentioned, by this UCSD group. And this structure successfully verified a negative index of refraction by demonstrating refraction according to modified Snell's law.

And you can see here in Figure 4B. So, here they are plotting the transmitted power at 10.5 gigahertz as a function of the refraction angle. For both a Teflon sample, which is the dotted one, and also a left-hand material sample, which is the solid curve. Now these two curves are basically normalized such that you know the magnitude of both the peaks are matched at one. So here you can observe that for the Teflon sample, the refracted power peak was measured to be 27 degrees.

Somewhere here corresponds to the positive index of refraction, which is 1.4 + or - 0.1. But on the other hand, if you look at the left-hand material sample, the peak was recorded at - 61. Okay, from which you can deduce the index of refraction to be -2.

7 ± 0.1. So, this is telling you that negative index materials are going to give you negative refraction. So, the beam width was set by the diffraction at the exit of the incident channel and the angular sensitivity of the detector and is similar to the bandwidth that is measured without a sample in place. So, that is how the experiment was performed. So, this was the first one, and later on, you know a three-dimensional negative index material cube that was made using a similar design. also confirmed this kind of findings removing any doubt regarding the feasibility of fabricating negative index materials.

So, this wear and splitting resonator combination became a standard prototype for most early research in this particular field. Now, they also performed the calculation of the refractive index. So, the index of refraction for this left hand material sample was also measured and they confirmed it to be negative within a specific frequency band as you can see in this particular figure ok. So, here you can see the blue curve which basically corresponds to the data of the teflon sample ok and then you have a black curve.

So, that is basically for the left-hand material data. And there in the black curve you see some dotted portion that is basically indicating that here the index is expected to be either outside the limit of detection which is basically + - 3 in this case and should be within 3 + - 3. Or it is dominated by the imaginary component, and therefore, it cannot be reliably determined experimentally. So, they are given as dotted lines. Along with that, the red ones basically show the theoretically calculated value of the left-hand materials. There also in the red you see the dotted lines they basically tell you the imaginary component of the critical expression of the refractive index.

Now, what does this comparison with Teflon tells us that if you have a Teflon sample, it basically shows a constant flat refractive index across the entire measured frequency band. So, it is basically the x band here, okay. In the case of left hand material, the first thing that you notice is the dispersive nature that is it is frequency dependent. So, this negative index behavior is highly dispersive. So, it changes significantly with frequency and this particular finding is consistent with the theoretical prediction as well.

And why theoretical prediction was needed because you know this basically shows that you know the frequency dependent models that we discussed earlier in the lecture. Lectures are okay; they can also predict the material permittivity and permeability to a good extent. So, all these things confirm that by using metamaterials, you can realize negative index materials that are not found in nature. So, with that, we conclude this lecture. If you have got any query regarding this lecture, you can drop an email to this email address mentioning the course name and the lecture number on the subject line. Thank you.

Slides inserted by fallback (review if needed):