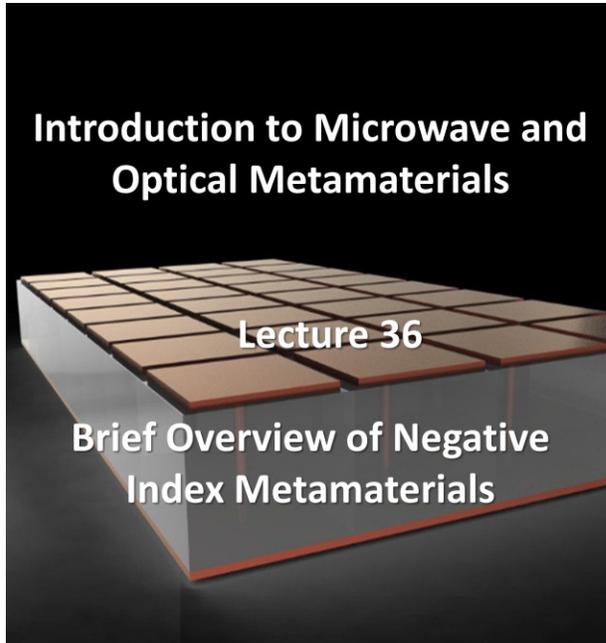


Course Name: Introduction to Microwave and Optical Metamaterials
Professor Name: Dr. Debabrata Sikdar
Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-8
Lecture-36

Lec 36: Brief Overview of Negative Index Metamaterials



Dr. Debabrata Sikdar

Department of Electronics and Electrical Engineering
Indian Institute of Technology Guwahati

Web: <https://www.iitg.ac.in/deb.sikdar>
Email: deb.sikdar@iitg.ac.in



NPTEL

NPTEL ONLINE CERTIFICATION COURSE
AN INITIATIVE OF MoE, GOVT. OF INDIA

Lecture Outline

- Negative Index Media: Introduction
- Wave Propagation
- Double-Positive (DPS) Medium
- Single-Negative (SNG) Medium
- Double-Negative (DPS) Medium
- Left-Handed Metamaterials (LHMs): An in-depth analysis

Hello, students. Welcome to Lecture 36 of the online course "

Negative Index Media: Introduction

- The propagation of an electromagnetic wave through a linear, isotropic medium is governed by the electric permittivity ϵ and magnetic permeability μ of the material.
- In general, these quantities are frequency-dependent and complex-valued.
- Wave properties, such as the propagation constant, velocity, attenuation coefficient, impedance, and dispersion relation, can be readily determined from ϵ and μ .
- The signs of the real and imaginary components of ϵ and μ at a given frequency govern the various regimes of wave propagation.

Negative Index Media: Introduction

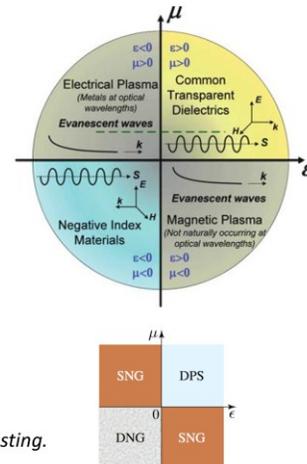
- For media in which μ is real and positive (indicating that there is neither magnetic absorption nor amplification), the wave propagation characteristics depend on the signs of the real and imaginary parts of ϵ .
- Similarly, for media with real and positive ϵ , the magnetic properties, described by μ , dictate the nature of wave propagation.
- Magnetic media, including media with metal components that carry induced electric currents and generate magnetic fields, generally have complex values of μ , with real and imaginary parts that may be either positive or negative.
- In the most general case, the manner in which the signs of the real and imaginary components of ϵ and μ dictate the characteristics of wave propagation is more subtle.

Introduction to Microwave and Optical

Single and Double Negative Media

- Single-negative (SNG) materials (either ϵ or μ is negative)
 - These materials are opaque but they support optical surface waves at boundaries with double positive (DPS) materials.
 - Metals such as gold and silver exhibit negative ϵ while maintaining positive μ in the infrared and visible spectral regions.
- Double-negative (DNG) metamaterials (both ϵ and μ are negative)

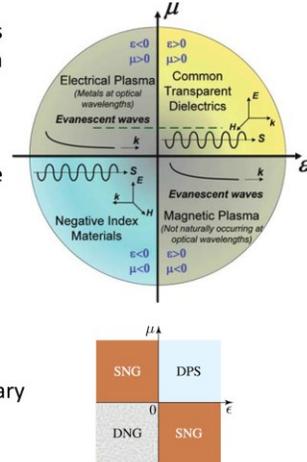
These materials, also called left-handed media are transparent and have negative refractive index, signifying that the application of Snell's law at a DPS-DNG boundary results in a negative angle of refraction.



The implications of this property for materials with multiple boundaries are most interesting. Metamaterials may be designed to exhibit such properties in specific frequency bands.

Single and Double Negative Media

- Let's confine ourselves principally to lossless and passive media, in which there is neither absorption nor gain, indicating that we are, for example, away from dielectric and magnetic resonances.
- Under these circumstances, both ϵ and μ are real, and their signs may be positive or negative at a given frequency.
- Double-positive (DPS) materials (both ϵ and μ are positive)
- These materials are transparent and have positive refractive index. Ordinary dielectric media fall into this category.



Metamaterials." Today we will be discussing a brief overview of negative index metamaterials. So, here is the lecture outline, we will give a brief introduction to negative index media, we will discuss about its wave propagation characteristics. So, before we come down to negative index medium, we will also discuss double-positive medium. Single negative medium, okay, and then we will go for double negative medium, that's okay.

And finally, we will discuss the left-hand metamaterials and go for an in-depth analysis. So, the propagation of an electromagnetic wave through a linear isotropic medium we have known that it is basically governed by the electric permittivity ϵ and the magnetic permeability μ of the material right. So, in general, these quantities are frequency-dependent, and they are complex. When you look for wave properties such as propagation constant, velocity, attenuation constant, impedance, and dispersion relation, they all can be readily determined from permittivity and permeability.

And because they're complex, the sign of the real and the imaginary components of this ϵ and μ okay at a given frequency they basically govern the various regimes of wave propagation so for media in which you will see μ is real and positive that indicates that there is neither you know magnetic absorption nor amplification The wave property or the wave propagation properties solely depend on the signs of the real and imaginary parts of the permittivity, ϵ . Similarly, for media with the real and positive ϵ , the magnetic properties which are described by μ , they basically dictate the nature of wave propagation. So, when you have magnetic media, it includes media with metal components that carry induced electric currents and can generate magnetic fields. They generally have complex values of μ with real and imaginary parts that may be either positive or negative. So, in most general cases, the manner in which the signs of the real and the imaginary components of ϵ and μ they basically dictate the characteristics of the wave propagation in a more subtle way.

So, let us now look into the single and double negative media. Okay. So, here you see the axis;

the coordinate shows that you have ϵ on the horizontal axis and μ on the vertical axis. This origin, so this part is positive for ϵ , this is positive for μ , this is negative for ϵ , this is negative for μ right. So, we are basically considering single-negative materials.

So, either ϵ or μ is negative. So, which are those two regions? These are basically these and these quadrants, okay? The second and fourth quadrants, this one and this one. So, these materials are basically opaque, but they support optical surface waves at boundaries with double positive material, which are basically the common transparent dielectrics, okay? The first quadrant's materials. So, here when you consider single negative materials. you can name a few something like you know metal such as gold or silver they exhibit negative permittivity while maintaining a positive permeability right in the infrared and optical ranges ok.

So, that is where you get the normal materials and single negative materials. Now, when you come here, okay? This one also has a single negative material, but this basically shows that your permittivity is positive and permeability is negative. So, this is basically magnetic plasma, but no naturally occurring material at optical frequencies or wavelengths shows this kind of characteristic. Next, we have this third quadrant, which is particularly interesting because it contains Dng materials. DNG means double negative

Wave Propagation

- For simplicity, we consider a monochromatic plane wave with electric and magnetic complex-amplitude vectors given by $\mathbf{E}(r) = \mathbf{E}_0 \exp(-j\mathbf{k} \cdot r)$ and $\mathbf{H}(r) = \mathbf{H}_0 \exp(-j\mathbf{k} \cdot r)$, respectively, and with wave vector \mathbf{k} .

- Maxwell's equations then require: $\mathbf{k} \times \mathbf{H}_0 = -\omega\epsilon\mathbf{E}_0$ (1)

$$\mathbf{k} \times \mathbf{E}_0 = \omega\mu\mathbf{H}_0 \quad (2)$$

- The associated wavenumber (magnitude of the vector \mathbf{k}) is: $k = \omega\sqrt{\mu\epsilon}$ (3)

- The impedance (ratio of the magnitudes of \mathbf{E}_0 and \mathbf{H}_0) is given by:

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}} \quad (4)$$

metamaterial, where both permittivity and permeability are negative.

So, these materials are also called left-handed media, and they are transparent. Transparent means they allow wave propagation and have a refractive index, but they have a negative

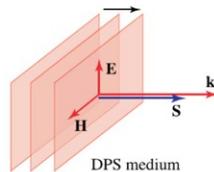
Wave Propagation

- Since k is in general complex, we write it as $\beta - j\gamma$, where β and γ are real, so that

$$\beta - j\gamma = \omega\sqrt{\mu\epsilon}$$

- The propagation constant $\beta = \omega/c$ determines both the wave velocity $c = c_0/n$ and the refractive index n , whereas γ represents the field attenuation coefficient.
- Now consider the implications of these equations for media in which ϵ and μ are real, where either or both may be negative.

- Double-positive (DPS) medium:**



Plane wave propagating in an ordinary double positive (DPS) medium. The vectors \mathbf{E} , \mathbf{H} , and \mathbf{k} form a right-handed set and the wavefronts travel in the same direction as the power flow.

refractive index. So they can satisfy the application of Snell's law at a double positive and a double negative interface that will give you negative angle of refraction. So, the implication of this property for materials with this multiple boundaries are most interesting and you can make metamaterials that can know exhibit such amazing properties in your desired frequency band. So, let us first confine ourselves to the lossless and the passive media in which there is neither absorption nor gain okay indicating that we are basically away from the dielectric and magnetic resonance.

So, under these kinds of circumstances, you can see that both ϵ and μ are basically real. And their signs can be either, you know, positive or negative at a given frequency. Right. If you consider double-positive metamaterials, you are basically considering that both ϵ and μ are positive. So, these are the materials which are basically transparent that means they allow light to propagate right and they have positive refractive index.

So, all the ordinary dielectric media basically fall into this category, which is in the first quadrant. So, you can also see the E and the H axis this is the K there is the wave vector and also you can see there is a wave propagation shown here with the pointing vector of the wave in the same direction of k. Now, that is something you need to understand that for the third one, third quadrant that is for the double negative

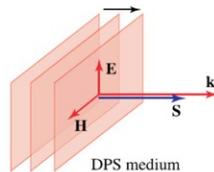
Double-Positive (DPS) Medium

- **Double-positive (DPS) Medium:**

The double-positive (DPS) medium provides a simple and familiar benchmark. Both ϵ and μ are positive, so that k and η are real, whereupon:

$$\gamma = 0; \quad \beta = nk_0; \quad n = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}}; \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

Support transverse electromagnetic (TEM) waves for which the vectors \mathbf{E}_0 , \mathbf{H}_0 , and \mathbf{k} are mutually orthogonal and form a right-handed system.



metamaterials, something is different here. You have E and H, but you have k pointing in the other direction, whereas the pointing vector goes in the same direction ok, we will come to that. So let us consider the wave propagation characteristics.

So, for simplicity we can consider a monochromatic plane wave with electric and magnetic complex amplitude vectors which are given by $\mathbf{E} e^{-j\mathbf{k} \cdot \mathbf{r}}$ and you can have magnetic field also in the same form. So, here \mathbf{k} is basically the wave factor. So, Maxwell's equation can then be written in this form that $\mathbf{k} \times \mathbf{H}$ will be equal to $-\omega \epsilon \mathbf{E}$ and $\mathbf{k} \times \mathbf{E}$ will be nothing but $\omega \mu \mathbf{H}$ ok. So, this first equation basically indicates that your \mathbf{E} is orthogonal to both the \mathbf{k} and \mathbf{H} while your second equation indicates that \mathbf{H} is orthogonal to both \mathbf{k} and \mathbf{E} , right. So, that means these three vectors E, H, and k are basically forming a mutually orthogonal set.

Now, the fixed orthogonal directions are okay if you choose them for the fields \mathbf{E} and \mathbf{H} . Then the wave factor \mathbf{k} is just you know orthogonal to the plane that is defined by the wave those two vectors right. And you can actually see that the actual direction basically will depend on the signs of ϵ and μ ok, which we will observe shortly ok. Here also you can get some idea that the associated wave number which is nothing but the magnitude of the wave factor that comes from $k = \omega \sqrt{\mu \epsilon}$. So, the value or the sign of this μ and ϵ plays an important role.

Another important parameter is impedance, which is basically the ratio of the magnitudes of the electric and non-magnetic fields. So, that you can define as η that is $\omega \mu / k$ which is nothing but square root of μ/ϵ ok. So, these two things play a very important role, and since we have understood that the \mathbf{k} wave vector or wave number is generally a complex quantity. You can always write it as $\beta - j\gamma$ where both β and γ are considered to be real ok and you can write $\beta - j\gamma$ will be equal to $\omega \sqrt{\mu \epsilon}$. Now, what is this β ? β is basically called the propagation

constant which is nothing but ω/c .

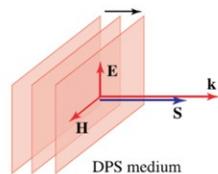
So, that basically determines both the wave velocity that is $c = c$ naught by n , c naught is the vacuum speed of light or speed of light in vacuum and $/n$ is basically the wave velocity in a refractive index of n that is the medium right. So, another term remains here, which is γ , that is basically giving you the field attenuation coefficient. So, now consider the implications of these equations for media in which you have both ϵ and μ to be real, but they can be either positive or both may be negative. So, let us consider the first case, which is a double positive medium; that is, where both ϵ and μ are positive. So, as we have discussed briefly there in the previous slide that if you have E and H which are orthogonal to each other, K is also orthogonal to them.

So, you can see that the plane wave propagating in an ordinary double positive medium looks like this, where S shows the

Double-Positive (DPS) Medium

- **Double-positive (DPS) Medium:**

The Poynting Vector $S = \frac{1}{2} \mathbf{E}_0 \times \mathbf{H}_0^*$ points along the same direction as the wave vector \mathbf{k} , and the intensity of the wave (power flow per unit area) is given by $I = \text{Re}\{S\} = |E_0|^2/2\eta$.



The vectors \mathbf{E} , \mathbf{H} , and \mathbf{k} form a right-handed set and the wavefronts travel in the same direction as the power flow.

Poynting vector that is the flow of energy also happening in the same direction of k . So, what you can understand from here, these are basically the face front plane wave. So, it looks like this. ok and you can also see that E , H and K are basically forming a right handed set ok and the wave front basically travels in the same direction of the power flow. So, that is your double positive medium, okay.

So, this basically

Single-Negative (SNG) Medium

- **Single-Negative (SNG) Medium:**

In a SNG medium, either ϵ or μ is negative so that k and η are both imaginary provides:

$$\gamma = \omega \sqrt{|\epsilon||\mu|}; \quad \beta = 0; \quad n = j \sqrt{\frac{|\mu|}{|\epsilon|}}$$

- These parameters correspond to an exponentially decaying field that behaves as $\exp(-\gamma z)$, where z is the propagation distance.
- Since $\beta = 0$, a SNG medium does not support propagating waves.

provides a simple and familiar benchmark. So, both ϵ and μ are positive. So, that you also get you know k that is the wave number and η the impedance they are also real. And here, γ is 0 because that is the attenuation coefficient and the imaginary part, okay. So, that goes to 0, β is simply can be written as $n k_0$, n is nothing but square root of $\epsilon \mu / \epsilon_0 \mu_0$ and η that is the impedance of this medium can be given by square root of μ/ϵ .

So, we understood that to support transverse electromagnetic wave for which the vectors E or E naught, H naught and k should be also they should be all mutually orthogonal. So, that is the case for TEM transverse electromagnetic waves, and they basically form a right-handed system, okay. So, we also discussed the power flow. So, that is basically coming from the pointing vector S . So, you can calculate the pointing vector S half E naught cross product with H naught conjugate okay.

So, if you calculate this that basically points in same direction of the wave vector k okay and the intensity of the wave that is basically the power flow per unit area will be given as I which is nothing but the real part of the pointing vector S and that can be calculated as modulus of E naught square that is modulus of the electric field square that is the electric field magnitude square / 2η . Now, let us look into that is the simplest medium that is the double positive medium which we encounter very closely every day because these are the ordinary dielectric medium. Now we move on to the other type, which is a single negative medium. So, in a single medium, either ϵ or μ can be negative, as you understand. So, that will make your k and η both imaginary ok, because you will also get γ that is having you know equation of ω square root of modulus of ϵ * modulus of μ .

β will be 0; that means there is no wave propagation allowed in this case, okay. It will only attenuate and then n can be obtained as j square root of modulus of $\mu /$ modulus of ϵ . So, these parameters basically correspond to a exponentially decaying field that basically behaves as

exponential $- \gamma z$, where z is basically the propagation constant. So, this attenuation constant tells you that the wave is basically going to decay exponentially in this kind of single negative medium. And obviously, as I told you that β is 0 here, that means the single negative medium does not support any propagation waves. Because the wave is

Single-Negative (SNG) Medium

- **Single-Negative (SNG) Medium:**

- The optical intensity is attenuated by the factor e^{-1} at a depth $d_p = 1/2\gamma = \lambda_0/4\pi\sqrt{|\epsilon/\epsilon_0||\mu/\mu_0|}$

d_p = penetration/skin depth

- The imaginary impedance η indicates that there is a $\pi/2$ phase shift between the electric and magnetic fields.
- Moreover, the Poynting vector $\mathbf{S} = \frac{1}{2}\mathbf{E}_0 \times \mathbf{H}_0^*$ is imaginary so that the intensity $I = \text{Re}\{S\} = 0$, indicating that no power is transported through such a medium.

attenuated in this kind of medium, the optical intensity, you can see that it will get attenuated by a factor of $1/e$ at a depth of d_p , which is given by $1/2\gamma$.

that can be written as $\lambda_0 / 4\pi \sqrt{|\epsilon/\epsilon_0||\mu/\mu_0|}$. So, here d_p is basically called the penetration depth or the skin depth. Now, the imaginary impedance η also indicates something that tells you there is basically a $\pi/2$ phase shift between the electric and magnetic fields. And another important factor is what is happening with the energy flow. We already have an understanding that β is 0.

So, the wave does not propagate. So, even if you calculate the pointing vector as \mathbf{S} , okay, which is half \mathbf{E}_0 crossed with \mathbf{H}_0 conjugate, it will come out to be imaginary. That means if you find out the intensity, which is basically the real part of the pointing vector, it will come out to be 0. That means there is no power being transmitted through such a medium. Now we go to the third quadrant which is basically the double negative medium D and G. So in this medium both ϵ and μ are basically So, you get k the wave vector or the wave number the amplitude is given as $\omega \sqrt{|\epsilon/\epsilon_0||\mu/\mu_0|}$.

So, this is real ok and then in this medium because both are negative you get a real quantity, but then you get a negative sign along with that. So, what you also obtain you can also see that γ the

Double-Negative (SNG) Medium

▪ Double-Negative (DNG) Medium

- In a double-negative (DNG) medium, both ϵ and μ are negative $k = \omega\sqrt{|\epsilon||\mu|}$, which is real, whereupon:

$$\gamma = 0; \quad \beta = nk_0; \quad n = -\sqrt{\frac{|\epsilon|}{\epsilon_0} \frac{|\mu|}{\mu_0}}; \quad \eta = \sqrt{\frac{|\mu|}{|\epsilon|}}$$

- Indicating that the refractive index is negative.
- Since $\gamma = 0$, the medium sustains wave propagation without attenuation.
- The choice of signs for the square roots is established by examining the directions of the vectors \mathbf{E}_0 , \mathbf{H}_0 , and \mathbf{k} , which may be determined directly from Maxwell's equations.

attenuation constant will come out to be 0 because it is a real quantity. And β is nk_0 where n can be written as - square root of modulus ϵ/ϵ_0 modulus μ/μ_0 naught. So, you are getting a negative refractive index and you are getting an impedance that is given by square root of modulus μ by modulus ϵ . So, what you obtain here as I mentioned that you are getting the refractive index to be and more importantly the

Double-Negative (SNG) Medium

▪ Double-Negative (DNG) Medium

- The DNG medium yield:

$$\mathbf{k} \times \mathbf{H}_0 = \omega|\epsilon|\mathbf{E}_0 \quad (5)$$

$$\mathbf{k} \times \mathbf{E}_0 = -\omega|\mu|\mathbf{H}_0 \quad (6)$$

- As with the DPS medium:

$$\mathbf{k} \times \mathbf{H}_0 = -\omega\epsilon\mathbf{E}_0 \quad (1)$$

$$\mathbf{k} \times \mathbf{E}_0 = \omega\mu\mathbf{H}_0 \quad (2)$$

- The reversal of signs shows exchange in roles of the electric and magnetic fields.

medium has got you know 0 attenuation that means the medium can sustain propagation of wave without any attenuation.

Now if you look into the sign of the square roots okay you can understand that you know the direction of the \mathbf{E} \mathbf{H} and \mathbf{k} must be determined directly from the Maxwell's equation. So, if you go back to the Maxwell's equation the previous ones and here you write it specifically for the double negative material you will write $\mathbf{k} \times \mathbf{H}$ will be μ modulus of ϵ \mathbf{E} okay and then you have $\mathbf{k} \times \mathbf{E}$ that will be $-\omega$ modulus μ \mathbf{H} okay. So, you are basically replacing the double positive media if you compare. So, this ϵ is now replaced by the modulus of ϵ ; this μ is also replaced by the modulus of μ . So, because of that, what you are also obtaining is nothing but a sign reversal.

So, this basically tells you that there is an exchange in the role of the electric and the magnetic field between the DNG double negative and the double positive medium. So, how do you interpret it? Ok. So, you can see that in the double positive medium, these are not the double positive medium. So, normally in the double positive medium, we have seen that \mathbf{E} , \mathbf{H} , and \mathbf{k} basically form a right-handed set. So, in the double-negative medium, they will basically form a left-handed set.

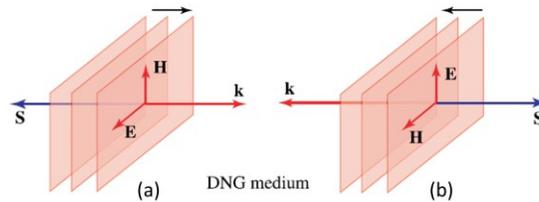
So, if you do $\mathbf{E} \times \mathbf{H}$, okay, you have to use the left hand. So, your \mathbf{k} vector will be along this direction, right? But your energy flow will be in the right direction. earlier direction. So, you can actually follow this one, or you can try to interpret this figure.

So, here what we are doing is that \mathbf{E} and \mathbf{H} . So, the \mathbf{E} field is basically now exchanged with \mathbf{H} ; that is what the equations gave us, right? So, the \mathbf{E} and \mathbf{H} are swapped. So, here you can also see that $\mathbf{E} \times \mathbf{H}$ is basically giving you this kind of \mathbf{k} vector. So, again, you can use the left-handed rule, and you can find the \mathbf{k} vector to be here if the \mathbf{E} field is in this direction, right? So, the wave front basically travels in the opposite direction of the power flow in either case, okay. These are the two orientations that you can see. So, these two things are basically identical representations, except that this one is rotated 90 degrees with respect to the other.

Right. So, what is important here to notice that the

Left-Handed Metamaterials

- It is apparent that \mathbf{E}_0 , \mathbf{H}_0 , and \mathbf{k} in a DPS medium form the usual right-handed set of vectors, whereas in a DNG medium they form a left-handed set (the medium is then said to be **left-handed**).
- Since the impedance η positive, the wavenumber k is taken to be negative, and therefore so too is the refractive index n . The DNG material is therefore a **negative-index metamaterial (NIM)**.



impedance η is positive the wave number that is k is taken to be

An in-depth analysis

- What happens when both μ and ε are negative?

$$\begin{aligned} \nabla \times \vec{E} &= -j\omega(-\tilde{\mu})\vec{H} & \longrightarrow & \nabla \times \vec{E} = -j\omega\tilde{\mu}(-\vec{H}) \\ \nabla \times \vec{H} &= j\omega(-\tilde{\varepsilon})\vec{E} & & \nabla \times (-\vec{H}) = j\omega\tilde{\varepsilon}\vec{E} \end{aligned}$$

The system is left-handed!

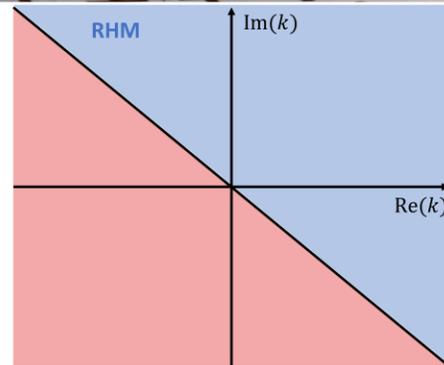
$$\vec{k} \perp \vec{E} \perp (-\vec{H}) \quad \longrightarrow \quad -\vec{k} \perp \vec{E} \perp \vec{H}$$

\mathbf{E} , \mathbf{H} and \mathbf{k} form a "left-handed" system

negative and you also get the refractive index to be negative and that is why this dng medium that is double negative medium is called negative index

Overall picture

- Complex wave vector (\mathbf{k}) plane for electromagnetic waves in different types of materials, *distinguishing right-handed and left-handed metamaterials.*
- Axes
 - $Re[k]$ (horizontal axis): Real part of the wave vector corresponds to wave propagation.
 - $Im[k]$ (vertical axis): Imaginary part of the wave vector corresponds to wave attenuation or growth.



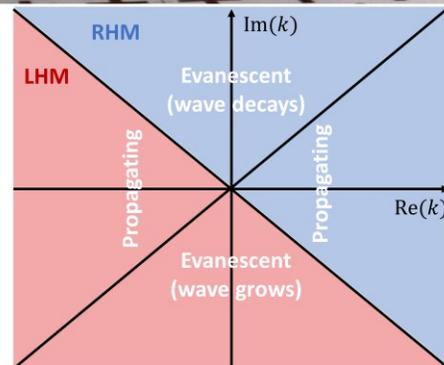
- Right-Handed Materials (RHM)**

- Located on the positive $Re[k]$ axis.
- Characterized by both permittivity and permeability are positive.
- Waves propagate normally (phase velocity and energy flow are in the same direction).

Overall picture

- Left-Handed Materials (LHM)**

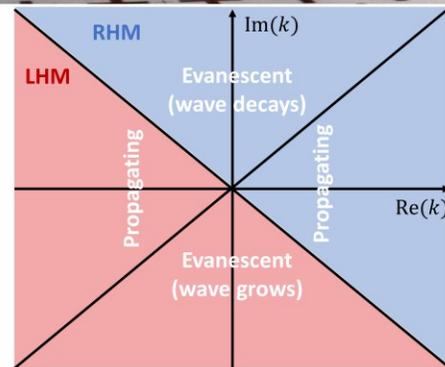
- Located on the negative $Re[k]$ axis.
- Characterized by both permittivity and permeability are negative.
- Waves propagate with a negative refractive index (phase velocity and energy flow are opposite).



- Propagating Waves (horizontal direction):** Occur when both permittivity and permeability have the same sign (both positive \rightarrow right-handed; both negative \rightarrow left-handed).
- Evanescient Waves (vertical direction):** Occur when permittivity and permeability have opposite signs \rightarrow the wave decays or grows exponentially without true propagation.

Overall picture

- **Propagating Waves (horizontal direction):** Occur when both permittivity and permeability have the same sign (both positive \rightarrow right-handed; both negative \rightarrow left-handed).
- **Evanescent Waves (vertical direction):** Occur when permittivity and permeability have opposite signs \rightarrow the wave decays or grows exponentially without true propagation.
- $\epsilon'\mu'' + \epsilon''\mu' > 0 \rightarrow$ Right-handed propagation (energy propagates in the expected direction).
- $\epsilon'\mu'' + \epsilon''\mu' < 0 \rightarrow$ Left-handed propagation (negative index, reversed phase velocity).



metamaterial. So, now let us do an in-depth analysis of what happens when both μ and ϵ are negative. So, we first of all we understood from the Maxwell's equation that when you put μ and ϵ both are negative that signs can be adjusted into the fields okay and that will basically change your system from a right hand system to a left hand system. So, you are basically getting E, H, and K to form a left-handed system.

So, that is basically coming from this. So, this is the right-hand system; this is the left-hand system. So, let us discuss the overall picture of this. So, let us first look into the complex wave vector plane. So, here is the complex wave vector for electromagnetic waves in different types of materials. So, we will be distinguishing the right-handed and the left-handed materials.

So, we have plotted the two axis the real part of the k comes in the horizontal axis that is the real part of the wave vector that corresponds to wave propagation. Imaginary of k is in the vertical axis that is the imaginary part of the wave vector that basically corresponds to wave attenuation if that is positive if it is negative it gives you growth. Now, we can define right-hand material, which is basically located on the positive real part of the k-axis. It is characterized by both permittivity and permeability being positive, right? So, here the wave can propagate normally, which means the phase velocity and the energy flow will be in the same direction. Now comes the other case, which is of left-hand materials.

So, that is located on the negative real part of the K axis, right? So, here it is characterized by both positive permittivity and permeability becoming negative. So, here also wave can propagate, but it propagates with a negative refractive index that means the phase velocity and the energy flow are basically opposite to each other. So, in both cases, you can see you are having propagation along the horizontal direction, right? So, here both permittivity and permeability have the same sign. So, here both are positive, here both are negative. When both are positive, you have right-hand material; when both are negative, you have left-hand material.

Now there are also evanescent waves that are in the vertical direction, as you can see here. So, when the permittivity and permeability have opposite signs, you are basically experiencing evanescent waves. The wave will decay if the imaginary k is positive and the wave will grow exponentially if the imaginary k is negative. Negative, okay, and in both cases, propagation is not allowed. So, propagation just the final summary is that the propagation is allowed along this horizontal direction that happens when both permittivity and permeability are of the same sign.

So, if both are positive, you get right-handed; if both are negative, you get left-handed. Evanescent waves are along this vertical direction when permittivity and permeability have opposite signs. The wave either decays or grows, depending on the sign of the imaginary part of k , and there is no true propagation here. So, this is the condition that ϵ' that is the real part of permittivity and μ'' that is the imaginary part of the permeability + ϵ'' and μ' . If this is positive, that means it gives you right-handed propagation, which means energy propagates in the expected direction.

And on the other hand if you have this term becoming negative you are getting left handed propagation which indicates negative index and a reversed phase velocity. So, with that, we conclude this lecture. If you have got any query regarding this lecture, you can drop an email to this email address mentioning the course name and the lecture number on the subject line. Thank you.



Thank You

Slides inserted by fallback (review if needed):



Thank You