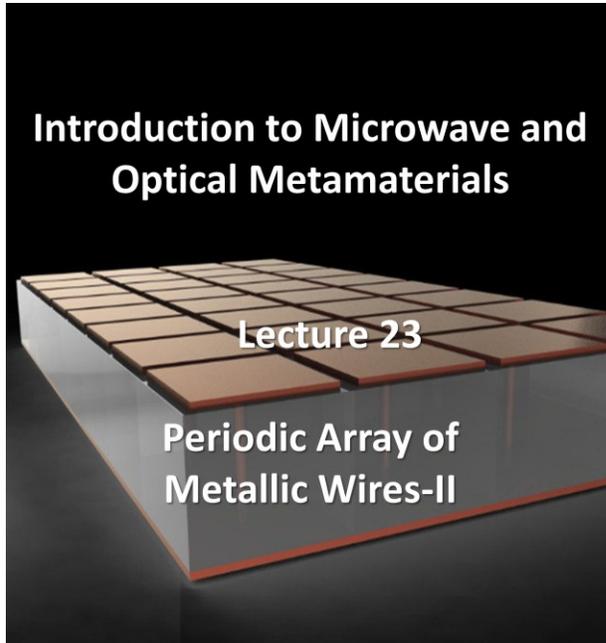**Course Name: Introduction to Microwave and Optical Metamaterials**
Professor Name: Dr. Debabrata Sikdar
Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-5
Lecture-23

Lec 23: Periodic Array of Metallic Wires-II
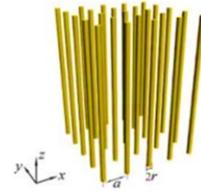
- Example of a Wire Medium

- Wire Medium at Other Wavelengths and its
  Applications

- Semicontinuous Metal Films

Hello students, welcome to lecture 23 of the online course on Introduction to Microwave and Optical

Example of a Wire Medium

- Continued example – A silver wire medium with a plasma frequency in the gigahertz range with wire radius: $r = 5\,\mu m$ & lattice constant: $a = 40\,mm$.

- Results of plasma frequency:
  - Pendry's model : $\omega_p = 2\pi \times 1.0$ GHz
  - Alternative models: $\omega_p = 2\pi \times 1.1$ GHz

- As for the dissipation in the wire medium, from earlier equation we obtain a damping constant of $\Gamma = 0.018\omega_p$

- This gives a loss factor similar to that of conventional conductors:
  - For comparison: $\Gamma/\omega_p = 0.011$ for copper

- Like a Drude metal, the wire array metamaterial exhibits:
  - Negative permittivity for frequencies below the plasma frequency
  - Positive permittivity above the plasma frequency, acting as a transparent dielectric

- This behavior supports applications in filters and polarizers.

- <u>Example:</u> A plasmonic high-pass filter using a 2D cubic lattice of gold wires for terahertz waves has been reported by Wu *et al*.

- In practice, wires need not be pure metal:
  - A dielectric skeleton coated with metal is equally effective
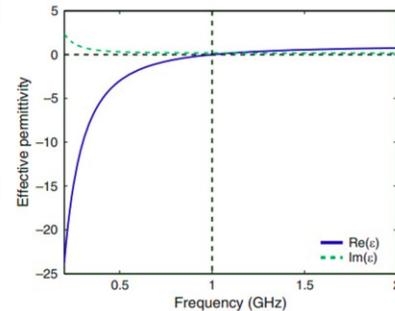 - Metal coating must exceed the skin depth at the operating frequency

Fig. The dielectric function of a silver wire array with r = 5 μm and a = 40 mm.

- This method is especially useful when working with expensive metals like silver or gold.

- The wire array in the example fits well within the effective-medium approximation:
  - Lattice constant $a \approx \lambda/10$ at 1 GHz (where $\lambda = 0.3$ m)

- The metal filling fraction is extremely low—only a few parts per million.

- Despite this, a significant change in effective permittivity $\varepsilon(\omega)$ is observed.

- This is explained by the near-zero depolarization factor when:
  - The incident electric field is aligned with the wires, resulting in minimal screening

- Even small metal concentrations significantly affect the effective dielectric response in this configuration.

- The permittivity remains equal to that of the host medium – as for electric field perpendicular to the wires have negligible interaction.

- The effective permittivity $\varepsilon_{\text{eff}}(\omega) = 1 - \dfrac{\omega_p^2}{\omega(\omega + i\varepsilon_0 a^2 \omega_p^2/\pi r^2 \sigma)}$ for the wire array assumes wires are made of a quasi-perfect conductor: with bulk permittivity $\rightarrow \infty$

- Finite conductivity introduces the damping constant Γ.

- Initially, the effective plasma frequency ω_p appears independent of metal properties.

Metamaterials. In this lecture, we will continue discussing the periodic array of metallic wires. So, here is the lecture outline; we will continue with the example of the wire medium. We will discuss about wire medium at other wavelengths and its applications and then we will discuss about different aspects of semi continuous metal films. So, here we continue with our example. Of the wire medium, where you can see a 2D silver array.

So, this has a plasma frequency in the gigahertz range. So, if you look at the structure carefully you can see that the wire radius which is r here taken to be 5 micron ok and the lattice constant a is of the order of 40 millimeter. Right. So, we also consider like the electron density in this case will be around 5.

8 into 10 to the power 28 per meter cube and the conductivity was around 6.3 into 10 to the power 7 centimeters per meter. So, this is something we saw in the previous lecture. So, when you calculate the plasma frequency ah from ah Penrith's model you could achieve something like this ω p will be equal to 2 π into ah 1 gigahertz ok. And then the alternative models gave you a similar range, but slightly different.

So, it was something like ω p = 2 π * 1.1 gigahertz, right? So, as for the dispersion in the wire medium ok ah the earlier equation can give us the damping constant of γ = 0.018 ω p. Now, this is something very interesting because this gives a loss factor which is similar to the conventional conductors. Why I am saying that because you can see for comparison if you see this factor γ / ω p for copper that is around 0.

011. And for this artificial dielectric, you are actually getting something of the same order, okay. So, that makes it very interesting that you get an artificial dielectric with a similar kind of loss factor to that of a conventional conductor. Now, the

- At visible frequencies, metals follow the Drude model:

$$\varepsilon_m = 1 - \frac{\omega_{p,m}^2}{\omega^2}$$

- Here, metal permittivity has limited bandwidth, and ω_p depends on the bulk metal plasma frequency $\omega_{p,m}$

- Adjusted plasma frequency expression:

$$\omega_p^2 = \frac{\pi r^2 \omega_{p,m}^2}{a^2}$$

- This equation indicates that the effective plasma frequency ω_p is proportional to the wire volume fraction.

effective permittivity of this example wear medium can be seen in this particular figure. So, similar to a Drude model this where array metamaterial also has got some negative permittivity for the electromagnetic frequencies which are lower than the effective plasma frequency this is the plasma frequency ok. You can normalize the graph, okay.

So, here it was ω p, which was 2 π * 1 gigahertz. So, you can understand that 1 is the linear frequency, okay, where you have this plasma frequency happening, right? Above this plasma frequency, you can understand that the effective

- This aligns with intuition:
  - As the metal filling factor increases, ω_p approaches the bulk metal plasma frequency $\omega_{p,m}$
  - When the filling factor nears unity, the wire medium behaves more like bulk metal

- At optical frequencies, wire arrays are used in a wide variety of structures and devices.

- In negative-index metamaterials:
  - Metal wire arrays create a tunable negative permittivity background essential for achieving negative index behavior

- Wire media are also used to design materials with ultralow refractive indices (< 1).

- These occur at frequencies just above the effective plasma frequency.

permittivity is basically positive and the medium will act as a transparent dielectric, right? Now, this particular behavior can support applications such as in filters and polarizers right. So, below this frequency it will basically reflect. Below this frequency, the incident light can actually pass through the material, the rodded medium, or the wire medium, right? So, you can take an example of a plasmonic high pass filter that is made using a 2D cubic lattice of gold wires that is working for the terahertz waves and that was reported by group of U. Now, it is also noteworthy that in practical applications, these wires do not necessarily consist of pure metals.

So, instead, you can think of a dielectric skeleton. forming the lattice and it is coated with the desired metal and that will work equally well as the pure metal case as long as the metallic coating is thicker than the metal's skin depth at that particular operating frequency. So why do we need to do that? Because this particular method will ensure that your material cost is kept low. Because when you are working with expensive material like gold and silver, it is difficult to, it will be expensive to have entire wear made of that pure metal. You can actually go with that kind of coated structure.

So, the where array in this particular example is comfortably within the applicability regime of the effective medium theory because the lattice constant is considered to be you know of the order of λ by 10 at 1 gigahertz. So, their λ is around 0.3 meters. So, you can see that you know the lattice constant was 40 mm, okay. So, it is on the order of 0.

3 meters by 10. So, you can calculate that okay, and you will see there is the same order. Okay, and another important factor is that the metal filling fraction is extremely low, which means only a few parts per milliliter, right? Now the finite conductivity in the metal will give rise to the damping if constant of the effective medium while the effective plasma frequency at first appears to be independent of the metal properties right. So, the assumption the effective

Wire Medium at Other Wavelengths and its Applications

- Experimental demonstrations of low-index wire media include:
    - Total external reflection
    - Air-core planar dielectric waveguides
    - Power splitters for air-core waveguides

- These analysis applies specifically to the ideal case:
    - Where the electric field is parallel to the wires

- If the external field has an arbitrary orientation, the wire mesh may support additional modes.

- In such cases, a local permittivity model is insufficient.

- The medium exhibits non-local (spatially dispersive) behavior:
    - Induced polarization depends not just on local electric fields, but also on field variations across space

permittivity that we have seen is typically independent of the metal properties and this assumption holds only for long wavelengths that is beyond the infrared. where the novel metals typically have very large negative permittivity.

Now, at the visible frequencies, the metals basically follow the Drude model. So, it follows this simple relation: $\varepsilon_m = 1 - \omega^2_{pm} / \omega^2$. So, this m specifically tells you that you are now talking about metal ok. So, here you can see that the metal permittivity has a limited bandwidth and ω_p depends on the plasma frequency, right? So, you can adjust because you are now from in the metal wire array you will see that you will have a adjusted plasma frequency expression where this term can be replaced by ω_p square because instead of this you are now having a volume fraction. okay, that is πr square / a square * this metal plasma frequency, right.

So, this basically gives you the effective plasma frequency, which you can see is proportional to the wire volume fraction. Now, this aligns perfectly with the intuition that as the metal filling fraction increases, ω_p will approach the bulk metal plasma frequency, which is ω_{pm}. That makes sense, and when the filling fraction just nears unity or is almost unity, that means the wear medium will behave like the bulk metal, right? So, at optical frequencies the wear arrays are used in a wide variety of structures and devices such as in negative index

Semicontinuous Metal Films

- Most literature metamaterial designs exhibit some form of periodicity.

- A different class – random metal-dielectric composites:
    - These are disordered metamaterials without defined meta-atoms

- Exhibit distinct optical properties from bulk materials.

- This type of nanostructured material have drawn significant interest due to:
    - Rich physics and unique properties in disordered metal-dielectric systems

- Potential in miniaturized electronic and optoelectronic devices.

- Examine their local optical properties at the nanometer scale.

- Random metal-dielectric composites are typically fabricated using:
  - Thermal evaporation
  - Electron-beam evaporation
  - Sputtering
  - Electroplating

- These methods generally produce thin films or coatings on 2D or 3D substrates.

- As a result, such composites are usually modeled as 2D systems.

- The typical fabrication process:
  - Metal is deposited onto a dielectric substrate

- Initially, small, isolated metal islands nucleate → low metal filling fraction (see Fig. a).
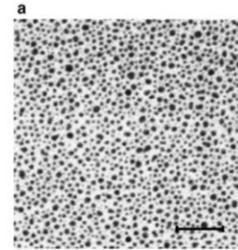
metamaterials where the metal wear arrays can create a tunable negative permittivity background. which is essential for achieving this negative index behavior. You will see that on top of the metallic wire, you now have to bring in another metamaterial design that gives you negative permeability.

So that typically comes from an array of split resonators. And when they are combined, you have negative permittivity and negative permeability occurring / the same frequency band. And that is where you get your negative index behavior. Now this wire media is also used to design materials with an ultra-low refractive index that is less than 1. And this could occur at frequencies just above the effective plasma frequency.

So you get a lot of tuning parameters for this effective plasma frequency because it purely depends on the physical design of the wire and the array. So, people have shown experimental demonstrations of this kind of low-index wear media, and applications include total external reflection and air core planar. Dielectric waveguides; people have designed power splitters for air core waveguides. Now, this analysis basically applies specifically to the ideal case where the electric field is considered to be parallel to the wires. Now assume that the external field has some arbitrary orientation, which means it is not purely parallel to the length of the wires.

So, in that case, the wire mesh will support some additional modes, okay. So, in such cases, a local permittivity model will be insufficient. So, people have to go beyond this local model, okay. And you have to think of non-local which is spatially dispersive behaviour that means you have to understand that now the induced polarization depends not just on the local ah electric fields, but also on the field variation present across the space ok. So, that is the non-local effect.

So, previous discussions. So, now we will go for semi-continuous metallic films. So, until now, we have seen that you know about the electric

- With further deposition, islands grow and merge into fractal-shaped clusters (see Fig. b).

- As deposition continues, at certain point, the metal concentration f can exceed a critical value f_c (percolation threshold).

- Below f_c: the composite acts as a dielectric.

- When the metal filling fraction reaches the value f_c, an infinite "backbone cluster" spanning the entire substrate is formed.

- Above f_c: the system begins to behave like a conductor.

- The percolation threshold represents more than just a transition from insulator to metal.

- Near this threshold:
  - Small morphological changes in the film can lead to large shifts in optical behavior
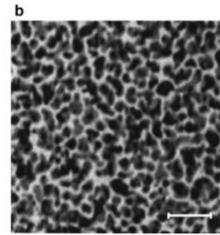
metamaterials with an ordered metal-dielectric kind of arrangement. And these metamaterial designs actually exhibited some form of periodicity, right? Now, we will look into another class of metal dielectric composites ok, where you do not have a well defined meta atom right. So, these are basically disordered metamaterials, so they also show some distinct optical properties that are not found in the bulk materials.

And this type of nanostructure material has drawn significant interest due to its rich physics and unique properties. In disordered metal-dielectric systems, right. So, the random metal dielectric composites are basically regarded as the disordered

- Percolation films have fractal geometry:
  - Structures look similar across different length scales (self-similar)
  - Common natural fractals: coastlines, snowflakes, lightning, fern leaves, cauliflowers, blood vessel systems

- These composites contain clusters of all sizes, including:
  - Tiny individual metal particles to an infinite cluster spanning the film

- This fractal structure gives rise to localized plasmonic resonances or "hot spots".
  - These occur over a wide electromagnetic spectrum

- This can be understood qualitatively based on very simple arguments.

metamaterials ok and they they are unique to the other ones we have seen till now. They have potential in miniaturized electronic and optoelectronic devices and such systems are likely to become even more important with the miniaturization of electronics and optoelectronics component ok and you can examine their local optical properties at the nanoscale.

Right. So, now let us look into them and understand how their behavior is defined and how they can provide you with some unique responses. So, first of all, how do you make this kind of random metal-dielectric composites? So, you can fabricate them using standard methods like thermal evaporation, e-beam evaporation, sputtering, or electroplating, right? So, these kinds of methods are typically used to produce thin films or coatings on 2D or 3D substrates. Now, as a result, such composites can also be modeled as 2D systems. So, a typical fabrication process will follow like this that you have metal deposited on dielectric substrate Okay and you can see that during the deposition process small and isolated islands nucleate first on the substrate and a composite with a low metal filling fraction similar to this kind of sample begins to develop. So, this figure is basically a TEM image, or a transmission electron microscopy image, of a typical metal-dielectric composite.

That follows the Maxwell-Carnot geometry we have seen earlier, right? So, this are basically nothing, but small isolated metal islands the dark spots that you can see they are corresponding to low metal filling fraction. Now, with additional deposition and subsequent increase in the metallic you will see that this metallic islands will grow in size and they will start getting merged to each other and forming irregular shaped clusters of fractal geometry on the substrate as you can see here. So, this is basically the TEM image of a typical metal dielectric composite that follows the Brackman geometry that you have seen earlier. Not in this lecture, a couple of lectures back, okay? So, the dark and the bright spots here basically represent the metal and the dielectric component respectively and the scale bar is typically around 200 nanometer and that is for both this image and the other one. So, you can understand comparatively that how the islands have grown in size and then they merge together to give this kind of a structure.

so as the deposition continues at a certain point you will see that the metal concentration f can exceed a critical value called fc which is known as

- Near the percolation threshold:
  - The system behaves as a scale-invariant fractal set

- At optical frequencies, noble metals show high conductivity and low losses, determined by the imaginary part of permittivity.

- Metal grains/clusters in the composite can be modeled as inductors (L) in series with resistors (R).

- Dielectric gaps are modeled as capacitors (C).

- The overall composite behaves like a network of RLC circuits.

- Let l represent the metal and dielectric grain size:
  - Both inductance (L) and capacitance (C) proportionally scale with l
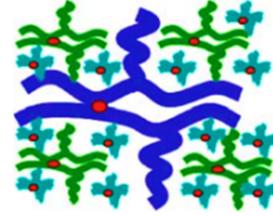
- The resonant frequency of these circuits: $\omega_r = 1/\sqrt{LC} \propto l^{-1}$

- This implies:
  - Smaller clusters resonate at higher frequencies
  - Larger clusters resonate at lower frequencies

- Therefore, metal-dielectric films function as a collection of optical RLC resonators.

- Due to geometric disorder – R, L, and C have random values.

- This randomness leads to a broad resonance spectrum ranging from UV to mid-IR.

- Light-induced plasmon modes in percolation metal-dielectric films can cause significant enhancement of optical responses across a broad spectral range.

percolation threshold so what is that you can consider that below this fc the composite can act as a dielectric and when the metal filling fraction reaches that critical value of fc an infinite backbone cluster spanning that entire substrate is basically formed so you have basically have a backbone of metal covering the substrate so you can say that above fc the system begins to behave like a conductor and below fc it is basically acting as a dielectric So, the percolation threshold is nothing but a representation of more than just a transition from the insulator to the metallic phase. So few interesting things happen near this threshold. like now small

morphological changes in the film can actually lead to large shift in the optical behavior because you understand that the behavior of a dielectric is very different to that of a metal so there is a concentration fc where this changeover is taking place so percolation films are basically characterized by a fractal geometry that will appear similar at different length scales So, the similarity, the self-similarity in these fractals implies that a fractal system exhibits the same overall shape as one of its parts. Therefore, the fractal shows the same statistical properties at various scales. So, here are some examples of fractals in nature.

that includes coastlines, snowflakes, lightning, fern leaves, cauliflowers, and even the blood vessel systems. So these composites contain clusters of all sizes, including tiny individual metal particles to an infinite cluster that spans across the film. So, you can have, you know, all kinds of variations. So, the fractal structures basically give rise to localized surface plasmon resonances as well. So, those are working as hotspots because you know they can concentrate the electromagnetic field between their gaps ok.

So, this can occur / a wide electromagnetic spectrum, and this can be understood qualitatively through very simple arguments, okay. So, we will try to see what happens near the percolation threshold. So, in the vicinity of the percolation threshold, the system is basically a scale-invariant fractal set. As you can see in this particular figure. So, here the figure basically highlights the, you know, self-similar features in the local film morphology.

So, at optical frequencies, noble metals show high conductivity and low losses that you can determine from the negative part of the permittivity, right? So when you look into this particular structure, you can see that there are grains and clusters in this semi-continuous metallic films that you can model as inductors in series with some resistances. And the gaps that you see can be modeled as capacitors. So what do you see is that the overall composite, which is a semi-continuous metallic film, basically behaves like a network of RLC circuits, okay. So, here you can consider L, which represents the metal and the dielectric grain size. So, you can see that both inductance as well as capacitance C and inductance L, they proportionately scale with this size L, okay.

So, you can consider the resonant frequency for this kind of circuits $\omega r = 1$ by square root of capital LC that is proportional to 1 by L that is size of those grids. So, what does it imply? It tells you that the smaller clusters would resonate at higher frequencies, and obviously, the larger clusters would resonate at lower frequencies. Therefore, you can say this metal dielectric film basically functions as a collection of optical RLC resonators. Now as there are so many different sizes, there are geometric disorder, you can say that this RLNC will have very random values and this randomness will basically give you a broad resonance spectrum that ranges from you know ultraviolet to mid IR. So, light-induced plasmon modes in percolation metal-dielectric films can cause a significant enhancement of optical responses.

across a broad spectral range, and that is basically coming because of this factor, right? And we also mentioned hotspot formation. So, hotspots are the points where strong local field concentration takes place. So, the formation of these

Semicontinuous Metal Films

- Electromagnetic "hot spots" formation in semicontinuous metal films is related to Anderson localization:
  - A phenomenon where electron wave functions localize due to disorder, originally predicted for electrons in semiconductors by P. W. Anderson

- Numerical models simulate field fluctuations and higher-order moments in semicontinuous films:
  - Predict localization of the surface plasmons and strong local field enhancement

- Experiments confirm theoretical predictions:
  - Local fields can be enhanced by factors of 10^5 (linear responses) and 10^{20} (nonlinear efficiencies)

electromagnetic hotspots happens in a semi-continuous metallic film, and they can be related to Anderson localization. So, this is basically a phenomenon where, in semiconductors, the electron wave function is localized due to disorder. So, this was originally predicted for electrons in semiconductors by Anderson, and that is known as Anderson localization.

So, the similar kind of concept appears in this metallic film and we get electromagnetic hot spots which are also very vital for you know sensing kind of applications. There are numerical models that can simulate field

Semicontinuous Metal Films

- Figure illustration:
  - Shows local field enhancement in a percolating metal-dielectric at 1.5 μm wavelength
  - The electric field concentrates in nanometer-sized regions with sharp peaks, much greater than the applied field

- Random metal-dielectric composites are increasingly influential in optics.

- Near the percolation threshold, giant electric field fluctuations lead to:
  - Significant enhancement of nonlinear optical effects
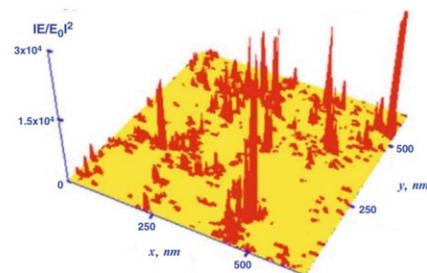
fluctuations, and you know higher-order moments in such semiconductor films, okay. They can predict the localization of the surface + bonds and also show a strong local field enhancement. Okay, because of the capacitance forming between the two small islands. And people have also conducted experiments to confirm these theoretical predictions, and they have seen that local fields can be enhanced by a factor of 22 or 5.

or even 10 to the power 20 if there are you know 10 to the power 5 in the case of linear responses if there are some non-linearities involved you can go even up to 10 to the power 20. So, that that is like you know huge enhancement and that you can actually take help of that local field enhancement and do sensing of trace amount of molecules typically using surface enhanced Raman scattering kind of

Semicontinuous Metal Films

- Applications include:
  - Surface-enhanced Raman spectroscopy (SERS) – Extremely high detection sensitivity, as Raman scattering ∝ fourth moment of the electric field
  - Enhancements in optical Kerr effect and four-wave mixing
  - Use in surface-enhanced infrared absorption
  - Applications in harmonic generation
  - Integration in photovoltaic devices

- Semicontinuous metal films are utilized in optical data recording and mid-infrared filtering – enabled by a frequency-selective photomodification technique.

applications okay where you can do ppb parts per billion level of sensing which is otherwise impossible to do. So, here in this figure it shows the spatial distribution of the normalized local intensity of a typical silver silica percolating film at an arbitrary wavelength of 1.5 micrometer. So, what has been plotted here is basically the intensity because the normalized intensity E/E naught modulus squared.

So, that tells you you know how much is the local field enhancement taking place. So, you can see the spikes basically show you that there are some nanometer sized regions between the islands where you can see this kind of sharp peaks which are way much greater than the incident field. So, you can see the scale here; this is like 3 * 10 to the power of 4, okay, something like that. So, random metal dielectric composites have had a growing impact in various fields of optics because of this kind of impact. extraordinary features and what we have seen that near the percolation threshold giant electric field fluctuations happen and that can lead to significant enhancement of non-linear optical effects and that has got a lot of applications.

Now, if I need to name a few applications, the first one that comes to my mind is this Raman and surface-enhanced Raman spectroscopy. That is SCRS and that is as I already mentioned that is used for extremely high detection sensitivity because the Raman scattering is typically proportional to the fourth moment or fourth order of the electric field. So, if the local electric field is enhanced by say a factor of 10 to the power 2 ok, the Raman scattering will have a proportionality of 10 to the power 8 ok. So, that much it can scale up. So, you can also use semi continuous metallic films in optical data recording ok and mid infrared filters ok.

You can use them for optical car effect 4 wave mixing; this kind of non-linear effect is okay. They can be used for surface-enhanced infrared absorption. You can use them for harmonic generation and integration with photovoltaic devices. And all these things allow you another plethora of kind of applications which I already discussed that you can use them for optical data recording and also mid infrared filtering. And that is basically enabled by a frequency-selective photomodification technique.

So, semi continuous films can be made very easily using the normal ah deposition techniques that we discussed, but near the critical fraction ok where it changes from a dielectric to a metallic kind of a ah material. So, that is where a lot of interesting optical phenomena take place, and you can use those for making all these different applications. Thank you. So, with that, we will stop in this particular lecture. If you have got any queries regarding this, you can drop an email to this email address mentioning the course title and the lecture number in the subject line.

So, in the next lecture, we will start discussing inhomogeneous and chiral metamaterials. See you then.

Slides inserted by fallback (review if needed):

Thank You

Send your queries at deb.sikdar@iitg.ac.in