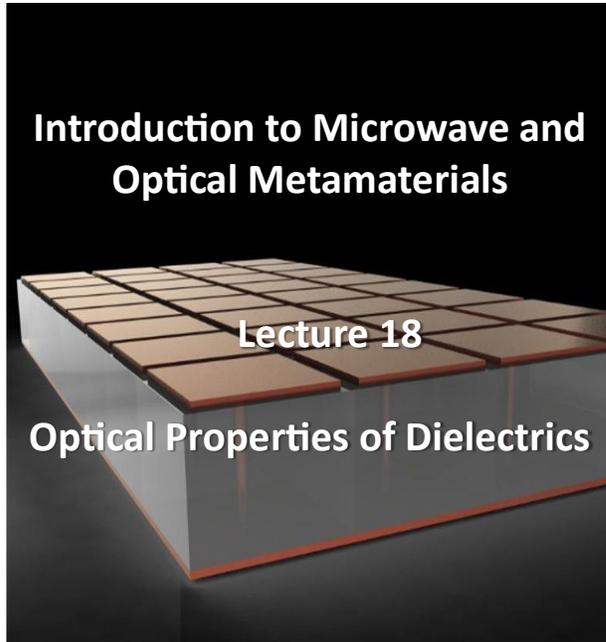Course Name: Introduction to Microwave and Optical Metamaterials
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Week-4
Lecture-18

Lec 18: Optical Properties of Dielectrics

Introduction to Microwave and Optical Metamaterials

Lecture 18

Optical Properties of Dielectrics

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Hello, students. Welcome to Lecture 18 of the online course. An introduction to microwave and optical metamaterials. Today's lecture will be on the optical properties of dielectrics. So, here is the lecture outline. We will see the optical properties of dielectric materials and discuss the Selmayr dispersion formula, absorption coefficient, and properties of bulk crystals.

So, let us begin with the optical properties of dielectric materials. And we can see that the real part of the permittivity, which is plotted in blue, typically exhibits a range. Line shape around each resonance. So, you can see there is a resonance in the infrared and also in the blue wavelength range, okay.

So, this is higher energy; this is lower energy. So, you can say that this longer wavelength and this shorter wavelength, and in between these ultraviolet and infrared, lies a visible range where the red dashed line represents the absorption, or that basically represents the imaginary part of the permittivity, which is almost 0 or negligible. So, you just have to know the real part dominating the permittivity in the visible range, right? So, another important thing to see here is that in the case of resonance.

- Optical Properties of Dielectric Materials

- Sellmeier Dispersion Formula

- Absorption Coefficient

- Properties of Bulk Crystals

So, you are basically having the resonance where this particular absorption is maximized or the imaginary part is maximized or you can say where the real part undergoes a change in the sign. So, the real part which is real of ε that is large and positive as you can see at the low frequency side of the resonance okay and then it quickly becomes negative when the frequency is slightly higher than that of the resonance.

Optical Properties of Dielectric Materials

- The real part of $\varepsilon(\omega)$ exhibits a Lorentz line shape around each resonance.

- The real part $\text{Re}(\varepsilon)$ is large and positive at the low-frequency side of the resonance, and it has a negative value when the frequency is slightly higher than that of the resonance.
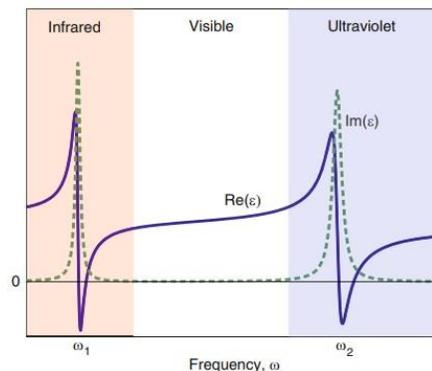

Figure: The dielectric function $\varepsilon(\omega)$ for a typical dielectric material with the lattice resonance and electron transition resonance marked as ω_1 and ω_2, respectively.

So, this is exactly where the resonance frequency lies, okay. So, here what again we are actually showing the dielectric function of a typical dielectric material with you have got the lattice resonance and the electron transition resonance these are marked as ω_1 and ω_2 respectively okay. Now when you get this negative dielectric constant that is somewhat counterintuitive because the transparent materials they have positive dielectric function in the visible range. Okay. And when you have a negative value of the real power of the permittivity that

Optical Properties of Dielectric Materials

- A negative dielectric constant is somewhat counterintuitive, because transparent materials have positive dielectric functions in the visible range.

- A negative value of $\text{Re}(\varepsilon)$ implies that the response (in this case, the polarization density **P** or the electric displacement **D**) is directed opposite to the electric field **E**.
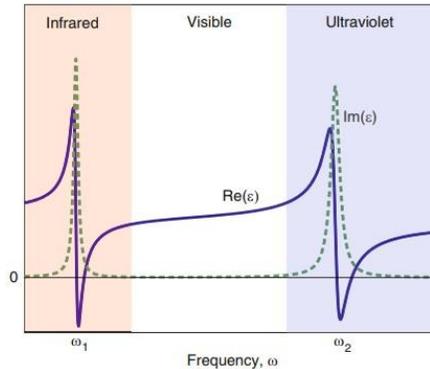

Figure: The dielectric function $\varepsilon(\omega)$ for a typical dielectric material with the lattice resonance and electron transition resonance marked as ω_1 and ω_2, respectively.

basically implies that the polarization density P or you can say the electric displacement D in those kind of material at that particular given frequency will be directed in the direction opposite to that of the incident electric field.

So, that is how you can understand what this negative real part of the permittivity means. Now, the electromagnetic response of a material around the resonance can be seen here. So, this is where the resonance happens. So, you can actually normalize the frequency axis to the resonance frequency ω_0. So, you can simply write ω as ω_0.

Optical Properties of Dielectric Materials

- The electromagnetic response of a material around a resonance is illustrated in Figure, where a spring-mass oscillator is used to facilitate understanding.

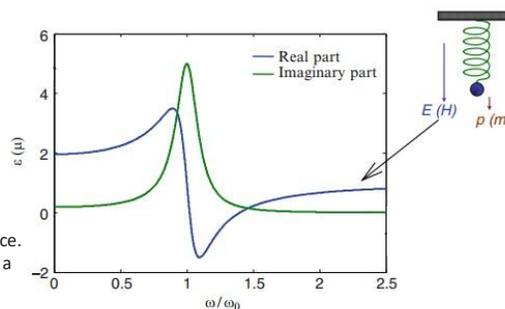- Away from the resonance, the response is able to follow the driving force with no delay.

Figure. The positive and negative electromagnetic responses around a resonance. The three spring-mass oscillators are used as a mechanic analogue of the scenario.

So, ω_0 is basically your resonant frequency. so when $\omega/\omega_0 =$ one that is where when this imaginary part is speaking okay so this is where the transition takes place okay for the real part of the permittivity as well now this particular resonance okay you can actually modulate like a spring mass oscillator that can help you In better understanding. So, you have the electric or the magnetic field this way, okay, and you can have. So, we are mainly talking about the electric field okay and you can see that the mass or the spring is or the polarization is also in the same direction. Now when you are basically here which is far away from the resonance okay in that case the response that you see here will be able to follow the driving force with no delay.

Optical Properties of Dielectric Materials

- When the frequency is slightly lower than the resonance frequency ω_0, the function (ε or μ) is increasingly positive and the response (**P** for electric resonances and **M** for magnetic ones) is enhanced.

- On the other side of the resonance peak, the function can take a negative value, thus the response is opposite to the driving force.
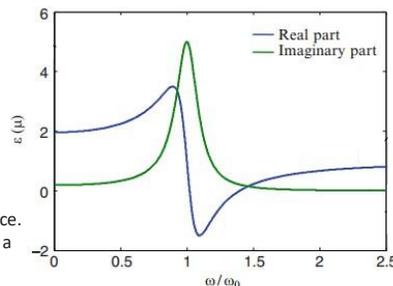

Figure. The positive and negative electromagnetic responses around a resonance. The three spring-mass oscillators are used as a mechanic analogue of the scenario.

However, when you are at frequency which is slightly lower than that of the resonance frequency ω_0 say you are kind of here okay. In that case what you see that your ε or μ for that matter okay we are just let us talk only in terms of permittivity right now ε. So, you can see that the μ is basically an increasingly positive function. That means the response which is polarization for electric resonances and if you consider μ you will talk in terms of magnetization for the magnetic ones okay that is also getting enhanced okay. So, in this case the response is slightly getting enhanced which you can see from the mass spring kind of arrangement as well.

Now, when you are on the other side of the resonance peak that is where your response your ε or μ is showing a negative value that means the response is basically opposite to the driving force which can be seen here. So, you can write electric field and you will see the polarization is in the opposite direction or when you talk about magnetic field you will see the magnetization is in the opposite direction. So, this is what this particular graph tells you. Now, the simple square root relationship that you have seen between the refractive index and the permittivity of any material in the frequency domain will just hold good ok. So, now if you have a complex permittivity that means, it has got a real and the imaginary part in that case your refractive index the frequency dependent refractive index will also become complex and you can say it has got two part the real part is n prime ω And then you have an imaginary part, which is n double prime ω.

And that can be correlated to the square root of ε ω, which itself is kind of complex, right? So, we understood that the real part of the complex number n ω this part can be called as the refractive index or index of refraction the same thing. And the imaginary part n double prime

- The simple square-root relationship between the refractive index and the permittivity still holds in the frequency domain.

- Therefore the frequency-dependent refractive index $n(\omega)$ being a complex value in the general case, is related to the dielectric function $\varepsilon(\omega)$ by:

$$n(\omega) = n^{'}(\omega) + in^{''}(\omega) = \sqrt{\varepsilon(\omega)}$$

- The real part of the complex number $n(\omega)$ is simply called the refractive index or the index of refraction, while the imaginary part $n^{''}(\omega)$ is referred to as the absorption index.

ω is referred to as the absorption index ok. So, that basically quantifies the amount of absorption in that material. Now, the simple square root relationship is between the refractive index. So, if you do some basic algebra you can correlate this equation and you can find out how this real and the imaginary part will be correlated to the permittivity.

So, the some basic algebra can lead you to some useful relationships where you can write down the real and the imaginary part of the permittivity in terms of the real and the imaginary part of the refractive index like ε prime can be written as n prime square - n double prime whole square and ε double prime can be written as 2 n prime n double prime. So, in different books they also just write it as n n kappa or k. So, you just need to know the different notations people use throughout the literature, but they all can actually give the same relation between permittivity and refractive index. So, if you do the reverse that how if the real and the imaginary part of the permittivity from that how can you obtain the real and the imaginary part of the refractive index you can basically use this particular equation. Now, we move on to Sellmeier formula.

- The simple square-root relationship between the refractive index and the permittivity still holds in the frequency domain.

- Therefore the frequency-dependent refractive index $n(\omega)$ being a complex value in the general case, is related to the dielectric function $\varepsilon(\omega)$ by:

$$n(\omega) = n'(\omega) + in''(\omega) = \sqrt{\varepsilon(\omega)}$$

- Some basic algebra can lead to the useful relations between the real and imaginary parts of the refractive index and those of the dielectric function:

$$\varepsilon' = (n')^2 - (n'')^2 \qquad (n')^2 = \left[\varepsilon' + \sqrt{(\varepsilon')^2 + (\varepsilon'')^2}\right]/2$$
$$\varepsilon'' = 2n'n'' \qquad (n'')^2 = \left[-\varepsilon' + \sqrt{(\varepsilon')^2 + (\varepsilon'')^2}\right]/2$$

So, we will tell you what the significance of this formula is. So, we have seen that the frequency dependence of refractive index that is n ω is basically having a very similar kind of plot that is given earlier for the permittivity. So, this is the plot for permittivity except that there will be some changes that the curve of the real part of the refractive index will be raised. That means this will have some offset and everything will be a positive value because in real material you will not see any negative value in the real part of the index. So, in the permittivity part this part shows negative value of the permittivity whereas, in the refractive index part this entire graph the solid graph will be shifted upwards so that you can avoid the negative value right.

Sellmeier Formula

- The frequency dependence of $n(\omega)$ is very similar to the plot given in Figure for the permittivity $\varepsilon(\omega)$, except for the fact that:

  - the curve for the real part $n'(\omega)$ is raised (offset) along the vertical axis to avoid a negative value in the real part of the index.

- The imaginary part $n''(\omega)$ is negligibly small (usually less than 10^{-5}) for common transparent dielectrics in the visible range.

- The refractive index of such materials can be modeled in a form similar to:

$$\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

without involving the imaginary part or the damping constants.
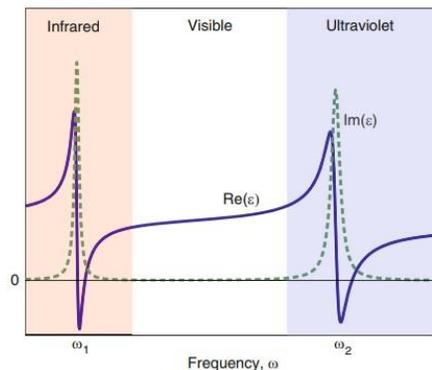
and the imaginary part that is n double prime ω is negligibly small typically of the order of 10^{-5} that is for the common dielectrics in the visible range which is also can be seen from here that it is very very small right. So, in such case the refractive index of this type of material can be modeled

in a form which is so, you can write ε ω can be written as 1 + summation / j S j ω j ² / ω j ² - ω ² - i ω γ j. So, that basically Without involving the imaginary part or the damping constants, right? So, with if you do not take the imaginary part of the damping constants you can just remove this particular part okay. So, the whole thing will look like this then okay and you can replace the frequencies with the corresponding wavelength. So, everything you can convert in terms of refractive index and wavelength, right? So, when you say that there is no damping associated, the imaginary   part   also   goes   to   0.

So, you can simply write that in this equation, the permittivity will be more or less real. So, when it is real you can simply write it as n ² λ you are now moving from the frequency domain to the wavelength domain ok. So, you have 1 + summation / j Sj now instead of ω j

Sellmeier Formula

- The refractive index of such materials can be modeled in a form similar to:

$$\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

without involving the imaginary part or the damping constants.

- In this situation, aforementioned equation reduces to the widely used Sellmeier dispersion formula:

$$n^2(\lambda) = 1 + \sum_j \frac{S_j \lambda^2}{\lambda^2 - \lambda_j^2}$$

- Some modified forms of the Sellmeier formula also exist in the literature in which an empirical approximation for $n(\lambda)$ instead of $n^2(\lambda)$ is provided.

² / ω j ² - ω ² you will simply have λ ² / λ ² -. λ j ². So, they are basically inversely proportional to each   other   I   am   sure   you   all   of   this   particular   relation   like   f   =   c/λ.

Sellmeier Formula

- The refractive index of such materials can be modeled in a form similar to:

$$\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

without involving the imaginary part or the damping constants.

- In this situation, aforementioned equation reduces to the widely used Sellmeier dispersion formula:

$$n^2(\lambda) = 1 + \sum_j \frac{S_j \lambda^2}{\lambda^2 - \lambda_j^2}$$

- Power series approximations to the Sellmeier formula are also used:

  - where the index $n(\lambda)$ or $n^2(\lambda)$ is expressed as the sum of powers of wavelength as $\sum_j A_j \lambda^j$

  - where the index j can be both positive and negative.

So, from there you can convert f is linear frequency when you do it * 2 π you get ω and from that you can always find out this relation. Right. So, some modified form of this Sellmeier formula also exist okay where you will see that there is now empirical approximation of n λ directly provided instead of talking in terms of n ² λ. So, the power series approximation to Sellmeier formula can also be used where the refractive index n λ or the ² of it can be expressed as sum of power of wavelength something like summation / j a j λ j where j can be both positive and negative right. standard representation of the Selmayer dispersion formula n square λ can be written as 1 + summation / j Sj λ ² / λ ² - λ j ² right.

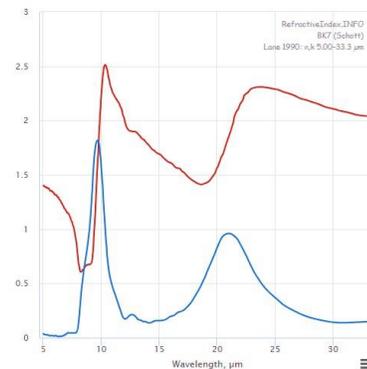# Sellmeier Formula

Sellmeier Dispersion Formula

$$n^2(\lambda) = 1 + \sum_j \frac{S_j \lambda^2}{\lambda^2 - \lambda_j^2}$$

- For characterization of glasses, the equation consisting of three terms is commonly used:

$$n^2(\lambda) = 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}$$

- As an example, the coefficients for a common borosilicate crown glass known as BK7 are shown below:

$B_1 = 1.04$ $C_1 = 6 \times 10^{-3}\,\mu m^2$

$B_2 = 0.23$ $C_2 = 2 \times 10^{-2}\,\mu m^2$

$B_3 = 1.01$ $C_3 = 1.03 \times 10^2\,\mu m^2$

Now, for the characterization of glasses. Okay, the equation basically consisting of three terms is commonly used. So, you can see that this is a particular graph from the refractive index dot info

the red one shows the real part of the refractive index and the blue one shows the imaginary part which is basically k okay. So, you can find out that if you take a common borosilicate crown glass, which is also known as BK7. So, you can use this particular constants B_1, B_2 and B_3 okay as this and C_1, C_2, C_3 as this that will closely model this particular characteristics ok.

Sellmeier Formula

- For common optical glasses, the refractive index calculated with the three-term Sellmeier equation deviates from the actual refractive index by less than 5×10^{-6} over the wavelengths' range of 365 nm to 2.3 μm, which is of the order of the homogeneity of a glass sample.

- Additional terms are sometimes added to make the calculation even more precise.

- Sometimes the Sellmeier equation is used in two-term form:

$$n^2(\lambda) = A + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2}$$

- Here, the coefficient A is an approximation of the short-wavelength (*e.g.*, ultraviolet) absorption contributions to the refractive index at longer wavelengths.

- Other variants of the Sellmeier equation exist that can account for a material's refractive index change due to temperature, pressure, and other parameters.

So, for common optical glasses the refractive index calculated with this 3 term Sellmeier equation deviates from the actual refractive index by less than 5 * 10 $^{-6}$ / the wavelength range of 365 to 2.3 micron. That means you have a very good approximation, right? So, this is of the order of the homogeneity of the glass samples. So, you can always try to add additional terms that can make the calculation even more precise, okay. But sometimes you will see the Sellmeier equation is also used in two term form.

So, in that case n ^{2} λ can be simply written as a + B_1 λ ^{2} / λ ^{2} - C_1 + B_2 λ ^{2} / λ ^{2} - C_2 right. So, you will have only two terms here. So, you can see that there is a new coefficient a that is being used as an approximation of the short wavelength that is like ultraviolet absorption contribution okay and that contributions to the refractive index at longer wavelengths. Other variants of the Sellmeier equation also exist that can account for the materials refractive index change due to temperature, pressure and any other parameters. So, now let us look at the absorption coefficient.

Absorption Coefficient

- For weakly absorptive media, the absorption coefficient α is routinely used to characterize the attenuation of light propagating in the material.

- The absorption coefficient is the exponential index appearing in Beer's law, which says that the intensity (I_0) of light will decrease as the light travels a distance z in a medium:

$$I = I_0 \exp(-\alpha z)$$

- Since the electric field **E** changes along the propagation direction following the function:

$$\exp\left[\frac{i2\pi(n' + in'')z}{\lambda_0}\right]$$

- The intensity is proportional to $|\mathbf{E}|^2$, it is easy to see that the absorption coefficient α is connected to the imaginary part the refractive index by:

$$\alpha = 4\pi n'' / \lambda_0$$

Dimensions of reciprocal length as cm^{-1}.

So, for weakly absorptive media the absorption coefficient which is α ok that is regularly used to characterize the attenuation of light when it is propagating through the material. Now the absorption coefficient is the exponential index that appears in the Beer's law or Beer-Lambert's law which says that the intensity I_0 of the light will decrease exponentially as the light will travel through a medium and at a distance z it can be calculated as I_0 sorry this will be I okay yeah it can be written as I which is I_0 okay initial intensity exponential - or you can say intensity when z = 0 exponential - α z. Now, since the electric field E changes along the propagation direction that also follows this function exponential i_2 π n prime + i n double prime z / λ_0 since the electric field E changes along the propagation direction following the term. So, you have exponential i_2 π * n prime + i n double prime that is the complex refractive index * z / λ_0 right. So, you will see that the intensity this one is basically proportional to modulus of E ^{2}.

Properties of Bulk Crystals

- Optical anisotropy and birefringence are intrinsic properties of crystalline dielectrics.

- In an anisotropic medium such as uniaxial or biaxial crystals, the optical response to the external field is directionally dependent.

- Moreover, the polarization field **P** is not necessarily aligned with the electric field, so tensors instead of scalar values must be used for the electric susceptibility χ and the permittivity ε.

- In optical metamaterials, however, the length scale of the continuous dielectric portion of the unit structure is so small – being much smaller than the light wavelength – that all dielectric components tend to have an "amorphous" state without a preferred direction.

So, it is easy to say that the absorption coefficient α can be connected to the imaginary part of the refractive index and you can write $\alpha = 4\pi n$ double prime $/ \lambda_0$ ok. So, typically your absorption coefficient α will have the dimension of reciprocal of length that is centimeter inverse. Now we'll look into the properties of bulk crystals, shall we? So optical anisotropy and birefringence are basically intrinsic properties of crystalline dielectrics. So, in an anisotropic medium which is which can be a uniaxial or biaxial crystal, the optical response to the electric field is basically direction dependent. So, and we will see that the polarization field P is not necessarily aligned with the external field.

So, you have to use tensors instead of scalar values that must be used in the case of electric susceptibility and also the permittivity ε. In optical metamaterials however, when the length scale of the continuous dielectric portion of the unit structure is so small that means, it is very sub wavelength or much smaller than the wavelength of light. Then you will see that all the dielectric components will tend to have an amorphous state without any preferred direction. So, therefore, what you can do that in the design and simulation of optical metamaterials, you can assume a scalar value and that will be typically sufficient for the permittivity or the refractive index of the dielectric constituents. So, as for the non-linear properties of the dielectrics. they are normally neglected because most of today's optical metamaterials are operated in the linear response region ok. So, even when the nonlinearity of the metamaterials is included the nonlinear effects will typically result from metamolecules as a whole + the surface states of the metallic part while the nonlinearity of the dielectric itself will play a very minor role right. So, conventionally the use of metals in optical systems is limited in very few applications something like mirrors or optical thin films. So in optical metamaterials, you will see that most of the designs which will be studied will incorporate metal in the unit cell of the metamaterial itself. And the sharp contrast between the optical response of metals and that of the dielectric will play an important role in making the meta atom the functional elements.

Properties of Bulk Crystals

- Therefore, in the design and simulation of optical metamaterials, a scalar value is usually sufficient for the permittivity or the refractive index of the dielectric constituent.

- As for the nonlinear properties of dielectrics, they are normally neglected because most of today's optical metamaterials are operated in the linear response region.

- Even when the nonlinearity of metamaterials is included, the nonlinear effects will typically result from the meta-molecules as a whole plus the surface states of the metallic part, while the nonlinearity of the dielectric itself plays a very minor role.

- Conventionally, the use of metals in optical systems is limited to only a few applications like mirrors and optical thin films.

- In optical metamaterials, however, most of the designs being studied incorporate metals in the unit structure of the metamaterial.

- The sharp contrast between the optical response of metals and that of dielectrics is essential to making meta-atoms functional elements.

So, most of the compositional units used in making the meta atoms of photonic metamaterials are delicate sub wavelength structures and they contain both metal and dielectric components. And the response of this metamaterial to any electromagnetic field is dependent on its specific architecture of this meta atoms. So, how you design that will tell you how the electromagnetic field is going to behave with that meta atom. Therefore, it is hard to develop a simple and unified method for analyzing the properties of any arbitrary structural unit of a metamaterial. So, you cannot have a simplified generalized theory for all the metamaterial meta atoms right.

Metal-Dielectric Composites and Mixing Rules

- Most of the compositional units used in making the meta-atoms of photonic metamaterials are delicate, subwavelength structures containing both metal and dielectric components.

- The response of a metamaterial to an electromagnetic field is critically dependent on the specific architecture of the meta-atoms.

- Therefore, it is hard to develop a simple and unified method for analyzing the properties of an arbitrary structural unit of a metamaterial.

- However, for composite materials without well-structured building blocks, there are generalized analytical approaches that allow us to estimate their average electromagnetic response.

- In such randomly structured media, the metal and dielectric components are arranged in a disordered fashion.

- The overall optical properties of a metal-dielectric composite can be significantly different from those of its constituent materials.

However, for composite materials which does not have well structured building blocks, you can use some generalized analytical approach that will give you the luxury of estimating the average electromagnetic response. So, in such randomly structured media, the metal and dielectric

components are basically arranged in a disordered fashion. So, the overall optical properties okay of such a metal dielectric composite will be significantly different to those of its constituent molecules. So, that is where the metal dielectric composites and different mixing rules will come into picture and we will continue that in the next lecture. So, thank you with this we will stop here if you have got any queries you can always drop an email to this particular email address mentioning the course number.

This is the lecture number, then the course name in the subject line. So, in the next lecture, we will see the effective medium theory, which will be a continuation of this discussion.