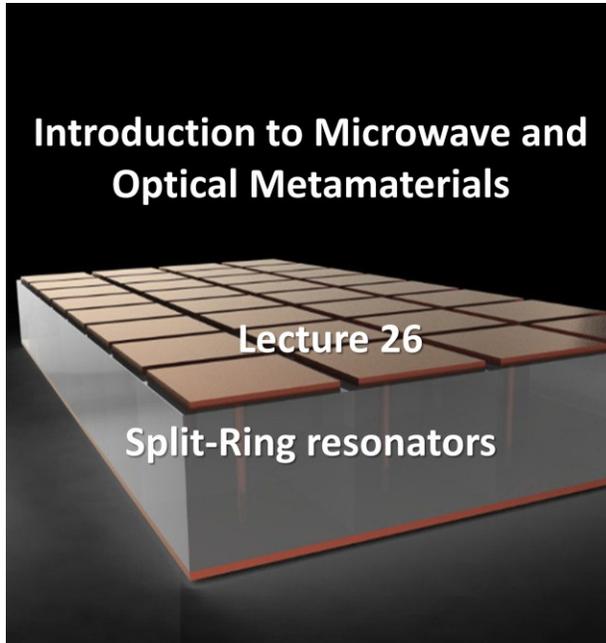**Course Name: Introduction to Microwave and Optical Metamaterials**
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Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-6
Lecture-26

Lec 26: Split Ring Resonators

Lecture Outline

- Negligible Optical Magnetism in Nature

- Split-Ring Resonators

- Equivalent Circuit of Split-Ring Resonator

- Split-Ring Resonators at Higher Frequencies

Hello everyone, welcome to Lecture 26 of the online course on Introduction to Microwave and Optical

Negligible Optical Magnetism in Nature

- At optical frequencies, materials typically exhibit different dielectric permittivity (ε) than in vacuum.

- Noble metals often show negative permittivity below their plasma frequency.

- Magnetic permeability (μ) in natural materials remains nearly equal to that of free space in the optical range.

- Optical magnetism is largely absent in nature; materials do not show a significant magnetic response at these frequencies.

- Landau and Lifshitz emphasized that magnetic permeability loses its physical relevance at optical frequencies.

- Thus, magnetic susceptibility is typically disregarded in optical frequency discussions, assuming $\mu = 1$.

Negligible Optical Magnetism in Nature

- Optical magnetism is largely absent because light's magnetic field interacts weakly with atoms compared to its electric field.

- Magnetic coupling is proportional to the Bohr magneton $\mu_\mathrm{B} = e\hbar/2m_\mathrm{e}c = \alpha e a_0/2$ while electric coupling is ea_0.

- The magnetic interaction includes the fine structure constant ($\alpha \approx 1/137$), making effect of light on the magnetic permeability α^2 times weaker than electric permittivity.

- As a result, only the electric component of light effectively interacts with atoms in conventional materials.

- The magnetic component of light has an indirect role governed by Maxwell's equations.

- Natural materials' magnetic responses fade above a few gigahertz.

Metamaterials. In today's lecture, we will be discussing splitting resonators and why they play a very important role in

Negligible Optical Magnetism in Nature

- Lack of free magnetic monopoles prevents forming magnetic plasmas, unlike electron-based electric plasmas.

- Achieving magnetic responses, especially negative permeability, at microwave or higher frequencies remains extremely challenging.

- Metamaterials have revolutionized the possibility of achieving magnetism at optical frequencies.

- Unlike natural materials, metamaterials use artificial, subwavelength structures called "meta-atoms."

- These meta-atoms allow light's magnetic field to interact effectively, enabling artificial magnetism.

- This interaction is not restricted to atomic spin states, which are typically ineffective at optical frequencies.

- Various structures enable this effect, including split-ring resonators (SRRs), metallic staples, coupled nanorods, nanoplates, and nanostrips.

metamaterial design. So, first we will talk about negligible optical magnetism in nature and then we will look into split ring resonators which allow you to have some considerable amount of permeability in this optical range. Ok, or even in the microwave range, mainly in the optical range. And then we will see the equivalent circuit of a splitting resonator and how splitting resonators can, you know, be designed at higher frequencies. So, let us discuss negligible optical magnetism in nature.

So, at optical frequency it has been found that all natural materials typically exhibit different dielectric permittivity ε ok than in vacuum right. We have also seen that noble metals often show negative permittivity below their plasma frequencies. Now, when you talk about the magnetic response, talk about magnetic permeability, μ. In natural materials, it has been observed that it remains nearly equal to that of free space in the optical range.

That means optical magnetism is largely absent in nature, and materials do not show a significant magnetic response at these frequencies. Right. So, indeed in the famous textbook by Landau and Lifshitz there is a quite general statement on the reason why a magnetic resonance resulting from orbital current in atoms should be negligible at the optical frequencies. And consequently, the magnetic permeability ceases to have any physical meaning at those frequencies, right? So, typically, people you know disregard magnetic susceptibility in discussions of optical frequencies, assuming that μ is 1. Now, optical magnetism is largely absent because light's magnetic field interacts very weakly with atoms compared to its electric field.

So, the problem comes from the magnetic coupling, right? Because the magnetic coupling is basically proportional to the Bohr magneton. So, you have μ B which = E h power / 2 ε E c. Now, μ bar is the natural unit for measuring the magnetic moment of an electron, okay. E is the charge; ℏ is the reduced Planck constant, okay. I am the electron mass, and A naught here is basically the Bohr radius, right? So, the magnetic coupling is proportional to the Bohr magneton ok, while the electric coupling is basically simply you can write it as E A naught ok.

So, now what happens is that there is the proportionality of this α; okay, the magnetic

Split-Ring Resonators

- Achieving magnetic activity at high frequencies is a key goal in metamaterial research.

- The weak interaction of light's magnetic field with natural materials can be sidestepped by using artificial metamaterials.

- Pendry proposed – concentric split-ring resonators (SRRs) with subwavelength dimensions, oriented in opposite directions.

- These SRRs generate an effective permeability different from unity, allowing magnetic behavior at microwave frequencies.

- The term "split-ring resonator" predates Pendry's work, originally described by Hardy in the 1980s.

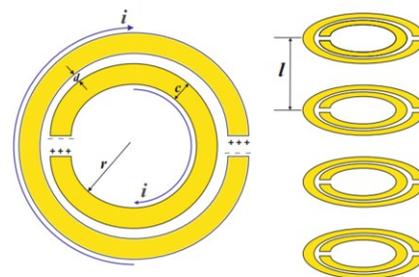- Pendry's reinvention of SRRs laid the foundation for many magnetic elements in modern metamaterials.

Fig. The double split-ring resonator as the building block of a metamaterial with magnetic response at microwave frequencies.

interaction basically includes this α. The fine structure constant, which is the α value, is small; it is 1 / 137. So, because of that, it makes the effect of light on the magnetic

- The SRR is the foundational magnetic meta-atom – central to artificial magnetism at microwave frequencies.

- Understanding its development starts with examining a simple metal plate in an oscillating electromagnetic wave.

- When the magnetic field is perpendicular to the metal plate, a circular current is induced.

- This current generates a magnetic flux that opposes the external field, demonstrating Lenz's law.

- Consequently, a metal plate is weakly diamagnetic.

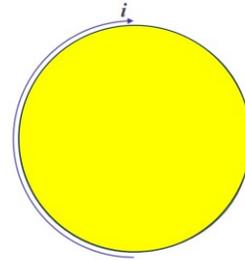- Arrays of such metal plates show an effective permeability just below one.

Fig. A metal plate with magnetic field perpendicular to it

permeability α squared * weaker than that of the electric permittivity. So, you can now understand that why the magnetic permeability because of light is very weak where else you know the interaction in terms of electric field is much stronger. So, as a result, only the electric component of the light can effectively interact with the atoms in conventional materials.

 And However, you know you it is not that you know the magnetic field is not playing any role. The magnetic field component of the light basically plays an auxiliary role or a indirect role through its relation with the electric field which is governed by the Maxwell's equation we have all seen. So, the magnetic response which is mainly due to the electronic spin states ah in naturally occurring materials diminishes at frequencies which are higher than a few gigahertz. So, below that, you can have some magnetic response, but after that, beyond a few gigahertz, the magnetic response is very weak, okay. Moreover, there are no free magnetic monopoles, and thus it is not feasible to obtain a magnetic plasma as we can obtain for electrons, right? So, it is indeed challenging issue to achieve any magnetic response in the microwave or higher

- Circular current in a metal plate is mainly around the perimeter – removing the inner part creates a metallic ring.

- A metallic ring alone is inductive and lacks resonance.

- Introducing a gap creates capacitance – enabling resonance.

- Metallic ring prevents the formation of a complete circular current, and charges accumulate across the gaps.

- The combination of inductance and capacitance makes the split-ring resonator (SRR) a resonant magnetic element.

- Double SRRs (two concentric rings with opposing gaps) enhance capacitance more effectively than single SRRs.
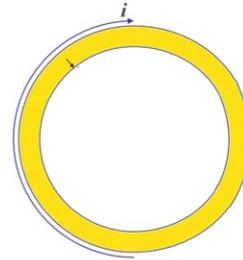
Fig. Metallic ring with a slit - split-ring resonator as the building block of a metamaterial with magnetic response at microwave frequencies.

frequencies ok, let alone you know extreme values like negative permeability and all.

So, you are not at all getting any simple response from the magnetic part of the electromagnetic field at these high frequencies. So, the metamaterials have basically revolutionized the possibility of achieving magnetism at optical frequencies. So, unlike natural materials, we all know that metamaterials are basically based on artificial sub-wavelength structures, which are known as meta-atoms. And these meta atoms basically allow the magnetic field of the light to interact more effectively with the meta atom, and that can give rise to artificial magnetism.

Okay. So, this interaction is not restricted to the atomic spin states, which are typically ineffective at optical

- Meta-atom – the pair of rings in a double SRR.

- Double SRRs are preferred as they:
  - Minimize electric polarizability
  - Reduce electric dipole effects due to charge accumulation
  - Allow the magnetic dipole moment to dominate

- Therefore, a standard double SRR structure is most often used in SRR-based metamaterial designs.

- Each split-ring resonator (SRR) must be much smaller than the free space wavelength.

- Ensuring subwavelength SRR size is critical for achieving desired metamaterial behavior at the target frequency range.
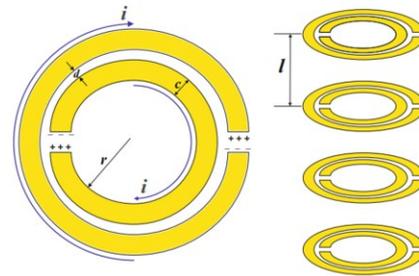

Fig. The double split-ring resonator as the building block of a metamaterial with magnetic response at microwave frequencies.

frequencies, right? So, in this particular discussion we will focus on a series of magnetic elements which are developed during the last few decades in the metamaterial research, including the splittering resonators, you know, metallic staple arrays, ok, coupled nano rods, nano plates, nano strips, ok. So, the magnetic response in this kind of structure ranges from, say, the microwave frequencies up to the entire visible spectrum. So, you can see exactly how artificial magnetism can be made real using this kind of metamaterials. In the pursuit of magnetically active structures at high frequencies, which is one of the core area of the metamaterial research, people have seen that splittering resonators are very important. So, why are they interesting? Because you can design a structure that can boost the interaction of the elements with the light's magnetic field.

So, in this regard for the microwave range a recipe was suggested by Sir John Pendry where he predicted that a pair of concentric splittering resonators ok these are also called double splittering resonators with sub wavelength dimension and they are facing in the opposite direction They can create this kind of effect. So, these splitting resonators generate an effective permeability that is different from unity, which means they exhibit magnetic behavior or response at microwave frequencies. Now, the concept and design of splitting resonators predates Pendry's work. Originally, it was described by Hardy in 1990s, but you know John Pendry's reinvention of SRR ah in the ah 19 it is almost like you know close to year 2000 ok 1999 or 2000 that is when he could laid the foundation of many ah magnetic elements in the modern metamaterial research and from there you know the metamaterial research has gained momentum. So, the split ring resonator is a foundational magnetic meta-atom that is central to the artificial magnetism at microwave frequencies.

So, understanding its development will basically start with examining a simple metallic plate in an oscillating electromagnetic field like this. So, you can think of a metallic plate where the magnetic field is basically perpendicular to it, and a circular current like this will be induced in the plate. Now, this current will basically generate a magnetic flux that opposes the external field

and is described well by Lenz's law. So that means this metallic plate behaves like a weakly diamagnetic material. Now, an array of such metallic plates can show effective permeability just below 1.

Now, in the circular current in a metallic plate, you will see that it is mainly around the perimeter. So you can safely remove the inner part and create a metallic ring. Now, if you just think of the metallic ring alone, that is inductive, isn't it? And it lacks resonance. So if you introduce a gap in this ring, that can introduce

Equivalent Circuit of Split-Ring Resonator

- Each SRR acts like a quasistatic LC circuit under external field excitation.

- The ring structure provides the inductive element, with estimated inductance $L \approx 2\mu_0 r$.

- Capacitance arises from both the inter-ring slot and the cuts in individual rings.

- The effective current flow and charge distribution create a complex path, involving conductive and displacement currents across both rings.
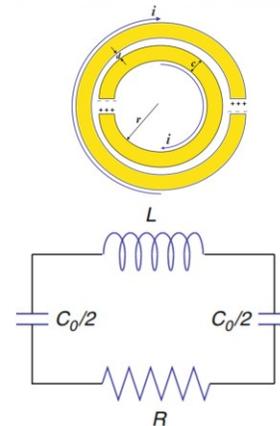
Fig. The equivalent circuit of a double SRR.

- Dominant capacitance arises between the two rings while gap capacitance within each ring is minimal.

- Total capacitance formula: $C_0 \approx 2\pi r \varepsilon_0 (2c)/d$
  (but not used directly for resonance calculations).

- Because the total capacitance of a double SRR – series connection of two separated capacitors

- The effective capacitance of a double SRR is approximated as $C = C_0/4$, since C_0 splits as two series capacitors.

- Ohmic losses are modeled as resistance $R \approx \pi r / c\delta\sigma$ where δ is skin depth and σ is metal conductivity.
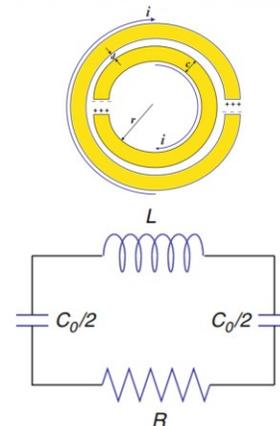
Fig. The equivalent circuit of a double SRR.

capacitance, and you have L and C, so you can enable resonance, okay. So, as you can see in the figure, a gap in the metallic ring prevents the formation of a complete circular current, and charges will accumulate across the gap.

So, you will get a capacitance here, right? So, the capacitance is effectively introduced. You can also place two rings next to each other. So, you can also introduce, you know, the mutual capacitance between the two, right? So, first you need to understand one element of how it is working. So, you can see that this inductance and capacitance make the split ring resonator work like a magnetic element. Now, when you bring another one, a concentric split ring.

ok that can enhance the capacitance effect more effectively as compared to ah the splitting resonator because here only this was contributing to the capacitance. But here, when you have the induced current, you put it in the opposite direction. So, the current induced is in this this direction as this one is also in this direction. So, you can actually have this two giving you the mutual capacitance and that can actually help you tune the resonance and we will see that in the subsequent slides. So, let us consider this particular design, which is a double splitting resonator that is nothing but a pair of rings considered as the meta atom.

So, why a double spectric resonators are preferred because they minimize the electric polarizability and they also reduce the effect of electric dipole because there are two opposite dipoles created and in that case the magnetic dipole moment will be able to dominate. Right. So, that is why you know as standard double splittering resonator is mainly used as this metamaterial meta atom not a single one because once again in a single SRR the cumulative charges you will see that will be around the gap and that will give you a pronounced electric dipole moment. And that may overshadow the desired magnetic dipole moment. But when you put the two SRRs in this particular way, what will happen? You have a dipole moment like this; you have a dipole moment in the opposite direction.

So they will basically cancel each other out, okay? They will try to cancel each other out, and then the magnetic moment will dominate, and that is what you want, right? So one thing it's important to remember is that the goal of the design is to create artificial magnetic atoms for a metamaterial. So, you have to ensure that the size of each splitting resonator is much smaller than the free space wavelength, okay? So, the effective current flow and the charge distribution you can see can create a complex path. So, you have conductive and displacement current across both rings. Now you have to see that the dominant capacitance basically arises between the two rings because you have a much larger length that is being covered. And the gap capacitance within each ring is basically much smaller than that.

So, if you consider the total capacitance to be c naught which can be approximated as 2 π r ε naught 2 c/d. So, each ring is basically having width of c, d is basically the gap between the 2 rings So, they cannot be directly used for resonance calculation though because the total capacitance of the double splitting resonator is basically a series combination of the two separated capacitors. So, the effective capacitance will be like you can put C naught / 2 and C naught. So, they are in series. So, the effective capacitance of this double splitting resonator is basically C naught by 4 right and you can consider the ohmic losses because of the metallic these are like metallic strips right.

So, the conductivity and the skin depth of the material, I will tell you about the resistance this guy is going to offer. So, you can estimate the resistance as π r / c δ σ, δ is the skin depth and σ is basically the metal conductivity. So, with that, you get an

Equivalent Circuit of Split-Ring Resonator

- The full RLC circuit has a resonance frequency approximated by
$$\omega_0 = \sqrt{1/(L + R/j\omega_0)C}$$

- Despite being a simplified model, this LC circuit analogy effectively estimates resonance and helps understand geometry-dependent behavior.

- Magnetic response strength and effective permeability in an SRR array can be approximated using the magnetic moment of each unit $m_H = \pi r^2 I$.

- Magnetic moment expression:
$$m_H = \pi^2 r^4 \mu_0 H/(\omega_0^2/\omega^2 - 1)L$$

  where H: external magnetic field; r: SRR radius; L: inductance; ω_0: resonance frequency; ω: operating frequency.
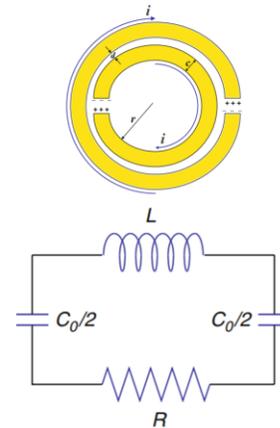
Fig. The equivalent circuit of a double SRR.

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- Effective permeability of the array: $\mu_{\text{eff}} = 1 + m_H/V\boldsymbol{H}$
  where V is the unit cell volume.

- Response type (diamagnetic or paramagnetic) depends on whether ω is below or above ω_0.

- Hence, μ_{eff} is different from unity, even though the natural materials comprising the SRR array have unity permeability at frequencies at about a gigahertz or higher.

- This estimation ignores key behaviors like electric coupling and bianisotropy; more accurate modeling requires analytical and numerical methods.
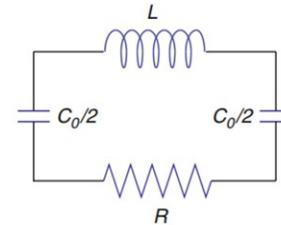
Fig. The equivalent circuit of a double SRR.

- SRRs in the GHz range were first implemented by Smith *et al.*, with ring diameters of several millimeters.

- Double SRRs for microwave applications are commonly fabricated using PCB (Printed Circuit Board) technology.

- These SRRs serve as magnetic components in various microwave-frequency metamaterial applications.

- Notably, SRRs enabled the first experimental realization of negative-index materials.

- The double SRR design's publication (Figure) marked the first explicit use of the term "metamaterial" in literature.
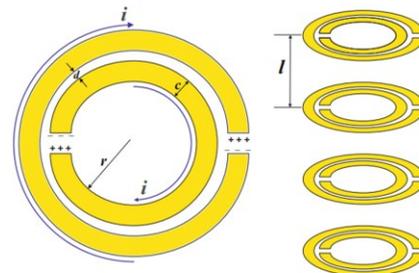
Fig. The double split-ring resonator as the building block of a metamaterial with magnetic response at microwave frequencies.

equivalent circuit model of this double split-ring resonator, and the full RLC circuit will give you the resonance frequency estimation. So, that comes out to be ω naught which square root of 1 by L + r/j ω naught * C. So, this is a very simple model, but now with this kind of LC circuit analogy you can effectively estimate the resonance and it also allows you to understand that how geometric parameters like C then d this is the gap okay the overall length which is which can be obtained from this radius then you have c then you have d again you have c.

So, all these parameters can tell you that how it is getting correlated with the resonance frequency. Now the magnetic response strength and the effective permeability in an array of this kind of splittering resonators can be approximated using the magnetic moment of each unit. So you can calculate m h magnetic moment as π r square i. So you can write this moment expression as m h can be also written as in this case π square r to the power 4 μ naught h / ω naught square by ω square - 1 * L. So, here you can see that h is basically the external magnetic field which is introducing the current I and then the radius of the splitting resonator is considered to be R, L is basically the inductance, ω naught is the resonance frequency and ω is the operating frequency right.

So, the effective permeability of the array, μ effective, can be estimated as 1 + m h / v h, where v is basically the unit cell volume. Now the type of response whether it is diamagnetic or paramagnetic that depends on whether your ω is above or below the plasma that resonance frequency right ω naught. So here you can see that you know you are getting ε effective different from unity even though the natural materials comprising the SRR array have unity permeability at the

Split-Ring Resonators at Higher Frequencies

- After the microwave-range success of artificial magnetism, efforts shifted to infrared and optical magnetic metamaterials.

- Theoretically, SRR designs can be scaled down to reach higher frequencies, leveraging Maxwell's equation scalability.

- Practically, metals exhibit different behaviors in optical frequencies (e.g – plasmon resonance) versus microwave frequencies.

- Simple downscaling of double SRRs is limited by the complexities and challenges in nanoscale fabrication.

- As SRR sizes shrink, standard PCB methods are inadequate – optical or electron-beam lithography is required.

- Lithographic fabrication of submicron metal structures typically follows a layer-by-layer approach.

- SRRs need their plane to align with the incident wave-vector for effective magnetic field interaction.

- Flat substrate placement of SRRs complicates this alignment.

- Oblique incidence of electromagnetic waves helps ensure the magnetic field penetrates SRR planes effectively.

- Scaling and inclined incidence techniques have enabled SRR resonance frequencies to reach up to 1 THz.

- A sample with terahertz SRRs and the associated polarization condition is illustrated in the figure.
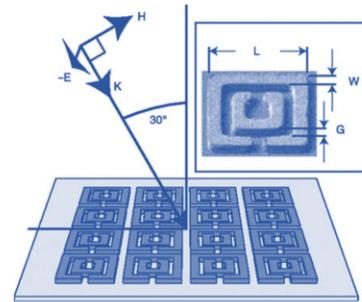
Fig. Double SRR structure with terahertz magnetic response using oblique incidence.

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Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

- A terahertz SRR sample features elements around $40\ \mu m$ in size.

- Near resonance frequency, the effective permeability shows a Lorentzian spectral profile.

- The permeability ranges from a minimum of -1 to a maximum of 4 at resonance.
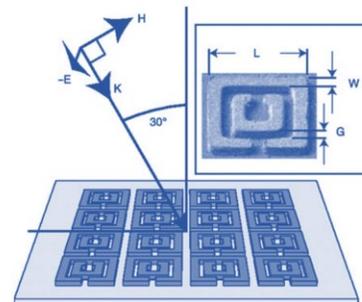
Fig. Double SRR structure with terahertz magnetic response using oblique incidence.

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Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

frequencies at about you know gigahertz or anything above. So, the constituent materials they do not have any permeability, they have unit permeability whereas the effective permeability from the structure that is your μ effective is different from unity. So, definitely this estimation is giving you something new, although it is ignoring some key behaviors, such as electric coupling by anisotropy and all these things.

So, you can always do more accurate modeling using some kind of analytical and numerical

methods that have already been extensively done in the literature. Now the split ring resonators designed in the gigahertz range were first implemented by Smith's group, I believe in Duke University, where the ring diameters were several millimeters length. So, double split resonators were first designed for microwave applications, which were commonly fabricated using printed circuit board technology. So, this kind of split ring resonator serves as a magnetic component in various microwave frequency metamaterial applications. And that enabled people to also demonstrate the first experimental realization of a negative index metamaterial.

So, you can make an array of this kind of splitting resonators and couple it to the array where the array is okay. So, I believe we have already discussed those. So, if the wire array can give you negative permittivity and this array of splitting resonators can give you negative permeability, when you can have both permeability and permittivity negative / the same frequency range you can get negative refractive index. So, this double splittering resonator marked the first explicit use of the term metamaterial in the literature and that is where from there the field of metamaterial just took off. Now, what is important to notice that after microwave range success of artificial magnetism people working in the infrared and optical field they also you know tried shifting this towards the infrared and the optical range.

So, you have infrared and optical metamaterials So, in that case what you have to do theoretically when the frequency is going up you have to scale down the split-trig resonator design. So, that allows you to know Maxwell's equation scalability, okay; frequency scaling, you can say. Now, metals exhibit very different behavior in optical frequencies; especially, you know, they show + 1 resonance. So, as compared to the micro frequencies, right when you are there, you do not have all those special effects coming in. So, simply downscaling double splitting resonators will not work because there will be some complexities and challenges also coming from nanoscale fabrication.

Right, and as the size of the splitting resonators becomes smaller, you cannot even use the standard PCB printing method. So, you have to go for optical or e-beam lithography, right? So, people have done, you know, lithography fabrication of submicron metallic structures using, you know, a layer-by-layer approach. So, splitting resonators need their planes to align with the incident wave vector for this effective magnetic field interaction. However, it is hard to accomplish when the splitting resonators are basically lying flat on a substrate. One easy method to partially solve this issue is to use oblique incidence for an external electromagnetic field.

So, this will provide a magnetic field that obliquely penetrates the splitting resonator plane and using scaling techniques along with inclined incidence, the resonance frequency of double splitting resonators have been pushed up to 1 gigahertz. So, here is the sample of terahertz splitting resonators with the oblique incidence technique that is shown here. So, that was giving a magnetic response in the terahertz regime, right? So, in this particular figure, you can see that the dimensions of these elements are typically 40 microns. The resonance and property were coming around, I think, at 1 terahertz. Near the resonance frequency, the effective permeability shows a typical Lorentzian spectral profile.

and the permeability ranges from a minimum of -4 to a maximum of 4 at resonance. So, you are able to get both negative permeability at some particular frequencies and you are also able to get permeability which is much larger than unity using this kind of a structure. So, with that, we will stop here for this lecture. So, we will continue our discussion of optical magnetic elements in the next lecture and you will see that how this elements can be used for making metamaterials working at higher frequencies. So, with that we will stop here if you have got any query on this lecture drop an email to this email address mentioning the course title and lecture number in the subject line. Thank you.

Thank You

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Thank You