**Course Name: Introduction to Microwave and Optical Metamaterials**
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**Institute Name: Indian Institute of Technology, Guwahati**
**Week-4**
**Lecture-16**

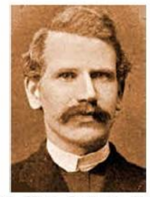
Lec 16: Optical Properties of Metals-I


Hello everyone, welcome to Lecture 16 of the online course on Introduction to Microwave and Optical Metamaterials. Today's lecture will be on the optical properties of metals. So, here is the lecture outline. We will briefly move towards metal optics and introduce a new topic called plasmonics. We will also see Lorentz oscillator model which will be important to describe their optical property of dielectrics and then we will see that how you can make modification in that model and describe the o ptical properties of an electron gas in metal ok and that will give rise to Drude model. So on the right, you can see the picture of Drude, whose full name is Paul Carl Ludwig Drude.

He is a German physicist who worked to integrate optics with Maxwell's electromagnetism. So he basically forged a theory which is commonly known as this Drude model that is useful for describing the behavior of electrons in metals. So, until now we have seen that the majority of the optical components are basically based on dielectrics, right? So, what are the pros and cons? So, the pros with dielectric is that they support high speed, you will have less loss ok, you can also support high bandwidth, but there are certain cons ok. The first thing is that they do not scale well, okay? So, you need to require you need to have some material that allows you large scale integration, but dielectric is definitely not those.

## Lecture Outline

- Towards Metal Optics — Plasmonics
- Lorentz Oscillator Model
- Optical Properties of an Electron Gas (Metal) — Drude Model



**Paul Karl Ludwig Drude (1863–1906)**, a German physicist, worked toward integrating optics with Maxwell's electromagnetics. He forged a theory, commonly known as the Drude model, for describing the behavior of electrons in metals.

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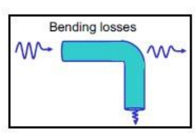
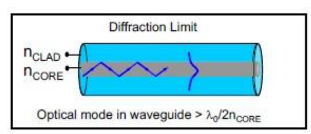
ok and there are some other kind of losses which is something like a bending loss that you have seen with fibres when you bend the mode basically leaks out of the fibre. Also there is a inherent limitation called the diffraction limit which is basically telling you that say if you consider a waveguide or a optical fiber like this. So, the optical mode in the waveguide should be larger than $\lambda_0$ by 2 n core. So, that is basically saying that you cannot actually differentiate the two points which are smaller than this $\lambda$ by 2 $n_0$ or n core. So, that is a fundamental limit called the diffraction limit of light in dielectrics.

## Towards Metal Optics — Plasmonics

- The majority of optical components are based on dielectrics:
  - **Pros:** High speed, high bandwidth (w), but…
  - **Cons:** Does not scale well Needed for large scale integration




- **Solution** → **Plasmonics**
  - Plasmonics *forms a major part of the fascinating field of nanophotonics, which explores how electromagnetic fields can be confined over dimensions on the order of or smaller than the wavelength.*

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So, these are the two problems that tell you that you cannot do very large-scale integration using this dielectric material. So, what is the solution? The solution is to move toward plasmonics. Now, what is plasmonics? It is basically a field of nanophotonics which basically explores electromagnetic fields that can be confined / dimensions which are of the order of or much smaller than the wavelength. So, it can basically solve this problem of the diffraction limit and allow you to go for miniaturized devices. So, once again, it is basically a field of nanophotonics.

## Towards Metal Optics — Plasmonics

- **In the past**, devices were relatively slow and bulky.
- The semiconductor industry has performed an incredible job in scaling electronic devices to nanoscale dimensions.
- Unfortunately, interconnect delay time issues provide significant challenges for electronic circuits operating above ~10 GHz.

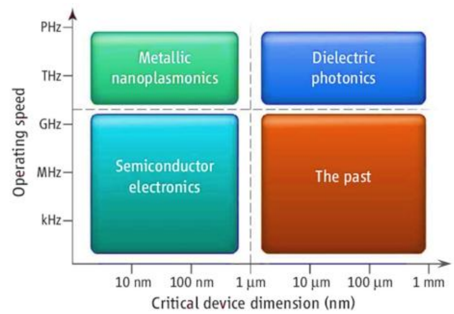


**Figure.** Operating speeds and critical dimensions of various chip-scale device technologies, highlighting the strengths of the different technologies.



Source: R. Zia, J. A. Schuller, A. Chandran, and M. L. Brongersma, "Plasmonics: the next chip-scale technology," Materials Today, vol. 9, no. 7–8, pp. 20–27, Jul. 2006.

Now, to understand how plasmonics is going to help us, we have to see where the devices currently stand. So, in the past, we have seen that the devices were really slow and bulky. And then with the progress in the semiconductor industry, we have seen that you know the electronic devices have moved down to nanoscale dimensions and that has allowed lot of miniaturization all the devices are now becoming lightweight and compact. So, electronics have done extremely well towards miniaturization aspect, but there is a limit coming from the RC time constant or the resistive capacitive time constant in the electronic circuits. So, you know that they cannot go beyond a certain speed.

## Towards Metal Optics — Plasmonics

- **Photonic devices** possess an enormous data-carrying capacity (bandwidth).
- Unfortunately, **dielectric photonic components** are limited in their size by the laws of diffraction, preventing the same scaling as in electronics.
- Finally, **Plasmonics** offers precisely what we need:
  - **Size** of **electronics**
  - **Speed** of **photonics**

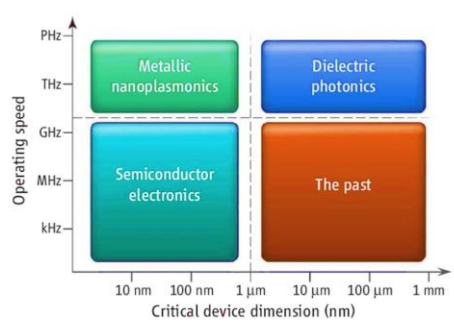


**Figure.** Operating speeds and critical dimensions of various chip-scale device technologies, highlighting the strengths of the different technologies.



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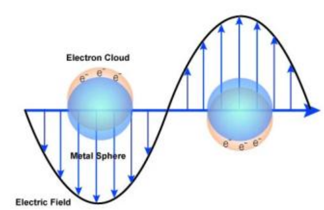
So, typically the interconnect delay time that that is a big issue and it it is a challenge in the electronic circuits which are operating above 10 gigahertz ok. So, in that aspect if you compare dielectric photonics Okay, those are actually able to operate at a much higher speed or they can support much larger bandwidth. So, here come the photonic devices, which are a bit bulky. So, this is what the critical device dimension looks like, and this is where the operating speed is plotted. So, you can see that photonic devices can allow for huge data capacity.

Okay, but the problem is that the size is kind of restricted ok and that is coming from the laws of diffraction or I have told you already is that it is coming from the fundamental limit also known as the diffraction limit of light. So, we basically want to move here that you want miniaturization as well as high speed and this is what plasmonics or you can say metal nanoplasmonics or you can simply say metallic nanophotonics that offers you this luxury of getting the best of both the worlds okay. So, plasmonics enjoy the size of electronics and the speed of photonics. Now, when I am talking about plasmonics, you have to understand that it comes from the word "plasmon." So, what is a plasmon? So, one can squeeze optical signals into minuscule wires by using light to produce electron density waves and this electron density waves are plasmons.

## Towards Metal Optics — Plasmonics

- What is a **Plasmon**?
- One can squeeze optical signals into minuscule wires by using light to produce electron density waves called **plasmons**.
- Compare electron gas in a metal and real gas of molecules, metals are expected to allow for electron density waves: **plasmons**
- **Plasmons** are quasi-particles that are collective oscillations of conduction electrons in a material, excited by electromagnetic radiation.
- They are also referred to as a 'quantized plasma' (charge density) wave.
- These oscillations are similar to the electronic plasma oscillations in a gaseous discharge, which led to Pines coining the term "*plasmon*" to describe the phenomena in 1956.

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So, just compare the electron gas in a metal and the real gas of molecules, okay? You will see that the metals are expected to allow for the electron density waves, which are nothing but plasmons. So, plasmons are basically quasi particles that are collective oscillations of the conduction electrons in a material excited by an electromagnetic radiation. You can refer to them as quantized plasma, which is nothing but the charge density wave, right? And these oscillations are very similar to the electronic plasma oscillations in a gaseous which has led to pines coining this particular term + 1 to describe this phenomena in 1956. So, here you can see a metallic sphere or a nanoparticle ok and the size is much smaller than the wavelength of the light it is interacting with you can see. When the electric field is this way the electron cloud is pushed above ok leaving behind a positive charge So, that again pulls back the electron cloud when the you know electric field changes direction and that is how there is a normal oscillation in the metallic sphere in the presence of this oscillating electromagnetic field.

## Towards Metal Optics — Plasmonics

- Three types of **Plasmons**:

**Bulk plasmon**

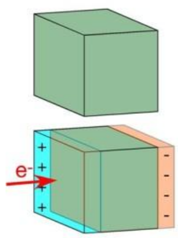
Metals allow for EM wave propagation above the plasma frequency

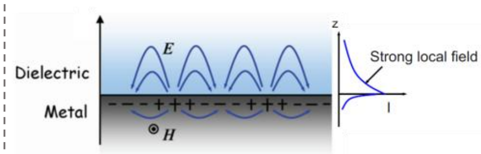
**Surface Plasmon**

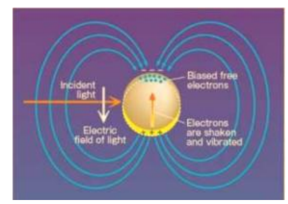
Propagating surface plasmons at metal/dielectric interface: TM wave

**Particle Plasmon**

Localized plasmon oscillating around a sub-wavelength nanostructure



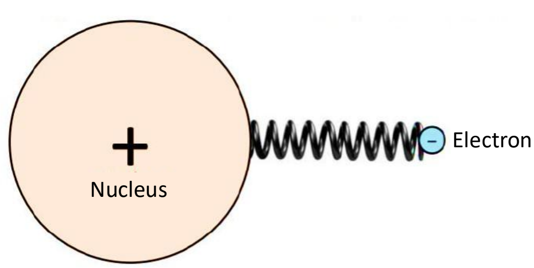



Source: R. Zia, J. A. Schuller, A. Chandran, and M. L. Brongersma, "Plasmonics: the next chip-scale technology," Materials Today, vol. 9, no. 7–8, pp. 20–27, Jul. 2006.

Now, there are three types of plasmons; one is called bulk plasmon. So, that comes in bulk with the material. So, metals you will see allow electromagnetic wave propagation through them above the plasma frequency, right? So, normally metals are reflective, but there is a frequency called plasma frequency beyond which the metal becomes transparent and it is also known as UV transparency okay. So, bulk plasmons as you can see here are basically a longitudinally polarized charge wave okay that means the electron oscillations are basically along the direction of propagation. On the other hand, you have surface + bond, which are basically seen at the interface between a metal and a dielectric.

## Lorentz Oscillator Model

- Modeling electrons in materials as spring-mass systems.
- **Assumption:**
  - Lattice ions do not move because it is much heavier than electrons. Otherwise, use reduced or effective mass.
  - The binding force behaves like a spring.






Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

So, these are basically propagating surface + bond in the form of tm waves and you will see the penetration is more towards the dielectric side and less towards the metallic side and this is a TM wave that propagates along the interface ok. So, these are all transverse waves, and the third category is called particle plasmon. So, that we saw in the previous slide that you have a small metallic nanoparticle where the plasmons are localized, but they are allowed to oscillate in the according to the oscillating incident electric field. So, that happens in sub-wavelength nanostructures. So, this is a transverse wave, this is a longitudinal wave, and this is an oscillating phenomenon, right? So, now let us look into a model a oscillator model that can describe the motion of electrons in this kind of scenario.

So, here our objective is to model the electrons in materials as a spring-mass kind of system. So, that is nothing but your Lorentz oscillator model. So, there are certain assumptions that you have to make. The first thing is that the lattice ions do not move because they are much heavier compared to electrons. Otherwise you have to use reduced or effective mass and the binding force that basically behaves like a spring the binding force between the nucleus and the electron ok.

So, here is your mass-and-spring kind of arrangement. So, the driving force can be written as F t which is $Re\{F_0 e^{-i\omega t}\}$. Now, there is a damping which can be written as $-b\dot{x}$. So, you can simply write the equation of motion. So, x is basically the displacement; the damping is - b * x dot, which is proportional to the velocity.

## Lorentz Oscillator Model

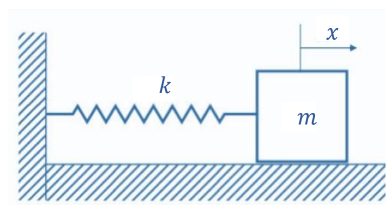
- Driving Force: $F(t) = Re\{F_0 e^{-i\omega t}\}$
- Viscous damping: $-b\dot{x}$
- Equation of Motion:

$$m\ddot{x} = -kx - b\dot{x} + Re\{F_0 e^{-i\omega t}\}$$

- By substituting:

$$\omega_0^2 = \frac{k}{m}, \gamma = \frac{b}{m}, \text{and } f_0 = \frac{F_0}{m}$$

$$\ddot{x} = -\omega_0^2 x - \gamma\dot{x} + \mathrm{Re}\{f_0 e^{-i\omega t}\}$$

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Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

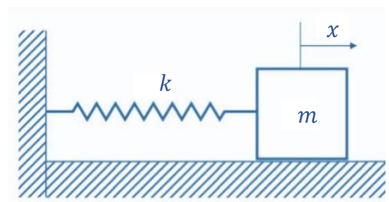
So, you can write the equation of motion as F = ma. So, you can write $-kx - b\dot{x}$ okay. The amount of displacement then the damping factor and then you have the driving force okay. So you can write this equation of motion, you have to substitute all these different constants that you see. So k is nothing but you can relate it to the frequency of oscillation that is $\omega_0$ square can be written as k/m okay, $\gamma$ the damping constant can be related to b/m and $f_0$ is basically capital $F_0$/m. So, all these things are basically getting normalized with the mass that comes from here because you are dividing this equation by m on both sides.

So, you are just left with x double dot. - k/m, which is nothing but $\omega_0$ squared, okay. So, - b/m, which is nothing but - γ, and then you have small $f_0$, right? So you have this particular equation. Now, if we assume a time harmonic driving field then to obtain the frequency domain equations you can use the Fourier transform with an e to the power i ω t time dependence where ω is basically the angular frequency.

## Lorentz Oscillator Model

$$\ddot{x} = -\omega_0^2 x - \gamma \dot{x} + \mathrm{Re}\{f_0 e^{-i\omega t}\}$$

- If we assume a time-harmonic driving field, then, to obtain the frequency-domain equations, we use the **Fourier transform** with an $e^{i\omega t}$ time dependence (where, $\omega$ is the angular frequency).
- The derivative of $e^{i\omega t}$ with respect to time is $i\omega e^{i\omega t}$.
- Thus, we can easily convert the time-domain equations to the frequency-domain by replacing $\delta/\delta t$ with $i\omega$ and $\delta^2/\delta t^2$ with $-\omega^2$.
- Thus, $\ddot{x} \Rightarrow -\omega^2 x$ and $\dot{x} \Rightarrow i\omega x$




$$\ddot{x} = -\omega_0^2 x - \gamma \dot{x} + \mathrm{Re}\{f_0 e^{-i\omega t}\} \rightarrow (\omega_0^2 - \omega^2 - i\gamma\omega)A = f_0$$

$$\therefore A(\omega) = \frac{f_0}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

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Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

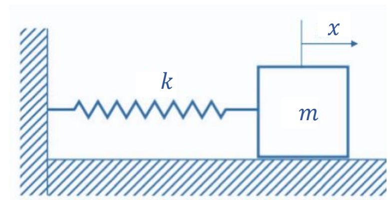
Right. So, if you look into the derivative of e to the power i ω t with respect to time that is nothing, but i ω e to the power i ω t. So, you can easily convert the time domain equations into frequency domain equation by replacing the first derivative that is dot dot t/i ω and the second derivative that is dot square dot t square. with - ω square. So, the - is coming from, you know, i ω i ω.

So, i squared is - 1. So, from that, you get it. So, you can simply write x double dot as - ω squared x. and x dot will be i ω x. So, with this you go back to your previous equation substitute these terms here and you will be left with $(\omega_0^2 - \omega^2 - i\gamma\omega)A = f_0$. So, from that you can obtain what is a that is A(ω) that is basically $f_0$ by this clear.

So now, you can clearly write the restoring force that is given by the spring. If spring is nothing but - m $\omega_0$ squared x, the damping that is coming from the electron scattering. So, damping will be - m / tau; tau is the relaxation time. So, 1 / tau is nothing but γ. So, you can also write this as - m γ x dot and the driving force which is basically the local electric field in the case of the particle will be d driving which is - e, e local.

## Lorentz Oscillator Model

- Restoring Force: $\mathbf{F}_{\text{spring}} = -m\omega_0^2\mathbf{x}$




- Damping (electron scattering):

$$\mathbf{F}_{\text{damping}} = -\frac{m}{\tau}\dot{x} = -m\gamma\dot{\mathbf{x}}$$

$\tau$: relaxation time

- Driving Force (local electric field):

$$\mathbf{F}_{\text{driving}} = -e\mathbf{E}_{\text{loc}}(t) = -e\mathbf{E}_0 e^{-i\omega t}$$

- Equation of motion:

$$m\ddot{\mathbf{x}} = \mathbf{F}_{\text{spring}} + \mathbf{F}_{\text{damping}} + \mathbf{F}_{\text{driving}} \rightarrow m\ddot{\mathbf{x}} = -m\omega_0^2\mathbf{x} - m\gamma\dot{\mathbf{x}} - e\mathbf{E}_0 e^{-i\omega t}$$

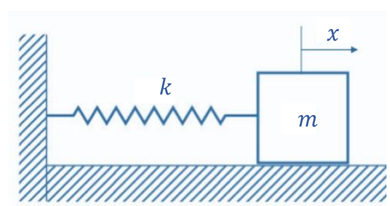


Okay as a function of time. So, when you convert this, you will get - e $E_0$ $e^{-i\omega t}$, okay. So, once you have everything in the all the forces ready you can write the equation of motion as m x double dot that will be F spring + F damping + F driving. So, you get m x double dot = - m $\omega_0$ this one - m $\omega_0$ square x, then you have - m $\gamma$ $x_0$ and then you have this particular term - e $E_0$ e to the power - i $\omega$ t. So, from that you can find out what is the displacement x t that can be written in terms of - e/m / $\omega_0$ square - $\omega$ square - i $\gamma$ $\omega$ * E local right. Now, this field is going to induce a dipole moment that can be written as $\mathbf{p} = -e\mathbf{x}$.

## Lorentz Oscillator Model

- Displacement:

$$x(t) = -\frac{\frac{e}{m}}{\omega_0^2 - \omega^2 - i\gamma\omega}\mathbf{E}_{\text{loc}}(t)$$




- Induced dipole moment:

$$\mathbf{p} = -e\mathbf{x}$$

- Polarization ($n$: the number of electrons per unit volume)

$$\boldsymbol{P} = n\langle\mathbf{p}\rangle = -ne\langle\mathbf{x}\rangle = \frac{ne^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega}\langle\mathbf{E}_{\text{loc}}\rangle$$
$$= \varepsilon_0\chi\mathbf{E}$$

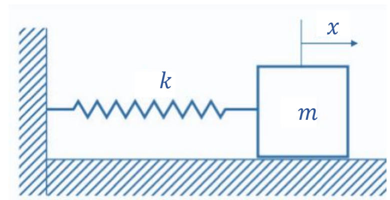
- In general, $\langle\mathbf{E}_{\text{loc}}\rangle \neq \mathbf{E}$ because $\langle\boldsymbol{E}_{\text{loc}}\rangle$ is usually an average over atomic sites, not over regions between sites.
- However, in metals, conduction electrons are not bound, so they feel macroscopic field $\mathbf{E}$ on average.

So, x you have already obtained, okay. So, the polarization is basically for n number of electrons per unit volume. So, the polarization capital P can be written as n, and then you have the average value of these things, okay. So, you get $-ne\langle \mathbf{x} \rangle$. So, x you can obtain from here which is nothing but $\frac{ne^2/m}{\omega_0^2-\omega^2-i\gamma\omega}$ and then you have the mean value of the electric local electric field.

## Lorentz Oscillator Model

- Therefore, we arrive at the desired result, the dielectric function of the material :

$$\varepsilon(\omega) = 1 + \chi(\omega) = 1 + \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$$




$$\mathrm{Re}\{\varepsilon\} = 1 + \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$

$$\mathrm{Im}\{\varepsilon\} = \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$

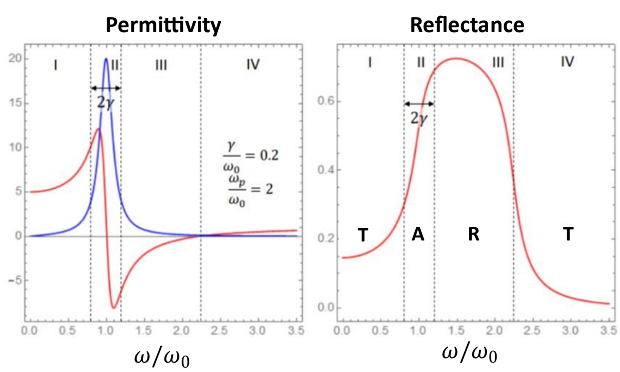


So, that gives you this polarization is nothing but $\varepsilon_0$ chi e. So, once you correlate these two you can also find out what will be your chi. So, in general it is seen that the mean value of the local electric field is not exactly equal to this because this is typically an average by the atomic sites not by region between the sites ok. So, that is how these two values are not exactly same, however in the case of metal. The conduction electrons are not bound; they are free to move in the lattice.

## Lorentz Oscillator Model

$$\mathrm{Re}\{\varepsilon\} = 1 + \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$

$$\mathrm{Im}\{\varepsilon\} = \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$




- Region I ($\omega \ll \omega_0$): $\varepsilon_2 \approx 0$, $\varepsilon_1 > 1 \rightarrow$ transparent
- Region II ($\omega \sim \omega_0$): large $\varepsilon_2 \rightarrow$ absorptive

So, you fill the microscopic electric field only on average. Therefore, we can derive the desired result, which is that we want to obtain the dielectric function of the material. So, you can write ε ω which is the dielectric function of the material as 1 + chi ω and chi you can obtain from the previous equation by comparing these two you can obtain what is chi that is n square by m $\varepsilon_0$ * 1 / $\varepsilon_0$ $\omega_0$ square - ω square - i γ ω. So, from this, you can obtain what the real part of the permittivity and the imaginary part of the permittivity are, right? So, if you plot the real part of the permittivity in red and the imaginary part in blue So, this is how the permittivity looks like and then you can also see based on and this is normalized frequency basically $\omega/\omega_0$ ok. So, you can see that there are 4 regions.

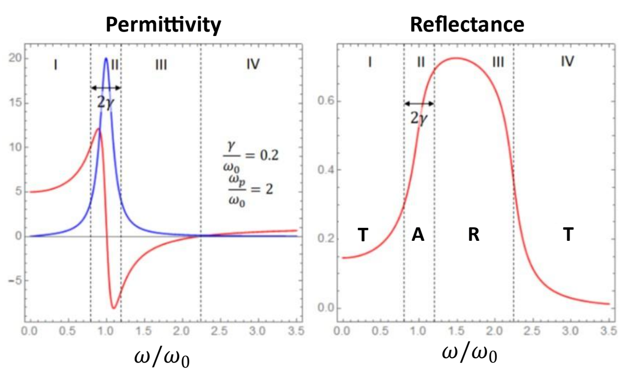
I will explain what are those 4 regions ok and while plotting we have considered γ to be 0.2 $\omega_0$ or you can say γ by $\omega_0$ = 0.2 and ω p/$\omega_0$ = 2. So, these are the two values that are being used here, okay.

So, now let us look at the first region. So, the first region is where your $\omega/\omega_0$ is much much lesser than 1 that means ω is much much lesser than $\omega_0$. So, that is the case when your $\varepsilon_2$, which is basically the imaginary part, is almost 0. That means you are only dictated by this particular real part, which is $\varepsilon_1$. So, real ε can be taken as $\varepsilon_1$, and imaginary ε can be taken as $\varepsilon_2$. So, here you can see that it is more than one.

## Lorentz Oscillator Model

$$\mathrm{Re}\{\varepsilon\} = 1 + \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$

$$\mathrm{Im}\{\varepsilon\} = \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$




- Region III ($\omega \gg \omega_0$): small $\varepsilon_2$, $\varepsilon_1 < 0$ → reflective
- Region IV $\left(\omega > \sqrt{ne^2/m\varepsilon_0}\right)$: $\varepsilon_2 \approx 0$, $\varepsilon_1 > 0$ → transparent

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Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

So, it is just behaving like a transparent material. So, if you plot the reflectance curve, you will see that in this particular region, the material is transparent. Now, let us go into this region where ω is equivalent to $\omega_0$ that is you are basically in this region where it is $\omega/\omega_0$ is close to 1. So, here you can see that in this particular region, you have a very strong or large value of $\varepsilon_2$.

That means it is an absorbing region, okay. So, your material is basically absorbing, right? In the third region where ω is now getting larger with respect to $\omega_0$ here, you see your $\varepsilon_2$ is again getting smaller But here what is happening to the $\varepsilon_1$ which is the red one the real part of the permittivity that is negative. So, it is behaving like a negative means field that will not be able to enter here, and it will be reflected. So, this particular regime tells you about reflection ok. And then you come to the fourth region where again you will see that $\varepsilon_2$, the blue curve, is almost 0, okay.

but your red curve is greater than 0. So, it is again behaving like transparent right. So, here you can consider ω greater than the square root of n squared by m $\varepsilon_0$. So, we will find out what this is basically, ω p. So, this is where the material again become transparent. So, these are the four regions from which you can understand how the material behaves.

Sometimes it is transparent, sometimes it is absorbing, sometimes it is again reflecting and then again transparent. So, all depends on the relationship of the frequency that is basically falling on the frequency of the light that is falling on the material. Now, with that understanding, we can move on and see the optical properties of an electron gas in metal, okay. And we will see how this model can be used to describe the electrons that are free in metals.

## Optical Properties of an Electron Gas (Metal) — Drude Model

- In metals, most electrons are free because they are not bound to a nucleus.
- For this reason, the restoring force is negligible and there is no natural frequency (*i.e.* $\omega_0 = 0$).
- The Drude model for metals is derived from the Lorentz model by setting $\omega_0 = 0$.

$$\varepsilon(\omega) = 1 + \chi(\omega) = 1 + \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{1}{\cancel{\omega_0^2} - \omega^2 - i\gamma\omega}$$

Lorentz Model

$$\varepsilon(\omega) = 1 - \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{1}{\omega^2 + i\gamma\omega}$$

Drude Model



Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

and none of them are bound to the nucleus right. So, what you can understand that in metal you can simply take the restoring force to be negligible in that case there will be no natural frequency of oscillation. So, $\omega_0$, which is a natural frequency, can be set to 0. So, the Drude model for metals can be simply derived from the Lorentz oscillator model by setting $\omega_0$ equal to 0. So, if you remember this was the previous thing that you have saw we have seen that $\varepsilon(\omega) = 1 + \chi(\omega) = 1 + \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$. So, you can simply remove this term, and this is what you are left with.

So, this simplified model of the Lorentz one is known as the Drude model. So, what have we done? We have just set $\omega_0$ to 0 in this case. Right. So, this is the Drude model, okay? So, ε ω can be written as 1 - n e square m $\varepsilon_0$ * 1 / ω square + i γ ω and if you substitute the plasma frequency ω p which is given by square root of n e square by m $\varepsilon_0$. So, n is the electron concentration, e is the electron charge, m is the mass of the electron, and $\varepsilon_0$ is the vacuum permittivity.

## Optical Properties of an Electron Gas (Metal) — Drude Model

- Drude Model: $\varepsilon(\omega) = 1 - \left(\frac{ne^2}{m\varepsilon_0}\right)\frac{1}{\omega^2 + i\gamma\omega}$

- Substituting plasma frequency: $\omega_\mathrm{p} = \sqrt{\frac{ne^2}{m\varepsilon_0}}$

$$\varepsilon(\omega) = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2 + i\gamma\omega}$$

- The real and imaginary components of this complex dielectric function $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$ are given by:

$$\varepsilon_1 = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2 + \gamma^2} \qquad \varepsilon_2 = \left(\frac{\gamma}{\omega}\right)\frac{\omega_\mathrm{p}^2}{(\omega^2+\gamma^2)}$$

$$\Longleftrightarrow$$

$$\varepsilon_1 = 1 - \frac{\omega_\mathrm{p}^2\tau^2}{1 + \omega^2\tau^2} \qquad \varepsilon_2 = \frac{\omega_\mathrm{p}^2\tau}{\omega(1 + \omega^2\tau^2)}$$

$$\therefore \gamma = 1/\tau$$

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Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

I believe all of this, so I am not actually explaining this in the previous one also. So, you can see that ω p, this term is basically ω p squared. So, you can write $\varepsilon(\omega) = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2+i\gamma\omega}$. Right. So, from this, you can obtain what the real part and the imaginary part are. So, you can write ε ω = $\varepsilon_1$ + i $\varepsilon_2$. So, the real part is called $\varepsilon_1$ ω, and the imaginary part is $\varepsilon_2$ ω. So, how can you do that? You can, you know, multiply this fraction by its conjugate, okay? in the numerator and denominator and then separate out the real and the imaginary part and compare it on the both sides. Because the left side you will be writing as this, from that you can obtain the real part and the imaginary part. You will see that the real part $\varepsilon_1$ looks like $1 - \frac{\omega_\mathrm{p}^2}{\omega^2+\gamma^2}$, whereas the imaginary part that is $\varepsilon_2$ looks is given by $\left(\frac{\gamma}{\omega}\right)\frac{\omega_\mathrm{p}^2}{(\omega^2+\gamma^2)}$.

Now, if you consider γ equal to 1 by tau, you can write these equations again using tau. So, $\varepsilon_1$ will be 1 - ω p squared tau squared / 1 + ω squared tau squared. E 2 will be ω p squared tau / $\omega_1$ + ω squared tau squared. So, these are very simple mathematics you can do on your own to see how you can obtain the Drude model from the Lorentz model and then you separate out the real and the imaginary part ok and you can obtain this $\varepsilon_1$ and $\varepsilon_2$ right.

## Optical Properties of an Electron Gas (Metal) — Drude Model

- **Drude Model:** $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ $\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$ $\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$ $\varepsilon_2 = \left(\frac{\gamma}{\omega}\right)\frac{\omega_p^2}{(\omega^2+\gamma^2)}$

- $\omega \ll \gamma: \varepsilon_2 \gg \varepsilon_1 \rightarrow$ absorptive
- $\gamma \ll \omega < \omega_p : \varepsilon_1 \gg \varepsilon_2, \varepsilon_1 < 0 \rightarrow$ reflective $\varepsilon_1 \approx 1 - \frac{\omega_p^2}{\omega^2}$

- **Why metals are shiny?**
  - Electrons in metals follow the oscillating electric field & cancel it. As a result, the electromagnetic wave can't enter a metal & gets totally reflected.



Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

So, this is what we have seen so far. that this is the Drude model, this is the plasma frequency, this is the real part of the permittivity and this is the imaginary part of the permittivity. Now, if you consider ω to be much, much smaller than γ, that means this fraction is very large. Okay. In that case what will happen $\varepsilon_2$ will be much larger as compared to $\varepsilon_1$ and the material will be highly absorptive. If you consider ω to be much much larger than γ, but yet ω is larger than smaller than the plasma frequency, you will see that this particular fraction will be now much larger.

## Optical Properties of an Electron Gas (Metal) — Drude Model

- **Drude Model:** $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ $\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$ $\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$ $\varepsilon_2 = \left(\frac{\gamma}{\omega}\right)\frac{\omega_p^2}{(\omega^2+\gamma^2)}$

- When the external field oscillates too fast for electrons to follow, metals loses its reflectivity.

- $\omega_p < \omega$: $\varepsilon_1 \gg \varepsilon_2,\ 0 < \varepsilon_1 \rightarrow$ transparent

- **Alkali metals (e.g., lithium & sodium)** actually exhibit ultraviolet transparency, but noble metals don't due to interband absorption.



Source: S. A. Maier, Plasmonics: Fundamentals and Applications. New York, NY: Springer US, 2007.

So, so not fraction this this real part will be much larger as compared to $\varepsilon_2$ which is the imaginary part. And also, you can see from here that this fraction will be negative, right? So, that means the material or the metal will be highly reflective. So, you can simply write $\varepsilon_1$ is equivalent to 1 - ω p squared / ω squared. So, you can simply ignore this because it is much smaller compared to ω. Now, that explains why metals are shiny: because the electrons in the metal will follow the oscillating electric field, and they will cancel it.

So, the electromagnetic field will not be allowed to enter the metal and it will hence get totally reflected right and that is why metals appear shiny. Now, what if the external field starts oscillating too fast? okay for the electrons in the metal to follow that okay in that case at a but after a particular frequency the electrons in the metal will give up oscillating and that is how they will just allow the incident field to pass through and that is the point called plasma frequency right so that is the frequency at which the metals will lose its reflectivity So, we can see from this equation that if you consider $\omega$, which is larger than $\omega$ p, the plasma frequency. You will see that $\varepsilon_1$ will be, in that case, much, much larger than $\varepsilon_2$, and also $\varepsilon_1$ will be positive. That means this metal will now become transparent and all the alkali metals like lithium, sodium they exhibit ultraviolet transparency, but typically noble metals like gold, silver they do not because of the interband absorption. But you can see this ultraviolet transparency in lithium and sodium.

So, with that, we will conclude this particular lecture. We will discuss this optical property of the material further in the next lecture. If you have got any query regarding this one, drop an email to this email address mentioning the lecture title and the course number in the subject line. Thank you.