Course Name: Introduction to Microwave and Optical Metamaterials
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Week-7
Lecture-31

Lec 31: 2D Model of Microwave Antenna with One Layer Metamaterial

Lecture Outline

- 2D Model of Microwave Antenna With One-layer Wire Composite/Metamaterial Substrate
- Different Design Models of Microwave Antenna

Hello everyone, welcome to Lecture 31 of the online course on Introduction to

2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

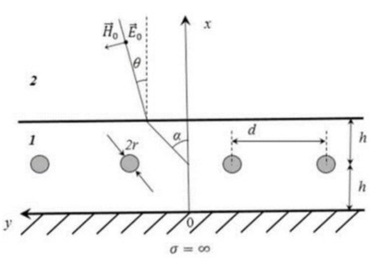
- The approximations for the magnetic intensity vector in the 1st and 2nd space domains are to be defined using Faraday's law.
- First domain ($0 < x < 2h$) expression gives:

$$\boldsymbol{H}_1(x,y) = -\frac{E_0}{Z_1}[E^{1+}(x,y)\cdot \sin\alpha \cdot \boldsymbol{x}_0 - E^{1-}(x,y)\cdot \cos\alpha \cdot \boldsymbol{y}_0] \quad (1)$$

- Second domain ($x > 2h$) expression gives:

$$\boldsymbol{H}_2(x,y) = -\frac{E_2(x,y)}{Z_2}[\sin\theta \cdot \boldsymbol{x}_0 + \cos\theta \cdot \boldsymbol{y}_0] \quad (2)$$

where $E^{1\pm}(x,y) = e^{ik_1 y \sin\alpha}\left(e^{ik_1 x\cos\alpha} \pm Re^{-ik_1 x \cos\alpha}\right)$ (3)



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Microwave and Optical Metamaterials. Today's lecture we will be discussing on 2D model of microwave antenna with one layer metamaterial. So, it is basically a continuation of the previous lecture. Here is the lecture outline. So, we will complete our discussion on this 2D model of microwave antenna with one layer wire composite or the metamaterial structure as a substrate that we studied earlier. And then we will also look into different design models of microwave antennas.

So, in the previous lecture, we studied the electric intensity vector in region 1 and region 2, okay? So, now in this lecture, we will start with the magnetic intensity vectors, okay. So, you will see that the approximation for the magnetic intensity vector in the first and second space domains is defined using Faraday's law. So, in the first domain that is with x within 0 and 2 h that is here where the wires are basically embedded in the substrate. So, in that medium you can write the magnetic field h_1 xy = - E naught that is the incident electric field / z_1 that is the impedance in the first medium * E_1 + xy into sin α this angle * x naught that is basically x cap - that is the unit vector along the x direction - e_1 - xy * cos α into y naught or y cap.

In the second domain here where x is greater than 2 h the expression can be written as h_2 x y = - e_2 x y/z_2. So, Z_2 will be the impedance in the second domain. So, if it is free space, you can consider this as z naught, okay. For this example. So, that is sin θ * x naught + cos θ * y naught.

Okay, these are basically the unit vectors. Now, what is E_1 + - x y? So, you can see that that can be written as e to the power i k_1 y sin α * sin e to the power i k_1 x cos α + - r that is basically the reflection okay and coefficient and then you have e to the power - i k_1 x cos α okay. So, this is basically the reflection term, okay, that has to be considered. Now, let us present the current flow on the patch in the form of j that can be written as j δ x - 2 h * z naught that is basically the z cap right. So, you can write this as - j naught δ x - 2 h into e to the power - k_2 y sin θ * z naught.

So, here what is this z naught it is basically a constant to be

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- Let us present the current flow on the patch in the form :

$$\boldsymbol{j} = j \cdot \delta(x-2h) \cdot \boldsymbol{z}_0 = -j_0 \cdot \delta(x-2h) \cdot e^{-ik_2 y \sin\theta} \cdot \boldsymbol{z}_0 \quad (4)$$

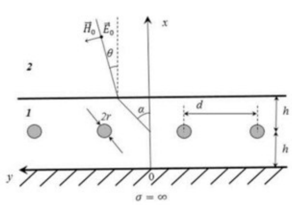
  where j_0 is a constant to be predetermined, $k_2 = k_0$ is the wave number in the 2nd space domain $(x > 2h)$ which is the free space, and $\boldsymbol{z}_0$ is the unit vector of the z-axis.

- Here $\delta(x-2h)$ is the Dirac delta function, which is zero everywhere except at $x = 2h$, that implies that the current density $\boldsymbol{j}$ is confined to the plane $x = 2h$.

- The unknowns E_0, A and α can be calculated by imposing the boundary conditions for the tangential electric and magnetic fields to be continuous at the interface $x = 2h$, i.e:

$$\boldsymbol{E}_1|_{x=2h} = \boldsymbol{E}_2|_{x=2h} \quad (5)$$

$$[\boldsymbol{x}_0 \times (\boldsymbol{H}_2 - \boldsymbol{H}_1)]|_{x=2h} = j \cdot \boldsymbol{z}_0 \quad (6)$$



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predetermined and then you have k_2 appearing that is basically k_0 which is the wave factor or wave number you can say in the second space domain that is for x greater than 2 h which is free space in this particular example and then this guy is nothing but your Unit vector in the x-axis, right? So, this rotation is basically taken from this book. Now, what is this δ function, right? You see a δ x - 2 h that is a direct δ function which tells you that it is 0 everywhere except for x = 2 h. So, that implies that the current density is basically confined in the plane x = 2 h right in this particular plane only. Now, there are a couple of unknowns, like E naught, A, and α. So, these are to be calculated by imposing the boundary conditions for the tangential electric and magnetic fields to be continuous at the interface at x = 2 h.

So, when you put those boundary conditions okay they will look like E_1 you calculate at x = 2 h that should be equal to E_2 as well at the boundary. Okay, and also, okay from the magnetic field, the curl, okay. So, you see x cap or x naught ok and then you take the difference in the magnetic field ok h_2 - h_1 at this particular plane will be nothing but the current density ok or you can say this is basically the current flow right in and that will be in the z naught direction. Or the Z cap. So, Z is basically the direction of the length of the wires that are placed in the substrate.

Now, once you substitute those equations and the equations from the electric intensity vector this ones that we have seen in the previous lecture for both the domains into this expressions 5 and 6 that is your boundary condition you will be able to obtain what is E naught. So E naught can be written as J naught z_2 z_1 / z_1 cos θ E_1 +. And you are taking it at x = 2, h okay, and y is considered 0. So, that is x y basically + z_2 cos α e_1 -, and then at the same point, ok. So, what is α here or you can say cos α can be written as square root of 1 - n square θ / ε m μ m.

So, that is how you can obtain what is α right. So, from that, you can also obtain what the

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- Substituting Eqs. (1–4) and equations of electric intensity vector of both the domains into Eq. (5) and (6) gives:

$$E_0 = \frac{j_0 Z_2 Z_1}{Z_1 \cos\theta \cdot E^{1+}(2h,0) + Z_2 \cos\alpha \cdot E^{1-}(2h,0)} \quad (7)$$

where $\cos\alpha = \sqrt{1 - (\sin^2\theta / \varepsilon_m \mu_m)}$

$$\boldsymbol{E}_1 = E_1(x,y) \cdot \boldsymbol{z}_0 = E_0 e^{ik_1 y \sin\alpha}\left(e^{ik_1 x \cos\alpha} + Re^{-ik_1 x \cos\alpha}\right) \cdot \boldsymbol{z}_0$$

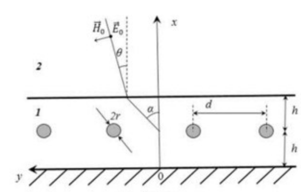
$$\boldsymbol{E}_2 = E_2(x,y) \cdot \boldsymbol{z}_0 = A \cdot e^{-ik_2 y \sin\theta} e^{-ik_2 (x-2h)\cos\theta} \cdot \boldsymbol{z}_0$$

$$R = -1 + \frac{(2Z_1/\cos\theta)\sin^2(k_1 h \cos\theta)}{(Z_1/2\cos\theta)(1 - e^{-2ik_1 h \cos\theta}) + Z_g}$$

- The efficiency of the antenna is to be evaluated by the formula

$$\eta = \frac{P_{rad}}{P_T} = \frac{\mathrm{Re}(P_r)}{P_T} \quad (8)$$

where P_{rad} is the total radiated power, P_r is the complex input power of the antenna and P_T is the total power delivered to the antenna terminal via a transmission line.



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efficiency of the antenna is. So, you can evaluate this using this particular formula: η = p red / p t. So, what is a period? Period is basically the total radiated power, and Pt is basically the total power delivered to the antenna terminal via the

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- The complex input power of the antenna (P_r), which is determined as:

$$P_r = -\frac{1}{2}\int_S E_2 \cdot \boldsymbol{j}^* ds = \frac{j_0 S}{2} A \quad (9)$$

- It is assumed that 'Imag(P_r)' is due to energy that is mostly stored in the vicinity of the wire grid and is just partially stored in the vicinity of antenna patch.

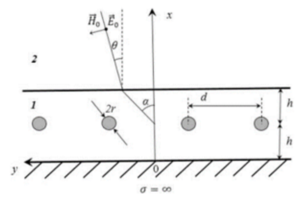
- The total power delivered to the antenna terminal via a transmission line (P_T) with $S = W \times L$ as the surface area of antenna patch is given by:

$$P_T = P_{rad} + P_{ohm} \quad (10)$$

- Here, P_{ohm} is the power dissipated due to ohmic losses, given by:

$$P_{ohm} = \frac{1}{2}\int_S |\boldsymbol{j}|^2 R_p ds \quad (11)$$

where R_p is the ohmic resistance of the antenna patch.



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transmission line. So, this can also be written as real of P r that tells you that P r is basically the complex input power of the So, this complex input power of the antenna P r can be determined as P r = - half surface integral.

The patch you have, E_2, is basically the incident electric field in that region, okay? Then you have J conjugates ds. So, if you work it out, it can be written as J_0 s by 2a. Now, if you compare this, you will say that A is nothing but - 1 by J0's surface integral of E_2 * J conjugate, okay. So, you can say that A is basically the normalizing coupling coefficient between the incident electric field E_2 and the complex conjugate of the current distribution J. So, you have this J conjugate here which is calculated / the surface of the patch S.

Now, it is assumed that the imaginary part of this P r okay is due to energy that is mostly stored in the vicinity of this wire grid and is just you know partially stored in the vicinity of the antenna patch okay. So, the imaginary part mainly lies with the wires. So, in that case, the total power is delivered to the antenna terminal via a transmission line. So, you can calculate that as Pt the total power and S is nothing but the you know area of the patch that is W/W by L okay or W cross L you can say and from that you can obtain that the total power Pt can be written as P red + P ohm. What is this, P ohm? p ohm is basically the power dissipated due to ohmic loss and that you can calculate as using this formula which is p ohm = half again surface integral of the you know current distribution j.

So, you have mod j square ok into R ok. So, Rp is basically the ohmic

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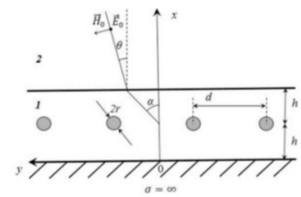
- Throughout the study, we consider a perfectly thin copper patch along with $R_p = R_s S$ where: $R_s = \sqrt{\omega\mu_0/2\sigma_i}$ is the surface resistance of a highly conductive non-magnetic metal in the microwave frequency range.
- Substituting Eqs. (9) and (11) in Eq. (10) finally gives:

$$P_T = \frac{j_0 S}{2} \cdot \mathrm{Re}(A) + \frac{j_0^2 S^2}{2}\sqrt{\frac{\omega\mu_0}{2\sigma_i}} \quad (12)$$

- Substituting Eqs. (9) and (12) in Eq. (8) finally yields:

$$\eta = \left(1 + \frac{j_0 S}{\mathrm{Re}(A)}\sqrt{\frac{\omega\mu_0}{2\sigma_i}}\right)^{-1} \quad (13)$$



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resistance of the antenna patch. So, this is typically nothing but I squared R, okay. So, you are calculating the surface integral of the current density, right? Now, throughout this study we consider a perfectly thin copper patch along with you know R p which is the ohmic resistance that you have seen given as R s * s. So, you can take R s to be equal to square root of ω μ naught / 2 σ i.

So, R s will be nothing but your sheet resistance or surface resistance ok. You can say this is the surface resistance of a highly conductive non-magnetic metal in the microwave frequency range. So, once you substitute you know 9 and 11 into equation 10. So, you can see here 9 and 11 into equation 10 because you are interested in finding out P t. You will see the P t takes this particular form J not s by 2 * real of a + J square s square by 2 square root of you know this term which is ω μ not by 2 σ i and substituting this.

You can find out by substituting 9 into equation 8. So, in equation 8, I will just show you what that is. So, this was your equation 8 that was giving you the efficiency of the antenna where you require this real power of P r/P t. So, now you have obtained both. So, you can get the efficiency looking like this which is 1 / 1 + j naught s real of a square root of ω μ naught by 2 σ i.

So, from the conductivity of the material also you can relate to the efficiency of the patch antenna. Now we need to calculate the normal edge gain and the

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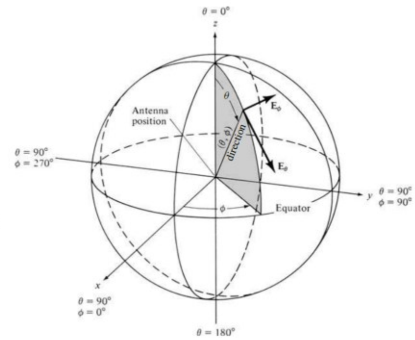
- To calculate both the normalized gain and directivity, the radiated far-field representation in the spherical coordinate system is required.

- As per Balanis, the electric field components are given by:

$$E_\theta = ik_0 \frac{e^{-ik_0 r}}{2\pi r} \frac{1+\cos\theta}{2} f(\theta,\varphi)\sin\varphi \quad (14)$$

$$E_\varphi = ik_0 \frac{e^{-ik_0 r}}{2\pi r} \frac{1+\cos\theta}{2} f(\theta,\varphi)\cos\varphi \quad (15)$$

where where r is the radial distance, φ is the azimuthal angle (0° from x-axis in x–y plane) and θ is the polar angle (0° from z-axis).



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directivity. So, to do that, the radiated fulfillment intensity needs to be plotted in the spherical coordinate system, which is shown here. So, you can see that this is the spherical coordinate system. So, at any point this is the radial direction coming out and this is this angle is called θ and this is the azimuthal angle φ.

So, you can consider this direction as θ = 0, which is along the z direction. So, on the opposite side, you have θ = 180 degrees; again, this is considered to be the θ = 90 degree plane, and then φ = 0. So, when you come to this side here θ is also 90 degree φ is also 90 degree when you go to the other side your φ is 270 degree right and θ remains 90 degree only. So, we can assume antenna to be here and you can use the spherical coordinate system to find out all these parameters like

With One-layer Wire Composite/Metamaterial Substrate

- The expression for the radiation pattern $f(\theta, \varphi)$ is to be evaluated as follows:

$$f(\theta,\varphi) = \int_S \mathrm{E}_2(2\mathrm{h},\mathrm{y}) \cdot \mathrm{e}^{\mathrm{i}k_{2y}\mathrm{y}+\mathrm{i}k_{2z}\mathrm{z}}\mathrm{dydz} =$$

$$4|\mathrm{A}| \cdot \mathrm{S} \cdot \mathrm{sinc}\left(\frac{\mathrm{L}}{\lambda_2}(\cos\varphi - 1)\sin\theta\right) \cdot \mathrm{sinc}\left(\frac{\mathrm{L}}{\lambda_2}\sin\theta\sin\varphi\right) \cdot \mathrm{sinc}\left(\frac{\mathrm{W}}{\lambda_2}\sin\theta\cos\varphi\right) \quad (16)$$

where $\lambda_2 = \lambda_0$ is the wavelength in the 2nd space domain, and $\mathrm{sinc}\,(x) = \sin(\pi x)/\pi x$ (normalized sinc function).

- Then, the expression for the radiation density is given by:

$$U(\theta,\varphi) = \frac{k_0^2}{8\pi^2 Z_0}\frac{(1+\cos\theta)^2}{4}|f(\theta,\varphi)|^2 \quad (17)$$

normalized gain and

With One-layer Wire Composite/Metamaterial Substrate

- Expression for the directive gain is given as:

$$D(\theta,\varphi) = 4\pi\frac{U(\theta,\varphi)}{\mathrm{Re}(P_r)} = \frac{(2k_0 S|A|)^2}{\pi Z_0}\frac{(1+\cos\theta)^2}{\mathrm{Re}(A)}.$$

$$\mathrm{sinc}^2\left(\frac{L}{\lambda_0}(\cos\varphi - 1)\sin\theta\right) \cdot \mathrm{sinc}^2\left(\frac{L}{\lambda_0}\sin\theta\sin\varphi\right) \cdot \mathrm{sinc}^2\left(\frac{W}{\lambda_0}\sin\theta\cos\varphi\right) \quad (18)$$

- As can be seen from Eq. (18), the maximum value of directive gain is achievable for $\theta = 0°$, and its value, notably, the directivity, is given as:

$$D_{\max} = \frac{8k_0^2}{\pi Z_0}\frac{(S|A|_{\theta=0})^2}{\mathrm{Re}(A)|_{\theta=0}} \quad (19)$$

directivity. So, as per Balanis, the electric field components are given by E_θ and E_φ.

So, e θ can be written as i k_0 e to the power - i k_0 r / 2 π r * 1 + cos θ by 2 f which is a function of θ and φ both and then you have sin φ. For E φ, everything else remains the same; just that you have the cos φ component here. So, what is R? r is basically telling you the radial distance; φ is basically giving you the azimuthal angle. So, it starts with the x-axis. So, you make basically measure it from the x axis in the xy plane ok.

So, you are basically in this xy plane and you are measuring it θ is called the polar angle and that you measure from the z axis ok. Now, the

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- At the same time, the maximum gain can be written as:

$$G_{\max} = \eta D_{\max} = \frac{8k_0^2}{\pi Z_0}\frac{(S|A|_{\theta=0})^2}{\mathrm{Re}(A)|_{\theta=0}}\left(1 + \frac{j_0 S}{\mathrm{Re}(A)|_{\theta=0}}\sqrt{\frac{\omega\mu_0}{2\sigma_i}}\right)^{-1} \quad (20)$$

- Taking into account Eq. (16), we are able to derive the expression for the normalized gain in the form:

$$\mathrm{g}(\theta,\varphi) = \frac{(1+\cos\theta)^2}{4}\frac{|\mathrm{f}(\theta,\varphi)|^2}{|\mathrm{f}(\theta,\varphi)|^2_{\max}} = \frac{(1+\cos\theta)^2}{4}\mathrm{sinc}^2\left(\frac{\mathrm{L}}{\lambda_0}(\cos\varphi - 1)\sin\theta\right)\cdot \mathrm{sinc}^2\left(\frac{\mathrm{W}}{\lambda_0}\sin\theta\cos\varphi\right) \quad (21)$$

expression for the radiation pattern is given. So, here you will see that f(θ, φ) is basically your radiation pattern and that you can calculate it using this formula. So, f θ φ can be obtained as you know again integral / the surface of the patch e_2 you are calculating at x = 2 h plane ok y. So, these are the function this is basically x y right and then you have e to the power i k_2 y and + e to the power i k_2 z.

Okay. So, then you have dy and dz. So, when you compute this you basically get 4 modulus of A * S that is the W cross L and then you have 3 sinc functions multiplying each other. So, sinc L/λ by 2 cos φ - 1 sin θ then you have sinc L/λ by 2 sin θ sin φ and then you have sin w/λ/λ_2 sin θ which is sin θ cos φ. So, here you can see that λ_2 is basically nothing, but the λ naught in the second space domain right and we are using sinc function here it you consider this as sinc x will be equal to sin π x/π x. So, that is basically the

With One-layer Wire Composite/Metamaterial Substrate

- Now, the expressions for the normalized gain along two principle planes are given as follows.
- For the H-plane, we set $\varphi = 0^0$, which gives $E_\theta = 0$ in Eq. (21), i.e.,

$$g_{\mathrm{H}}(\theta) = \lim_{\varphi \to 0} g(\varphi, \theta) = \frac{(1+\cos\theta)^2}{4} \mathrm{sinc}^2\left(\frac{\mathrm{W}}{\lambda_0}\sin\theta\right) \qquad (22)$$

and for the E-plane, we set $\varphi = 90^0$, which gives $E_\varphi = 0$ in Eq. (21), i.e.,

$$g_E(\theta) = \lim_{\varphi \to 90^0} g(\varphi, \theta) = \frac{(1+\cos\theta)^2}{4} \mathrm{sinc}^2\left(\frac{L}{\lambda_0}\sin\theta\right) \qquad (23)$$

- Performance Assessment Focus – Primarily evaluates normalized gain along two principal planes (H-plane and E-plane).
- Also includes estimation of antenna efficiency and maximum power gain.

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normalized sinc function ok and this value.

So, these are basically k_2 y So, k_2 y is nothing, but k_0 sin θ sin φ ok and k_2 z is nothing, but k_0 cos θ. So that you can obtain this from the previous figure as well. So, with that, you can write down the expression for radiation density that can be given in this particular form. So, you can write u θ φ will be equal to k_0 square / 8 π square z naught * 1 + cos θ whole square by 4 and then you have modulus.

This radiation pattern is okay. So, the modulus of f θ φ squared. From that, you can obtain the expression for the directive gain as d θ φ. So, that will take this radiation density into account. So, you have 4 π u θ π / real of P r. So, when you expand it you will get 2 k_0 S modulus of A whole square / π z_0 * 1 + cos θ whole square / real of A.

* you know sin squared, you have this 3 sin function that is L/λ naught cos φ - 1 * sin θ. So, here you basically have sin squared, okay? So, the 3 functions are basically sin squared. So, this is sin square L/λ naught sin θ sin φ * sin square W/λ naught sin θ cos φ. So as you can see from this equation that the maximum value of the directive gain can be achieved when you have θ = 0 degree right and its value can be written as you know d max. So that basically gives you the directivity, and that comes down to this equation, which is 8 k_0 squared / π z naught.

8 k_0 square / π z naught and then you have S a

Different Design Models of Microwave Antenna

- Example-
- Let us create a simple design of rectangular patch antenna with a dielectric substrate
  - Substrate Material: RT duroid-5880 ($\varepsilon_r = 2.2$)
  - Resonant Frequency: $f_r = 7.3$ GHz
  - Substrate Thickness: $d = 0.0008$ m
- Then, using these equations, the calculated patch dimensions are:
  - Length $L = 0.0132$ m
  - Width $W = 0.016$ m

$$\varepsilon_{\text{reff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{d}{W}\right]^{-1/2}, W = \frac{c}{2f_r}\sqrt{\frac{2}{1+\varepsilon_r}}$$

$$L = \frac{c}{2f_r\sqrt{\varepsilon_{\text{reff}}}} - 0.824 \cdot d\frac{(\varepsilon_{\text{reff}} + 0.3)(W/d + 0.264)}{(\varepsilon_{\text{reff}} - 0.258)(W/d + 0.8)}$$

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calculated at θ = 0 whole square of that / real of a calculated at again θ equal 0 degree right. So, from that, you can find what the maximum directive gain is, okay. So, you can also find out the maximum gain that can be written as g max which is nothing, but the efficiency of the antenna * the maximum directivity. So, that turns out to be 8 k_0 square / π z naught and then you have S modulus of A calculated at θ = 0 whole square of that / real of A.

Θ = 0. So, that is basically your d max and then you put η which is 1 / 1 + j naught s / real of a at θ = 0 square root of ω μ naught / 2 σ i ok. So, if you take into account equation 16, that is this particular expression for the radiation pattern. Okay. You can derive the expression for the normalized gain in the form of g(θ, φ), which will be, you know, this is the normalized. So, you are basically dividing it by the maximum radiation pattern.

So, you see (1 + cos θ) whole square by 4 / this fraction, okay. So, that is the modulus of f θ φ squared / f θ π max squared. So, it gives you 1 + cos θ whole square by 4 divided into sin square L/λ naught * cos φ - 1 into sin θ * another factors sin square W/λ naught sin θ cos φ. So, now that the expression for the normalized gain can be obtained along the two principal planes, it can be given in this particular form. So, if you consider the h-plane, you can set φ equal to 0.

So, that will basically give you e θ = 0 in equation 21 that is here, okay. So, you will get only for the h plane; you can write g, h, and θ because φ is 0 in this case. So, that is basically where you are getting it from. So, you have you are applying a limit of φ going to 0 on this particular expression that we have seen in equation 21 and this gives you 1 + cos θ whole square / 4 into sin square W/λ naught into sin θ. And if you want to find out the normalized gain for the E plane, you have to set φ equal to 90 degrees.

In that case, you will get E φ = 0. So, you can do the same exercise again. So, you can write g e θ. So, in this case the limit is basically φ going to 90 degree you apply it on this generalized gain ok g φ θ and this gives you 1 + cos θ whole squared by 4. So, the same fraction remains; here it is sin squared, but then here it is L / λ naught into sin θ, okay. So, the performance assessment will primarily, you know, evaluate this normalized gain.

Along the two principal planes, which are the H plane and the E plane, to assess the overall performance of the antenna. And this also includes the estimation of antenna efficiency and maximum power gain. So, now let us look into different design models of microwave antennas. So, let us create a simple design for a rectangular patch antenna with a

Different Design Models of Microwave Antenna

- Composite Substrate Design:
  - Uses same antenna dimensions as the dielectric substrate version
  - Material Inclusions: Copper wires with circular cross-section
  - Evaluation Approach: Effective Medium Theory (EMT)
- The appropriate calculations finally give:
  - Relative Permittivity $\varepsilon_m \approx 2$
  - Relative Permeability $\mu_m \approx 1$
  - Wire Radius $r = 0.00009$ m

Source: Choudhury, P.K , Metamaterials: Technology and Applications, CRC Press, 2021

Different Design Models of Microwave Antenna

- Study Approach:
  - Utilizes FDTD simulations and analytical modeling
  - Evaluates: S_{11}-spectra, far-field patterns (E- and H-planes), efficiency, power gain, and near-field intensity

- Antenna Design Models Considered:
  1. Standard Homogenous Numerical Model:
     - ✓ Patch antenna with a dielectric substrate simulated using FDTD
  2. Standard Inhomogeneous Numerical Model:
     - ✓ Patch antenna with a composite substrate simulated using FDTD

Source: Choudhury, P.K , Metamaterials: Technology and Applications, CRC Press, 2021

dielectric substrate. So, it is assumed that the substrate can be created from RT durate 5880 which has got a relative permittivity of 2.

2 ok and the antenna operates at the resonance frequency of 7.3 gigahertz. And we can also assume the substrate thickness to be D = 0.0008 meters. Now using these equations okay that tells you about the ε are effective that is the effective relative permittivity which correlates the width and the length of the antenna.

So, you can do that and you can obtain the length to be you know 0.0132 meter and width to be 0.16 meter ok that is obtained from this equations for this

Different Design Models of Microwave Antenna

3. Miniaturized Inhomogeneous Numerical Model:
   - ✓ Miniaturized patch antenna with a composite substrate simulated using FDTD
4. Standard Inhomogeneous Analytical Model:
   - ✓ Patch antenna with a composite substrate simulated analytically using the derived equations

- Simulation Tool:
  - Numerical simulations conducted using PLANC FDTD
- Simulation Parameters:
  - Input values include: $r, d, \varepsilon_m, \mu_m, f_r, L,$ and W
  - These values are used to set up the FDTD simulations of antenna performance

performance parameter right. So, D is already given; you know the resonance frequency, okay, that is f r, and ε r is also given.

So, you can obtain this. Now, the dimensions of the antenna are with a composite substrate. is same as the dimensions of the antenna with

Different Design Models of Microwave Antenna

- S_{11}-Spectra Simulation Results (Figure):
  - First peak in each S_{11} parameter spectrum is termed the main peak
  - All main peaks are shifted from the theoretical 7.3 GHz due to:
    - ✓ Fringe effects on patch edges
    - ✓ Spatial dispersion in inhomogeneous models
- Resonant Frequencies:
  - Dielectric substrate antenna: 7.307 GHz
  - Composite substrate antenna: 7.385 GHz

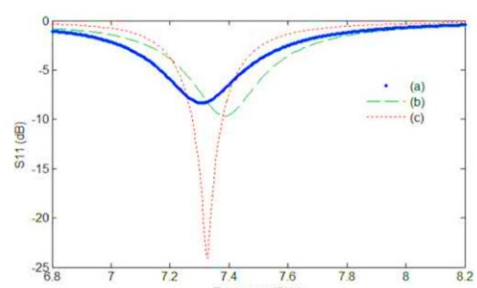

Fig. Simulated S_{11}-spectra for (a) standard homogenous numerical model, (b) standard inhomogeneous numerical model and (c) miniaturized inhomogeneous numerical model.

dielectric substrate right. So, let us evaluate the material parameters of the composite substrate, which are, you know, the ε m, μ m, and r, okay? So, all those things are basically the wear radius, okay. So, we can calculate those assuming that you know the copper wires have a circular cross section, okay. And in order to do that, you can take into account the mentioned use of composites or metamaterials.

So, this is due to the idea of tuning the effective frequency dependent relative permittivity and relative permeability of these artificial materials by individually adjusting the dimensions of the electric and magnetic resonance

Different Design Models of Microwave Antenna

- Analytical Model Limitation:
  - Does not account for fringe effects on edges of patch
- Impact of Composite/Metamaterial Substrate:
  - Redistributes reactive impedance between patch and metal inclusions
  - Enhances energy storage in the EM field radiated by antenna
  - More effective use of magnetic component of EM radiation
  - Better impedance matching than with dielectric substrate

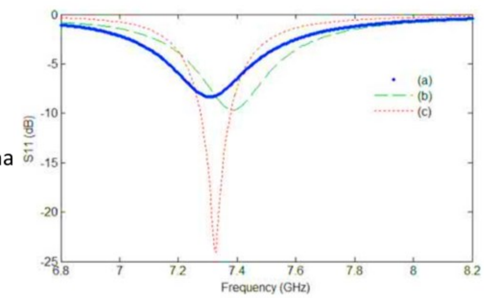

Fig. Simulated S_{11}-spectra for (a) standard homogenous numerical model, (b) standard inhomogeneous numerical model and (c) miniaturized inhomogeneous numerical model.

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inclusions made in the unit cell. So, this is why you know that the values of ε m, μ m, and r can be evaluated using the effective medium theory. So, if you use that you can calculate the relative permittivity to be equal to 2, relative permeability will be close to 1 and you can obtain the wear radius to be 0.00009 meter. So, in this study they actually used finite difference time domain that is MTTD simulations of S_{11} spectra and the fur fields were calculated in E and H plane also that evaluated the efficiency power gain and the near field intensity around them.

Now, what does the antenna design model consider? So, four different design models were considered in this study. The first model considers the FDDT simulation for a patch antenna with a dielectric substrate. So, let us call it a standard homogeneous numerical model. The second model basically corresponds to FDDT simulation for patch antenna with composite substrate. So, we will call it a standard inhomogeneous numerical model or design.

The third model basically assumes the FDTD simulations for the miniaturized patch antenna. With the composite substrate obtained, which we will discuss in the next lecture. So, we will call this a miniaturized inhomogeneous numerical model. And finally, the fourth model considers the analytical simulations using derived equations for a patch antenna with a composite substrate. So, we will call it a standard inhomogeneous analytical model or design.

So, we will now use the above-given classification of antennas throughout the study. So, the commercial FDD software Planck FDD was basically used in this particular study, as reported in this book, for making the numerical simulations. So, in fact, the earlier obtained values of radius you know then this pitch d ε m μ m fr l and w are being used in the parameter settings of the software. For performing this above mentioned you know FDDT simulations for evaluating the antenna performance. The results you can see here, okay? The results of the FDDT simulations of S_{11} spectra for all the considered models are shown here.

So, here you can see that the blue line basically tells you about the standard homogeneous numerical model. This dashed line, which is B, tells you about the standard inhomogeneous numerical model, and the dotted line is basically from the miniaturized inhomogeneous numerical model. So, what you see here that there is a peak and throughout this study we are going to call this any peak that is appearing first as the main peak and as you can see the first peak in the S_{11} spectrum of each of this numerical model is basically shifted from the theoretical peak of 7.3 So, it basically comes from the effect that you know there is this fringe effect on the patch edges which are not accounted for also there are special dispersion in the case of homogeneous models. So, indeed, the numerical model with the dielectric substrate resonates at 7.

307 gigahertz, and the composite substrate basically resonates at 7.385 gigahertz, right? So, theoretically, they were supposed to be close to 7.3 gigahertz. Now, here note that this the proposed analytical model does not take into account the fringe effects on the edges of the patch and Therefore, you know by replacing a homogeneous high permittivity dielectric substrate with a inhomogeneous metal dielectric composite or metamaterial with the same value of both effective relative permittivity and permeability. will lead to redistribution of the effective will lead to redistribution of the reactive part of the load antenna impedance.

Therefore, by replacing a homogeneous high permittivity dielectric substrate with an inhomogeneous metal dielectric composite metamaterial with the same values of both effective relative permittivity and permeability leads to redistribution of the reactive part of load antenna impedance between patch and metal inclusions ok. So, it results in increasing the energy stored in the EM field radiated by the antenna due to a more effective use of the magnetic component of the radiated EM field compared to the case of the antenna with dielectric substrate. We can also observe from the figure that the antenna with a composite substrate is better impedance-matched. than the antenna with the dielectric substrate. So, you can see a better dip in the S_{11} spectrum, right? So, you need to keep these points in mind as they are important.

And with this, we will stop here. And in the next lecture, we will talk about miniature edge design and far-field patterns in the H-plane and E-plane for all these antenna models. So thank you if you have got any query regarding this lecture drop an email to this email address mentioning the course name and the lecture number on the subject line.

Slides inserted by fallback (review if needed):