Course Name: Introduction to Microwave and Optical Metamaterials
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Department Name: Electronics and Electrical Department
Institute Name: Indian Institute of Technology, Guwahati
Week-5
Lecture-22

Lec 22: Periodic Array of Metallic Wires-I

Lecture Outline

- Rodded Medium
- Drude Model for Periodic Array of Metallic Wires
- Example of a Wire Medium

Hello students, welcome to lecture 22 of the online course on Introduction to Microwave and Optical

Rodded Medium

- "Rodded medium" – thin metallic wires arranged periodically in vacuum or dielectric.
- 1D, 2D, and 3D wire mesh networks were systematically studied by Rotman about half a century ago.
- Figure illustrates different grid structures:
  - (a) 2D wire array for z-polarized electric field plasma resonance.
  - (b) Layered grid effective for waves with wave vector normal to the grid plane.
  - (c) 3D cubic lattice forms a quasi-isotropic medium with plasma resonance for any polarization.

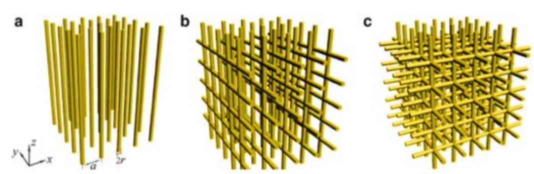

Fig. (a) A 2D wire array for z-polarized electric field. (b) A 3D lattice for any electric field polarized within the yz plane. (c) A 3D quasi-isotropic wire grid for arbitrary polarization.

Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

Rodded Medium

- These artificial wire composites behave like dilute plasmas, creating a negative electric response with adjustable strength.
- Initially known in radio engineering as artificial dielectrics with tunable properties.
- The concept was revived and popularized in recent physics and materials science by Sir John Pendry, giving it broader relevance and application.
- The rodded medium exhibits plasmonic behavior within a specific frequency band.

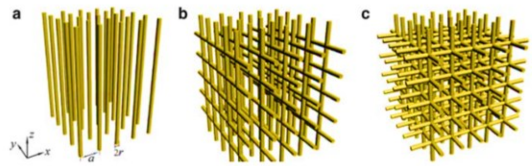

Fig. (a) A 2D wire array for z-polarized electric field. (b) A 3D lattice for any electric field polarized within the yz plane. (c) A 3D quasi-isotropic wire grid for arbitrary polarization.

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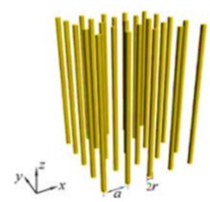
Rodded Medium

- This response depends primarily on geometric parameters:
  - Wire diameter: $2r$
  - Unit cell length (lattice constant): a
- When unit cell length ≪ wavelength, the wire array behaves as an electric metamaterial.
- The main objective is to derive the effective dielectric function $\varepsilon(\omega)$ of the wire grid.
- For simplicity, a 2D wire array from Fig. (a) is considered.
- The medium is anisotropic and focuses on a plane wave in the xy-plane with z-polarized electric field.
- Thus, $\varepsilon(\omega)$ represents the $\varepsilon_{zz}(\omega)$ component of the 3×3 permittivity tensor.



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Metamaterials. In today's lecture, we will be discussing periodic arrays of metallic wires. So, here is the outline we will continue from the recorded medium that we discussed. We will see how you can derive Drude model for periodic array of this metallic wires and we will take an example of wire medium and discuss in details. So the lecture basically starts with the discussion of a rotted medium which is made of nothing but thin metallic wares arranged periodically in a vacuum or dielectric. So 1D, 2D, and 3D were mesh networks systematically studied by Rotman about half a century ago, right? So, these are the different types of structures.

As you can see, a two-dimensional array is basically shown in this first figure, okay. So, this is a 2D wire array. So, these are the axes x, y, and z. Here, you can see this is for the z-polarized electric field, which means the electric field is along the length of the wire.

And then you have this second one, which is basically a 3D lattice again for any electric field that is polarized within the yz plane. So, you have added ah this third one third dimension by adding this ah wires here and this is basically a kind of periodic cubic lattice ok. So, this is basically when your metallic wires are forming a three-dimensional mesh, okay. So, the wire medium basically becomes quasi-isotropic. and it can exhibit a plasma resonance for field for any arbitrary polarization angle.

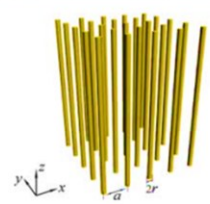
So, this is where things become interesting in this case. However, this is also not bad this is like you know you have you need to maintain the electric field to be polarized along the y z plane ok like this plane y z you can see here. So, these artificial wear composites basically behave like dilute plasmas, okay. So, they basically can create negative electric response with some adjustable strength. So, they were initially known in radio engineering as artificial dielectrics with tunable

Drude Model for Periodic Array of Metallic Wires

- The effective permittivity $\varepsilon_{\text{eff}}(\omega)$ of the wire medium follows a Drude-like model:

$$\varepsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_{\text{p}}^2}{\omega(\omega + i\Gamma)}$$

- Unlike real Drude metal, ω_{p} and Γ are not intrinsic but depend on the wire array geometry and metal properties.
- A key goal is to approximate ω_{p} and Γ using known quantities.
- Multiple models exist for estimating ω_{p} in a rodded medium.
- In bulk metals, plasma frequency depends on free electron density and mass, and typically situated in the UV/visible spectrum for good conductors.



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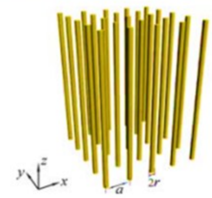
Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

properties.

This same concept was done long back, but then it was revived and it got popular in recent physics and metamaterial science when Sir John Pendry gave a broader relevance and application to this wear meshes that can be used for creating negative permittivity at a desired frequency range at will. And that can be combined with another metamaterial that can provide negative permeability also at the same frequency, and then you can obtain a left-handed material or left-handed metamaterial, right? So, this is the main purpose: the rotted medium could exhibit prismatic behavior within a specific

Drude Model for Periodic Array of Metallic Wires

- In wire media, the plasma frequency is much lower due to two main factors:
  - Effective electron density is diluted by a factor of $\pi r^2/a^2$:
    caused because free electrons being confined within the physical boundaries of the wires.
  - Increase in "effective electron mass":
    because of the induced current in the wire and the excited magnetic field.
- This is a conceptual effect tied to the context of described self-inductance effect, not actual mass increase.
- Modified equivalent plasma frequency accounts for the reduced effective electron density: $N_{\text{eff}} = N\frac{\pi r^2}{a^2}$



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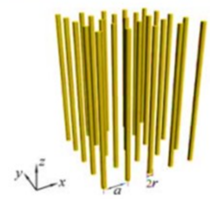
frequency band. So, as you can see in this figure, if you consider this particular wear grit or wear mesh, you will see there are two important physical parameters. The first one is the diameter of the wear, which is 2R, and then another important parameter is the periodicity of this wear lattice, which is A. So, the response, as you can see, will be completely dependent on these two important geometric parameters.

The first one is wire diameter r and the second one is unit cell length that is the latest constant a. So, what you have to what you have to pay attention to is that the u length unit length unit cell length that is the lattice periodicity or A should be way sub wavelength that is much much smaller than the wavelength of light. And that is when the wire array will behave as an electric

Drude Model for Periodic Array of Metallic Wires

- To modify the plasma frequency further, the "effective mass" of electrons from self-inductance is evaluated.
- This is done using the concept of generalized (canonical) momentum from quantum mechanics.
- Total momentum: $p = mv + q\boldsymbol{A}$
- The effective mass due to self-inductance is expressed as: $m_{\text{eff}} = \frac{e\boldsymbol{A}}{v}$
- For a mean velocity v and free electron density N in the metallic wires: The current in one wire is $\pi r^2 eNv$



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metamaterial. So, that is the case when electromagnetic radiation that will be falling on this material will not be able to you know resolve the wires and rather it will see some effective medium. So, the main objective here will be to derive this effective dielectric function, which can be written as ε of ω for this wire grid structure.

So, for simplicity, we will be considering this structure, which has, you know, only two important parameters: a and 2r, okay. So, as you can see, the composite medium is apparently anisotropic, and the polarization state that we will be interested in corresponds to a plane wave propagating in the xy plane, with its electric field polarized along the z-axis, that is, along the length of the thin wires. So you can understand that we are only interested in that particular dimension. So, ε permittivity or ε ω okay represents basically the ε z z ω that is the z z component of the 3 by 3 permittivity tensor. So, in this particular scenario the effective permittivity ε effective okay of this wear medium will follow a Drude like model.

So, you can write ε effective as a function of ω as 1 - ω p squared / ω * ω + i γ. So, what is ω p here, which is the effective plasma frequency, and γ is basically the effective damping constant? Now, unlike real druid metal, this ω p and γ here are not intrinsic properties of the material. They are basically dependent on the wear array geometry and metal properties. So, this is where you know you are creating your own artificial materials. Normally in any metal these are kind of fixed natural properties these are the intrinsic properties but here they are not okay so here you can do everything by designing this wire array and change So, the key goal here will be to estimate this ω p and γ using this known quantities and there are multiple models that exist for estimating this

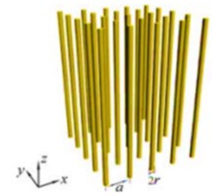
Drude Model for Periodic Array of Metallic Wires

- Ampere's law is used to compute the magnetic field $\boldsymbol{H}(R)$ at a distance R from the wire center: $\boldsymbol{H}(R) = \frac{\pi r^2 eNv}{2\pi R}$
- The magnetic field $\boldsymbol{H}(R)$ directed along the z axis in terms of the curl of a vector potential: $\boldsymbol{H}(R) = \nabla \times \boldsymbol{A}(R)/\mu_0$
- We choose the vector potential in the following form:

$$\boldsymbol{A}(R) = \frac{\mu_0 r^2 eNv}{2} \ln(a/R)$$

- For good conductors, electrons move on the wire surface, so all electrons experience $\boldsymbol{A}(r)$.
- Substituting into the effective mass formula gives:

$$m_{\text{eff}} = \frac{\mu_0 r^2 e^2 N}{2} \ln(a/r)$$



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effective plasma frequency in such a rodded medium.

But we will strictly follow the effective mass approach that was initiated by Sir John Pendry in his seminal paper. So, in bulk metals, you will see that the plasma frequency basically depends on free electron density. the mass and they are typically situated in the UV visible spectrum for the case of gold conductors right. So, below the plasma frequency, the material behaves like a metal. So, the electromagnetic wave is reflected above the plasma frequency; it basically gives way to the incident wave.

So, the material becomes transparent right. We will see that in such a wire medium the plasma frequency is much lower as compared to the normal metal mainly because of the two factors here that the effective electron density is basically getting diluted by a factor of π r square / a square. Because you assume that if you have a you know equivalent area of a square here the region within this a square that is having electron is basically the metallic wire and the area of that will be π r square. So, this is the factor that tells you what the effective electron density is, and this is caused by free electrons being confined within only the physical boundary of the wires. And the second one tells you about the increase in the effective electron mass.

Now, this second factor was described by surgeon Pendry ok and he described this effect of effective electron mass ok getting increased that is resulting from the reduced or that is basically coming from the induced current in the wire and the excited magnetic field. So the self inductance that is possessed in the metallic wires acts to oppose the rate of change in the current which is basically a direct result of the Lenz law that we all know. So, this is a conceptual effect that is basically tied to this context. Of described self-inductance effect, not actual mass increase. So, you can think of an increase in the effective electron mass, right? So, the modified equivalent plasma frequency accounts for these two factors.

So, if you consider for the reduced effective electron density, you can consider the new density as n effective which is n and then you have this factor that was calculated here π r square / a square. And if you want to modify the plasma frequency further considering the increase in the effective mass of the electrons that is coming from the self inductance okay you can also do that. This is done using the concept of generalized momentum, which is a canonical momentum from quantum mechanics. So, you can calculate. The total momentum p = mv + qa.

So, here v is basically the charge velocity, and a represents the vector potential. m is nothing but the static charge, and q represents the static mass. Sorry, and q is the charge, right? So, the effective mass you can obtain due to self-inductance is in the form of m effective, which is Ea / v. So, for a mean velocity v and free electron density n in the metallic wires, the current that flows through one wire can be given as πr^2 e n v, right? So, you can use Ampere's law to compute the magnetic field h r at a distance r from the wire centre and you can express this magnetic field h r = π r square e n v / 2 π r. So, the magnetic field, which is directed along the z-axis, is in terms of the curl of a vector potential, right? So, you can write hr, which is derived from this vector potential that is obtained by taking its curl.

So, the curl of a r / μ naught. Now, although you can although generally speaking the vector potential ar is not uniquely defined because of its issue of Gauss choices. So, you can make an assumption that your r is much much smaller than a and in that case you know due to lattice symmetry also you can consider Ar the vector potential in this particular form that you have μ naught r square Env / 2 ln A/r. So, for good I am not going into the details of this you can look into the papers or by Professor Pendry if you want the complete mathematical details for this. So, what I wanted to tell you that the vector potential is all dependent on the physical parameters Anr and for good conductors we have seen that the electrons basically move on the know wire surface ok.

So, in that case, it is safe to assume that all the electrons will basically experience the same vector field, right? So, substituting into the effective mass formula will give you M effective which is basically μ_0 R square E square N / 2 ln A/R. So, you can actually see what is the effective mass. So, once you know the effective mass you can also compute the plasma frequency of this wire medium ω p square will be nothing but n effective e square / ε naught m effective right. So, n effective you already know because of this factor how it will come down from n and then you can also calculate substitute the value of m effective ok and this is what you got. So, you can see your plasma frequency is dependent on a and r, while c_0 squared is nothing but c_0, which is basically the speed of light in a vacuum.

So, c_0 is 1 / the square root of μ_0 ε_0. So, this is how you can calculate the plasma frequency of this wear medium, which is completely dependent on the two physical parameters a and r. Shalev and Sarichev, okay, they also provided a similar kind of expression, okay. But they actually have gone for more complicated but precise derivation of this effective plasma frequency of the swear medium okay and you can see that this equation although they are dependent on the same two parameters it is more specific okay and even Maslowsky et al they also came up with an expression for plasma frequency which is slightly different, but again similar kind of parameter dependence is there you are just depending on the two physical parameters A and R. So, if you compare between these two models and you will see that all these approaches agree reasonably well with the transfer matrix simulation ok.

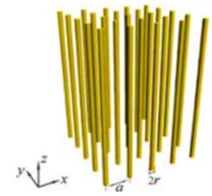
So, the Shalev and Sarichev expression as well as Maslowski improved model for plasma frequency although they give you better prediction for the plasma behavior. And, to complete the Drude model for effective permittivity, you can determine the next important parameter, which is the effective damping constant, right? Now, the damping constant γ basically represents the loss factor in the wire medium. So, if you consider γ equal to 0, that means you are considering the wires to be perfect conductors. So, in that case, you can write ε permittivity ε effective ω as 1 - ω p square / ω square. So, γ is basically 0, right? Now, in real metal, you do not have γ equal to 0.

So, you have a finite conductivity that is giving you some ohmic or resistive kind of losses. So, you have γ, which is non-zero. So, this is incorporated by introducing an imaginary part to the inductance, and you will see here how it is done. So, you consider the magnetic field HR, and for that, the unit inductance L. for a particular wire can be estimated as L = μ_0 integration / r_2 a by root π h r dr / π r square e and b.

So, that gives you the inductance is typically equivalent to μ_0 by 2 π ln a/r. So, here you can see the denominator basically = The current flow in the wire and the integration upper limit, which is by π, basically represent the effective radius of the unit cell in this wire array, right? So, what is the validation limit for this particular expression you will see that these are all valid for very thin wires where r is considered to be much much smaller than a. Now by comparing the plasma frequency expression with the inductance you got this relationship that ω p square can be related to the inductance as 1 by ε naught a square L. Now in this particular equation you have a direct correlation between the wear inductance and the plasma frequency of this overall system. So, when the metal has a finite conductivity σ, you can say that the inductance of the wire has a complex value, and you can substitute L as L + σ π r squared by i ω.

So, in that case you can if you put that here you can say that the effective permittivity of the wire medium in the Drude form will. So, the effective permittivity of the wire medium can now be

Drude Model for Periodic Array of Metallic Wires



- We obtain the plasma frequency of the wire medium:

$$\omega_p^2 = \frac{N_{\text{eff}} e^2}{\varepsilon_0 m_{\text{eff}}} = \frac{2\pi c_0^2}{a^2 \ln(a/r)}$$

- Shalaev and Sarychev expression:

$$\omega_p^2 = \frac{2\pi c_0^2}{a^2[\ln(a/\sqrt{2}r) + \pi/4 - 3/2]}$$

- Maslovski *et al.* show that the plasma frequency can be expressed as:

$$\omega_p^2 = \frac{2\pi c_0^2}{a^2\left[\ln(a^2/4r(a-r)\right]}$$

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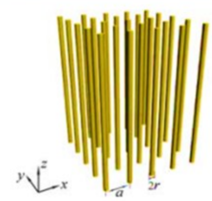
expressed in a Drude form as ε

Drude Model for Periodic Array of Metallic Wires



- A comparison between these models has been conducted and all these approaches agree reasonably well with transfer matrix simulations.
- Shalaev and Sarychev expression and Maslovski *et al.* improved model for ω_p indeed offer a better prediction of the plasma behavior.
- To complete the Drude model for effective permittivity $\varepsilon_{\text{eff}}(\omega)$, we need to determine the effective damping constant Γ.
- Γ represents the loss factor in the wire medium.
- $\Gamma = 0$ if wires are perfect conductors, yielding: $\varepsilon_{\text{eff}}(\omega) = 1 - {\omega_p}^2/\omega^2$

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effective = 1 - ω p square / ω * ω + i ε naught a square ω p square / π r square σ. So, this is obtained by combining the expression for the effective permittivity. So, all these things you can put here, and you can obtain this effective permittivity of the wire medium in the Drude form, right? So, what we see here is that if the wire array is basically embedded in a host medium that has a permittivity of ε h instead of vacuum. In that case the first term on the right side of the equation this this one will get replaced by the host medium permittivity ε h that is a very simple thing to do.

So, you can actually consider this metallic where array where array to be embedded in any other dielectric medium and in that case this will this one will be replaced by ε h. Now, the effective plasma frequency ah ω p in a wire medium thus can be ah we have seen that it can be derived via the mediums geometric parameter that is a and r. And that basically allows you to target specific frequency ranges from microwave all the way to optical frequencies. You can change the design of your wire array and put it anywhere in between.

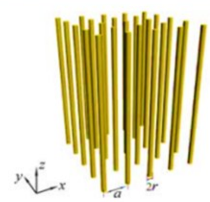
Right. So, by using thin wire arrays, the plasma frequency can be reduced by several orders of magnitude compared to bulk metal. So, that is also one big thing because typically the plasma frequencies of bulk metals lie in a much higher frequency range. So, if you use this kind of thin metallic wire instead of solid metal, you can bring down the plasma frequency, and that allows it to make microwave

Drude Model for Periodic Array of Metallic Wires

- In real metals, finite conductivity leads to ohmic (resistive) losses in the wires.
- This is incorporated by introducing an imaginary part to the inductance.
- Based on the magnetic field $\boldsymbol{H}(R)$, the unit inductance L of a wire is estimated as:

$$L = \frac{\mu_0 \int_r^{\frac{a}{\sqrt{\pi}}} \boldsymbol{H}(R)dR}{\pi r^2 eNv} \approx \frac{\mu_0}{2\pi} \ln\left(\frac{a}{r}\right)$$

- Here, the denominator equals the current flow in the wire.
- The integration upper limit $\frac{a}{\sqrt{\pi}}$, represents the effective radius of a unit cell in the wire array.
- The approximation in the above equation is valid for thin wires where $r \ll a$.



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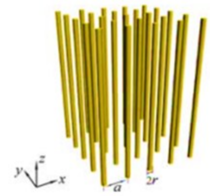
Drude Model for Periodic Array of Metallic Wires



- By comparing the plasma frequency expression with the inductance, we get a relationship:

$$\omega_p^2 = \frac{1}{\varepsilon_0 a^2 L}$$

- This equation, links plasma frequency ω_p to the wire inductance L.

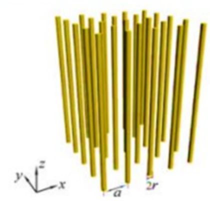
- When the metal has a finite conductivity σ, the inductance of the wire is taken as a complex value with the following modification:

$$L \rightarrow L + \frac{\sigma \pi r^2}{i\omega}$$

- Effective permittivity of the wire medium in a Drude form:

$$\varepsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma\right)}$$

Drude Model for Periodic Array of Metallic Wires



- If the wire array is embedded in a host medium with a permittivity ε_h instead of a vacuum, the first term in the right-hand side of equation $\varepsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma\right)}$ should be replaced by ε_h

- The effective plasma frequency ω_p in a wire medium can be tuned via the medium's geometric parameters.

- This allows targeting specific spectral regions, from microwave to optical frequencies.

- By using thin wire arrays, the plasma frequency can be reduced by several orders of magnitude compared to bulk metals.

Example of a Wire Medium

- Example – A wire medium with a plasma frequency in the gigahertz range
- Consider a two-dimensional wire array with
  - Wire radius: $r = 5\ \mu\text{m}$
  - Lattice constant: $a = 40\ \text{mm}$
- Material: Silver, with:
  - Free electron density: $N = 5.8 \times 1028\ m^{-3}$
  - Conductivity: $6.3 \times 10^{7}\ \text{S/m}$ (room temperature)
- Due to geometry, electron density is diluted by 8 orders of magnitude.
- From earlier equation, the effective electron mass m_{eff} is 2.1×10^{-25} kg,— which is "heavier" than a silver atom.

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Example of a Wire Medium

- These two effects cause the plasma frequency to drop by 6 orders of magnitude.
- Results:
  - The expression by Pendry $\omega_{\text{p}}^2 = \frac{N_{\text{eff}} e^2}{\varepsilon_0 m_{\text{eff}}} = \frac{2\pi c_0^2}{a^2 \ln(a/r)}$ gives $\omega_{\text{p}} = 2\pi \times 1.0$ GHz
  - On solving other alternative models

1. Shalaev and Sarychev expression: $\omega_{\text{p}}^2 = \frac{2\pi c_0^2}{a^2[\ln(a/\sqrt{2}r)+\pi/4-3/2]}$
2. Maslovski expression: $\omega_{\text{p}}^2 = \frac{2\pi c_0^2}{a^2[\ln(a^2/4r(a-r)]}$

- Both the above models give a slightly different value $\omega_{\text{p}} = 2\pi \times 1.1$ GHz

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metamaterials as well. Something like, for example, a wire medium with a plasma frequency in the gigahertz range is something amazing.

Okay, so let us take an example here. So, if you consider a_2 dimensional wire array which has got a wire radius of say 5 micron and the lattice constant to be 40 millimeter ok. So, you see, you are basically considering r to be much, much smaller than a. So, if you consider the wire to be made of silver which has got free electron density of n that = 5.8 into 10 to the power I think that is 10 to the power 28 meter cube per meter cube.

So, read it like that; just correct it. So, if you consider the material the wire is made of, silver, which has a free electron density n equal to 5.8 * 10 to the power of 28 per cubic meter. And it has a conductivity of 6.3 into 10 to the power of 7, 7 centimeters per meter at room temperature. Now you can see that because of this geometry the electron density can be diluted by 8 orders of magnitude and from the earlier equation that you have seen that the effective electron mass will increase and it is something like you know 2.

1 into 10 to the power - 25 kg which is basically heavier than a silver atom right. So, with these two effects, you can combine them, and you will see that the plasma frequency is basically dropping by six orders of magnitude. So, how is it working? Because if you look into the expression given by Serpentry, ω p square depends on n effective, e square / ε naught, m effective. So, this is increasing, and this is decreasing, both equivalently bringing it down, okay. So, that is bringing your plasma frequency down.

So, you can see that you get ω p in the order of gigahertz. So, that is 2π * 1.0 gigahertz. Now if you do not go with Professor Penry's model and if you try to solve it using the alternative models which we have already seen that Shalev and Sarichev expression which is this one ω p square = 2 π c naught square / a square ln of a by root π + π by 4 - 3 by 2. So, and the other one was Maslowski expression that is given as ω p square = 2 π c naught square / a square ln a square by 4 π into mnsr.

So, in these two cases, you get slightly different values; ω p will be 2 π * 1.1 gigahertz. You see, there is a slight variation, but more or less, they give you the same order of magnitude. So, it tells you that you can get the plasma frequency, which is way below the bulk metal plasma frequency. So, that is how you can make things work in the gigahertz range.

You can do that. So keep this in mind as well, because it will be important for our next lecture. So we will stop here, and we will continue with the periodic array of metallic wares in the next lecture. So if you have got any query regarding this one, Drop an email to this email address mentioning the course name and the lecture number and the subject line.

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