Course Name: Introduction to Microwave and Optical Metamaterials
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Week-4
Lecture-17

Lec17: Optical Properties of Metal and Dielectrics

Hello everyone, welcome to Lecture 17 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on the optical properties of metals and dielectrics; it is basically a continuation of the previous lecture. So, in this one, we will briefly look into the optical properties of metals that we have described earlier based on the Drude model. We will take up the dispersion relations of bulk plasmons, and we will discuss other dispersion relation examples in detail. And we will also look into the optical properties of other dielectric materials. So, this is what you have seen earlier: that in the Drude model, the permittivity can be described as $\varepsilon\ \omega$.

which is $1 - \omega\ p^{2}$ by $\omega^{2} + i\ \gamma\ \omega$. So, $\omega\ p$ is basically the plasma frequency, which is given as the square root of $n^{2} / m\ \varepsilon_0$. And in this particular model, if you separate out the real and the imaginary parts, you will get ε_1 and ε_2. This is how you can write them: ε_1 is $1 - \omega\ p^{2} / \omega^{2} + \gamma$.

Lecture Outline

- Optical Properties of an Electron Gas (Metal) — Drude Model
- Dispersion Relations — Bulk plasmon
- Dispersion Relations
- Dispersion Relations — Example
- Optical Properties of Dielectric Materials

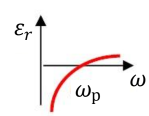
Square, whereas ω_2 is γ by ω * ω p square / ω square + γ square. Now this is how you can plot it, so you can say at ω = ω p, okay, the permittivity. So, you can put this if there is no damping involved. Okay, you can say that at ω = ω p. Your permittivity becomes 0, something like that, okay.

Optical Properties of an Electron Gas (Metal) — Drude Model

Drude Model: $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ $\qquad \omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$ $\qquad \varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$ $\qquad \varepsilon_2 = \left(\frac{\gamma}{\omega}\right)\frac{\omega_p^2}{(\omega^2+\gamma^2)}$

- At frequencies $\omega < \omega_p$, **metals retain their metallic character**.
- For large frequencies close to ω_p, the product $\omega\tau >> 1$, leading to negligible damping.
- Here, $\varepsilon(\omega)$ is predominantly real:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$



- Can be taken as the dielectric function of the **undamped** free electron plasma.

 Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

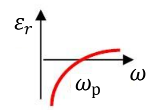
So, what do you understand when the frequency is below the plasma frequency? So, the permittivity is negative. So, that is why the metals retain their metallic characteristics: the metal will be reflective. But for larger frequencies that are close to the plasma frequency, we will see that, you know, the product ω tau. will be much much greater than 1 that leads to negligible damping. So, in that case, you will understand that this particular term will become predominantly real, and you will have ε ω = 1 - ω p ^{2} / ω ^{2}.

So, you are basically ignoring the damping in this particular case, and that is what is typically plotted here, right? So, you can take this as the dielectric function of undamped free electron plasma. So, this is what the expression will look like: 1 - ω p [2] by ω [2]. So, I understood that this is the case when damping is not present. So, typically the relaxation time tau is of the order of 10^{-14} seconds, which will give You have a γ ray which is around 100 terahertz, and that is 0.4 electron volts.

Optical Properties of an Electron Gas (Metal) — Drude Model

Drude Model: $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ $\quad \omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$ $\quad \varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$ $\quad \varepsilon_2 = \left(\frac{\gamma}{\omega}\right)\frac{\omega_p^2}{(\omega^2+\gamma^2)}$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$



Note:

- Relaxation time of most metal is $\tau \sim 10^{-14}$s $\rightarrow \gamma \sim 100THz\sim 0.4$eV
- $m \sim 9.1 \times 10^{-31}$ kg (electron mass)
- $n \sim 6 \times 10^{22}$ cm^{-3} $= 6 \times 10^{28}$ m^{-3} (Au and Ag)
- $e = 1.6 \times 10^{-19}$ C (elementary charge)
- $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (vacuum permittivity)
- $\omega_p = 10$ eV (ultravoilet) $\gg \gamma$

IIT Guwahati | NPTEL swayam — Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

The mass of the electron is $m \sim 9.1 \times 10^{-31}$kg ε0 $\varepsilon_0 = 8.85 \times 10^{-12}$; this is the vacuum permittivity and the electron concentration. So, typically, you know you have $n \sim 6 \times 10^{22}\text{cm}^{-3} = 6 \times 10^{28}\text{m}^{-3}$ for gold and silver. and elementary charge of electron is $e = 1.6 \times 10^{-19}$C . So, these are all the constraints that you should know. So, with that what we understood that this is the region which is of first interest because below which it behaves Like a shiny metal, beyond this frequency, the metal will become transparent. Right. So, this is what we will be considering for very low frequencies where, you know, ω is much, much lesser.

Than 1 by tau, that is your γ frequency. Okay, 1 by tau is nothing but γ, the damping frequency. In that case, you will see that you know the damping is much larger. So, your ε_2, which is nothing but the imaginary part of the permittivity, will be much larger than the real part. So, in that case, you can always try to write the relative permittivity.

Okay, ε r, say you write it in the real and the imaginary parts; you can also take this as ε_1 and ε_2. Why I am giving all these different notations is because you know in different books they use different kinds of notation. Some go with this kind of prime and double prime notation; a few books write about it. With ε_1 and ε_2 kind of notation, that is absolutely fine. So, this is the relative permittivity, and this is basically complex.

Optical Properties of an Electron Gas (Metal) — Drude Model

- **We consider next the regime of very low frequencies**, where $\omega << \tau^{-1}$.
- Hence, $\varepsilon_2 >> \varepsilon_1$, and the real and the imaginary part of the complex refractive index are of comparable magnitude with:

$$n \approx \kappa = \sqrt{\frac{\varepsilon_2}{2}} = \sqrt{\frac{\tau\omega_p^2}{2\omega}}$$

$$\varepsilon_r = \varepsilon_r^{'} + i\varepsilon_r^{''} = (n + i\kappa)^2$$

where $\varepsilon_r^{'} = n^2 - \kappa^2$ & $\varepsilon_r^{''} = 2n\kappa$

- In this region, metals are mainly absorbing, with an absorption coefficient of: $\alpha = \left(\frac{2\omega_p^2\tau\omega}{c^2}\right)^{1/2}$
- By introducing the dc-conductivity σ_0, this expression can be recast using: $\sigma_0 = \frac{ne^2\tau}{m} = \omega_p^2\tau\varepsilon_0$
- On substitution:

$$\alpha = \sqrt{2\sigma_0\omega\mu_0}$$

So, you can write it as $(n + i\kappa)^2$, where n is basically the real part. Of the refractive index, and this is giving you the absorption coefficient, right? So, n and kappa are nothing but $\sqrt{\frac{\varepsilon_2}{2}}$. So, ε_2 is basically ε r double prime; that's the same thing. you can put these values in this particular regime. So, you are getting the $\frac{\tau\omega_p^2}{2\omega}$. There are other correlations or relationships that can relate this real part and the imaginary part with n and kappa. In a few books, you will also see they are written as n_1 and n_2; those are all fine, okay. The real part of the convex refractive index can be written as n, and the imaginary part can be written as kappa or k. So, if you do n ^{2} - kappa ^{2}, that gives you the real part of the permittivity. and when you do 2 n k or 2 n kappa that will give you the imaginary part of the permittivity.

Optical Properties of an Electron Gas (Metal) — Drude Model

- The application of Beer's law of absorption implies that for low frequencies the fields fall off inside the metal as $e^{-z/\delta}$, where δ is the **skin depth.**

$$\delta = \frac{2}{\alpha} = \frac{c}{\kappa\omega} = \sqrt{\frac{2}{\sigma_0\omega\mu_0}}$$

- At higher frequencies ($1 \leq \omega\tau \leq \omega_p\tau$):

  The complex refractive index is predominantly imaginary (leading to a reflection coefficient $R \approx 1$), and σ acquires more and more complex character, blurring the boundary between free and bound charges.

- Our description up to this point has assumed an ideal free-electron metal, we will now briefly compare the model with an example of a real metal important in the field of plasmonics.
- In the free-electron model, $\varepsilon \rightarrow 1$ at $\omega >> \omega_p$.
- For the noble metals (e.g. Au, Ag, Cu), an extension to this model is needed in the region $\omega > \omega_p$ (**where the response is dominated by free electrons**).

Now, in this region where the damping is high, the metal mainly absorbs. So, there is an absorption coefficient, which is α, that is given as the square root of 2 ω^2 tau ω/c^2. Now, if you introduce the DC conductivity, which is σ_0, in this expression, You will see that σ_0 is given as n ^{2} tau by m. So, which is basically ω p ^{2} tau ε_0, okay? So, if you do the substitution here, you will see α can be written as the square root of 2 ε_0. sorry α can be written as square root 2 σ_0 ω μ_0.

So, c^2 is nothing but 1 by c^2, which is nothing but μ_0 ε_0. So, you can use that to find out what this α is, which is the absorption coefficient, right? Now, the application of Beer-Lambert's law of absorption would imply that for low frequencies, the field will fall off inside the metal. Following a relationship that is exponential decay given by $e^{-z/\delta}$. Δ is nothing but skin depth. So, δ is related to the absorption that we saw earlier.

So, $\delta = \frac{2}{\alpha} = \frac{c}{\kappa\omega} = \sqrt{\frac{2}{\sigma_0\omega\mu_0}}$. So, this is skin depth, right? Now, when you move to higher frequencies, it's something like, you know, you go for ω tau. which is much larger than 1, but smaller than, you know, ω p tau. So, when you are here in this region, you will see that the complex refractive index is predominantly imaginary.

Optical Properties of an Electron Gas (Metal) — Drude Model

- This residual polarization due to the positive background of the ion cores can be described by adding the term

  $\mathbf{P}_\infty = \varepsilon_0(\varepsilon_\infty - 1)\mathbf{E}$ where **P** now represents solely the polarization due to free electrons.

- This effect is therefore described by a dielectric constant for high frequency, ε_∞ (usually $1 \leq \varepsilon_\infty \leq 10$), and thus:

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

- The validity limits of the free-electron description are illustrated for the case of gold in the following Figure.
- Clearly, at visible frequencies the applicability of the free-electron model breaks down due to the occurrence of **interband transitions**, leading to an increase in ε_2.

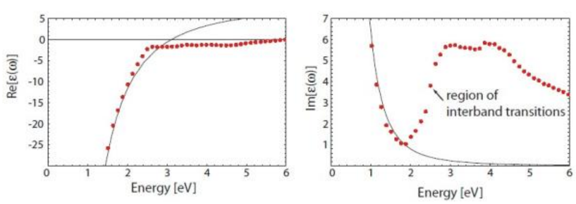


Figure. Dielectric function $\varepsilon(\omega)$ of the free electron gas (**solid line**) fitted to literature values of the dielectric data for gold (dots).

Interband transitions limit the validity of this model at visible and higher frequencies.

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So, that means it is going to give you a reflection coefficient r that = 1. The conductivity σ will acquire an increasingly complex character. So, that will basically blur the boundary between the free and bound charges. So, our discussion up to this point has assumed an ideal free electron metal. But now we will briefly compare the model with an example of real metal that is important in this field of plasmonics.

So, in the free electron model you can put you know this ε going to 1 and the frequency It is much larger than the plasma frequency, right? We have seen that. So, for the noble metals gold, silver, and copper, you can use an extension of this model. Okay, that is needed for the region when the frequency is much larger than ω p. where the response is mainly dominated by the free electrons right. So, in that case, the residual polarization that arises from the positive background of the ion cores can be described.

By adding a term something like p infinity that = ε_0 ε infinity - 1 * E. So, here the P represents the polarization due to the free electrons, right? So, what we are seeing here is that this effect is therefore described by a dielectric constant. For high frequency, that is this particular term ε infinity. So, typically this is a constant that ranges from 1 to 10, and you can use this as your high-frequency dielectric constant. So, you can modify your permittivity values for gold, silver, and copper to be ε ω.

Dispersion Relations — Bulk plasmon

- The physical significance of the excitation at ω_p:
  - Consider the collective longitudinal oscillation of the conduction electron gas versus the fixed positive background of the ion cores in a plasma slab.
  - A collective displacement of the electron cloud by a distance u leads to a surface charge density $\sigma = \pm neu$ at the slab boundaries.

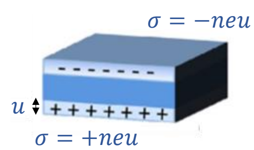

Figure. Longitudinal collective oscillations of the conduction electrons of a metal: Volume plasmon or Bulk plasmon

- Electric field inside the slab: $\mathbf{E} = \sigma/\varepsilon_0$
- Restoring force applied to an electron: $\mathbf{F} = -e\mathbf{E}$
- Equation of motion: $m\ddot{x} = -\frac{ne^2}{\varepsilon_0}x$
- Equation of motion: $\omega_p = \sqrt{ne^2/\varepsilon_0 m}$

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Which is ε infinity - ω p ^{2} / ω ^{2} + i γ ω. So, this model can be used in the case of gold, silver, and copper, okay. You cannot use one here because of this particular reason. Now, the validity limit for the electron description can be seen here; for this particular energy range, the value This model, which is the solid line, matches very well with the dotted lines. Here, you can also see, but beyond a particular point, the interband transitions take place.

and that basically limit the validity of this model typically at visible and other higher frequencies right. So, below that, it is doing a pretty good job, but after this, like in the visible. and the higher frequency range because of the inter band transitions it is falling failing ok. So, this is the dielectric function of the free electron gas, which is the solid line fitted to the Literature values of the dielectric data of gold, which were obtained from the Johnson and Christie paper in 1972, are correct. So, now let us look into the dispersion relations of different plasmons, okay.

So, let us start with bulk plasmons. So, the physical significance of excitation at the plasma frequency would be to allow us to consider the collective longitudinal. Oscillation of the conduction electron gas versus the fixed positive background of the ion cores in a plasma slab. So, this is the amount of displacement. So, you can see that the positive conductivity will be σ = + nu, where nu is the amount of displacement.

Dispersion Relations

- We now turn to a description of the thus-far omitted transparency regime $\omega > \omega_p$ of the free electron gas model.
- The dispersion relation of traveling waves can be obtained as:

$$\left.\begin{aligned} \omega^2 \varepsilon_r &= c^2 k^2 \\ \varepsilon_r &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned}\right\} \Rightarrow \omega^2 = \omega_p^2 + K^2 c^2$$

- This relation is plotted for a generic free electron metal.

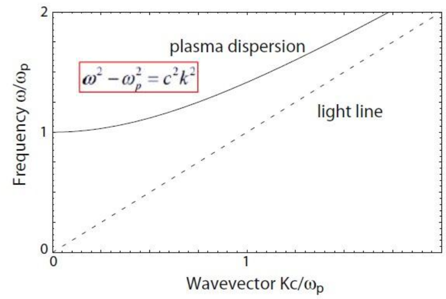

Figure. The dispersion relation of the free electron gas. Electromagnetic wave propagation is only allowed for $\omega > \omega_p$.

IIT Guwahati | NPTEL swayam Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

So, here it will be - neu. So, this is nothing but a longitudinal collective oscillation of the conduction electrons of a metal. So, what you see here is called volume plus one or bulk plus one. So, this is coming from a collective displacement of the electric cloud, which is displaced by a The distance of u has led to this surface charge density, which is σ. You can write as + neu on this side, okay, and that is because the electron cloud has moved away, okay. So, this is happening at the slab boundaries.

So, the electric field inside the slab can be written as E, which is σ / ε_0. You can understand that the restoring force applied on the electron, F, can be written as -

Dispersion Relations

- We now turn to a description of the thus-far omitted transparency regime $\omega > \omega_p$ of the free electron gas model.
- The dispersion relation of traveling waves can be obtained as:

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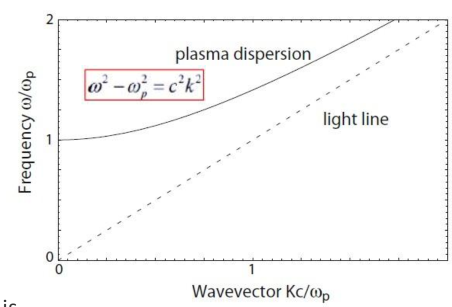
- As can be seen, for $\omega < \omega_p$
  - The propagation of transverse electromagnetic waves is forbidden inside the metal plasma.
- For $\omega > \omega_p$ however, the plasma supports transverse waves propagating with a group velocity $v_g = d\omega/d\mathbf{K} < c$.



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small e capital E. So, this is the electric field; this is the charge of the electron. The equation of motion is pretty simple here: m x double dot will be - n e square. By ε_0 * x, the amount of displacement.

So, here it is: u, or you can also take it as x; that is fine. And from this, you can obtain what is also called the plasma frequency. So, the plasma frequency will be the square root of n e^2 / ε_0 m. Here, you can see the plasma frequency. is mainly dependent on the electron concentration, electron charge, vacuum permittivity, and the mass of electrons.

So, this is pretty much, you know, the material-dependent parameter. Now, if you turn to a description of the transparency region, which is supposed to be at a frequency much larger than the plasma frequency of the free electron gas model, you will see that the dispersion relation of the ah travelling waves Can be obtained as $\omega^2 \varepsilon_r = c^2 \omega^2$, and ε r we have already written. as in the in the range of transparency it will be 1 - ω p square by ω square ok. So, if you put this guy here, you will see that you will obtain a relation $\omega^2 = \omega_p^2 + K^2c^2$.

So, if you try to plot this as a function of frequency and wave vector, both normalized with the plasma frequency. So, the frequency axis can be written as ω/ω_p, and the wave vector can be written as k c/ω_p. So, this is basically a light line to ω = ck; that is the light line. So, if you take $\varepsilon_r = 1$, you get the light line, and this is for, you know, plasma dispersion. So, you can obtain the equation as $\omega^2 - \omega_p^2 = k^2 c^2$.

Dispersion Relations

- The significance of the plasma frequency ω_p can be further elucidated by recognizing that in the small damping limit, $\varepsilon(\omega_p) = 0$ (for $\mathbf{K} = 0$).
- This excitation must therefore correspond to a collective longitudinal mode.
- In this case, $\mathbf{D} = 0 = \varepsilon_0\mathbf{E} + \mathbf{P}$.
- We see that at the plasma frequency the electric field is a pure depolarization field, with $\mathbf{E} = -\mathbf{P}/\varepsilon_0$.

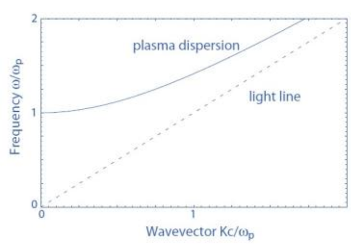

Figure. The dispersion relation of the free electron gas. Electromagnetic wave propagation is only allowed for $\omega > \omega_p$.

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So, this is how it looks. So, this is the dispersion relation of a free electron gas and the electromagnetic waves. Propagation is allowed only for ω greater than ω p, right? So, this is for free electron metal, right? Now, as can be seen, when you have ω less than ω p, okay. So, that is here the propagation is not allowed you can only propagate when your you know when you have Ω is greater than ω p. So, below this, the propagation of the transverse An electromagnetic wave is basically forbidden inside the metal plasma. So, if you have something more than ω p, that is good.

So, the plasma then supports the transverse wave that can propagate inside the material with a group velocity. That is given as vg, which will be nothing but the slope of this curve that is d ω by dk, okay. So, that will be less than c. This slope is obviously less than the light line. So, you will have a slower-moving wave in the material.

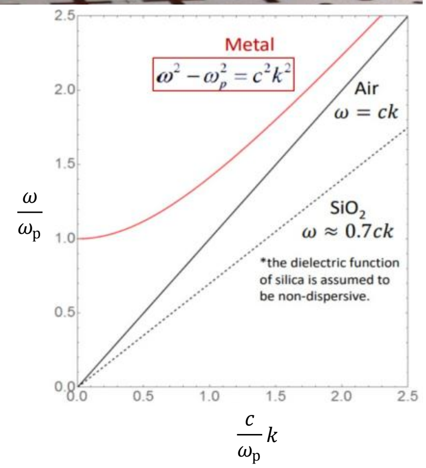
Now the significance of this plasma frequency can be further elucidated by recognizing that in the small damping limit, when You consider ε ω p = 0; that means, for k = 0, the excitation must therefore be a collective longitudinal mode. So, in this case, you can write d = 0, which is nothing but ε_0 e + p, right? And from this you can understand that the plasma frequency at plasma frequency the electric field is basically nothing But the depolarization field, because the electric field, will be simply given as - p/ε_0, right? Now let us take some further examples of these dispersion relations. So, when you have free space, you can write ε r = 1. So, you can put this equation k = square root ε/c into 1.

Dispersion Relations — Examples

- Free space ($\varepsilon_r = 1$): $k = \frac{\sqrt{\varepsilon}}{c}\omega \rightarrow \omega = ck$
- Silica ($\varepsilon_r = 2$): $k = \frac{\sqrt{\varepsilon}}{c}\omega \rightarrow \omega \approx 0.7ck$
- Metal $\varepsilon(\omega) \approx 1 - \frac{\omega_p^2}{\omega^2}$:

$$k = \frac{\omega}{c}\sqrt{\left(1 - \frac{\omega_p^2}{\omega^2}\right)} \rightarrow \omega^2 = \omega_p^2 + c^2k^2$$

- Slow group velocity: $v_g < c$
- Long wavelength: $k > k_0$



Asymptotically approaches to light line as ω → ∞

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So, this is one. So, you simply have ω = ck; that is basically your air or light line. When you have ε equal to that is in silica. So, this equation will become k = the square root of ε / c * ω.

So, that means ω will be 0.7 ck. So, this is how the slope of the curve reduces, okay. So, here you assume the silica to be non-dispersive and it has a refractive index that is the same throughout all the frequencies. For the metal, your ε ω can be written as 1 - ω p ^{2} by ω ^{2}. So, if you put it back into this equation, you get $k = \frac{\omega}{c}\sqrt{\left(1 - \frac{\omega_p^2}{\omega^2}\right)} \rightarrow \omega^2 = \omega_p^2 + c^2k^2$. Now we have also seen that the group velocity is slower than c. So, you are having a Slow group velocity can also allow a longer wavelength k to be larger than k_0. So, asymptotically you can see that this basically approaches the light line as ω approaches infinity. So, in this particular axis, just to remind you, this is at the normalized frequency, and this is the normalized wave vector.

Optical Properties of Dielectric Materials

- Dielectrics are by far the dominant materials used for optical components and devices.
- In conventional optical systems, almost all functional parts, except for some reflection surfaces, are made from crystalline and glassy materials.
- *The reason is simple:* light is effectively manipulated only when it can efficiently pass through a medium, meaning that the medium is, at least to some degree, a dielectric.
- The underlying physical background of light interacting with a dielectric can be analyzed using Maxwell's equations plus the following two constitutive relations:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 \mu_r \mathbf{H}$$

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Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

So, it is an ω k graph, okay. So, it is checked by omega P. this is the value here. So, now let us look at the optical properties of dielectric materials. Dielectrics are, by far, the dominant materials used for optical components and devices. In conventional optical systems, almost all functional parts, except for the reflection surfaces, are basically made from crystalline and glass materials.

The reason is simple: light is effectively manipulated only when it is efficiently passing through a Medium meaning that you know the medium should be, to at least some degree, a dielectric. The underlying physical background of light interacting with a dielectric can be analyzed. By using Maxwell's equations + the two constitutive relations we have seen. So, you can use this particular equation, okay. Here, the first equation tells you the relationship between the electric displacement, the electric field, and the polarization density P.

So, we have already seen how to write that $\varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}$. Okay. So, here ε_0 is the vacuum permittivity; this is the relative permittivity. Similarly, you can also know the relationship between B, H, and M.

So, we have seen earlier. So, this can also be written as $\mu_0 \mu_r \mathbf{H}$. H is the magnetic field and B is the magnetic flux density. So, the study of the linear interactions between light and matter, especially involving which involves you know determining the solutions to Maxwell's equation. It will be by using these constitutive relations along with appropriate boundary conditions. So, at optical frequencies, you can see that the magnetic susceptibility χ_m In any conventional material, typically, it vanishes.

Optical Properties of Dielectric Materials

- The study of the (linear) interactions between light and matter, essentially, involves determining the solutions to Maxwell's equations by using the constitutive relations along with appropriate boundary conditions.
- At optical frequencies, the magnetic susceptibility (χ_{m}) in any conventional material diminishes, and the relative permeability (μ_{r}) is taken normally to be unity.
- This condition substantially simplifies our description of optical materials – transparent ones in particular – by assigning a refractive index $n = \sqrt{\varepsilon_{\mathrm{r}}}$ to each medium.
- Even though all media are dispersive and absorbent in a strict sense, the use of real numbers for refractive indices is extremely convenient, and therefore it has become standard practice in the design and analysis of optical components and devices.
- Particularly, the refraction and reflection behaviors of dielectric systems, including magnitude and phase information, are well characterized by a set of Fresnel equations, which are simple algebraic formulas with refractive indices and incident angles as the only variables.

Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

So, you can safely assume the relative permeability μr to be 1 in all those cases. So, this condition simplifies our description of the optical materials which are basically transparent ones, to be very particular. So, we can simply take the refractive index to be n, which is the square root of ε r, because μ r is always 1. So, even though you know all media are dispersive and absorbent in a very strict sense. The use of real numbers for the refractive index makes it very convenient, and therefore, that has Become a standard practice in the design and analysis of optical components and devices.

And particularly when you think about the refraction and reflection behavior of dielectric systems, including magnitude. And phase information, these are well characterized by a set of Fresnel equations we have already seen. These are simple algebraic formulas that are based on the refractive indices and the incident angle as variables. But that describes the reflection and refraction from different interfaces of different media, right? Now, although the refractive index of isotropic, homogeneous, and linear dielectric materials It can be viewed as a real number in many cases. We should note that a rigorous treatment of light interacting with optical media needs to consider the frequency dependence.

Optical Properties of Dielectric Materials

- Although the refractive index of an isotropic, homogeneous, and linear dielectric material can be viewed as a real number in many cases, we should note that for a rigorous treatment of light interacting with optical media, the frequency dependence of the material properties should be carefully considered.
- At optical frequencies, the oscillation of the electric field is so fast that the bound charges in atoms or molecules are unable to follow the electric field in time.
- Consequently, $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0(1+\chi_e)\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E}$ does not hold in the time domain for the high frequencies of the optical range.
- Instead, the electromagnetic response of the medium described by $\mathbf{D}(t)$ at time t depends not only on the electric field $\mathbf{E}$ at that moment, but also on the value of $\mathbf{E}$ at all past times.

Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

So, typically at optical frequencies, the oscillation of the electric field is so fast that the bound electrons Atoms or molecules are usually unable to follow the electric field in time. So, consequently, you can think that the equation $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0(1+\chi_e)\mathbf{E}$ that has given us the relative permittivity. This particular equation does not hold in the time domain for high frequencies in the optical range.

Instead, the electromagnetic response of the medium should be better described as a function of time, considering dt. Because it not only depends on the electric field at that moment, but also on the electric field of all past *. So, ideally, the constitutive relationships should take into account this time operator or the convolution and the equation. should be written as dt = ε_0 et + ε_0 integration From - infinity to tau chi e t - tau e tau d tau ok. So, this is how you should actually obtain the constitutive relationship at optical frequencies.

Now, fortunately, the proportionality is still valid for the relationship between D and E in the frequency domain. As long as the material is considered to be a linear medium, right? That means the susceptibility remains independent of the strength of the electric field in such a medium. So, you can safely write the frequency-domain constitutive relationship as d ω. which is ε_0 εω e ω. So, ε ω can be written as 1 + chi e ω, right? So, this is the relationship in the frequency domain.

Optical Properties of Dielectric Materials

- The constitutive relation has to involve time operators (convolution) as follows:

$$\mathbf{D}(t) = \varepsilon_0 \mathbf{E}(t) + \varepsilon_0 . \int_{-\infty}^{\tau} \chi_e(t-\tau)\mathbf{E}(\tau)d\tau$$

- Fortunately, the proportionality is still valid for the relationship between $\mathbf{D}$ and $\mathbf{E}$ in the frequency domain, as long as the material being considered is a linear medium.

- Means the susceptibility is independent of the strength of the electric field.

- Therefore, write the frequency-domain constitutive relationship as:

$$\mathbf{D}(\omega) = \varepsilon_0 \varepsilon(\omega)\mathbf{E}(\omega) = \varepsilon_0[1 + \chi_e(\omega)]\mathbf{E}(\omega)$$

Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

Now, why are we bringing up the dependence on ω? Because most materials, or you can say metamaterials, which... They are basically metal dielectric composite units; they are strongly dispersive, which means the property depends on the frequency. So, in the visible or near-infrared wavelength range, the origin of the dispersion in a metamaterial can be mostly ascribed to the metallic part because metals are much more dispersive than the transparent dielectric medium.

Optical Properties of Dielectric Materials

- Most metamaterials with metal-dielectric composite units are strongly dispersive.

- In the visible or near-infrared wavelength region, the origin of the dispersion in a metamaterial is mostly ascribed to the metallic part, because metals are much more dispersive than transparent dielectric media.

- Nevertheless, it is very helpful to study the frequency sensitivity of the dielectric function $\varepsilon(\omega)$ in dielectric media.

- In particular, when we extend our interest to the whole optical spectrum ranging from the near-UV (200–400 nm) to the mid-infrared (tens of microns), the dispersion of even the best transparent material is no longer a negligible feature.

- The transparency window for most dielectric materials is bounded at the long-wavelength side by the infrared absorption mode of phonons due to lattice vibrations, while at the high-frequency side the window is bounded by inter-band electron-hole transitions.

Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

Nevertheless, it is helpful to study the frequency sensitivity of the dielectric function ε(ω) in a dielectric medium. And in particular when we extend our interest to the whole optical spectrum from, say, near UV to medium infrared. which are ranging from 200 to 400 nanometer to tens of microns, the dispersion of even The best transparent material can also no longer become negligible for such a wide range. You can think the same for the microwave frequency, which also ranges at that.

At different frequencies, the material may behave slightly differently. So, you should always take into account the dispersion. The transparency window for most dielectric materials is bounded on

Optical Properties of Dielectric Materials

- The approximate spectral transparency ranges for a number of important dielectric materials are shown in **Figure**.
- When designing an optical metamaterial, it is important to make sure that the selected dielectric constituent is transparent within the wavelength range of interest.
- Otherwise substantial loss may arise from electron or photon resonances in the dielectric, which are detrimental to the performance of the metamaterial in most cases.

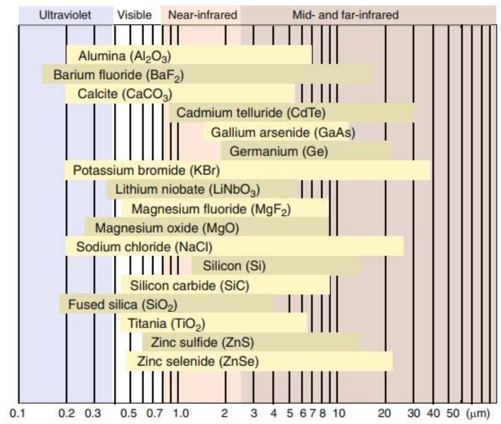


Figure. The spectral range of transparency for several important dielectric materials.

Source: W. Cai and V. Shalaev, Optical metamaterials, Springer US, 2011.

the long-wavelength side. By the infrared absorption mode of the phonons due to lattice vibrations. While the high frequency side ok the window is bounded by the inter band electron hole transition. So, on one side, you have low frequency that is coming from the infrared absorption of the phonons.

which mainly come from the lattice vibration on the high-frequency side On the short wavelength side, that is also bounded by interband electron-hole transitions. Right? So the approximate spectral transparency ranges for a number of important dielectric materials can be seen here. Starting from aluminum, calcite, barium fluoride, zinc selenide, zinc sulfide, and so on. So this is the spectral range given in micrometers, this is the wavelength, and this is the UV, visible, infrared, mid, and far infrared.

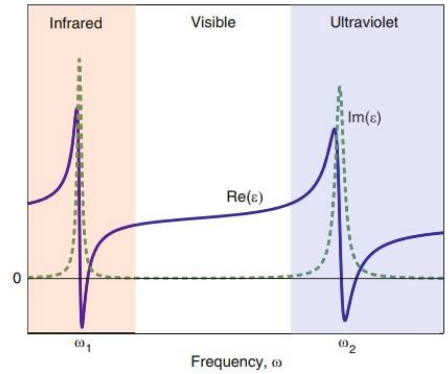
Optical Properties of Dielectric Materials

- The dielectric function $\varepsilon(\omega)$ can be expressed in a classical Helmholtz–Drude model:

$$\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

where ω_j is the resonance frequency of the j^{th} mode, S_j and j represent the strength, and the damping constant of the j^{th} mode, respectively.

- A typical frequency-dependent permittivity of a usual, transparent dielectric is plotted in Fig.

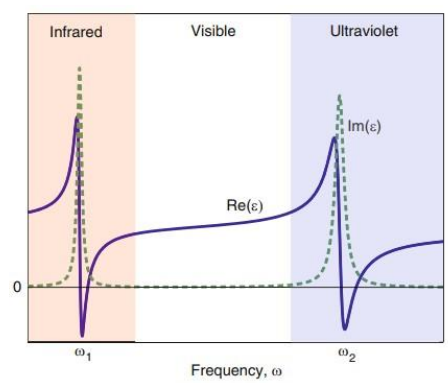


- Two resonances are included in the oscillator formula, with ω_1 representing the phonon resonance in the mid-infrared, and ω_2 corresponding to the electron transition in the UV range due to the bandgap of the crystal.

So this range tells you the transparency level. So, when designing an optical metamaterial, it is important to make sure that the selected The dielectric constituent is basically transparent within that wavelength range of interest. So, that is where you can refer to this particular chart. If you do not do that, there will be substantial loss arising from the electron absorption or, say, phonon resonances. In the dielectric, which is not good for the metamaterial performance. So, the dielectric function ε(ω) can be expressed in a classical Helmholtz-Drude model in the form of $\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$.

Optical Properties of Dielectric Materials

- The dielectric function exhibits a Lorentz line shape at each resonance along with a distinct peak in the imaginary part of $\varepsilon(\omega)$, which indicates the loss feature associated with the resonance.
- Figure illustrates that between the two resonance frequencies, the permittivity curve is rather flat with a negligible imaginary part.
- This explains why a common dielectric like quartz or alumina is transparent to the visible light.



So, here ω j is basically the resonance frequency of the jth mode. S_j and j basically represent the strength and the damping constant of the jth mode. Right. So, you can see that you can obtain there are different modes associated In a particular dispersion range or dispersion relation. So, a typical frequency-dependent permittivity of a usual transparent dielectric material can be seen like this. So, here it is completely transparent in the visible range, while in the ultraviolet and infrared, it is absorbing strongly.

So, you can see that these resonances can be included in this oscillator formula with ω_1 that represents the phonon resonance in the mid infrared okay that is basically from the lattice vibrations or and you can see ω_2 That is another one that tells you about the electron transition in the ultraviolet range due to the band gap of the crystal. So, the dielectric function here oscillates with a Lorentz line shape at each resonance. along with a distinct peak that you can see in the imaginary one. So, the red dotted one shows the imaginary power of the permittivity. which is referring to the significant loss associated with the resonance. And this figure tells you that between these two resonant frequencies, the permittivity curve is rather flat. And it is called a negligible imaginary part, and this is exactly where you should be operating. For that particular purpose of making optical metamaterials. So, if you take, you know, the common dielectrics like quartz or alumina, this is the reason why they They appear transparent to visible light, but they are not in the infrared and ultraviolet regions.

So, with that, we will conclude this lecture. So, if you have any queries regarding this lecture, drop an email to this email address. Mention the subject and the course number in the subject line.