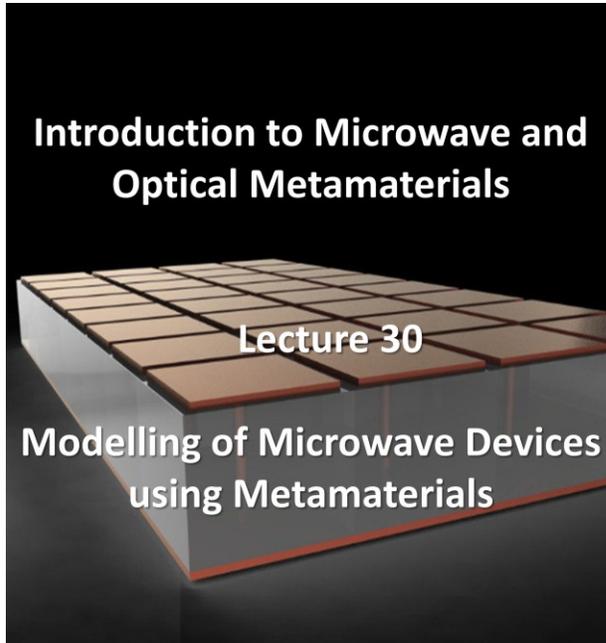


**Course Name: Introduction to Microwave and Optical Metamaterials**  
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**Department Name: Electronics and Electrical Department**  
**Institute Name: Indian Institute of Technology, Guwahati**  
**Week-6**  
**Lecture-30**

Lec 30: Modelling of Microwave devices using metamaterials



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## Lecture Outline

- Modelling of Microwave Devices using Metamaterials
- 2D Model of Microwave Antenna With One-layer Wire Composite/Metamaterial Substrate



Hello everyone, welcome to lecture 30 of the online course on

## Modelling of Microwave Devices using Metamaterials

- Microwave Device Evolution:
  - Advanced due to next-gen communication, sensing tech, and compact high-performance needs
- Role of Antennas:
  - Core components for efficient transmission and reception of electromagnetic waves
- Impact of Metamaterials:
  - Introduction of engineered electromagnetic properties not found in nature
  - Revolutionized modeling and performance of microwave antennas and devices
- Rapid Growth in Wireless Tech:
  - Driven by e-health, IoT, and smart buildings

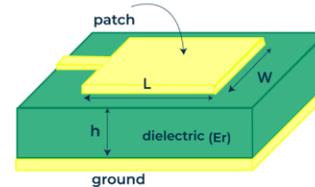


Source: Choudhury, P.K., Metamaterials: Technology and Applications, CRC Press, 2021

introduction to

## Modelling of Microwave Devices using Metamaterials

- Need for Smart Antennas:
  - Must handle broader spectrum efficiently
  - Should be energy-efficient, cost-effective, and compact
  - Must function in diverse environments: chip packaging, human body, matching networks, etc
- Patch Antennas: A Compact Wireless Solution:
  - Printed aperture antennas widely used for their:
    - ✓ Simple design
    - ✓ Low cost
    - ✓ Light weight
    - ✓ Low profile
    - ✓ Ease of integration with circuits and arrays



microwave and optical metamaterials. In this lecture, we will look into the modeling of microwave devices using metamaterials. So, here is the lecture outline we will go about discussing the modeling of microwave devices using metamaterials and we will take up some examples of how to demodel of a microwave antenna. with one layer where the composite metamaterial substrate looks like. So, we have seen microwave devices have evolved significantly / the past few decades mainly driven by the demands of the next generation communication systems, sensing technologies and compact high performance components. Among these devices, antennas serve as the fundamental building blocks responsible for the efficient transmission and reception of electromagnetic waves.

Now, with the emergence of metamaterials, which are basically engineered structures that exhibit unique electromagnetic properties not found in nature. You will see that the modeling and performance of microwave antennas and related devices have seen revolutionary enhancements. So, wireless technology has been growing exponentially since the beginning of this century due to applications in, you know, domains like e-health, IoT, and also smart buildings. So, this first growth can continue if we develop smart antenna systems that can combat a wider spectrum with energy efficiency.

So, they must be energy-efficient, cost-effective, and also compact, and must function in diverse environments. So, chip packaging, the human body, and, you know, matching networks, etc. So, they should be able to operate in all these different environments. So, one of the possible candidates for compact wireless antenna systems is the patch antenna, right? So, a patch antenna which is also known as a microstrip antenna ok is a type of antenna with low profile and is often printed directly onto a circuit board. It basically contains a metallic patch, as you can see here, typically rectangular or circular in shape.

It's mounted on a ground plane with a dielectric substrate in between. So the patch, as you can

see here, is basically the radiating element, which is made of a conductive material like copper etched into a specific shape. It can be a rectangle, square, or circle. Then you have a ground plane at the bottom. This is again a conductive layer that acts as a reflector and it provides reference for the radiating patch and then you have a substrate typically FR4 or Teflon is used ok that separates your patch and the ground plane.

Now, how does this patch antenna work? Patch antennas operate based on the principle of resonant cavities. So, when excited by a feed, this is the feed point, okay. So, it can be a coaxial feed or a microstrip line, which is shown here, okay. The patch acts as resonator creating a field so the patch antenna basically operate based on the principle of resonant cavities when excited by a feed typically a coaxial probe or a microstrip line which is shown here the patch acts as a resonator that creates a field between the patch and the ground plane so the edges of the patch this ones they basically you know radiate electromagnetic waves into space Creating the antenna's radiation pattern. Now, what are the advantages of using a patch antenna? The first thing is that it is low profile.

So, it is integrated into a flat substrate, making it suitable for mobile devices, GPS modules, and other compact applications. The another important factor is low cost because relatively inexpensive method for manufacturing such as you know, PCB printing can be used for this kind of Application. The third important point is that the design is simple, so it's easy to fabricate. You can simply go to a PCB manufacturer and get it done. And another important factor is versatility.

So it can be designed for various frequencies and polarization like linear, circular by adjusting the patch dimensions like length and width and also the feed method. Now, it has also come it comes with some disadvantages like the first thing would be narrow bandwidth. Typically, this kind of antenna has a narrow bandwidth, meaning that it operates well for a specific frequency band only. The gain is another factor, they may have typically bit lower gain as compared to other types of antennas and the feed network that is like where the signal gets connected to the patch can sometimes be complicated depending on the type of polarization. But overall, it has its own, you know, positives.

As I mentioned here, low design, simple design, low cost, lightweight, low profile, and easy to integrate with circuits and arrays. So, it makes it very popular. So, the design features of patch antennas make it tempting to miniaturize their volume profile by means of an increase in the dielectric constant of the substrate, right? So, indeed you can think of the simplest design of a rectangular patch antenna which is given by this formulas right so this is the same design but here with more technicalities so here you can see that this is the patch you have got  $l$  and this is the width  $w$  of the patch okay so you can see that the length of the patch can be calculated using this formula  $c / 2 f r c / 2 f r$  square root of  $\epsilon_r$  are effective -  $0.824 * D \epsilon_r$  effective +  $0.3 * W / D$  ok.

$D$  is basically the thickness of the dielectric slab, +  $0.264$ . The whole thing /  $\epsilon_r$  effective -  $0.258 * w / d + 0$ .

8, okay. So, what is this  $r \epsilon_r$  effective that is basically  $\epsilon_r + 1$  by  $2 + \epsilon_r - 1$  by  $2 *$ . Square root of

$1 + 12d$  by  $w$ , okay. I will not go into the derivation of this, okay, but here you can see  $w = c / 2 f_r$ . So, you can understand what is  $f_r$  that is the resonating frequency in the linear frequency right square root of  $10$  square root of  $2 / 1 + \epsilon_r$ . So, from that you can calculate what should be the you can also see what is  $L_a$  that is the pitch length ok or the periodicity or no  $L_a$ .

Sorry from here. So, from this you can also see what is  $L_a$  that is the length of the antenna that will be given by as  $L + 6 D$ . So,  $L_a$  is marked as this ok the overall length of the antenna. So,  $D$  is basically the particular thickness of the

## Modelling of Microwave Devices using Metamaterials

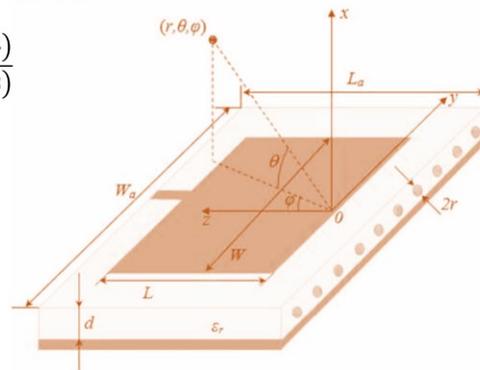
- Miniaturization Strategy:
  - Achieved by increasing the substrate's dielectric constant
  - Simplest rectangular patch antenna design follows defined formulas

$$L = \frac{c}{2f_r \sqrt{\epsilon_{\text{reff}}}} - 0.824 \cdot d \cdot \frac{(\epsilon_{\text{reff}} + 0.3)(W/d + 0.264)}{(\epsilon_{\text{reff}} - 0.258)(W/d + 0.8)}$$

$$\epsilon_{\text{reff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + 12 \frac{d}{W} \right]^{-1/2}$$

$$W = \frac{c}{2f_r} \sqrt{\frac{2}{1 + \epsilon_r}}$$

$$L_a = L + 6 \cdot d, W_a = W + 6 \frac{\log 16}{\pi} d$$



dielectric slab. in between and  $w_a$  that is the width of the antenna given as  $w$  that is the width of the patch +  $6 \log 16$  by  $\pi * d$ . So,  $L_a$  and  $L_w$  this basically gives you the length and width of the antenna.

$L$  and  $W$  gives you the length and width of the antenna right and  $\epsilon_r$  effective tells you about the effective relative effective index or you can say  $\epsilon_r$  effective is basically giving you the relative effective

## Modelling of Microwave Devices using Metamaterials

- Miniaturization via High Permittivity Substrates:
  - According to previous equations, increasing relative permittivity ( $\epsilon_r$ ) reduces patch antenna dimensions ( $L_a$  and  $W_a$ )
  - This principle inspired antenna miniaturization efforts in late 20<sup>th</sup> century

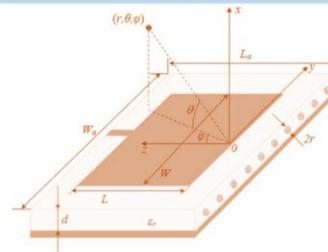
$$L = \frac{c}{2f_r \sqrt{\epsilon_{\text{reff}}}} - 0.824 \cdot d \cdot \left( \frac{\epsilon_{\text{reff}} + 0.3}{\epsilon_{\text{reff}} - 0.258} \right) (W/d + 0.264)$$

$$\epsilon_{\text{reff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + 12 \frac{d}{W} \right]^{-1/2}$$

$$W = \frac{c}{2f_r} \sqrt{\frac{2}{1 + \epsilon_r}}$$

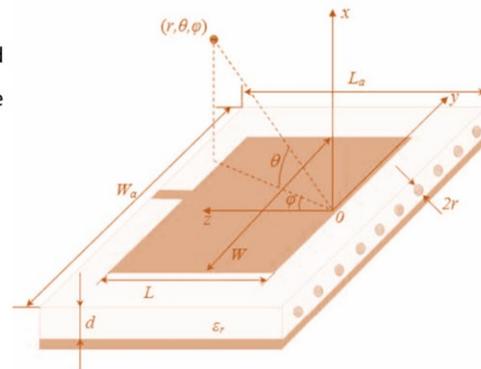
$$L_a = L + 6 \cdot d, W_a = W + 6 \frac{\log 16}{\pi} d$$

- Challenges Faced:
  - Performance degradation due to strong capacitive coupling between the antenna patch and ground plane



## Modelling of Microwave Devices using Metamaterials

- Solution: Use of Composites/Metamaterials:
  - Replace homogeneous high-permittivity materials with inhomogeneous composites/metamaterials
  - These allow tuning of effective permittivity and permeability through individual adjustments in the resonant inclusion dimensions within the unit cell
- Initial Approaches to Antenna Miniaturization:
  - Early designs used:
    - ✓ Double-negative metamaterials
    - ✓ Mu-negative ( $\mu$ -negative) materials
    - ✓ Low effective refractive index metamaterials



permittivity of the antenna. Now, looking from the equations one may state that the greater the value of the  $\epsilon_r$  that is the relative effective permeability, the smaller will be the linear dimensions that is  $L$  and  $W$  right. So, that means if you can increase the effective permeability of the dielectric material that is between the patch and the ground plane, you will get a overall smaller dimension of the patch antenna, right. And this principle leads to the idea of the miniaturization of the patch Indian antenna. So, people have tried different approaches to increase the relative permittivity of this antenna substrate to decrease the antenna volume profile, right? So, this has

required a lot of effort since we have seen significant efforts during the late 20th century for the miniaturization of antennas using this approach.

So, although miniaturization was achieved in this kind of study, the performance of such antennas was considerably degraded. And the reason is a strong capacitive coupling between the antenna patch and the ground plane. So you cannot ideally just keep on increasing the permittivity of the dielectric spacer in between, as that will also bring in side effects like this. So, what is the solution to this problem? So in order to decrease this kind of capacitive coupling ok, it is proposed to replace a

## Modelling of Microwave Devices using Metamaterials

- Less Common Yet Logical Alternative:
  - Metamaterials with an increase in the complex effective refractive index for creating the composite substrates
  - More logical substitute for homogeneous high-permittivity dielectric substrates, despite being less commonly used
- Historical Context:
  - The use of composites/metamaterials for miniaturized antennas was proposed early
  - Despite this, it remains a challenging problem in antenna design

homogeneous high

## Modelling of Microwave Devices using Metamaterials

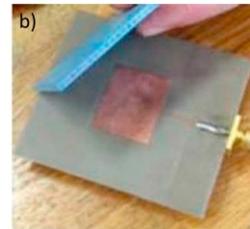
- Current Objective:
  - Develop a miniaturization concept for rectangular patch antennas using metal-dielectric composites/metamaterials
- Focus is on materials with:
  - Enhanced effective relative permittivity ( $\text{Re}(\epsilon_r) > 1$ )
  - Non-magnetic substrates ( $\text{Re}(\mu_r) \approx 1$ )

permittivity

## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

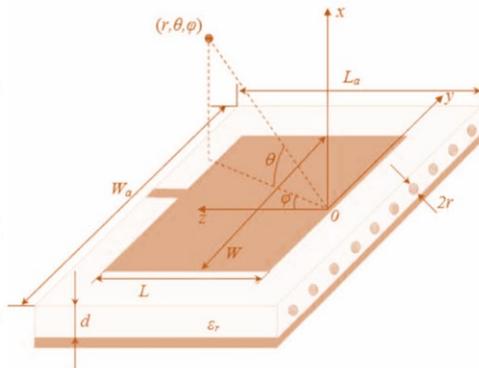
- Hypothesis Summary:
  - Tuning electric properties of composite substrates can enhance rectangular patch antenna performance
- Experimental Basis:
  - Zouganelis *et al.* showed that placing a metal-dielectric composite (dielectric matrix with wire grid) on top of a patch antenna improved its directivity
  - The composite:
    - ✓ Has a parallelepiped shape
    - ✓ Contains wires aligned parallel to the antenna patch



## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

- Theoretical Interpretation:
  - Initially explained via Snell's law by Rybin and Shulga
  - Rybin expanded the theory for wire grids embedded into the dielectric substrates
- Current Study:
  - Conducts full-wave analysis of a patch antenna with a one-layer wire grid embedded in the dielectric substrate
  - The analysis is based on the approach of given surface current distribution



dielectric by a inhomogeneous composite or metamaterial structure with the same value of constitutive parameters.

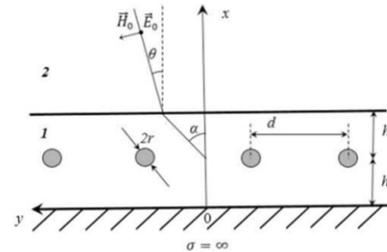
Okay. So, this will allow tuning of the effective permittivity and permeability through individual adjustments in the resonant inclusion dimensions within the unit cells. So, the initial approaches to antenna miniaturization involved trying some early designs using double negative metamaterials. Then, negative permeability materials have also been tried, including low effective refractive index metamaterials, and so on. So, metamaterials with an increase in the effective refractive index are much less commonly used for creating the composite substrate. The more logical substitute for homogeneous high permittivity dielectric substrate, but the less commonly used ones are basically those where you have you know a substrate with some sort of high value inclusions or high value permittivity inclusions right.

So, you can think of some composite metamaterials for this purpose. So, you can use composite metamaterials for miniaturized antennas, and that was proposed early. So, though there are some attempts made, it remains a challenging problem in antenna design. So, we will see some of those approaches today, okay. So, the current objective would be to develop a miniaturization concept for

## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

- Geometry Setup:
  - 2D rectangular patch antenna analyzed with respect to a Cartesian coordinate system (Figure)
  - Patch location: Positioned at  $x = 2h$ , spanning:
    - ✓ Width:  $-W/2 \leq y \leq W/2$
    - ✓ Length:  $-L/2 \leq z \leq L/2$
  - Patch is a perfect conductor with zero thickness
- Wire Grid Configuration:
  - Located at  $x = h$  and backed by a perfect conducting wall at  $x = 0$

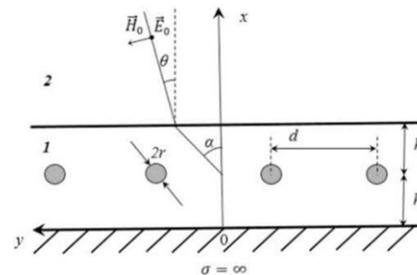


rectangular patch antennas using metal-

## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

- Wire details:
  - ✓ Made of copper, with circular cross-sections
  - ✓ Radius:  $r$
  - ✓ Aligned parallel to z-axis
  - ✓ Center-to-center spacing:  $d$
- Substrate Properties:
  - Dielectric region: Between  $0 \leq x \leq 2h$
  - Material properties:
    - ✓ Relative permittivity:  $\epsilon_m$
    - ✓ Relative permeability:  $\mu_m$



dielectric composites or metamaterials.

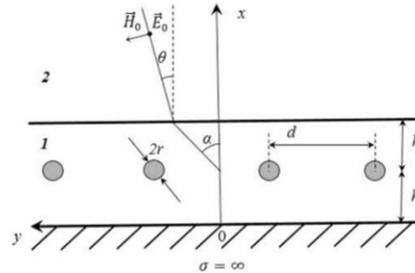
So, what are we going to do? We will focus on creating the concept of miniaturization of these rectangular patch antennas on metal dielectric composite or metamaterials with enhanced effective relative permittivity. In doing so, we are considering a non-magnetic substrate, which means you keep the real power of the permeability at 1. So, now let us look into the design more carefully, which I have been showing you for a long time. In this lecture, what is that one layer

where the composite metamaterial substrate is? So, that is basically the 2D model of the microwave antennas we will see now, okay. So, according to the hypothesis, tuning the electric properties of the composite

## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

- Relative Permittivity of Wires ( $\epsilon_i$ ):
  - Approximation:  $\epsilon_i(\omega) = 1 + \sigma_i / i\omega\epsilon_0$   
 where  $\epsilon_0$  is permittivity of vacuum and  $\sigma_i$  is conductivity of the wire material
- Relative Permeability of Wires ( $\mu_i$ ):
  - Approximation:  $\mu_i(\omega) = \frac{2 J_1(k_s r)}{k_s r J_0(k_s r)}$   
 where  $k_s(\omega) = k_0 \sqrt{\epsilon_i(\omega)}$ ,  $J_m(x)$  is  $m^{\text{th}}$  order Bessel function of a real variable  $x$  of the first kind,  $k_0$  is wave number in free space



substrates enables one to improve the performance of the rectangular patch antennas.

So, the idea of using a dielectric matrix with a inbuilt wire grid you can see here they were putting they are making some holes and putting some wires to make it a you know wire grid. So, the this kind of idea of using a dielectric matrix with a built in wire grid as composite basically rose when a relevant experiment was reported by Zoganelli's group ok. So, here you can see that putting a flat you know metal dielectric composite structure on the top and then on that you put this you know patch antenna basically can give you improved directivity of the antenna. So you get better performance from the antenna.

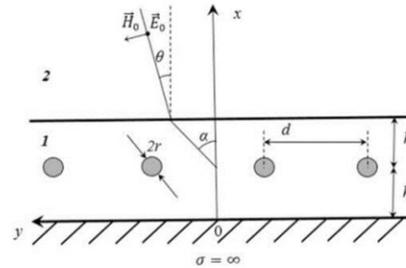
So, this is how it's done. So, you take the composite sample first ok and then you make the holes there and then in the same sample you basically put some iron wires into the hole ok and then you take this composite as your substrate and then print the patch antenna on top right. So, a few things to notice here are that this particular composite has a parallelepiped shape, okay. So, what is this? Parallelepiped is nothing but a three-dimensional figure with six parallelogram faces. And it contains wires that are aligned parallel to the patch antennas. So that is also an important requirement.

As you can see, this is the substrate, and then you have put the antenna patch on top of this substrate face. So, you have these wires which are parallel to the patch antenna. Now, the phenomena what is happening here was explained in short by Rubin and Shulga So, Rubin basically expanded the theory for wire grids embedded into dielectric

## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

- EM Wave Description:
  - A plane monochromatic EM wave of angular frequency  $\omega$
  - The electric field is parallel to the z-axis, with magnitude  $E_0$
  - Excited by a surface current  $j$  on the antenna patch
  - The wave strikes the wire grid at an angle of incidence  $\theta$



- The primary electric field intensity vector:

$$\mathbf{E}^{\text{in}} = E^{\text{in}}(x, y) \cdot \mathbf{z}_0 = E_0 e^{-ik_2(x \cos \theta + y \sin \theta)} \cdot \mathbf{z}_0$$

where  $k_2 = k_0$  is the wave number in the 2<sup>nd</sup> ( $x > 2h$ ) space domains which is the free space,  $E_0$  is amplitude of the incident electric field and  $\mathbf{z}_0$  unit vector along the z-axis

- The time factor  $e^{i\omega t}$  is omitted throughout the study.

substrates and in this current study now you can see what was this drawn I have never discussed about this. So, right now just focus, but until now we have seen this part, okay. Now, let us look into this part as well more carefully that your substrate is not basically a homogeneous substrate it has got wear holes with iron wires inserted through them and the axis is basically, these wires are basically parallel to your antenna patch.

So in this current study, we will basically present the full wave analysis of patch antenna with one layer wire grid embedded into the dielectric substrate as you can see here. And the analysis is based on the approach of the given surface current distribution, right? So, consider this 2D geometry of the rectangular patch antenna. which has a composite substrate, and we are currently showing you in the Cartesian coordinate system. So, the patch is positioned here, okay. So, this is the ground that has been marked.

So, you have this metal ground plane here. So, from 0 to 2 h, this is the height of the substrate, you know. So, your patch is basically positioned here. right and you are considering the width to be you know from w. So, you are basically having the y axis ranging from - w by 2 to w by 2 and the length will be along z which is into the plane of the screen ok.

So, the length is basically from - 1 by 2 to + 1 by 2 ok and we are considering the patch as a perfect conductor with 0 thickness Now, the wires are considered to be non-magnetic and this wire grid as you can see this is the metallic wire grid. So, they are a non-magnetic metallic wire grid, and it is located at a height of  $x = h$ . So, this is the height. So, each wire has a diameter of 2R, right? So, it is backed by a perfectly conducting wall that you can see here at  $x = 0$ . Now, that experiment was done with iron wires; here we are considering wires made of copper, and we have considered a circular cross section.

So, you can consider the radius of the wire to be r or the diameter is 2r and notice that they are all

aligned parallel to the z axis which is going into the screen. And another important parameter is the periodicity of these wires, which is the center-to-center spacing between the wires considered to be  $d$ . Now, what are the substrate properties? So, first thing is the substrate is in this region the dielectric is basically within the region of  $x = 0$  to  $x = 2h$ . So, that in between there and then it has got a material property like relative permittivity of  $\epsilon_m$  and relative permeability of  $\mu_m$ . Now, we also have to consider the permittivity of the wires that are included.

So, the microwave approximation of the relative permittivity of the wires  $\epsilon_i$  is given as you know  $\epsilon_i \omega = 1 + \sigma_i / i \omega \epsilon_0$  right. So, here  $\epsilon_0$  is basically the permittivity of the vacuum,  $\sigma_i$  is the conductivity of the wire material, right? And then you have an estimate of the relative permeability of the wires that is  $\mu_i$  and you can consider it to be you know  $\mu_i \omega = 2 / K_{sr}$   $J_1 K_{sr} / J_0 K_{sr}$ . So, here you can see  $K_{zs}$  is basically  $K_0 * \text{the square root of } \epsilon_i / \omega$ ; okay, that is the wave vector, or the wave number. to be precise and then  $J_m x$  is basically nothing but the  $m$ th order Bessel function of a real variable  $x$  ok. So, you can see the Bessel function of the first kind, and these are the orders.

$k_0$  here is basically nothing but the wave number. in the free space. So, first you have to estimate your  $\epsilon_i \omega$ ; from that, you can also estimate what  $\mu_i \omega$  is, right? Now, let us go into the description of electromagnetic waves. So, let us consider a plane monochromatic electromagnetic wave that has an angular frequency of  $\omega$ . An electric field component of magnitude  $E_0$ .

So, we are considering the electric field to be parallel to the z axis, right? So, the field is basically excited in such a way that you can see that the electric field vector is given as a dot. So, that is like going into the field into the screen that is along the Z axis ok and it excites you know a surface current  $J$  on the antenna patch and another important thing is that the wave is tracking the wave front at an angle of  $\theta$  right. So, after having these parameters, these are the two mediums marked 1 and 2. So, the primary electric field intensity vector  $E$  in can be written as  $E$  in as a function of  $x, y, z$  ok.

So, you are currently in this region. So, you can write  $E_0 e^{-i k_2 z}$ , which is  $E_0$ , as basically the incident electric field vector. So, you can write  $E_0 e^{-i k_2 z}$  that is where the propagation constant of the wave number in medium 2 is being used  $x \cos \theta + y \sin \theta * z$  ok. Now,  $k_2$  as I mentioned it is basically  $k_0$  only because it is considered to be air or free space ok and that is wave number in the second medium that is for the region  $x$  greater than  $2h$  this region ok. And  $E_0$  is basically the amplitude of the incident electric field, and  $z$  is the unit vector along the z axis. So, remember that in the study, the time you know factor  $e^{i \omega t}$  is omitted throughout.

So, we are not carrying that out in our work. So if you also neglect the

## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

- Neglecting evanescent waves and wire radius  $r \ll d$ ,  $r \ll h$  (thin wire approximation)
  - First Space Domain ( $0 < x < 2h$ ):

$$E_1 = E_1(x, y) \cdot z_0 = E_0 e^{ik_1 y \sin \alpha} (e^{ik_1 x \cos \alpha} + R e^{-ik_1 x \cos \alpha}) \cdot z_0$$

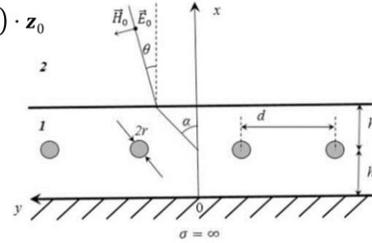
- Second Space Domain ( $x > 2h$ ):

$$E_2 = E_2(x, y) \cdot z_0 = A \cdot e^{-ik_2 y \sin \theta} e^{-ik_2 (x-2h) \cos \theta} \cdot z_0$$

- Here

$$R = -1 + G = -1 + \frac{(2Z_1 / \cos \theta) \sin^2(k_1 h \cos \theta)}{(Z_1 / 2 \cos \theta)(1 - e^{-2ik_1 h \cos \theta}) + Z_g}$$

where  $k_1 = \sqrt{\epsilon_m \mu_m} k_0$  and  $Z_1 = \sqrt{\mu_0 \mu_m / \epsilon_0 \epsilon_m}$



## 2D Model of Microwave Antenna

With One-layer Wire Composite/Metamaterial Substrate

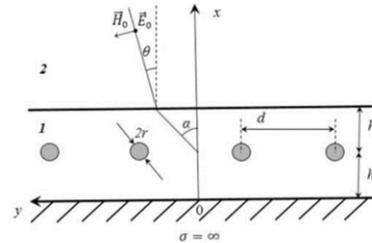
- The surface impedance of the wire grid ( $Z_g$ ) is given by:

$$Z_g = \frac{i\mu_0 \omega d}{2\pi} \left( \log \frac{d}{2\pi r} + \Delta \right) + (1 + i) \frac{d}{2\pi r} \sqrt{\frac{\mu_0 \omega}{2\sigma_i}}$$

where

$$\Delta = \frac{1}{2} \sum_{m=1}^{\infty} \left[ \frac{1 - e^{-4\pi(h/d) \sqrt{(m+(d/\lambda_1) \sin \theta)^2 - (d/\lambda_1)^2}}}{\sqrt{(m+(d/\lambda_1) \sin \theta)^2 - (d/\lambda_1)^2}} + \frac{1 - e^{-4\pi(h/d) \sqrt{(m-(d/\lambda_1) \sin \theta)^2 - (d/\lambda_1)^2}}}{\sqrt{(m-(d/\lambda_1) \sin \theta)^2 - (d/\lambda_1)^2}} - \frac{2}{m} \right]$$

- In the above equation,  $\lambda_1$  is the wavelength in the 1<sup>st</sup> space domain.



## 2D Model of Microwave Antenna

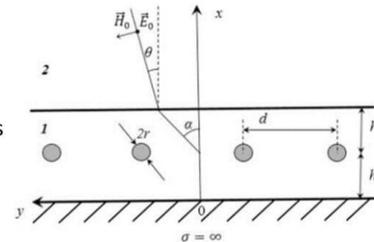
With One-layer Wire Composite/Metamaterial Substrate

- It is assumed throughout that  $d \ll \lambda_1$  and  $d = 2h$ , that is why  $\Delta$  is negligible compared with  $\log \frac{d}{2\pi r}$  and

$$\lim_{d/\lambda_1 \rightarrow 0} \Delta \Big|_{d=2h} = e^{-2\pi} / (e^{-2\pi} - 1)$$

- In this study, we neglect the evanescent waves.
- That is why it is unable to evaluate the antenna performance parameters associated with the near-field distribution.

$$Z_g = \frac{i\mu_0\omega d}{2\pi} \left( \log \frac{d}{2\pi r} + \Delta \right) + (1+i) \frac{d}{2\pi r} \sqrt{\frac{\mu_0\omega}{2\sigma_i}}$$



evanescent waves and you consider the wire radius to be much smaller compared to the periodicity of the wires and also the thickness of the substrate okay that means you are basically working in the thin wire approximation. you can safely write that in the first region that is the first space domain 1 that is where  $x$  is between 0 and  $2h$ . You can write  $E_1$  as  $E_1(x, y, z)$  naught. So,  $E_1$  you can write as  $E_1 e^{-k_1 y} \sin \alpha$ .

So, the wave makes an angle  $\alpha$  with the  $x$ -axis. So, you can also write  $e^{-k_1 y}$  to the power  $\cos \alpha$  +  $r$  will be basically the reflection that goes in the  $-x$  direction. So, you have  $e^{-k_1 x \cos \alpha}$ , right? In the second domain you can write that is  $x$  greater than  $2h$  domain you can write  $e_2 = e_2(x, y, z)$  naught that =  $a e^{-k_2 y} \sin \theta$ .  $e^{-k_2 y}$  to the power  $\cos \theta$  \*  $e^{-k_2(x-2h)}$  right. So, that is in this particular domain and here you can understand that  $r$  is basically the reflection coefficient which is given as  $-1 + g$  which is basically this particular constant ok. So, we can see here that all these parameters are known, other than this new parameter  $z$ , which I will tell you about, okay.

So, here we know how to calculate  $k_1$ ,  $k_1$  is basically square root of  $\epsilon_m \mu_m k_0$  and  $Z_1$  is basically the impedance of medium 1 that can be given as square root of  $\mu_m / \epsilon_m$ . So, what is that  $Z$  in the previous equation that is basically the surface impedance of the wire grid? And it is given by  $Z = i \mu_0 \omega d / 2\pi$  is all these parameters are known.  $\log \frac{d}{2\pi r} + \delta + 1 + i d / 2\pi r$  square root of  $\mu_0 \omega / 2\sigma_i$ . So,  $\sigma_i$  is basically the conductivity of the material of the wires and what is this  $\delta$ ?  $\Delta$  is basically this specific summation. So, half the summation from  $m = 1$  to infinity, and then you have these two parameters, right? I am not reading it out.

So, this derivation will not go into the details, but you if you can use this formula you can always compute what will be that two electric field as you can see in this case you can find out what is your  $E_1$  and  $E_2$  right. So, in this equation you can also see  $\lambda_1$  that is basically the

wavelength of the light or the wave that is in the first space region that is within this particular composite dielectric medium. So, in this equation, it is also assumed that  $D$ , which is the periodicity, is much, much smaller than this wavelength  $\lambda_1$ . And  $d$  is considered to be equal to  $2h$ , and that is why you know your  $\delta$  becomes negligible when compared to the log of  $d$  by  $2\pi r$ . And in that case you know you can find out the limit of  $\delta$  when the  $d/\lambda_0$  approaches 0 at  $d = h$  it can be written as  $e$  to the power  $-2\pi / e$  to the power  $-2\pi - 1$ .

So, things get much simpler here. So, we will continue this in the next lecture. and show you further how that can give you the better performance of the antenna by using this kind of a one layer wire composite structure. Remember that in this study we are also neglecting the evanescent wave and that is why it is unable to evaluate the antenna performance parameters associated with the near field distribution. So, with this we will stop here and in the next lecture we will continue from this part and further we will discuss about the 2D modeling of the microwave antenna that has got this kind of a one layer composite metamaterialized substrate right.

We will go into more detail in the next lecture. Thank you. So, if you have got any query regarding this lecture mention the course name and the lecture number on the subject line and drop an email to this particular email address. Thank you.



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*Thank You*